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Model building by coset space dimensional reduction scheme using eight-dimensional coset spaces

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Abstract: We investigate the twelve-dimensional gauge-Higgs unification models with an eight- dimensional coset space as the extra space. For each model, we apply the coset space dimensional reduction procedure and examine the particle contents of the resulting four-dimensional theory. All combinations of inputs to the procedure are exhaustively analyzed under several assumptions. As a result, some twelve-dimensional SO(18) gauge theories lead to models of the SO(10) × U(1) grand unified theory in four dimensions, where fermions of the Standard Model appear in multiple generations along with scalars that may break the electroweak symmetry. The representations of the obtained scalars and fermions are summarized.

Keywords: Grand Unification, Extra Dimensions

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1 Introduction

The Standard Model (SM) of particle physics is a highly successful theory that serves as a language to precisely describe phenomena related to elementary particles, especially those observed in accelerator experiments. The discovery of the Higgs scalar in the year 2012 fulfilled the last missing piece of the SM. This fact established the success of the SM and we are able to say that the SM is the standard model of elementary particle physics. However, unfortunately, the SM cannot predict the properties of the Higgs scalar, such as its mass and its coupling constant, and does not even refer to its theoretical origin. Therefore, a theory that explains the origin of the Higgs scalar beyond the SM has been intensely studied for many years.

One of the theories is called the gauge-Higgs unification (GHU) [1–3] (for further developments, see refs. [4–18]) that explains the origin of the Higgs scalar. In this framework, the theoretical origin of the Higgs scalar field is considered to be the extra-dimensional components of higher-dimensional (D-dimensional) gauge fields, and the properties of the Higgs scalar are attributed to the gauge symmetry and compactification scale of the higher-dimensional theory. After the dimensional reduction, we can obtain a theory that contains the Higgs scalar with desired properties. One such possible dimensional reduction is the scheme of the coset space dimensional reduction (CSDR) [19–25], where the extra space is assumed to be a coset space S/R of a compact Lie group S by its subgroup $R \subset S$, and symmetry transformations of the extra space are identified with gauge transformations in higher dimensions. These constraints determine nicely the gauge group and particle content in the resulting four-dimensional theory. Our main aim is to search for higher-dimensional models which describe, after CSDR, chiral theories in four dimensions as in the SM, which means that there are left-handed and right-handed fermions that belong to different representations of the gauge group.

The case of $D = 4n + 2$ dimensions ($n \in \mathbb{N}$) has been the most fascinating in this respect and has drawn much attention [26–28]. One can obtain a chiral theory in four dimensions after the dimensional reduction even if one chooses that vector-like fermions present in these dimensions. However, no phenomenologically promising models have been found in $D = 6, 10, 14$ dimensions [26, 28–34].

It, therefore, becomes necessary to consider the case of $D = 4n$, which is different from $D = 4n + 2$ because vector-like fermions in these dimensions cannot produce chiral fermions in four dimensions. Thus, one must introduce fermions belonging to complex representations of the gauge group in the higher dimensions. In our previous research [35], we have investigated the smallest dimension, i.e., $D = 8$ and we have shown the possibility of building phenomenologically realistic models even for $D \neq 4n + 2$. The purpose of the present study is to investigate the case of $D = 12$, the next smallest dimension after $D = 8$. Dimension twelve is also of interest from a purely theoretical perspective [36–38] and is another motivation for our study.

In this paper, we investigate twelve-dimensional gauge theories with an eight-dimensional coset space as the extra dimension and explore the possibility of obtaining the SM, Grand Unified Theories (GUTs), and their analogues through coset space dimensional reduction. First, we enumerate possible coset spaces S/R and gauge groups G in twelve dimensions under plausible assumptions. Then the four-dimensional gauge groups H arise from these settings. For each candidate, we introduce fermions and perform the dimensional reduction to obtain the resulting scalar and fermionic fields in four dimensions and determine their representations under the gauge group H.

This article is organized as follows. In section 2, we review the coset space dimensional reduction. Next, in section 3, we classify models aiming for a twelve-dimensional theory that gives rise to phenomenologically desirable models in four dimensions. Finally, in section 4, we summarize our results and discuss the implications of the models obtained in four dimensions.

2 Coset space dimensional reduction in twelve dimensions

In this section, we review the scheme of the coset space dimensional reduction and discuss its specific application in $D = 4 + 8$ dimensions [21].

2.1 Coset space dimensional reduction

Let S be a compact Lie group and $R \subset S$ be its compact subgroup. We denote their coset space by S/R. The group S is the symmetry group of S/R and R is the isotropy group (also known as the little group) of a certain point in S/R, which is set to be the coset corresponding to the identity element of S. The higher-dimensional coordinates are denoted by $X^M = (x^\mu, y^\alpha)$ for $M = 0, \ldots, D-1$, $\mu = 0, 1, 2, 3$, $\alpha = 4, \ldots, D-1$. The tangent space of S/R has a local symmetry group, $\mathrm{SO}(d)$, where d is the dimension of the coset space, i.e. $d = \dim S/R$, which includes R as a subgroup. We consider a Yang-Mills theory on

$\mathbb{R}^{1,3} \times S/R$, where S/R is added as a direct product to the four-dimensional Minkowski space $\mathbb{R}^{1,3}$. Furthermore, the gauge group of the theory is assumed to be a compact Lie group G. We introduce a higher-dimensional fermion that belongs to a representation F of the gauge group G. Under the settings above the higher-dimensional action $A^{(D)}$ is given by

$$\begin{aligned} A^{(D)} &= \int d^4x\, d^dy \sqrt{-g}\, \mathscr{L}^{(D)}(x,y) \\ &= \int d^4x\, d^dy \sqrt{-g} \left[-\frac{1}{8} g^{MK} g^{NL} \operatorname{Tr}(F_{MN}F_{KL}) + \frac{1}{2} i\bar{\Psi} D_M \Psi \right], \end{aligned} \tag{2.1}$$

where $g_{MN}(x,y)$ is the metric of the whole spacetime, $F_{MN}(x,y)$ is the field strength of the higher-dimensional gauge field $A_M(x,y)$, $\Psi(x,y)$ is the higher-dimensional fermion, and D_M is the covariant derivative, respectively.

We then impose symmetry conditions on fields in the theory as in refs. [19, 39–44]. These conditions require a transformation of the fields under a S symmetry $s : y \mapsto y^s$ is canceled out by a gauge transformation g of G. This is symbolically expressed as

$$\Phi^s(x,y) = \Phi^g(x,y), \tag{2.2}$$

where Φ denotes a field (A_μ, A_α, or Ψ). The infinitesimal forms of the symmetry conditions are the followings. For each Killing vector $\xi_A = (\xi_A^\alpha)$ $(A = 0, \ldots, \dim S)$ which represents an isometric transformation of S, there exists a matrix-valued parameter W_A for a gauge transformation that satisfies

$$\xi_A^\beta \partial_\beta A_\alpha + \partial_\alpha \xi_A^\beta A_\beta = \partial_\alpha W_A + [W_A, A_\alpha]. \tag{2.3}$$

The left-hand side of this equation represents the isometric transformation generated by ξ_A, and the right-hand side is the compensating gauge transformation with the parameter W_A. The constraint on Ψ has a similar infinitesimal expression, where the isometric transformation property is accompanied by the local Lorentz transformation [21]. The conditions require the fields to be covariant under S transformations, up to gauge transformations. We impose them by hand as the assumptions on the y-dependence of higher dimensional fields, which can also be regarded as the conditions that must be satisfied by four-dimensional fields.

As a result, the Lagrangian $\mathscr{L}^{(D)}(x,y)$ of the higher-dimensional theory loses its dependence on the extra-dimensional coordinates y, while the y-dependence of the fields remains. Integrating out the extra dimensions gives the Lagrangian $\mathscr{L}^{(4)}(x)$ of the four-dimensional theory. Moreover, the following representation-theoretic constraints are implied by CSDR to derive the content of the four-dimensional theory.

(i) The subgroup R of S is also a subgroup of G, which is denoted by $R_G \subset G$.

(ii) The gauge group H in four-dimensional theory is the centralizer of R in G, i.e., the set of all elements in G that commute with any element in R.

$$H = C_G(R_G) = \{g \in G \mid gr = rg \quad (\forall r \in R)\}. \tag{2.4}$$

(iii) The representation of the gauge group H for scalar fields in a four-dimensional theory is specified as follows. First, we decompose the adjoint representation $\operatorname{ad} S$ of the

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symmetry group of the coset space S/R under the subgroup R:

$$\begin{aligned} S &\supset R, \\ \operatorname{ad} S &= \operatorname{ad} R \oplus \left(\bigoplus_i \rho_i \right), \end{aligned} \tag{2.5}$$

where ρ_i are irreducible representations of R. We also write the decomposition of the adjoint representation $\operatorname{ad} G$ of the gauge group G under the action of $R_G \times H$ as follows (up to U(1) factors).

$$\begin{aligned} G &\supset R_G \times H, \\ \operatorname{ad} G &= \bigoplus_j (r_j, h_j) \\ &= (\operatorname{ad} R, \mathbf{1}) \oplus (\mathbf{1}, \operatorname{ad} H) \oplus \cdots . \end{aligned} \tag{2.6}$$

Here, r_j and h_j are irreducible representations of R and H, respectively. If equivalent irreducible R representations ρ_i and r_j appear in the decompositions, eq. (2.5) and eq. (2.6), the scalar fields belonging to h_j, appear in the four-dimensional theory.

In practice, it is useful to consider the R decomposition of SO(d) vector, as described below. The geometry of the coset space S/R is governed by its symmetry group S and its isotropy subgroup R. We write the generators of S as $\{Q_A\} = \{Q_i, Q_a\}$, where $A = 1, \ldots, \dim S$, $i = 1, \ldots, \dim R$, and $a = 1, \ldots, d$ with $d = \dim S/R$. $\{Q_i\}$ are the generators of R and $\{Q_a\}$ are the rest of the generators (coset generators). It is important that the coset generators Q_a span the tangent space of S/R, so they transform as vectors, denoted by $\boldsymbol{v}$, under the local symmetry SO(d) of the tangent space. Thus S, R, and SO(d) satisfy

$$\begin{aligned} S &\supset R, \\ \operatorname{ad} S &= \operatorname{ad} R + \boldsymbol{v}. \end{aligned} \tag{2.7}$$

From this relation and eq. (2.5), we see that the H representations for the four-dimensional scalar fields can be obtained by comparing the two decompositions. One is R decomposition of the SO(d) vector $\boldsymbol{v}$

$$\begin{aligned} \mathrm{SO}(d) &\supset R, \\ \boldsymbol{v} &= \left(\bigoplus_i \rho_i \right), \end{aligned} \tag{2.8}$$

and the other is the $R \times H$ decomposition of $\operatorname{ad} G$ shown in eq. (2.6).

(iv) The representations of the gauge group H for spinor fields in four-dimensional theory are derived according to the following procedure, as in ref. [45]. First, we decompose the spinor representation σ_d of the local symmetry group SO(d), under the subgroup R,

$$\begin{aligned} \mathrm{SO}(d) &\supset R, \\ \sigma_d &= \bigoplus_i \sigma_i. \end{aligned} \tag{2.9}$$

Here, σ_i are irreducible representations of R. The higher-dimensional spinor field belongs to the representation F of the gauge group G, as we have already mentioned. We write its $R_G \times H$ decomposition as

$$\begin{aligned} G &\supset R_G \times H, \\ F &= \bigoplus_j (r_j, h_j), \end{aligned} \tag{2.10}$$

where r_j and h_j are irreducible representations of R and H, respectively. If there are equivalent irreducible R representations σ_i and r_j, the spinor fields in the irreducible representation h_j appear in the four-dimensional theory.

To obtain a theory with chiral spinor fields, the following conditions should be imposed in addition to the rules above.

1. The rank of S, R must be the same: $\operatorname{rank} R = \operatorname{rank} S$. This is a geometric requirement based on a non-trivial result for the Dirac operator on the coset space S/R [46].

2. When $D = 4n$, the spinors introduced in higher-dimensional theories must be Weyl spinors, and in this case, the representation F of the gauge group G to which they belong must be a complex representation. Furthermore, the isotropy group R of the coset space S/R must also admit a complex representation [21].

Note that there is no guiding principle of the compactification scale. Thus we may choose it arbitrarily. For example, if we wish to construct a GUT model, we set it to be just above the GUT scale. Then readers may wonder if the hierarchy problem affects, but we consider it to be treated as in ordinary non-SUSY GUTs. One can also utilize coset spaces in a certain class (non-symmetric[1]) to handle multiple energy scales [21].

2.2 Application and remarks of CSDR on eight-dimensional coset spaces

For the case of $d = 8$ dimensional coset space, representations of the local SO(8) symmetry become important. The SO(8) group has three distinct self-conjugate representations, which are denoted as $\mathbf{8}_\mathrm{v}$, $\mathbf{8}_\mathrm{c}$, and $\mathbf{8}_\mathrm{s}$. The representation $\mathbf{8}_\mathrm{v}$ is the usual real eight-dimensional vector representation, while $\mathbf{8}_\mathrm{c}$ and $\mathbf{8}_\mathrm{s}$ are both real eight-dimensional spinor representations that describe Weyl spinors. We adopt the convention that $\mathbf{8}_\mathrm{c}$ has positive chirality and $\mathbf{8}_\mathrm{s}$ has negative chirality. The three eight-dimensional representations are related to each other by the outer automorphisms of SO(8) and in some contexts the vector $\mathbf{8}_\mathrm{v}$ and the spinors $\mathbf{8}_\mathrm{c}$ and $\mathbf{8}_\mathrm{s}$ can be considered equivalent. For example, it is possible that the decompositions of $\mathbf{8}_\mathrm{v}, \mathbf{8}_\mathrm{c}$, and $\mathbf{8}_\mathrm{s}$ under a subgroup of SO(8) may be interchanged for different choices of embedding. This equivalence of the three eight-dimensional representations is called triality (see "fun with SO(8)" in ref. [47]). We will now discuss some important notes regarding CSDR over the eight-dimensional coset space, using the representations $\mathbf{8}_\mathrm{v}$, $\mathbf{8}_\mathrm{c}$, and $\mathbf{8}_\mathrm{s}$.

[1] A coset space S/R is called symmetric if the associated Lie algebras $\mathfrak{s} = \operatorname{Lie} S, \mathfrak{r} = \operatorname{Lie} R$ and $\mathfrak{q}$ which corresponds to the coset generators satisfy $\mathfrak{s} = \mathfrak{r} \oplus \mathfrak{q}, [\mathfrak{r}, \mathfrak{r}] \subset \mathfrak{r}, [\mathfrak{r}, \mathfrak{q}] \subset \mathfrak{q}, [\mathfrak{q}, \mathfrak{q}] \subset \mathfrak{r}$.

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First, let us discuss scalars. A scalar field in four dimensions arises from the extra-dimensional components (ϕ_a) of a twelve-dimensional SO(1, 11) vector field $A = (A_M) = (A_\mu, \phi_a)$, where $a = 1, \dots, 8$. A twelve-dimensional vector is decomposed under the subgroup $\mathrm{SO}(1,3) \times \mathrm{SO}(8)$ as

$$\begin{aligned} \mathrm{SO}(1,11) &\supset \mathrm{SO}(1,3) \times \mathrm{SO}(8), \\ \mathbf{12} &= (\mathbf{4}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}_\mathrm{v}). \end{aligned} \tag{2.11}$$

Since $\phi = (\phi_a)$ behaves as a vector $\mathbf{8}_\mathrm{v}$ under SO(8), the representation of the gauge group H to which the four-dimensional scalar field belongs can be obtained by comparing the decomposition of $\mathbf{8}_\mathrm{v}$ under R,

$$\begin{aligned} \mathrm{SO}(8) &\supset R, \\ \mathbf{8}_\mathrm{v} &= \left(\bigoplus_i \rho_i \right), \end{aligned} \tag{2.12}$$

and the decomposition of $\mathrm{ad}\, G$ under $R \times H$ (see eq. (2.6)).

Next, we discuss spinors. As mentioned in the previous section, the Weyl spinors must be introduced in $D = 12$ dimensions. We denote the SO(1, 11) Weyl spinor with positive chirality as $\mathbf{32}$ and negative chirality as $\mathbf{32}'$, respectively. They transform under the subgroup $\mathrm{SO}(1,3) \times \mathrm{SO}(8)$ as follows:

$$\begin{aligned} \mathrm{SO}(1,11) &\supset \mathrm{SO}(1,3) \times \mathrm{SO}(8), \\ \mathbf{32} &= (\mathbf{2}, \mathbf{1}; \mathbf{8}_\mathrm{s}) + (\mathbf{1}, \mathbf{2}; \mathbf{8}_\mathrm{c}), \\ \mathbf{32}' &= (\mathbf{2}, \mathbf{1}; \mathbf{8}_\mathrm{c}) + (\mathbf{1}, \mathbf{2}; \mathbf{8}_\mathrm{s}), \end{aligned} \tag{2.13}$$

where $(\mathbf{1}, \mathbf{2})$ and $(\mathbf{2}, \mathbf{1})$ represent right-handed and left-handed Weyl spinors with positive and negative chirality in four dimensions, respectively. If $\mathbf{32}$ Weyl spinor is introduced in twelve dimensions, the H representation of the surviving right-handed spinor in four dimensions is derived by matching the representations that appear in the R decomposition in eq. (2.9) of $\sigma_d = \mathbf{8}_\mathrm{c}$ and in the $R \times H$ decomposition of the G representation F in eq. (2.10). For the left-handed spinor in four dimensions similar analysis applies with $\sigma_d = \mathbf{8}_\mathrm{s}$. Introducing $\mathbf{32}'$ in twelve dimensions yields right-handed spinors from $\mathbf{8}_\mathrm{s}$ and left-handed spinors from $\mathbf{8}_\mathrm{c}$. In this paper, we introduce $\mathbf{32}$ as the Weyl spinor.

3 Search for candidate models

In this section, we explore realistic models in CSDR using the eight-dimensional coset spaces. The coset spaces S/R for our analysis are listed, and candidates for S/R and G are narrowed down under some assumptions. Then combining them with fermion representations F of G up to $\dim F < 1000$, we perform CSDR for all possibilities and exhaustively search for models which lead to phenomenologically viable models in four dimensions.

First, we list in table 1 the eight-dimensional coset spaces satisfying the condition: $\mathrm{rank}\, R = \mathrm{rank}\, S$. As an ansatz for model building, the gauge group H in four dimensions should be the SM gauge group $G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, as well as the gauge groups SU(5), SO(10), and E_6 used in grand unified theories, and those with up to one extra U(1)

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Coset space	dimension	U(1)'s in R	R complex reps
$\mathrm{SU}(5)/\mathrm{SU}(4)\times\mathrm{U}(1)$	8	1	Yes
$\mathrm{SO}(9)/\mathrm{SO}(8)$	8	0	No
$\mathrm{Sp}(6)/\mathrm{SU}(2)\times\mathrm{Sp}(4)$	8	0	No
$\mathrm{G}_2/\mathrm{SU}(2)\times\mathrm{SU}(2)$	8	0	No
$\mathrm{Sp}(4)/\mathrm{U}(1)\times\mathrm{U}(1)_{\text{non-max}}$	8	2	Yes
$\mathrm{SU}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	8	1	Yes
$(\mathrm{SU}(4)/\mathrm{SU}(3)\times\mathrm{U}(1))\times(\mathrm{SU}(2)/\mathrm{U}(1))$	6+2	2	Yes
$(\mathrm{SO}(7)/\mathrm{SO}(6))\times(\mathrm{SU}(2)/\mathrm{U}(1))$	6+2	1	Yes
$(\mathrm{G}_2/\mathrm{SU}(3))\times(\mathrm{SU}(2)/\mathrm{U}(1))$	6+2	1	Yes
$(\mathrm{SU}(3)/\mathrm{U}(1)\times\mathrm{U}(1)_{\text{non-max}})\times(\mathrm{SU}(2)/\mathrm{U}(1))$	6+2	3	Yes
$(\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{U}(1)_{\text{max}})\times(\mathrm{SU}(2)/\mathrm{U}(1))$	6+2	2	Yes
$(\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{U}(1)_{\text{non-max}})\times(\mathrm{SU}(2)/\mathrm{U}(1))$	6+2	2	Yes
$(\mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1))^2$	4+4	2	Yes
$(\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2))\times(\mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1))$	4+4	1	Yes
$(\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2))^2$	4+4	0	No
$(\mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1))\times(\mathrm{SU}(2)/\mathrm{U}(1))^2$	4+2+2	3	Yes
$(\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2))\times(\mathrm{SU}(2)/\mathrm{U}(1))^2$	4+2+2	2	Yes
$(\mathrm{SU}(2)/\mathrm{U}(1))^4$	2+2+2+2	4	Yes

Table 1. The list of eight-dimensional coset spaces S/R of a compact simple Lie group S and its compact subgroup R that satisfy $\mathrm{rank}\, R = \mathrm{rank}\, S$ [48, 49]. The names of the coset spaces are followed by "max" or "non-max" to indicate whether R is a maximal subgroup of S or not. The column "U(1)'s in R" indicates the number of U(1) factors in R, and the column "R complex reps" indicates whether R has complex representations ("Yes") or not ("No").

factor. Thus, among the coset spaces listed in table 1, those with two or more U(1) factors in R are excluded from our search. The reason for this assumption is that the U(1) factors of R themselves enter the centralizer of R. This implies that the more U(1) factors are included in R, the more U(1) factors there are in H, leading to additional complexity from taking linear combinations of U(1) generators. In addition, as a requirement to obtain a chiral theory in four dimensions from a twelve-dimensional theory, R must have a complex representation. Considering these conditions, the coset spaces S/R to be examined in this study are summarized in table 2. In the table, the embedding of R into SO(8) obtained by eq. (2.7) is represented by the decomposition of the three eight-dimensional representations of SO(8) to R. Here is an example of how to choose the embedding. Let the extra space S/R be $S = \mathrm{SU}(5), R = \mathrm{SU}(4)\times\mathrm{U}(1)$. According to eq. (2.7), the embedding of R into SO(8) can be identified by observing the R-decomposition of the adjoint representation of S:

$$\begin{aligned} S = \mathrm{SU}(5) &\supset \mathrm{SU}(4)\times\mathrm{U}(1) = R, \\ \underbrace{\mathbf{24}}_{\mathrm{ad}\,S} &= \underbrace{\mathbf{15}(0)+\mathbf{1}(0)}_{\mathrm{ad}\,R}+\underbrace{\mathbf{4}(5)+\bar{\mathbf{4}}(-5)}_{v=\mathbf{8}_{\mathrm{v}}}. \end{aligned} \tag{3.1}$$

S/R	vector under R	spinors under R
$\mathrm{SU}(5)/\mathrm{SU}(4)\times\mathrm{U}(1)$	$\mathbf{8}_\mathrm{v}=\mathbf{4}(1)+\bar{\mathbf{4}}(-1)$	$\mathbf{8}_\mathrm{c}=\mathbf{4}(-1)+\bar{\mathbf{4}}(1)$ $\mathbf{8}_\mathrm{s}=\mathbf{6}(0)+\mathbf{1}(2)+\mathbf{1}(-2)$
$\mathrm{SU}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	$\mathbf{8}_\mathrm{v}=(\mathbf{2},\mathbf{2})(1)+(\mathbf{2},\mathbf{2})(-1)$	$\mathbf{8}_\mathrm{c}=(\mathbf{3},\mathbf{1})(0)+(\mathbf{1},\mathbf{3})(0)+(\mathbf{1},\mathbf{1})(2)+(\mathbf{1},\mathbf{1})(-2)$ $\mathbf{8}_\mathrm{s}=(\mathbf{2},\mathbf{2})(1)+(\mathbf{2},\mathbf{2})(-1)$
$\mathrm{SO}(7)/\mathrm{SO}(6)$ $\times$ $\mathrm{SU}(2)/\mathrm{U}(1)$	$\mathbf{8}_\mathrm{v}=\mathbf{6}(0)+\mathbf{1}(2)+\mathbf{1}(-2)$	$\mathbf{8}_\mathrm{c}=\mathbf{4}(1)+\bar{\mathbf{4}}(-1)$ $\mathbf{8}_\mathrm{s}=\mathbf{4}(-1)+\bar{\mathbf{4}}(1)$
$\mathrm{G}_2/\mathrm{SU}(3)$ $\times$ $\mathrm{SU}(2)/\mathrm{U}(1)$	$\mathbf{8}_\mathrm{v}=\mathbf{3}(0)+\bar{\mathbf{3}}(0)+\mathbf{1}(2)+\mathbf{1}(-2)$	$\mathbf{8}_\mathrm{c}=\mathbf{3}(-1)+\bar{\mathbf{3}}(1)+\mathbf{1}(-1)+\mathbf{1}(1)$ $\mathbf{8}_\mathrm{s}=\mathbf{3}(1)+\bar{\mathbf{3}}(-1)+\mathbf{1}(1)+\mathbf{1}(-1)$
$\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2)$ $\times$ $\mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1)$	$\mathbf{8}_\mathrm{v}=(\mathbf{2},\mathbf{1},\mathbf{1})(2)+(\mathbf{1},\mathbf{2},\mathbf{2})(0)+(\mathbf{2},\mathbf{1},\mathbf{1})(-2)$	$\mathbf{8}_\mathrm{c}=(\mathbf{1},\mathbf{2},\mathbf{1})(2)+(\mathbf{2},\mathbf{1},\mathbf{2})(0)+(\mathbf{1},\mathbf{2},\mathbf{1})(-2)$ $\mathbf{8}_\mathrm{s}=(\mathbf{1},\mathbf{1},\mathbf{2})(2)+(\mathbf{2},\mathbf{2},\mathbf{1})(0)+(\mathbf{1},\mathbf{1},\mathbf{2})(-2)$

Table 2. The list of the eight-dimensional coset spaces S/R that we are investigating. Each of them is picked out from table 1 under the conditions: R has at most one U(1) factor, and R admits complex representations. We also determine the embedding $R \subset \mathrm{SO}(8)$ by eq. (2.7) and display the decompositions of the vector ($\boldsymbol{v} = \mathbf{8}_\mathrm{v}$) and spinors ($\mathbf{8}_\mathrm{c}, \mathbf{8}_\mathrm{s}$) of SO(8) under R in the choice of embedding.

Thus the embedding is chosen so that $\mathbf{8}_\mathrm{v}$ is decomposed (up to U(1) charge rescalings) as

$$\begin{aligned} \mathrm{SO}(8) &\supset \mathrm{SU}(4)\times\mathrm{U}(1), \\ \mathbf{8}_\mathrm{v} &= \mathbf{4}(1)+\bar{\mathbf{4}}(-1). \end{aligned} \tag{3.2}$$

It is important to note that this embedding depends on S, even if R is isomorphic. For example, let us consider the case $S/R = S_1/R_1 \times S_2/R_2, S_1 = \mathrm{SO}(7), S_2 = \mathrm{SU}(2), R_1 = \mathrm{SO}(6), R_2 = \mathrm{U}(1)$. In this case, $R = R_1 \times R_2 = \mathrm{SO}(6)\times\mathrm{U}(1)$ is isomorphic to R in the previous example. However, the embedding $R \subset \mathrm{SO}(8)$ is different. Let us decompose the adjoint representation of $S = S_1 \times S_2 = \mathrm{SO}(7)\times\mathrm{SU}(2)$ under R:

$$\begin{aligned} S = \mathrm{SO}(7)\times\mathrm{SU}(2) &\supset \mathrm{SO}(6)\times\mathrm{U}(1) = R, \\ \underbrace{(\mathbf{21},\mathbf{1})+(\mathbf{1},\mathbf{3})}_{\mathrm{ad}\,S} &= \underbrace{\mathbf{15}(0)+\mathbf{1}(0)}_{\mathrm{ad}\,R}+\underbrace{\mathbf{6}(0)+\mathbf{1}(2)+\mathbf{1}(-2)}_{\mathbf{8}_\mathrm{v}}. \end{aligned} \tag{3.3}$$

It follows that the embedding $R \subset \mathrm{SO}(8)$ is specified (again up to U(1) rescalings) so that $\mathbf{8}_\mathrm{v}$ behaves as

$$\begin{aligned} \mathrm{SO}(8) &\supset \mathrm{SO}(6)\times\mathrm{U}(1), \\ \mathbf{8}_\mathrm{v} &= \mathbf{6}(0)+\mathbf{1}(2)+\mathbf{1}(-2). \end{aligned} \tag{3.4}$$

Hence, even if there are several possible embeddings of R into SO(8) (e.g. due to triality or a choice of U(1) generators), the choice is uniquely determined by eq. (2.7) for each S.

We examine the twelve-dimensional gauge group G such that the desired gauge group H is obtained for the coset space S/R. This is achieved by starting from a simple group G which has complex representations and then considering a sequence of maximal regular subgroups of G. In this paper we assume G to be a simple group so that U(1) charges

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S/R	G	H	result
$\mathrm{SU}(5)/\mathrm{SU}(4)\times\mathrm{U}(1)$	SU(9)	$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)$	$\times(1)$
	SU(9)	$\mathrm{SU}(5)\times\mathrm{U}(1)$	$\times(2)$
	SO(18)	$\mathrm{SO}(10)\times\mathrm{U}(1)$	✓
$\mathrm{SU}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	SO(14)	$\mathrm{SU}(5)\times\mathrm{U}(1)$	$\times(2)$
$[\mathrm{SO}(7)/\mathrm{SO}(6)]\times[\mathrm{SU}(2)/\mathrm{U}(1)]$	SU(9)	$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)$	$\times(3)$
	SU(9)	$\mathrm{SU}(5)\times\mathrm{U}(1)$	$\times(3)$
	SO(18)	$\mathrm{SO}(10)\times\mathrm{U}(1)$	✓
$[\mathrm{G}_2/\mathrm{SU}(3)]\times[\mathrm{SU}(2)/\mathrm{U}(1)]$	E_6	$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	$\times(4)$
	SU(8)	$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)$	$\times(1)$
	SU(8)	$\mathrm{SU}(5)\times\mathrm{U}(1)$	$\times(3)$
$[\mathrm{Sp}(4)/\mathrm{SU}(2)\times\mathrm{SU}(2)]\times[\mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1)]$	SO(14)	$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	$\times(4)$
	SO(18)	$\mathrm{SO}(10)\times\mathrm{U}(1)$	✓

Table 3. The list of candidate twelve-dimensional simple gauge groups G with complex representations and the resulting four-dimensional gauge groups H for the coset space S/R under investigation. H is given by the eq. (2.4) in section 2, where we require H to be $G_{\mathrm{SM}}, \mathrm{SU}(5)$, E_6, and their extensions by one U(1). Also, the embedding $R \times H \subset G$ is searched only in the sequence of regular subgroups of G. The last column indicates the result for each group G, taking into account the fermion representation F of G as described below.

would be quantized in H. By identifying R in the sequence, we determine the embedding $R_G \subset G$, and then the other factors in G times U(1) factors give H. We do not consider non-regular maximal subgroups (called special subgroups) to embed R into G, because we need a much larger representation of G for a special subgroup to realize SM contents than for a regular subgroup. Thus we consider sequences of maximal regular subgroups of G [50].

In table 3, we present the candidates for the twelve-dimensional gauge group G and the resulting four-dimensional gauge groups H. Note that no G gives rise to $H = \mathrm{E}_6$ in four dimensions because G is restricted to groups with complex representations, which cannot contain E_6 as a subgroup. The last column indicates the result for each group G, taking into account the fermion representation F of G as described below.

Next, we consider the G representations F of higher-dimensional spinors. The dimension of F is set to be less than 1000 to avoid the appearance of many unnecessary fermions in four dimensions. We then perform CSDR for all possible combinations of $(S/R, G, F)$ and search for cases where both scalars and at least one generation of fermions in the SM are obtained in four dimensions. Brute-force analysis is carried out semi-automatically using the power of computers, especially in the decompositions of the representations [51]. For the models with $H = G_{\mathrm{SM}}, \mathrm{SU}(5), \mathrm{E}_6$, and their extensions with one U(1), the exhaustive search proves that no candidates for twelve-dimensional models are quite promising because the particle content of the obtained four-dimensional theory is not enough to reproduce the SM. In the last column of table 3, we write the result for each model. The symbol "×" is for not obtaining a desired model with the number for the following reasons.

(1) No generation of SM fermions can be obtained.

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S/R	G	F	H	scalars	fermions
$\mathrm{SU}(5)/\mathrm{SU}(4) \times \mathrm{U}(1)$	$\mathrm{SO}(18)$	$\mathbf{256}$	$\mathrm{SO}(10) \times \mathrm{U}(1)$	$\mathbf{10}(1) + \overline{\mathbf{10}}(-1)$	$\mathbf{16}(2) + \mathbf{16}(0) + \mathbf{16}(-2)$ $+\mathbf{16}(1) + \mathbf{16}(-1)$
$\mathrm{SO}(7)/\mathrm{SO}(6)$ $\times$ $\mathrm{SU}(2)/\mathrm{U}(1)$	$\mathrm{SO}(18)$	$\mathbf{256}$	$\mathrm{SO}(10) \times \mathrm{U}(1)$	$\mathbf{10}(2) + \mathbf{10}(0) + \mathbf{10}(-2)$	$\mathbf{16}(1) + \mathbf{16}(-1)$ $+\mathbf{16}(1) + \mathbf{16}(-1)$
$\mathrm{Sp}(4)/\mathrm{SU}(2) \times \mathrm{SU}(2)$ $\times$ $\mathrm{SU}(3)/\mathrm{SU}(2) \times \mathrm{U}(1)$	$\mathrm{SO}(18)$	$\mathbf{256}$	$\mathrm{SO}(10) \times \mathrm{U}(1)$	$\mathbf{10}(2) + \mathbf{10}(0) + \mathbf{10}(-2)$	$\mathbf{16}(2) + \mathbf{16}(0) + \mathbf{16}(-2)$ $+\mathbf{16}(2) + \mathbf{16}(0) + \mathbf{16}(-2)$

Table 4. The inputs to CSDR, including the coset space S/R, the twelve-dimensional gauge group G, and the G representation F of twelve-dimensional fermions, along with the resulting content of the four-dimensional model (the gauge group H, the scalar representations, and the fermion representations).

(2) If scalars are obtained, fermions are not, and vice versa.

(3) No scalars are obtained.

(4) U(1) charges of SM fermions are not realized.

The analysis gives three twelve-dimensional models, as shown in table 3 denoted by the symbol "✓", which are potentially useful for phenomenology. The resulting content of the models is displayed in detail in table 4. We describe these models in the following subsections.

3.1 $S/R = \mathbf{SU(5)/SU(4) \times U(1)}, G = \mathbf{SO(18)}, F = \mathbf{256}$

Let us first consider the $H = \mathrm{SO}(10)\times\mathrm{U}(1)$ model obtained for the case $S/R = \mathrm{SU}(5)/\mathrm{SU}(4)\times \mathrm{U}(1)$, with $G = \mathrm{SO}(18)$ and $F = \mathbf{256}$. For the embedding of $R = \mathrm{SU}(4) \times \mathrm{U}(1)$ into $G = \mathrm{SO}(18)$, it is convenient to use the following subgroup decomposition:

$$\mathrm{SO}(18) \supset \mathrm{SO}(10) \times \mathrm{SO}(8)_G, \tag{3.5}$$

$$\mathrm{ad}\,\mathrm{SO}(18) = \mathbf{153} = (\mathbf{45}, \mathbf{1}) + (\mathbf{1}, \mathbf{28}) + (\mathbf{10}, \mathbf{8}_\mathrm{v}). \tag{3.6}$$

Here, $\mathrm{SO}(8)_G \subset G$ represents the SO(8) subgroup in G, and $\mathrm{SO}(8)_\mathrm{loc}$ is the SO(8) local symmetry of the tangent space of S/R. The embedding of R into G is specified by the choice for embedding R into $\mathrm{SO}(8)_G$, which has an ambiguity due to the fact that the three eight-dimensional representations of SO(8) behave differently under R. On the other hand, the embedding of R into $\mathrm{SO}(8)_\mathrm{loc}$ is fixed as

$$\begin{aligned} &\mathrm{SO}(8)_\mathrm{loc} \supset R = \mathrm{SU}(4) \times \mathrm{U}(1), \\ &\mathbf{8}_\mathrm{v} = \mathbf{4}(1) + \overline{\mathbf{4}}(-1). \end{aligned} \tag{3.7}$$

To obtain a four-dimensional scalar using the CSDR rule in eq. (2.12), we need to embed R into $\mathrm{SO}(8)_G$ in the same way as we embed R into $\mathrm{SO}(8)_\mathrm{loc}$ in eq. (3.7). Such an embedding, given by

$$\begin{aligned} &\mathrm{SO}(8)_G \supset R = \mathrm{SU}(4) \times \mathrm{U}(1), \\ &\mathbf{8}_\mathrm{v} = \mathbf{4}(1) + \overline{\mathbf{4}}(-1), \end{aligned} \tag{3.8}$$

leads to the decomposition for the adjoint representation **153** of $G = \mathrm{SO}(18)$ under $R \times H = \mathrm{SO}(10) \times \mathrm{SU}(4) \times \mathrm{U}(1)$ in eq. (3.6) as

$$\begin{aligned} &\mathrm{SO}(18) \supset \mathrm{SO}(10) \times \mathrm{SU}(4) \times \mathrm{U}(1), \\ &\mathbf{153} = \underbrace{(\mathbf{45},\mathbf{1})(0)}_{\mathrm{ad\,SO}(10)} + \underbrace{(\mathbf{1},\mathbf{15})(0) + (\mathbf{1},\mathbf{1})(0)}_{\mathrm{ad\,SU}(4)\times\mathrm{U}(1)} + (\mathbf{1},\mathbf{6})(2) + (\mathbf{1},\mathbf{6})(-2) \\ &\qquad + (\mathbf{10},\mathbf{4})(1) + (\mathbf{10},\overline{\mathbf{4}})(-1). \end{aligned} \tag{3.9}$$

Thus, comparing this and the decomposition of $\mathrm{SO}(8)_{\mathrm{loc}}$ vector $\mathbf{8}_{\mathrm{v}}$ under R in eq. (3.7), we obtain $\mathbf{10}(1)$ and $\mathbf{10}(-1)$ of $H = \mathrm{SO}(10) \times \mathrm{U}(1)$ as the four-dimensional scalar fields.

Furthermore, the R decompositions of the $\mathrm{SO}(8)_{\mathrm{loc}}$ spinors $\mathbf{8}_{\mathrm{c}}$ and $\mathbf{8}_{\mathrm{s}}$ under the embedding fixed by eq. (3.7) are given by:

$$\begin{aligned} \mathrm{SO}(8)_{\mathrm{loc}} \supset\ & \mathrm{SU}(4) \times \mathrm{U}(1), \\ \mathbf{8}_{\mathrm{c}} =\ & \mathbf{4}(-1) + \overline{\mathbf{4}}(1), \\ \mathbf{8}_{\mathrm{s}} =\ & \mathbf{6}(0) + \mathbf{1}(2) + \mathbf{1}(-2). \end{aligned} \tag{3.10}$$

Comparing this and the $R \times H$ decomposition of $F = \mathbf{256}$ of $G = \mathrm{SO}(18)$,

$$\begin{aligned} \mathrm{SO}(18) &\supset \mathrm{SO}(10) \times \mathrm{SO}(8)_G \\ &\supset \mathrm{SO}(10) \times \mathrm{SU}(4) \times \mathrm{U}(1), \\ \mathbf{256} =& (\mathbf{16},\mathbf{8}_{\mathrm{s}}) + (\overline{\mathbf{16}},\mathbf{8}_{\mathrm{c}}) \\ =& (\mathbf{16},1)(-2) + (\mathbf{16},\mathbf{1})(2) + (\mathbf{16},\mathbf{6})(0) \\ &+ (\overline{\mathbf{16}},\overline{\mathbf{4}})(1) + (\overline{\mathbf{16}},\mathbf{4})(-1), \end{aligned} \tag{3.11}$$

we obtain four-dimensional left-handed spinor fields in $\mathbf{16}(2), \mathbf{16}(0), \mathbf{16}(-2)$ of $\mathrm{SO}(10)\times\mathrm{U}(1)$, while the right-handed spinors in $\overline{\mathbf{16}}(1), \overline{\mathbf{16}}(-1)$ also appear in four dimensions. Taking the charge conjugation of the right-handed spinor fields yields five $\mathbf{16}$'s, each of which contains the fermions of one generation of the SM.

3.2 $S/R = [\mathrm{SO}(7)/\mathrm{SO}(6)] \times [\mathrm{SU}(2)/\mathrm{U}(1)], G = \mathrm{SO}(18), F = \mathbf{256}$

Next, we discuss the $H = \mathrm{SO}(10) \times \mathrm{U}(1)$ model obtained when the extra space is $S/R = [\mathrm{SO}(7)/\mathrm{SO}(6)] \times [\mathrm{SU}(2)/\mathrm{U}(1)]$ and $G = \mathrm{SO}(18)$ with $F = \mathbf{256}$. Due to the Lie algebra isomorphism $\mathrm{SO}(6) \sim \mathrm{SU}(4)$, the embedding of $R = \mathrm{SO}(6) \times \mathrm{U}(1)$ into $G = \mathrm{SO}(18)$ can be identified by choosing $R = \mathrm{SO}(6) \times \mathrm{U}(1) \subset \mathrm{SO}(8)_G$ in the same way as $R \subset \mathrm{SO}(8)_{\mathrm{loc}}$, which we have done in the previous section. Since the $\mathrm{SO}(8)_{\mathrm{loc}}$ vector transforms under R as

$$\begin{aligned} \mathrm{SO}(8)_{\mathrm{loc}} \supset\ & \mathrm{SO}(6) \times \mathrm{U}(1), \\ \mathbf{8}_{\mathrm{v}} =\ & \mathbf{6}(0) + \mathbf{1}(2) + \mathbf{1}(-2), \end{aligned} \tag{3.12}$$

the embedding $R \subset \mathrm{SO}(8)_G$ is specified as the same as above, which gives the decomposition for the adjoint representation of SO(18) as

$$\begin{aligned} &\mathrm{SO}(18) \supset \mathrm{SO}(10) \times \mathrm{SO}(6) \times \mathrm{U}(1), \\ \mathrm{ad\,SO}(18) = \mathbf{153} =& (\mathbf{45},\mathbf{1})(0) + (\mathbf{1},\mathbf{15})(0) + (\mathbf{1},\mathbf{1})(0) + (\mathbf{1},\mathbf{6})(2) + (\mathbf{1},\mathbf{6})(-2) \\ &+ (\mathbf{10},\mathbf{6})(0) + (\mathbf{10},\mathbf{1})(2) + (\mathbf{10},\mathbf{1})(-2). \end{aligned} \tag{3.13}$$

Comparing these two decompositions gives $\mathbf{10}(0), \mathbf{10}(2), \mathbf{10}(-2)$ of $\mathrm{SO}(10) \times \mathrm{U}(1)$ as the four-dimensional scalars.

As for the spinors, the decompositions of the $\mathrm{SO}(8)_{\mathrm{loc}}$ spinors under R, according to the embedding in eq. (3.12), are:

$$\begin{aligned} \mathrm{SO}(8)_{\mathrm{loc}} \supset\ & \mathrm{SO}(6) \times \mathrm{U}(1), \\ \mathbf{8}_{\mathrm{c}} =\ & \mathbf{4}(1) + \overline{\mathbf{4}}(-1), \\ \mathbf{8}_{\mathrm{s}} =\ & \mathbf{4}(-1) + \overline{\mathbf{4}}(1). \end{aligned} \tag{3.14}$$

If this is compared to the decomposition of $F = \mathbf{256}$ of $G = \mathrm{SO}(18)$ under $R \times H$,

$$\begin{aligned} \mathrm{SO}(18) \supset\ & \mathrm{SO}(10) \times \mathrm{SO}(8)_G \\ \supset\ & \mathrm{SO}(10) \times \mathrm{SO}(6) \times \mathrm{U}(1), \\ \mathbf{256} =\ & (\mathbf{16}, \mathbf{8}_{\mathrm{s}}) + (\bar{\mathbf{16}}, \mathbf{8}_{\mathrm{c}}) \\ =\ & (\mathbf{16}, \mathbf{4})(-1) + (\mathbf{16}, \overline{\mathbf{4}})(1) + (\overline{\mathbf{16}}, \mathbf{4})(1) + (\overline{\mathbf{16}}, \overline{\mathbf{4}})(-1), \end{aligned} \tag{3.15}$$

we obtain the four-dimensional left-handed spinors in $\mathbf{16}(\pm 1)$ of $\mathrm{SO}(10) \times \mathrm{U}(1)$ and the right-handed spinors in $\overline{\mathbf{16}}(\pm 1)$. Note that the embedding of R into $\mathrm{SO}(8)$ in eq. (3.12) differs from that in eq. (3.7), which gives different decompositions in eq. (3.10) and eq. (3.14). The expression above is displayed as the decomposition performed via $\mathrm{SO}(8)$ for clarity. In other words, four generations of fermions in $\mathbf{16}$ of $\mathrm{SO}(10)$ appear in four-dimensional theory if we take the charge conjugation of the right-handed spinors.

3.3 $S/R = [\mathrm{Sp}(4)/\mathrm{SU}(2) \times \mathrm{SU}(2)] \times [\mathrm{SU}(3)/\mathrm{SU}(2) \times \mathrm{U}(1)], G = \mathrm{SO}(18), F = 256$

Finally, we consider the case of the $H = \mathrm{SO}(10) \times \mathrm{U}(1)$ model obtained from $G = \mathrm{SO}(18)$ and $F = \mathbf{256}$ with the coset space $S/R = [\mathrm{Sp}(4)/\mathrm{SU}(2) \times \mathrm{SU}(2)] \times [\mathrm{SU}(3)/\mathrm{SU}(2) \times \mathrm{U}(1)]$. The embedding of $R = \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ into $G = \mathrm{SO}(18)$ is also provided by adjusting the embedding $R \subset \mathrm{SO}(8)_G$ to match $R \subset \mathrm{SO}(8)_{\mathrm{loc}}$:

$$\begin{aligned} \mathrm{SO}(8)_{\mathrm{loc}} \supset\ & \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1), \\ \mathbf{8}_{\mathrm{v}} =\ & (\mathbf{2}, \mathbf{1}, \mathbf{1})(2) + (\mathbf{1}, \mathbf{2}, \mathbf{2})(0) + (\mathbf{2}, \mathbf{1}, \mathbf{1})(-2). \end{aligned} \tag{3.16}$$

Then, $R \subset \mathrm{SO}(8)_G$ is chosen to be the same as above, which gives the $R \times H$ decomposition of the adjoint representation $\mathbf{153}$ of $G = \mathrm{SO}(18)$ as

$$\begin{aligned} & \mathrm{SO}(18) \supset \mathrm{SO}(10) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1), \\ & \quad 153 = (45, \mathbf{1}, \mathbf{1}, \mathbf{1})(0) + (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1})(0) + (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{1})(0) + (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3})(0) + (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(0) \\ & \qquad + (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{2})(2) + (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{2})(-2) + (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(4) + (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(-4) \\ & \qquad + (\mathbf{10}, \mathbf{1}, \mathbf{2}, \mathbf{2})(0) + (\mathbf{10}, \mathbf{2}, \mathbf{1}, \mathbf{1})(2) + (\mathbf{10}, \mathbf{2}, \mathbf{1}, \mathbf{1})(-2). \end{aligned} \tag{3.17}$$

Thus, we obtain the four-dimensional scalars that belong to $\mathbf{10}(0), \mathbf{10}(2), \mathbf{10}(-2)$ of $H = \mathrm{SO}(10) \times \mathrm{U}(1)$.

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In this embedding, the spinors of $\mathrm{SO}(8)_{\mathrm{loc}}$ transform under R as

$$\begin{aligned}\mathrm{SO}(8)_{\mathrm{loc}} \supset\ & \mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1),\\ \mathbf{8}_{\mathrm{c}} &= (\mathbf{1},\mathbf{2},\mathbf{1})(2)+(\mathbf{2},\mathbf{1},\mathbf{2})(0)+(\mathbf{1},\mathbf{2},\mathbf{1})(-2),\\ \mathbf{8}_{\mathrm{s}} &= (\mathbf{1},\mathbf{1},\mathbf{2})(2)+(\mathbf{2},\mathbf{2},\mathbf{1})(0)+(\mathbf{1},\mathbf{1},\mathbf{2})(-2),\end{aligned} \tag{3.18}$$

while $F=\mathbf{256}$ of $G=\mathrm{SO}(18)$ decomposes under $R\times H$ as

$$\begin{aligned}\mathrm{SO}(18) &\supset \mathrm{SO}(10)\times\mathrm{SO}(8)_G\\ &\supset \mathrm{SO}(10)\times\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1),\\ \mathbf{256} &=(\mathbf{16},\mathbf{8}_{\mathrm{s}})+(\overline{\mathbf{16}},\mathbf{8}_{\mathrm{c}})\\ &=(\mathbf{16},\mathbf{1},\mathbf{1},\mathbf{2})(2)+(\mathbf{16},\mathbf{2},\mathbf{2},\mathbf{1})(0)+(\mathbf{16},\mathbf{1},\mathbf{1},\mathbf{2})(-2)\\ &\quad+(\overline{\mathbf{16}},\mathbf{1},\mathbf{2},\mathbf{1})(2)+(\overline{\mathbf{16}},\mathbf{2},\mathbf{1},\mathbf{2})(0)+(\overline{\mathbf{16}},\mathbf{1},\mathbf{2},\mathbf{1})(-2).\end{aligned} \tag{3.19}$$

If the two decompositions are compared, we obtain $\mathbf{16}(\pm2), \mathbf{16}(0)$ as $\mathrm{SO}(10)\times\mathrm{U}(1)$ representations for the left-handed spinor fields, and $\overline{\mathbf{16}}(\pm2), \overline{\mathbf{16}}(0)$ for the right-handed spinor fields. Taking the charge conjugation of the right-handed spinor fields, six generations of $\mathbf{16}$'s of $\mathrm{SO}(10)$ in total appear in the four dimensions.

From each of these three models, it is possible to obtain several generations of the fermions in the SM. This is because the $\mathbf{16}$ representation of $\mathrm{SO}(10)$ includes one generation of the SM fermions.

4 Summary and discussion

In this article, we have utilized the coset space dimensional reduction (CSDR) to analyze twelve-dimensional gauge-Higgs unified models with an eight-dimensional coset space under appropriate assumptions. Then we built the twelve-dimensional models that lead to phenomenologically interesting models in four-dimensional space-time.

First, we have made a list of inputs for CSDR, including the extra space S/R, the twelve-dimensional gauge group G, and the G representation F of twelve-dimensional fermions. The eight-dimensional coset spaces S/R are classified by requiring $\operatorname{rank} R = \operatorname{rank} S$, and displayed in table 1. We have summarized the coset spaces shown in table 2 that permit complex representations for R and at most one additional $\mathrm{U}(1)$ factor in the resulting four-dimensional gauge group obtained via CSDR. Second, it is required that the twelve-dimensional gauge group G must contain complex representations and should lead to a favorable four-dimensional gauge group H that includes G_{SM}, $\mathrm{SU}(5)$, $\mathrm{SO}(10)$, and their possible extension by an extra $\mathrm{U}(1)$ factor. The candidates of twelve-dimensional gauge groups G that satisfy all requirements have been summarized in table 3. We have also limited the fermion representation F to a complex representation of up to one thousand dimensions.

Then CSDR is performed for each set of inputs, and we have studied the resulting particle content of the four-dimensional models. The twelve-dimensional models that lead to an $\mathrm{SO}(10)\times\mathrm{U}(1)$ model in four dimensions are listed in table 4 as phenomenologically interesting for model building. Note that with our assumptions we do not find models that lead to G_{SM}, $\mathrm{SU}(5)$, or E_6 gauge groups or their $\mathrm{U}(1)$ extensions. The $\mathrm{SO}(10)\times\mathrm{U}(1)$

GUT-like models include spinors in the representations **16** of SO(10) that contain one generation of the SM fermions and scalars in **10** which can be interpreted as Higgs fields that spontaneously break electroweak symmetry.

Among the results, we are particularly interested in the model with $S/R = \mathrm{SU}(5) \times \mathrm{SU}(4) \times \mathrm{U}(1)$, $G = \mathrm{SO}(18)$, and $F = \mathbf{256}$. This model results in five generations (odd generations) of fermions that include three generations of left-handed fermions. This is due to the triality of SO(8), where the SO(8) spinor $\mathbf{8}_\mathrm{s}$ behaves like a six-dimensional vector representation of $\mathrm{SU}(4) \sim \mathrm{SO}(6)$ under $R = \mathrm{SU}(4) \times \mathrm{U}(1)$, while the other spinor $\mathbf{8}_\mathrm{c}$ behaves differently from $\mathbf{8}_\mathrm{s}$. In addition, the extra U(1) charges for the three generations of left-handed fermions are 0 and ± 2. This charge assignment is reminiscent of the charges in the $\mathrm{U}(1)_{L_\mu - L_\tau}$ model [52–55] that has been extensively studied, for instance, in the contexts of the neutrino physics [56–59], cosmology [60–73], and realization of the $\mathrm{U}(1)_{L_\mu - L_\tau}$ symmetry from high-energy theory [74]. Although this model itself predicts five generations of fermions, by manipulating the two generations that appear in excess, we may be able to obtain left-handed fermions with $\mathrm{U}(1)_{L_\mu - L_\tau}$ charge for three generations, which could potentially explain the generation structure of the fermions in the SM.

After applying our currently investigated method, when a GUT model in four dimensions is obtained, this GUT symmetry must be broken down to the SM. The scalars in the considered model are supposed to break the electroweak symmetry spontaneously and cannot be used to break the GUT symmetry. We should consider other methods to break the GUT symmetry, for instance, using extra space $(S/R)/F$, where F is a freely acting discrete group of S/R, to modify H and scalar contents obtained in four dimensions [20, 23, 75–78].

In this current study, we have taken an ansatz that only one direct product factor of the U(1) in R is considered, just for simplicity. Thus, if we allow for two or more U(1) factors which generally introduce new degrees of freedom by linear combinations of the U(1) generators in embedding R into G. It is obvious that the analysis becomes very complicated and therefore we have to find a new way to study these issues.

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