Doubly heavy tetraquarks: Heavy quark bindings and chromomagnetically mixings

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We introduce an enhanced binding energy B_{QQ} between heavy-heavy quarks QQ and a flux-tube correction into the chromomagnetic interaction model to study nonstrange doubly heavy tetraquarks T_{QQ} ($Q = c, b$). A simple relation in terms of baryon masses is proposed to estimate the binding energies B_{QQ} and thereby map the flux-tube corrections in doubly heavy tetraquarks T_{cc}, T_{bb}, and T_{bc}. Our computation via diagonalization of chromomagnetic interaction predicts the doubly charmed tetraquark T_{cc} (in color $\bar{3}_c \otimes 3_c$) and T^*_{cc} (in $6_c \otimes \bar{6}_c$) with $IJ^P = 01^+$ to have masses of 3879.2 and 4287.6 MeV, respectively, with the former being in consistent with the measured mass 3874.7 ± 0.05 MeV of the doubly charmed tetraquark $T_{cc}(1^+) = cc\bar{u}\bar{d}$ discovered by LHCb. Further mass predictions are given of the doubly bottom tetraquarks T_{bb} and the bottom-charmed tetraquarks T_{bc} with $J^P = 0^+, 1^+, 2^+$, and $I = 0, 1$. A chromomagnetic mixing between the color configurations $\bar{3}_c \otimes 3_c$ and $6_c \otimes \bar{6}_c$ is noted for the bottom-charmed states T_{bc} with $IJ^P = 01^+$ and $IJ^P = 1(0^+, 1^+)$.

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I. INTRODUCTION

Multiquarks like tetraquarks (with quark configuration $q^2\bar{q}^2$) and pentaquarks ($q^4\bar{q}$) had been suggested earlier at the birth of the quark model [1,2] long before the advent of quantum chromodynamics (QCD). The multiquarks are considered to be exotic as they do not fit into conventional scheme of hadron classification via meson and baryons whereas they are allowed by QCD in principle. The first explicit calculation of multiquark states was carried out in the 1970s based on the MIT bag model [3,4]. These and other early theoretical explorations triggered a lot of experimental searches with no conclusive results until 2003 when the Belle collaboration discovered the first exotic hadron $X(3872)$ [5].

Since the discovery of the $X(3872)$, which was confirmed by other experiments [6], many tetraquark candidates have been observed later, which include the charmoniumlike states such as the $Z_c(3900)$ [7] and many others [6]. Recently, the LHCb collaboration reported a discovery of the first tetraquak with two charm quarks, the $X(3875)$ state ($IJ^P = 01^+$) [8,9]. This observation arouses interest of the doubly heavy (DH) tetraquarks $T_{QQ}(Q = c, b)$ and led to a set of studies on them based on pictures of hadronic molecular [10–13] and of the compact tetraquark [14–22]. See Refs. [23–25] for reviews.

The purpose of this work is to introduce a binding energy B_{QQ} between heavy-heavy quarks to study DH tetraquarks T_{QQ} in the framework of chromomagnetic interaction model. We use a simple relation connecting the measured masses of baryons to estimate B_{QQ} and compute the ground-state masses of the nonstrange DH tetraquarks T_{cc}, T_{bb}, and T_{bc} via diagonalization of the chromomagnetic interaction (CMI). For the masses of the doubly charmed tetraquark we predict $M(T_{cc}, 1^+) = 3879.2$ MeV and $M(T^*_{cc}, 1^+) = 4287.6$ MeV, with the former being in consistent with that of the LHCb-measured mass 3874.7 ± 0.05 MeV. The further computation of masses of the DH tetraquarks T_{bb} and T_{bc} is performed similarly. A discussion is given for the chromomagnetic mixing among the color-spin states of the tetraquarks with color configurations $\bar{3}_c \otimes 3_c$ and $6_c \otimes \bar{6}_c$, mainly occurred for the bottom-charmed states T_{bc} with $IJ^P = 01^+$ and $IJ^P = 1(0^+, 1^+)$.

After introduction, in Sec. II, we describe the CMI model for the DH tetraquarks and classify wave functions of the nonstrange DH tetraquarks via their symmetry in color and spin-flavor spaces. In Sec. III, three sets of the parameters involved in the CMI model are determined, including the binding energies. The numerical results are given in details

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for all nonstrange DH tetraquarks T_{cc}, T_{bb}, and T_{bc} in Sec. IV. We end with conclusions and remarks in Sec. V.

II. CHROMOMAGNETIC INTERACTION MODEL

For ground state of a multiquark $T_{QQ} = QQ\bar{q}\bar{q}$ ($Q = c, b, q = u, d$), the chromomagnetic interaction model is given by the Sakharov-Zeldovich formula [26,27]

$$M = \sum_i (m_i + E_i) + B_{QQ} + \langle H_{\rm CMI} \rangle, \qquad (1)$$

where m_i ($i = 1, \ldots, 4$) is the effective mass of the ith quark in hadron which can be the charm (c), bottom (b), or the light nonstrange ($q = u, d$) quark, E_i is the effective energy of color flux attached to the ith quark, B_{QQ} stands for enhanced binding energy between two heavy quarks $QQ(=cc, bb, bc)$, which enters here to account for an extra binding between QQ compared to that among the light quarks [28], and $H_{\rm CMI}$ is the chromomagnetic interaction in hadron, given by the Hamiltonian [29]

$$H_{\rm CMI} = -\sum_{i<j} (\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j)(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \frac{A_{ij}}{m_i m_j}. \qquad (2)$$

Here, $\boldsymbol{\lambda}_i$ is the Gell-Mann matrices and $\boldsymbol{\sigma}_i$ stand for the Pauli matrices associated with the ith quark, or the ith antiquark for which $\boldsymbol{\lambda}_i$ should be replaced by $-\boldsymbol{\lambda}_i$. The averaged matrix $\langle H_{\rm CMI} \rangle$ of the CMI in Eq. (1), evaluated in tetraquark configurations, has been simplified to the Hamiltonian equation (2) in color-spin space, with the parameter A_{ij} describing the effective CMI coupling between the ith (anti)quark and the jth (anti)quark.

As the flux tube energy E_i (associated with the quark i) in our flux-tube model stems from gluon dynamics, it is believed to be flavor independent in the chiral limit ($m_n \to 0$) and heavy quark limit ($m_Q \to \infty$). In the real world where quarks in hadrons have finite masses, however, E_i (scales with the size of hadrons) acquires its average value in hadrons that slightly depend on the flavor content of the hadrons, as the latter are of multiscales in dynamics and asymmetric in quark configuration. So, despite that E_i and the ensuing tetraquark mass nearly degenerate, we keep in our approach E_i a dynamical variable rather than a parameter having common value for different flavor contents of hadrons.

In order to find the spin multiplets and their state mixings due to the CMI in Eq. (1), one has to exhaust all possible spin and color wave functions of a tetraquark system and combine them appropriately with the flavor configurations so that they satisfy the constraint of the flavor-color-spin symmetry from Pauli principle. Obtaining the wave functions, one can calculate the matrix elements of $H_{\rm CMI}$ using the approach illustrated in Refs. [30,31].

For the spin configuration of the tetraquark $QQ\bar{q}\bar{q}$, the possible wave functions (denoted by χ^T) are

$$\begin{aligned}
&\chi_1^T = |(Q_1Q_2)_1(\bar{q}_3\bar{q}_4)_1\rangle_2, \quad \chi_2^T = |(Q_1Q_2)_1(\bar{q}_3\bar{q}_4)_1\rangle_1, \\
&\chi_3^T = |(Q_1Q_2)_1(\bar{q}_3\bar{q}_4)_1\rangle_0, \quad \chi_4^T = |(Q_1Q_2)_1(\bar{q}_3\bar{q}_4)_0\rangle_1, \\
&\chi_5^T = |(Q_1Q_2)_0(\bar{q}_3\bar{q}_4)_1\rangle_1, \quad \chi_6^T = |(Q_1Q_2)_0(\bar{q}_3\bar{q}_4)_0\rangle_0, \qquad (3)
\end{aligned}$$

where the subscripts of numbers denote spins of the heavy diquark, the light antidiquark, and the tetraquark state.

Based on the color $SU(3)_c$ symmetry, one can obtain two combinations of color singlets $6_c \otimes \bar{6}_c$ and $\bar{3}_c \otimes 3_c$ for tetraquarks, which are

$$\begin{aligned}
\phi_1^T = |(Q_1Q_2)^6(\bar{q}_3\bar{q}_4)^{\bar{6}}\rangle &= \frac{1}{\sqrt{6}}(rr\bar{r}\bar{r} + gg\bar{g}\bar{g} + bb\bar{b}\bar{b}) \\
&+ \frac{1}{2\sqrt{6}}(rb\bar{b}\bar{r} + br\bar{b}\bar{r} + gr\bar{g}\bar{r} + rg\bar{g}\bar{r} + gb\bar{b}\bar{g} + bg\bar{b}\bar{g} \\
&+ gr\bar{r}\bar{g} + rg\bar{r}\bar{g} + gb\bar{g}\bar{b} + bg\bar{g}\bar{b} + rb\bar{r}\bar{b} + br\bar{r}\bar{b}), \qquad (4)
\end{aligned}$$

$$\begin{aligned}
\phi_2^T &= |(Q_1Q_2)^{\bar{3}}(\bar{q}_3\bar{q}_4)^3\rangle, \\
&= \frac{1}{2\sqrt{3}}(rb\bar{b}\bar{r} - br\bar{b}\bar{r} - gr\bar{g}\bar{r} + rg\bar{g}\bar{r} + gb\bar{b}\bar{g} - bg\bar{b}\bar{g} \\
&+ gr\bar{r}\bar{g} - rg\bar{r}\bar{g} - gb\bar{g}\bar{b} + bg\bar{g}\bar{b} - rb\bar{r}\bar{b} + br\bar{r}\bar{b}). \qquad (5)
\end{aligned}$$

For the nonstrange tetraquarks with given heavy pair QQ, the isovector states $QQ\{\bar{n}\bar{n}\}_f$ and the isoscalar states $QQ[\bar{n}\bar{n}]_f$ do not mix since we ignore isospin breaking effects. So, we are position to combine the flavor, color, and spin wave functions together provided that the constraint from the Pauli principle is imposed. In the diquark-antidiquark picture, there are 12 possible bases for the wave function, which are in terms of the notation $|(QQ)^{Color}_{\rm spin}(\bar{q}\bar{q})^{Color}_{\rm spin}\rangle$ in diquark-antidiquark picture,

$$\begin{aligned}
\phi_1^T\chi_1^T &= |(Q_1Q_2)_1^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_2 \delta_{12}^A\delta_{34}^A, \\
\phi_2^T\chi_1^T &= |(Q_1Q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_2 \delta_{12}^S\delta_{34}^S, \\
\phi_1^T\chi_2^T &= |(Q_1Q_2)_1^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_1 \delta_{12}^A\delta_{34}^A, \\
\phi_2^T\chi_2^T &= |(Q_1Q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_1 \delta_{12}^S\delta_{34}^S, \\
\phi_1^T\chi_3^T &= |(Q_1Q_2)_1^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_0 \delta_{12}^A\delta_{34}^A, \\
\phi_2^T\chi_3^T &= |(Q_1Q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_0 \delta_{12}^S\delta_{34}^S, \\
\phi_1^T\chi_4^T &= |(Q_1Q_2)_1^6(\bar{q}_3\bar{q}_4)_0^{\bar{6}}\rangle_1 \delta_{12}^A\delta_{34}^S, \\
\phi_2^T\chi_4^T &= |(Q_1Q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_0^3\rangle_1 \delta_{12}^S\delta_{34}^A, \\
\phi_1^T\chi_5^T &= |(Q_1Q_2)_0^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_1 \delta_{12}^S\delta_{34}^A, \\
\phi_2^T\chi_5^T &= |(Q_1Q_2)_0^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_1 \delta_{12}^A\delta_{34}^S, \\
\phi_1^T\chi_6^T &= |(Q_1Q_2)_0^6(\bar{q}_3\bar{q}_4)_0^{\bar{6}}\rangle_0 \delta_{12}^S\delta_{34}^S, \\
\phi_2^T\chi_6^T &= |(Q_1Q_2)_0^{\bar{3}}(\bar{q}_3\bar{q}_4)_0^3\rangle_0 \delta_{12}^A\delta_{34}^A, \qquad (6)
\end{aligned}$$

where we employ a set of delta notations to reflect the requirement of the flavor symmetry. The notation $\delta^A_{34} = 0$ ($\delta^S_{34} = 0$) if two light quarks q_3 and q_4 are symmetric (antisymmetric) in flavor space, and $\delta^A_{12} = 0$ if two heavy quarks $Q_1 = Q_2$ are symmetric in flavor space. In all other cases, all these delta notations are one: $\delta^S_{34} = \delta^A_{34} = 1$, $\delta^S_{12} = \delta^A_{12} = 1$.

In order for the tetraquarks T_{QQ} in the ground states, we consider two classes: (i) the isospin $I = 0$. In flavor space, light antiquark pair $\bar{n}\bar{n}$ is antisymmetric (isosinglet), and two heavy quarks are symmetric if $QQ = cc$ or bb, while they have no certain symmetry if $QQ = bc$; and (ii) the isospin $I = 1$. In flavor space, light antiquark pair $\bar{n}\bar{n}$ is symmetric (isovector), and two heavy quark are symmetric if $QQ = cc$ or bb, and they have no certain symmetry if $QQ = bc$.

With the corresponding wave functions, we then use Eq. (2) to evaluate the CMI matrices for all ground-state configurations. Using notation $V_{ij} \equiv A_{ij}/(m_i m_j)$, the calculated results of the CMI matrices for the involved color-spin configurations of the DH tetraquark $QQ\bar{n}\bar{n}$ are shown in Table I in detail.

III. DETERMINATION OF PARAMETERS

As a mass formula for hadrons in ground state, Eq. (1) should be applicable to the conventional hadrons for which most measured or empirical data are available and the numbers of quarks in hadrons become less and some of them may be antiquarks. Further, the binding energy term will be absent when the hadron considered contains only one heavy quark. In this section, we demonstrate that the measured mass data from the conventional hadrons are sufficient to determine three parameters in the formulas (1) and (2): E_i, the flux-tube energy, B_{QQ}, the enhanced binding energy for the heavy pair QQ in Eq. (1), A_{ij}, the CMI coupling between the ith quark and the jth quark in Eq. (2).

We note that the binding energy term B_{QQ} is absent when Eq. (1) applies to mesons and baryons with only one heavy quark. With respect to the measured masses of the ground-state hadrons with one or two heavy quarks, we summarize their mass expressions, given by Eq. (1), in Table II for the charmed sector and in Table III for the bottom sector. There, the masses of the Ξ^*_{cc} are taken from the estimate

TABLE I. The CMI matrices for the tetraquarks $cc\bar{n}\bar{n}$, $bb\bar{n}\bar{n}$, and $bc\bar{n}\bar{n}$ with respective quantum numbers, color-spin wave functions. The parameters $V_{ij} \equiv A_{ij}/(m_i m_j)$.

System	IJ^P	Wave function	$H_{\rm CMI}$
$cc\bar{n}\bar{n}$	01^+	$(\phi_2^T\chi_4^T, \phi_1^T\chi_5^T)$	$\begin{pmatrix} \frac{8}{3}V_{cc} - 8V_{nn} & -8\sqrt{2}V_{cn} \\ -8\sqrt{2}V_{cn} & 4V_{cc} - \frac{4}{3}V_{nn} \end{pmatrix}$
	10^+	$(\phi_2^T\chi_3^T, \phi_1^T\chi_6^T)$	$\begin{pmatrix} \frac{8}{3}(V_{cc} - 4V_{cn} + V_{nn}) & 8\sqrt{6}V_{cn} \\ 8\sqrt{6}V_{cn} & 4(V_{cc} + V_{nn}) \end{pmatrix}$
	11^+	$(\phi_2^T\chi_2^T)$	$\frac{8}{3}(V_{cc} - 2V_{cn} + V_{nn})$
	12^+	$(\phi_2^T\chi_1^T)$	$\frac{8}{3}(V_{cc} + 2V_{cn} + V_{nn})$
$bb\bar{n}\bar{n}$	01^+	$(\phi_2^T\chi_4^T, \phi_1^T\chi_5^T)$	$\begin{pmatrix} \frac{8}{3}V_{bb} - 8V_{nn} & -8\sqrt{2}V_{bn} \\ -8\sqrt{2}A_{bn} & 4V_{bb} - \frac{4}{3}V_{nn} \end{pmatrix}$
	10^+	$(\phi_2^T\chi_3^T, \phi_1^T\chi_6^T)$	$\begin{pmatrix} \frac{8}{3}(V_{bb} - 4V_{bn} + V_{nn}) & 8\sqrt{6}V_{bn} \\ 8\sqrt{6}V_{bn} & 4(V_{bb} + V_{nn}) \end{pmatrix}$
	11^+	$(\phi_2^T\chi_2^T)$	$\frac{8}{3}(V_{bb} - 2V_{bn} + V_{nn})$
	12^+	$(\phi_2^T\chi_1^T)$	$\frac{8}{3}(V_{bb} + 2V_{bn} + V_{nn})$
$bc\bar{n}\bar{n}$	00^+	$(\phi_2^T\chi_6^T, \phi_1^T\chi_3^T)$	$\begin{pmatrix} -8(V_{bc} + V_{nn}) & 4\sqrt{6}(V_{bn} + V_{cn}) \\ 4\sqrt{6}(V_{bn} + V_{cn}) & -\frac{4}{3}(V_{bc} + 10V_{bn} + 10V_{cn} + V_{nn}) \end{pmatrix}$
	01^+	$(\phi_2^T\chi_4^T, \phi_1^T\chi_2^T, \phi_1^T\chi_5^T)$	$\begin{pmatrix} \frac{8}{3}V_{bc} - 8V_{nn} & -8(V_{bn} - V_{cn}) & -4\sqrt{2}(V_{bn} + V_{cn}) \\ -8(V_{bn} - V_{cn}) & -\frac{4}{3}(V_{bc} + 5V_{bn} + 5V_{cn} + V_{nn}) & -\frac{20\sqrt{2}}{3}(V_{bn} - V_{cn}) \\ -4\sqrt{2}(V_{bn} + V_{cn}) & -\frac{20\sqrt{2}}{3}(V_{bn} - V_{cn}) & 4V_{bc} - \frac{4}{3}V_{nn} \end{pmatrix}$
	02^+	$(\phi_1^T\chi_1^T)$	$-\frac{4}{3}(V_{bc} - 5V_{bn} - 5V_{cn} + V_{nn})$
	10^+	$(\phi_2^T\chi_3^T, \phi_1^T\chi_6^T)$	$\begin{pmatrix} \frac{8}{3}(V_{bc} - 2V_{bn} - 2V_{cn} + V_{nn}) & 4\sqrt{6}(V_{bn} + V_{cn}) \\ 4\sqrt{6}(V_{bn} + V_{cn}) & 4(V_{bc} + V_{nn}) \end{pmatrix}$
	11^+	$(\phi_2^T\chi_2^T, \phi_2^T\chi_5^T, \phi_1^T\chi_4^T)$	$\begin{pmatrix} \frac{8}{3}(V_{bc} - V_{bn} - V_{cn} + V_{nn}) & -\frac{8}{3}\sqrt{2}(V_{bn} - V_{cn}) & -8(V_{bn} - V_{cn}) \\ -\frac{8}{3}\sqrt{2}(V_{bn} - V_{cn}) & -8V_{bc} + \frac{8}{3}V_{nn} & -4\sqrt{2}(V_{bn} + V_{cn}) \\ -8(V_{bn} - V_{cn}) & -4\sqrt{2}(V_{bn} + V_{cn}) & -\frac{4}{3}(V_{bc} - 3V_{nn}) \end{pmatrix}$
	12^+	$(\phi_2^T\chi_1^T)$	$\frac{8}{3}(V_{bc} + V_{bn} + V_{cn} + V_{nn})$

TABLE II. Quark model description of the charmed hadrons, in which $m_u = m_d = m_n = 230$ MeV, $m_c = 1440$ MeV [32]. The masses are in MeV.

Hadron	J^P	Mass expression of hadrons	Experiment [6]
η_c	0^-	$2(m_c+E_c)+B_{c\bar{c}}-16\frac{A_{cc}}{m_c m_c}$	2983.9
J/Ψ	1^-	$2(m_c+E_c)+B_{c\bar{c}}+\frac{16}{3}\frac{A_{cc}}{m_c m_c}$	3096.9
Λ_c^+	$\frac{1}{2}^+$	$(m_c+E_c)+2(m_n+E_n)-8\frac{A_{nn}}{m_n m_n}$	2286.5
Σ_c	$\frac{1}{2}^+$	$(m_c+E_c)+2(m_n+E_n)-\frac{32}{3}\frac{A_{cn}}{m_c m_n}+\frac{8}{3}\frac{A_{nn}}{m_n m_n}$	2454.0
Σ_c^*	$\frac{3}{2}^+$	$(m_c+E_c)+2(m_n+E_n)+\frac{16}{3}\frac{A_{cn}}{m_c m_n}+\frac{8}{3}\frac{A_{nn}}{m_n m_n}$	2518.4
Ξ_{cc}	$\frac{1}{2}^+$	$2(m_c+E_c)+(m_n+E_n)+B_{cc}-\frac{32}{3}\frac{A_{cn}}{m_c m_n}+\frac{8}{3}\frac{A_{cc}}{m_c m_c}$	3621.6
Ξ_{cc}^*	$\frac{3}{2}^+$	$2(m_c+E_c)+(m_n+E_n)+B_{cc}+\frac{16}{3}\frac{A_{cn}}{m_c m_n}+\frac{8}{3}\frac{A_{cc}}{m_c m_c}$	3728.1

TABLE III. Quark model description of the bottomed hadrons in the groundstate, in which $m_u = m_d = m_n = 230$ MeV, $m_b = 4480$ MeV [32]. The masses are in MeV. The data marked with a are from the lattice data [33].

Hadron	J^P	Mass expression of hadrons	Experiment [6]
η_b	0^-	$2(m_b+E_b)+B_{b\bar{b}}-16\frac{A_{bb}}{m_b m_b}$	9398.7
Υ	1^-	$2(m_b+E_b)+B_{b\bar{b}}+\frac{16}{3}\frac{A_{bb}}{m_b m_b}$	9460.3
Λ_b^0	$\frac{1}{2}^+$	$(m_b+E_b)+2(m_n+E_n)-8\frac{A_{nn}}{m_n m_n}$	5619.6
Σ_b	$\frac{1}{2}^+$	$(m_b+E_b)+2(m_n+E_n)-\frac{32}{3}\frac{A_{bn}}{m_b m_n}+\frac{8}{3}\frac{A_{nn}}{m_n m_n}$	5813.1
Σ_b^*	$\frac{3}{2}^+$	$(m_b+E_b)+2(m_n+E_n)+\frac{16}{3}\frac{A_{bn}}{m_b m_n}+\frac{8}{3}\frac{A_{nn}}{m_n m_n}$	5832.5
Ξ_{bb}	$\frac{1}{2}^+$	$2(m_b+E_b)+(m_n+E_n)+B_{bb}-\frac{32}{3}\frac{A_{bn}}{m_b m_n}+\frac{8}{3}\frac{A_{bb}}{m_b m_b}$	10091.0^a
Ξ_{bb}^*	$\frac{3}{2}^+$	$2(m_b+E_b)+(m_n+E_n)+B_{bb}+\frac{16}{3}\frac{A_{bn}}{m_b m_n}+\frac{8}{3}\frac{A_{bb}}{m_b m_b}$	10103.0^a

$M(\Xi_{cc}^*) = M(\Xi_{cc}) + \Delta M(\Xi_{cc}) = 3728.1$ MeV, where $\Delta M(\Xi_{cc}) = 3/4[M(D^{*0}, 1^-) - M(D^0, 0^-)] = 106.5$ MeV are obtained by the heavy quark-diquark symmetry [34,35]. In Table III, the masses of the doubly bottom baryons, $M(\Xi_{bb}) = 10091.0$ MeV and $M(\Xi_{bb}^*) = 10103.0$ MeV are from the lattice data [33].

A. The coupling paramters A_{ij}

We consider first the CMI coupling A_{ij} in Eq. (2) which is related to the mass splitting between hadrons with same flavor content but different spin configurations. We evaluate all couplings A_{ij} via the hyperfine spin splittings of the conventional mesons and baryons provided that these couplings are approximately same to the CMI couplings for the DH tetraquarks with color configuration $\bar{3}_c \otimes 3_c$ in Eq. (5). For the tetraquark with the $6_c \otimes \bar{6}_c$ configuration, for which chromodynamic is not directly available in the conventional mesons or baryons associated with the configuration 3_c (or $\bar{3}_c$), we employ the ratios of the color factors of the two configurations to scale the CMI coupling A_{ij} and B_{QQ} for the configuration $\bar{3}_c \otimes 3_c$ of the tetraquarks to that for configuration $6_c \otimes \bar{6}_c$.

Application of Eq. (1) to the charmed and bottom hadrons of the system $c\bar{c}$, $b\bar{b}$, cnn, ccn, bnn, and bbn enables us to rewrite the mass expression of them explicitly. We list these mass expressions for the charmed hadrons $J/\Psi, \eta_c, \Lambda_c, \Sigma_c, \Sigma_c^*, \Xi_{cc}$, and Ξ_{cc}^* in Table II and the bottom hadrons $\Upsilon, \eta_b, \Lambda_b, \Sigma_b, \Sigma_b^*, \Xi_{bb}$, and Ξ_{bb}^* in Table III. Using these mass expressions of the J/Ψ and the η_c, the Ξ_{cc} and Ξ_{cc}^*, one can find the mass splitting between them. Same computation applies to the hadrons Υ and η_b, Ξ_{bb} and Ξ_{bb}^*, $B_c^+$and B_c^{*+}, as well as the light nucleon systems of the N and Δ. The results for the mass splittings are

$$M(J/\Psi) - M(\eta_c) = \frac{64}{3}\frac{A_{cc}}{m_c m_c}, \tag{7}$$

$$M(\Upsilon) - M(\eta_b) = \frac{64}{3}\frac{A_{bb}}{m_b m_b}, \tag{8}$$

$$M(B_c^+) - M(B_c^{*+}) = \frac{64}{3}\frac{A_{bc}}{m_b m_c}, \tag{9}$$

$$M(\Xi_{cc}^*) - M(\Xi_{cc}) = 16\frac{A_{cn}}{m_c m_n}, \tag{10}$$

$$M(\Xi_{bb}^*) - M(\Xi_{bb}) = 16\frac{A_{bn}}{m_b m_n}, \tag{11}$$

$$M(\Delta) - M(N) = 16\frac{A_{nn}}{m_n m_n}, \tag{12}$$

where masses of the nucleons have the form, in terms of Eq. (1),

$$M(N) = 3(m_n + E_n) - \frac{8A_{nn}}{m_n m_n} = 939.6 \text{ MeV}, \quad (13)$$

$$M(\Delta) = 3(m_n + E_n) + \frac{8A_{nn}}{m_n m_n} = 1232.0 \text{ MeV}. \quad (14)$$

Given the measured masses [6] of the hadrons J/Ψ, the η_c, the Υ, η_b and other heavy baryons in Eqs. (7)–(12), as well as the nucleon masses given in Eqs. (13) and (14), the $M(B_c^{*+}) = 6332.0$ MeV in Ref. [39], one can use Eqs. (7)–(12) to determine the CMI coupling parameters A_{cc}, A_{bb}, A_{bc}, A_{cn}, A_{bn}, A_{nn}. The obtained results for these couplings are collected in Table IV. Here, we show how the parameters A_{ij} are solved via taking A_{cc} and A_{bb}, for instance, as example. One can employ Eqs. (8) and (9) to solve them, in which the data for the lhs for these two equations are provided by the first and second lines of Tables II and III, respectively, while the quark masses $m_c = 1440$ MeV and $m_b = 4880$ MeV on the rhs of the Eqs. (8) and (9) are provided in the captions of the Tables II and III. As such, one can rewrite Eqs. (8) and (9)

$$3096.9 \text{ MeV} - 2983.9 \text{ MeV} = \frac{64}{3} \frac{A_{cc}}{(1440 \text{ MeV})^2},$$

$$9460.3 \text{ MeV} - 9398.7 \text{ MeV} = \frac{64}{3} \frac{A_{bb}}{(4880 \text{ MeV})^2},$$

and solve them. The obtained results are $A_{cc} = 0.01099$ GeV and $A_{bb} = 0.05800$ GeV, respectively, as listed in Table IV. The same procedure applies to solve the values of the other parameters A_{ij}, using the data listed also in the Tables II and III, and the data $M(B_c^{*+}) = 6332.0$ MeV in Ref. [39].

During solving the parameters A_{ij} via Eqs. (7)–(12) we explain briefly why we use the doubly heavy baryons Ξ_{QQ} and Ξ^*_{QQ} to solve A_{Qn} instead of using the singly heavy baryons Σ_{QQ} and Σ^*_{QQ}. In order to address the doubly heavy tetraquarks, which are, in framework of our model, more similar in color dynamics to the doubly heavy baryons than to the singly heavy baryons, we employ the former in Eqs. (11) and (12) to solve A_{Qn}. Further, we use a unified mass scheme ($m_n = 220$ MeV, $m_c = 1440$ MeV, and $m_b = 4880$ MeV) for the quark involved since this set of quark mass inputs are in consistent with Regge phenomenology of excited hadrons (see Appendix A of Refs. [32,37]), which are close to the values (around 220 MeV for m_n) in relativistic models like Ref. [38] but generally smaller than other inputs (more than 300 MeV for m_n), mostly in nonrelativistic quark models. We remark that redefining $m_i + E_i$ as effective mass or some parameter solely may lead to two different schemes of quark masses separately for mesons and baryons [28,36] which are essential to manifest the variation of the flux energy in two systems.

In Table IV, the CMI couplings A_{ij} describe strength of the effective color-charge (coupling) of the interaction between the magnetic moments of quark i and j, which are normally proportional to $lim_{x_i - x_j \to 0} |\Psi_{ij}(x_i - x_j)|^2 = |\Psi_{ij}(0)|^2$, the overlap of probability amplitude of quark i at position x_i and j at position x_j near the zero distance $x_i - x_j \to 0$. This overlap is enhanced ($\sim 10^{-2}$ GeV^3) for the heavy quark pairs $ij = cc, bb$, and bc as heavy quarks tend to be closer to each other most of the time while the overlap is to be suppressed when the pairs ij contain light quark n ($ij = cn, bn$, or nn) as n tend to be apart averagely from other quarks due to its relativistic motion. As a relativistic corrections to the chromoelectric interaction (spin-dependent interactions), the CMI interaction in Eq. (2) should drop rapidly as $x_i - x_j$ increase.

Though it is small some "muck" effect may exist for the charm quark c, which smears the locality of the charm quark and the overlap $|\Psi_{ij}(0)|^2$ is relatively larger for the pair $ij = cn$ than for the pair $ij = bn$, where b is nearly local and the overlap of the quarks is suppressed. In the case of the pair nn, the CMI coupling A_{nn} (overlap) is suppressed due to both of the relativistic motion and "brown muck" effect: n tends to be away averagely from other light quark.

B. The binding energy $B_{QQ}(Q = c, b)$

To extract the binding energy $B_{QQ}(Q = c, b)$ for two heavy quarks QQ and $B_{Q\bar{Q}}$ for a heavy-antiheavy pair $Q\bar{Q}$ from heavy conventional hadrons (mesons, baryons), we utilize a set of simple relations relating the measured masses of the established hadrons. The main idea of the

TABLE IV. The extracted coupling parameters for the CMI and comparison with that in Ref. [36].

Coupling	$ij = cc$	$ij = cn$	$ij = bb$	$ij = bn$	$ij = bc$	$ij = nn$
A_{ij} (GeV^3)	0.01099	0.00221	0.05800	0.00077	0.01742	0.00097
$A_{ij}/m_i m_j$ (MeV)	5.30	6.66	2.89	0.75	2.70	18.34
$\frac{8}{3} A_{ij}/m_i m_j$ (MeV)	14.13	17.76	7.71	2.00	7.20	48.91
a_{ij} (MeV) [36]	0.04155	0.00658	0.19841	0.00659	0.07333	0.00659
$a_{ij}/m_i m_j$ (MeV) [36]	14.20	10.60	7.80	3.60	8.50	50.00

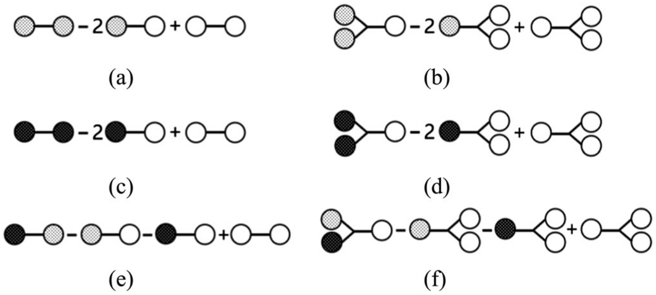

FIG. 1. The binding energies $B_{Q\bar{Q}}$ and B_{QQ} between one heavy and one antiheavy quarks and between two heavy quarks are evaluated by the sum of the mean masses of the first and the last hadrons minus that of others. The black (gray) solid circle stands for the bottom (charm) quark $b(c)$. The hollow circle stands for the nonstrange (u, d) quarks.

relations for the $B_{Q\bar{Q}}$ is illustrated in Fig. 1. Take Fig. 1(a) for instance, it shows that the binding energy ($B_{c\bar{c}}$) between the charm quark c and the anticharm quark $\bar{c}$ is measured to be the sum of mean masses of the $c\bar{c}$ system ($M_{c\bar{c}}$) and the light (nonstrange) meson $n\bar{n}$ systems ($M_{n\bar{n}}$), subtracted the mean mass of the singly charmed mesons ($M_{c\bar{n}}$) twice, which conceals all charm (anticharm) quark masses and that of the light quarks, left only with binding energy. The procedures for $B_{b\bar{b}}$ in Fig. 1(c) and for $B_{b\bar{c}}$ in Fig. 1(e) are similar: the pair $c\bar{c}$ and the $c\bar{n}$ are replaced by the $b\bar{b}$ and the $b\bar{n}$, or by the pair $b\bar{c}$ and both of the $c\bar{n}$ and the $b\bar{n}$, respectively. In the cases of the binding B_{QQ} between pair QQ, it is measured to be the sum of mean masses of the DH baryons (M_{QQn}) and the light nucleons (M_{nnn}), subtracted the mean mass of singly heavy baryons (M_{Qnn}) twice, which conceals all heavy quark masses and that of the light quarks. This relation is shown in Fig. 1(b) for the system ccn, Fig. 1(d) for the bbn, and Fig. 1(f) for the bcn, where the black (gray) solid circle stands for the bottom (charm) quark while the hollow circle stands for the light quarks. Corresponding to these subfigures, the relations are listed in Table V collectively.

Given the relations in Table V, one can estimate B_{QQ} (and $B_{Q\bar{Q}}$) using the measured (mean) masses [6] of the observed mesons and baryons involved, with the results listed in the second column of Table V. One exception is B_{bc} for which the masses of the baryons Ξ_{bc} and Ξ_{bc}^* are from the lattice computation, $M(\Xi_{bc}) = 6943.0$ MeV, $M(\Xi_{bc}^*) = 6985.0$ MeV [40]. Notice that in Table V the notation M with quark subscript represents the mean masses of the hadrons in the sense that the CMI are minimized. All mean masses for the hadrons involved are evaluated from the measured data of the hadrons and shown in Table VI explicitly, with exception for the pion mesons with $M(\pi) = 154.0$ MeV predicted by the relativistic quark model [41]. We choose the value that is slightly larger than measured data (140 MeV) for the pion since the pion is believed to be suppressed in mass normally by chiral symmetry (the Nambu-Goldstone mechanism) [42]. For $B_{Q\bar{Q}}$, our values are in consistent that predicted by Ref. [36], for instance, our value $B_{c\bar{c}} = -257.9$ MeV is close to the prediction $B_{c\bar{c}} = -258.0$ MeV in Ref. [36].

TABLE V. The binding energies between one heavy quark and another heavy quark or antiquark and their values computed via the relations in the first column.

The binding energy	Value (MeV)
$B_{c\bar{c}} = M_{c\bar{c}} - 2M_{c\bar{n}} + M_{n\bar{n}}$	−257.9
$B_{b\bar{b}} = M_{b\bar{b}} - 2M_{b\bar{n}} + M_{n\bar{n}}$	−562.0
$B_{b\bar{c}} = M_{b\bar{c}} - M_{b\bar{n}} - M_{c\bar{n}} + M_{n\bar{n}}$	−349.1
$B_{cc} = M_{ccn} - 2M_{cnn} + M_{nnn}$	−166.8
$B_{bb} = M_{bbn} - 2M_{bnn} + M_{nnn}$	−418.6
$B_{bc} = M_{bcn} - M_{bnn} - M_{cnn} + M_{nnn}$	−217.5

C. The flux energy E_i

We assume in this work that the DH baryons and the singly heavy baryons are similar in static dynamics to the DH tetraquarks with the same heavy flavor. In this approximation, one can estimate the flux-tube energy E_i tied to the ith quark in Eq. (1). As hadrons formed via the QCD force have masses which depend on the flavor explicitly, one may expect that the flavor-dependence enters somehow for hadrons to break the flavor symmetry underlying in hadron dynamics, for instance, one can expect that it enters mainly via kinematics of (flavored) quarks and gluons in hadrons. So, the flux tubes formed in hadrons can acquire different energies (scale like the lengths of the flux tubes, which depend on the inverse reduced masses $1/\min[\mu_{\mathrm{Reduced}}]$, with μ_{Reduced} the reduced mass of the subsystems of hadrons) and thereby they vary with the flavor content of hadrons considered. By the way, since we have considered the heavy-flavor binding energy $B_{QQ'}$ in Sec. III B and the light quark n (forming brown muck) is of relativistic, each of the light antiquarks $\bar{n}\bar{n}$ in the DH tetraquark T_{QQ} is away from the core system QQ as one light quark n does in heavy baryons, one can reasonably apply the flux tube energy E_Q determined from heavy baryons to tetraquarks.

Note here that E_Q differs slightly for the charmed and bottom hadrons (including the charmed-bottom tetraquarks) due to asymmetry of the flux tubes tied to the heavy quarks $Q = b$ and $Q = c$. We then consider the three cases:

(i) The doubly charmed baryons $\Xi_{cc} = ccn$: In this situation, one can apply Eq. (1) to the Ξ_{cc} and Ξ_{cc}^* and find the mean mass M_{ccn} for them to be

$$2(m_c + E_c) + (m_n + E_n) + B_{cc} + \frac{8}{3}\frac{A_{cc}}{m_c m_c} = M_{ccn}. \tag{15}$$

TABLE VI. The expressions and their values of the mean masses of hadrons in their ground states.

The spin-averaged mass	Value (MeV)	The spin-averaged mass	Value (MeV)
$M_{c\bar{c}} = [M(\eta_c) + 3M(J/\Psi)]/4$	3068.7	$M_{ccn} = [2M(\Xi_{cc}) + 4M(\Xi^*_{cc})]/6$	3692.6
$M_{c\bar{n}} = [M(D^\pm) + 3M(D^{*\pm})/4$	1973.2	$M_{cnn} = [2M(\Sigma_c) + 4M(\Sigma^*_c)]/6$	2496.9
$M_{n\bar{n}} = [M(\pi) + 3M(\rho)]/4$	620.0	$M_{nnn} = [2M(N) + 4M(\Delta)]/6$	1134.5
$M_{b\bar{b}} = [M(\eta_b) + 3M(\Upsilon)]/4$	9444.9	$M_{bbn} = [2M(\Xi_{bb}) + 4M(\Xi^*_{bb})]/6$	10099.0
$M_{b\bar{n}} = [M(B) + 3M(B^*)]/4$	5313.4	$M_{bnn} = [2M(\Sigma_b) + 4M(\Sigma^*_b)]/6$	5826.1
$M_{c\bar{b}} = [M(B^+_c) + 3M(B^{*+}_c)]/4$	6317.6	$M_{bcn} = [2M(\Xi_{bc}) + 4m(\Xi^*_{bc})]/6$	6971.0

Further, the same method applied to the charmed baryons Σ_c, Σ^*_c, and Λ^+_c leads to the relation for M_{cnn},

$$(m_c + E_c) + 2(m_n + E_n) = M_{cnn}. \quad (16)$$

With the values $M_{ccn} = 3692.6$ MeV and $M_{cnn} = 2444.3$ MeV in Table VI, Eqs. (15) and (16) allow one to solve for two parameters E_c and E_n. The results are $E_c = 308.7$ MeV, $E_n = 117.8$ MeV.

(ii) The doubly bottom baryons $\Xi_{bb} = bbn$: For this case, one can apply Eq. (1) to the Ξ_{bb} and Ξ^*_{bb} and thereby find

$$2(m_b + E_b) + (m_n + E_n) + B_{bb} + \frac{8}{3}\frac{A_{bb}}{m_b m_b} = M_{bbn}. \quad (17)$$

Similar method applied to the baryons Σ_b, Σ^*_b, and Λ^0_b give

$$(m_b + E_b) + 2(m_n + E_n) = M_{bnn}. \quad (18)$$

Combining $M_{bbn} = 10099.0$ MeV and $M_{bnn} = 5774.4$ MeV given in Table VI, Eqs. (17) and (18) lead to the solutions: $E_b = 601.7$ MeV, $E_n = E^*_n \equiv 116.3$ MeV. Here, the superscript star is used to denote the flux-tube energy E_n in the singly bottom baryons.

(iii) The bottom-charmed baryons $\Xi_{bc} = bcn$: In this case, application of Eq. (1) to the Ξ_{bc} and Ξ^*_{bc} leads to the following relation for M_{bcn},

$$(m_b + E_b) + (m_c + E_c) + (m_n + E_n) + B_{cb} + \frac{8}{3}\frac{A_{bc}}{m_c m_b} = M_{bcn}, \quad (19)$$

from which using $M_{bcn} = 6971.0$ MeV in Table VI one can solve $E^*_c = 312.7$ MeV, $E^*_b = 602.8$ MeV, and $E_n = E^{**}_n = 115.8$ MeV. Here, the superscript star in $E^*_{c,b}$ are used to denote the values of $E_{c,b}$ in the bottom-charmed baryons, and the double star stands for the values of E_n in the bottom-charmed baryons.

One sees that the values of the E_b in the bottom baryons (601.7 MeV) and the bottom-charmed baryons (602.8 MeV) are almost same while the values of the E_c in the charmed baryons (308.7 MeV) and the bottom-charmed baryons (312.7 MeV) differ slightly (with uncertainty 4 MeV). The values of the flux-tube energies E_n of the up and down quarks are nearly identical ($E_n = 117.8$ MeV, $E^*_n = 116.3$ MeV, and $E^{**}_n = 115.8$ MeV). In this work, we choose to use the respective values of the flux-tube energies $E_{c,b}$ and E_n for the three cases above though they are close in three cases.

IV. MASSES OF T_{cc} AND OTHER DH TETRAQUARKS

The binding energies $B_{QQ} = B_{QQ}[\bar{3}_c \otimes 3_c]$ discussed in Sec. III B correspond to the color configuration $\bar{3}_c \otimes 3_c$ of the baryons we consider. According to the ratio $-1:2$ of the color factors for the configurations $6_c \otimes \bar{6}_c$ and $\bar{3}_c \otimes 3_c$, the binding energy for the pair QQ in the tetraquarks in $6_c \otimes \bar{6}_c$ should be given by $B_{QQ}[6_c \otimes \bar{6}_c] = -B_{QQ}/2$ [30], as we shall apply in this work.

Since B_{QQ} form a diagonal matrix in Eq. (1), the possible mixing of the color-spin states stems only from the CMI Hamiltonian $H_{\rm CMI}$ in Table I. Given the values of three parameters, A_{ij}, B_{QQ}, and $E_{c,b,n}$, determined in Sec. III, one can compute the ground-state masses of the doubly heavy tetraquarks T_{cc}, T_{bb}, and T_{bc} via diagonalizing the Sakharov-Zeldovich Hamiltonian (1) with the CMI (2). The numerical diagonalization of the matrix $B_{QQ} + H_{\rm CMI}$ are carried out and illustrated explicitly in the Table VII for the tetraquarks T_{cc}, T_{bb}, and T_{bc} with various quantum numbers (I, J^P), with the obtained eigenvalues, eigenvectors, and the corresponding masses given by Eq. (1) shown.

We take the $cc\bar{n}\bar{n}$ tetraquark with $IJ^P = 01^+$ as an example. As shown in Table I, the allowed color-spin basis are $(\phi^T_2\chi^T_4, \phi^T_1\chi^T_5)$, and the CMI matrix (in MeV)

$$\langle H_{\rm CMI}\rangle = \begin{bmatrix} \frac{8}{3}V_{cc} - 8V_{nn} & -8\sqrt{2}V_{cn} \\ -8\sqrt{2}V_{cn} & 4V_{cc} - \frac{4}{3}V_{nn} \end{bmatrix} = -\begin{bmatrix} 132.59 & 75.35 \\ 75.35 & 3.25 \end{bmatrix}, \quad (20)$$

TABLE VII. Computed H matrices of the nonstrange DH tetraquarks $T_{QQ} = QQ\bar{n}\bar{n}$ ($QQ = cc, bb, bc$) and ensuing diagonalization with the mixed weights and tetraquark masses (MeV) obtained. Three sets of E_i slightly different (see text) are employed: $E_c = 308.7$ MeV, $E_n = 117.8$ MeV, $E_b = 601.7$ MeV, $E_n^* = 116.3$ MeV, $E_c^* = 312.7$ MeV, $E_b^* = 602.8$ MeV, and $E_n^{**} = 115.8$ MeV.

Systems	IJ^P	H (MeV)	Eigenvalues (MeV)	Eigenvectors	M (MeV)
$cc\bar{n}\bar{n}$	01^+	$\begin{pmatrix} -299.4 & -75.4 \\ -75.4 & 80.1 \end{pmatrix}$	$\begin{bmatrix} -313.8 \\ 94.6 \end{bmatrix}$	$\begin{bmatrix} (-0.98, -0.19) \\ (0.19, -0.98) \end{bmatrix}$	$\begin{bmatrix} 3879.2 \\ 4287.6 \end{bmatrix}$
$cc\bar{n}\bar{n}$	10^+	$\begin{pmatrix} -174.8 & 130.5 \\ 130.5 & 178.0 \end{pmatrix}$	$\begin{bmatrix} 221.0 \\ -217.8 \end{bmatrix}$	$\begin{bmatrix} (0.31, 0.95) \\ (-0.95, 0.31) \end{bmatrix}$	$\begin{bmatrix} 4414.0 \\ 3975.2 \end{bmatrix}$
	11^+	−139.3	−139.3	1.00	4053.7
	12^+	−68.3	−68.3	1.00	4124.7
$bb\bar{n}\bar{n}$	01^+	$\begin{pmatrix} -557.6 & -8.5 \\ -8.5 & 196.4 \end{pmatrix}$	$\begin{bmatrix} -557.7 \\ 196.5 \end{bmatrix}$	$\begin{bmatrix} (-1.00, -0.01) \\ (0.01, -1.00) \end{bmatrix}$	$\begin{bmatrix} 10298.3 \\ 11052.5 \end{bmatrix}$
$bb\bar{n}\bar{n}$	10^+	$\begin{pmatrix} -370.0 & 14.7 \\ 14.7 & 294.2 \end{pmatrix}$	$\begin{bmatrix} -370.3 \\ -294.5 \end{bmatrix}$	$\begin{bmatrix} (-1.00, 0.02) \\ (-0.02, -1.00) \end{bmatrix}$	$\begin{bmatrix} 10485.7 \\ 11150.5 \end{bmatrix}$
	11^+	−366.0	−366.0	1.00	10490.0
	12^+	−358.0	−358.0	1.00	10498.0
$bc\bar{n}\bar{n}$	00^+	$\begin{pmatrix} -385.8 & 72.6 \\ 72.6 & -18.1 \end{pmatrix}$	$\begin{bmatrix} -399.6 \\ -4.3 \end{bmatrix}$	$\begin{bmatrix} (-0.98, 0.19) \\ (-0.19, -0.98) \end{bmatrix}$	$\begin{bmatrix} 7127.5 \\ 7522.8 \end{bmatrix}$
	01^+	$\begin{pmatrix} -357.0 & 47.3 & -42.0 \\ 47.3 & 30.3 & 55.7 \\ -41.9 & 55.7 & 95.1 \end{pmatrix}$	$\begin{bmatrix} -367.8 \\ 127.7 \\ 9.4 \end{bmatrix}$	$\begin{bmatrix} (0.99, -0.13, 0.11) \\ (-0.03, 0.49, 0.87) \\ (-0.17, -0.86, 0.48) \end{bmatrix}$	$\begin{bmatrix} 7159.3 \\ 7654.8 \\ 7536.5 \end{bmatrix}$
	02^+	130.1	130.1	1.00	7657.2
$bc\bar{n}\bar{n}$	10^+	$\begin{pmatrix} -200.9 & 72.6 \\ 72.6 & 192.9 \end{pmatrix}$	$\begin{bmatrix} -213.8 \\ 205.9 \end{bmatrix}$	$\begin{bmatrix} (-0.98, 0.18) \\ (-0.18, -0.98) \end{bmatrix}$	$\begin{bmatrix} 7313.3 \\ 7733.0 \end{bmatrix}$
	11^+	$\begin{pmatrix} -118.1 & 22.3 & 47.3 \\ 22.3 & -190.2 & -42.0 \\ 47.3 & -41.9 & 178.5 \end{pmatrix}$	$\begin{bmatrix} -204.2 \\ 189.7 \\ -115.3 \end{bmatrix}$	$\begin{bmatrix} (0.32, -0.94, -0.14) \\ (-0.14, -0.10, 0.98) \\ (0.94, 0.34, -0.10) \end{bmatrix}$	$\begin{bmatrix} 7322.9 \\ 7716.8 \\ 7411.8 \end{bmatrix}$
	12^+	−141.6	−141.6	1.00	7385.5

with $V_{ij} \equiv A_{ij}/(m_i m_j)$. The corresponding matrix of the binding energy (MeV) is

$$B_{QQ} = \begin{pmatrix} B_{cc} & 0 \\ 0 & -B_{cc}/2 \end{pmatrix} = \begin{bmatrix} -166.8 & 0 \\ 0 & 83.4 \end{bmatrix}, \quad (21)$$

for $QQ = cc$, which leads to a sum of two matrices in Eqs. (20) and (21),

$$H = B_{QQ} + \langle H_{\mathrm{CMI}} \rangle = \begin{bmatrix} -299.4 & -75.4 \\ -75.4 & 80.1 \end{bmatrix}. \quad (22)$$

The diagonalization of the matrix H in Eq. (22) gives rise to the eigenvalues and the respective eigenvectors,

$$\mathrm{Eig}(H) = \begin{bmatrix} -313.8 \\ 94.6 \end{bmatrix},$$
$$\mathrm{Eigv}(H)_v = \begin{bmatrix} (-0.98, -0.19) \\ (0.19, -0.98) \end{bmatrix}, \quad (23)$$

by which one can use Sakharov-Zeldovich formula (1) where $M = 2(m_c + E_n) + \mathrm{Eig}(H)$ to find the groundstate masses of the tetraquark T_{cc} with $IJ^P = 01^+$,

$$M(T_{cc}, 01^+) = 3879.2\ \mathrm{MeV}, \qquad \text{in } \bar{3} \otimes 3_c(96\%), \quad (24)$$

$$M(T_{cc}, 01^+) = 4287.6\ \mathrm{MeV}, \qquad \text{in } 6 \otimes \bar{6}_c(96\%), \quad (25)$$

with the chromomagnetic-mixing probability $0.98^2 = 96\%$.

The first value of the T_{cc} masses here is quite close to the $D^{*+}D^0$ threshold $M(D^{*+}D^0) = 3875.1 \pm 0.1$ MeV [6], being in agreement with the observed mass of about 3875.7 ± 0.05 MeV of the $J^P = 1^+$ tetraquark $T_{cc}^+(3875)$ [8,9] discovered LHCb at CERN. Notice further that the LHCb-reported tetraquark $T_{cc}^+(3875)$ is very narrow (width $= 410 \pm 165$ keV), one can conclude that the lowest state of the doubly charmed tetraquark is the $J^P = 1^+$ tetraquark $T_{cc}^+(3875)$ that was observed by LHCb, which is the resonance state of the tetraquark system $cc\bar{u}\bar{d}$ with main color configuration of the $\bar{3}_c \otimes 3_c$. One infers, by

TABLE VIII. Our results (M) and other computations for masses of the DH tetraquarks T_{QQ}. The lowest threshold (T) of two heavy mesons ($Q\bar{q}$) for the tetraquarks considered and $\Lambda = M - T$ are also given. All in MeV.

System	IJ^P	M	T	Λ	[43]	[44]	[45]	[46]	[47]
$cc\bar{n}\bar{n}$	01^+	3879.2	3875.9(DD^*)	3.3	3978.0	3997.0	3868.7		3877.0
		4287.6	4017.2(D^*D^*)	270.4			4230.8		
	10^+	3975.2	3875.9(DD^*)	99.3	4146.0	4163.0	3969.2	3870.0	
		4414.0	4017.2(D^*D^*)	396.8			4364.9		
	11^+	4053.7	4017.2(D^*D^*)	36.5	4167.0	4185.0	4053.2	3900.0	4021.0
	12^+	4124.7	4017.2(D^*D^*)	107.5	4210.0	4229.0	4123.8	3950.0	
$bb\bar{n}\bar{n}$	01^+	10298.3	10604.2(BB^*)	−305.9	10482.0	10530.0	10390.9		10353.0
		11052.5	10649.4(B^*B^*)	403.1			10950.3		
	10^+	10485.7	10559.0(BB)	−73.3	10674.0	10726.0	10569.3		
		11150.5	10649.4(B^*B^*)	501.1			11054.6		
	11^+	10490.0	10604.2(BB^*)	−114.2	10681.0	10733.0	10584.2		10403.0
	12^+	10498.0	10649.4(B^*B^*)	−151.4	10695.0	10747.0	10606.8		
$bc\bar{n}\bar{n}$	00^+	7127.5	7146.8(BD)	−19.3	7229.0	7268.0	7124.6		
		7522.8	7333.3(B^*D^*)	189.5			7459.0		
	01^+	7159.3	7192.0(B^*D)	−32.7	7272.0	7269.0	7158.0		
		7536.5	7333.3(B^*D^*)	203.2			7482.4		
		7654.8	7333.3(B^*D^*)	321.5			7584.9		
	02^+	7657.2	7333.3(B^*D^*)	323.9			7610.3		
	10^+	7313.3	7288.1(BD^*)	25.2	7461.0	7458.0	7305.6		
		7733.0	7333.3(B^*D^*)	399.7			7684.7		
	11^+	7322.9	7288.1(BD^*)	34.8	7472.0	7469.0	7322.5		
		7411.8	7333.3(B^*D^*)	78.5		7478.0	7367.3		
		7716.8	7333.3(B^*D^*)	383.5			7665.1		
	12^+	7385.5	7333.3(B^*D^*)	52.2	7493.0	7490.0	7396.0		

Eq. (25), that there exists a nontrivial color partner $T^+_{cc}(cc\bar{u}\bar{d}, 6_c \otimes \bar{6}_c)$ of the tetraquark $T^+_{cc}(3875)$, being a fully new tetraquark $cc\bar{u}\bar{d}$ with color configuration $6_c \otimes \bar{6}_c$, and with the same $IJ^P = 01^+$ but the mass about 4288 MeV. Differing only in color representation from the $T^+_{cc}(3875)$, we hope that a further experimental search similar to the LHCb experiment [8] in the some final states (e.g., the $D^0D^0\pi^+$) with invariant mass about 408 MeV higher than the $T^+_{cc}(3875)$ can test our prediction. The numerical diagonalization of the matrix $H = B_{QQ} + H_{\mathrm{CMI}}$ are also performed for the $I = 1$ tetraquark T_{cc} with $J^P = 0^+, 1^+$, and 2^+, resulting in small color-spin mixing for $J^P = 0^+$ state, as illustrated in the Table VII.

Further computations are performed similarly for other DH tetraquarks T_{bb} and T_{bc} with $J^P = 0^+, 1^+, 2^+$ and $I = 0$, 1, as shown in Table VII, resulting in a similar mass order and configurations, except that the inverse mass order (4414.0, 3975.2) MeV for the $(T^+_{cc}(\bar{3}_c \otimes 3_c), T^+_{cc}(6_c \otimes \bar{6}_c))$ are replaced by the normal order (10485.7, 11150.5) MeV for the $(T^+_{bb}(\bar{3}_c \otimes 3_c), T^+_{bb}(6_c \otimes \bar{6}_c))$. The $I = 1$ tetraquark T_{bb} with $J^P = 1^+$ and that with $J^P = 2^+$ becomes nearly degenerated (10490.0 MeV, 10498.0 MeV), in contrast with the splitted masses (4053.7 MeV, 4124.7 MeV) of the $I = 1$ tetraquark T_{cc} with $J^P = 1^+$ and 2^+. In the case of the bottom-charmed tetraquarks T_{bc}, the chromomagnetic mixing occurs between two color reps, for the $J^P = 0^+$ and among three color-spin states for the $J^P = 1^+$ case, deeply for the $IJ^P = 01^+$. Generally, the chromomagnetic-mixing enters more or less for the $IJ^P = 01^+$ and $IJ^P = 1(0^+, 1^+)$ states of all DH tetraquarks T_{QQ}, for which the novel $6_c \otimes \bar{6}_c$ color state of the tetraquarks occupies the higher mass levels.

In Table VIII, our results for the DH tetraquark masses (M) are compared to other computations via the quark model [43,44], the chromomagnetic models [45] and the QCD sum rules [46,47]. In the Table VIII, we also list the lowest thresholds (T) of two heavy mesons ($Q\bar{q}$) as decaying final states of T_{QQ} and the respective mass difference $\Lambda = M - T$.

V. CONCLUSIONS AND REMARKS

Inspired by recent experimental progress of the tetraquarks, we systematically study the nonstrange doubly heavy tetraquarks T_{cc}, T_{bb}, and T_{bc} in the chromomagnetic interaction model with unified quark masses and the flux-tube correction incorporated. A binding energy B_{QQ} between heavy-heavy quarks is introduced and estimated via a simple relation in terms of the measured masses of baryons, by which we extract the flux-tube corrections E_i associated with ith quark in doubly heavy tetraquarks T_{QQ} via matching the model with the (measured or lattice) mass data of the heavy hadrons. Using numerical diagonalization of chromomagnetic interaction we predict that the ground-state masses of the doubly charmed tetraquark T_{cc} (in color $\bar{3}_c \otimes 3_c$) with $IJ^P = 01^+$ is 3879.2 MeV, about 3 MeV above the D^*D^0 threshold, and that of the T^*_{cc} (in $6_c \otimes \bar{6}_c$) with $IJ^P = 01^+$ is about 4287.6 MeV. The former prediction is in consistent with the measured mass 3874.7 ± 0.05 MeV of the doubly charmed tetraquark $T_{cc}(1^+)$ discovered by LHCb [8]. The same method is applied to the systems T_{bb} and T_{bc} to compute of the groundstate masses of the doubly bottom tetraquarks T_{bb} and the bottom-charmed tetraquarks T_{bc} with $J^P = 0^+, 1^+, 2^+$, and $I = 0$, 1.

As the observed tetraquark $T^+_{cc}(3875)$ is very narrow and near threshold, we suggest that the $J^P = 1^+$ tetraquark $T^+_{cc}(3875)$ observed by LHCb is the lowest state of the doubly charmed tetraquark, being a resonance state of the four-quark system $cc\bar{u}\bar{d}$ with color configuration of the $\bar{3}_c \otimes 3_c$ dominately. Further, there exists a new color partner $T^+_{cc}(cc\bar{u}\bar{d}, 6_c \otimes \bar{6}_c)$ of the tetraquark $T^+_{cc}(3875)$ consisting of the $cc\bar{u}\bar{d}$ with dominate color configuration of $6_c \otimes \bar{6}_c$, having the same $IJ^P = 01^+$ but the mass about 4288 MeV. A further experimental search similar to the LHCb experiment [8] around the invariant mass of the final states about 408 MeV higher than the $T^+_{cc}(3875)$ can test our prediction.

The chromomagnetic mixing enters generally more or less for the $IJ^P = 01^+$ and $IJ^P = 1(0^+, 1^+)$ states of all DH tetraquarks T_{QQ}, for which the novel $6_c \otimes \bar{6}_c$ color state of the tetraquarks occupies the higher mass levels. For the bottom-charmed tetraquarks T_{bc} with $IJ^P = 01^+$, the chromomagnetic coupling are strong enough among three color-spin states that some of them are mixed substantially between the color configurations $\bar{3}_c \otimes 3_c$ and $6_c \otimes \bar{6}_c$.

We remark that the heavy quark pair in diquark QQ in $\bar{3}_c$ may (when they are very heavy) tend to stay close to each other to form a compact core due to the strong Coulomb interaction, while it is also possible (when m_Q is comparable with $1/\Lambda_{\rm QCD} \approx 3\ \mathrm{GeV}^{-1}$) that Q attracts $\bar{q}$ to bind them into a colorless clustering (in 1_c) and other pair binds into another colorless clustering (in 1_c), resulting in a molecular system $(Q\bar{q})(Q'\bar{q}')$. In the former case, DH tetraquarks mimic the heliumlike QCD atom, while they resemble the hydrogenlike QCD molecules in the later case. The possible molecule configurations of two heavy mesons or mixture of molecules in the tetraquark systems are of interest and remains to be explored.

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