Non-BPS black branes in M-theory over Calabi-Yau threefolds. (Non-)uniqueness and recombination of non-BPS black strings in single modulus CICY and THCY models

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ABSTRACT: We study extremal solutions arising in M-theory compactifications on Calabi-Yau threefolds, focussing on non-BPS attractors for their importance in relation to the Weak Gravity Conjecture (WGC); M2 branes wrapped on two-cycles give rise to black holes, whereas M5 branes wrapped on four-cycles result in black strings. In the low-energy/field theory limit one obtains minimal $N = 2$, $D = 5$ supergravity coupled to Abelian vector multiplets. By making use of the effective black hole potential formalism with Lagrange multipliers and of the Attractor Mechanism, we obtain the explicit expressions of the attractor moduli for BPS and non-BPS solutions, and we compute the Bekenstein-Hawking black hole entropy and the black string tension. Furthermore, by focussing on one modulus complete intersection (CICY) or toric hypersurface (THCY) Calabi-Yau threefolds, we investigate the possible non-uniqueness of the attractor solutions, as well as the stability of non-BPS black holes and black strings (restricting to doubly-extremal solutions, for simplicity's sake). In all models taken into consideration, we find that both BPS and non-BPS extremal black hole attractors are always unique for a given, supporting electric charge configuration; moreover, non-BPS black holes are always unstable, and thus they decay into constituent BPS/anti-BPS pairs: this confirms the WGC, for which macroscopic non-supersymmetric solutions are bound to decay. For what concerns extremal black strings, it is well known they are unique in the BPS case; we confirm uniqueness also for non-BPS strings in one-modulus CICY models. On the other hand, we discover multiple non-BPS extremal black string attractors (with different tensions) in most of the one-modulus THCY

models, and we determine the corresponding magnetic configurations supporting them; this indicates the existence of volume-minimizing representatives in the same homology class having different values of their local minimal volume. Moreover, we find that non-BPS (doubly-) extremal black strings, both for single and multiple solutions, are kinematically stable against decay into their constituent BPS/anti-BPS pairs; in Calabi-Yau geometry, this means that the volume of the representative corresponding to the black string is less than the volume of the minimal piecewise-holomorphic representative, predicting recombination for those homology classes and thus leading to stable, non-BPS string solutions, which for the WGC are microscopic with small charges.

Contents

1 Introduction

In recent years, knowledge at the borders between Physics and Mathematics has witnessed great advances, stemmed from an intensive investigation of compactifications of string theory on special holonomy manifolds, which preserve some amount of Supersymmetry, and whose most notable class is provided by Calabi-Yau manifolds. So far, this is the unique framework in which stable solutions of Quantum Gravity (in particular concerning holography in string theory) have been discovered, related to BPS objects, with intriguing

relations to the mathematical theory of invariants. In a mathematical perspective, calibrated cycles in special holonomy have been the subject of many studies, though it is now well known that many cycle classes do not admit calibrated representatives, thus begging for the investigation of minimal volume cycles, which is a topic hard to deal with, in which very few results are currently established.

Supersymmetry, if any, is broken at low energies in our Universe. Thus, string theory, as a candidate for a theory of Quantum Gravity, has to deal with non-supersymmetric solutions, which have however been hitherto plagued by the lack of stability. A way out to such a conundrum may be provided by the study of non-supersymmetric solutions in supersymmetric string theory: thus, the theory is itself stable, and all instability can be ascribed to the decay of non-supersymmetric objects in an otherwise stable background. Along the years, a remarkable success in string theory has provided a deep-rooted understanding of the microscopic origin of black hole entropy [1]. More recently, black holes have turned out to be important ingredients in investigating the intensely studied conjecture which goes under the name of Weak Gravity Conjecture (WGC) [2], stating that gravity is always the weakest force, and therefore that all macroscopic, non-supersymmetric (i.e., non-BPS) objects are finally bound to decay into microscopic objects with small charge, which may result in saturating or not the BPS bound. Moreover, the WGC has proved to be instrumental [3] in order to formulate the so-called "Swampland program" [4]. Therefore, in this framework, the study of non-BPS black holes and black strings and their evolution toward a final state may play a crucial role in confirming or disproving the WGC.

Compactifications of eleven-dimensional M-theory over Calabi-Yau threefolds provides an important playground to study Quantum Gravity, and their low-energy limit is given by minimal, $N = 2$ supergravity theories in 5 space-time dimensions. More specifically, extremal (electric) black holes (with $\text{AdS}_2 \times S^3$ near-horizon geometry) are realized by wrapping M2 branes on 2-cycles (in particular, on non-holomorphic curves) of the Calabi-Yau manifold, whereas (magnetic) black strings (with $\text{AdS}_3 \times S^2$ near-horizon geometry) are given by M5 branes wrapped on 4-cycles (namely, on non-holomorphic divisors) of such a space. A wrapped cycle is conjectured to be a connected, locally volume-minimizing representative of its homology class. Hence, non-BPS black holes and black strings may provide key clues in investigating the existence, stability, and asymptotic count for minimal-volume, 2- and 4-cycles in Calabi-Yau threefolds.

Extremal black holes and black strings are characterized by the so-called Attractor Mechanism [5–8], in which the moduli spaces of scalar fields may admit multiple basin of attractors [9]. As one extrapolates the scalar fields from spatial infinity, where they can take arbitrary initial values, to the black hole horizon, they run into one of the attractor points. Within the same basin of attraction, the values of these scalar fields at the attractor point is determined in terms of the black hole charges only. Thus, the macroscopic properties of a black hole, such as its Bekenstein-Hawking thermodynamical entropy, depend only on the conserved charges associated to the underlying gauge invariance. In this respect, it is here worth remarking that the attractor machanism is applicable to both BPS as well as non-BPS solutions, as long as they remain extremal [8, 10]. One of the most fruitful approaches to the Attractor Mechanism is the use of the so-called black hole / black string

effective potential, whose critical points determine the attractor values of the moduli (scalar) fields at the event horizon of the black object under consideration.

This has been recently exploited by Long, Sheshmani, Vafa and Yau in [11], in which the procedure of minimization of the effective potential has been shown to fix the moduli inside the Kähler cone, yielding to the determination of the black hole entropy or of the black string tension, and hinting to a conjectural formula for the volume of the non-holomorphic cycles wrapped by the M2 or M5 branes. In a given homology class, non-BPS, doubly-extremal configurations have been used to obtain non-calibrated cycles of minimal volume, as well as to compute the asymptotic volumes of the representative cycles which minimize the volume. By explicitly considering Calabi-Yau threefold compactifications with a few moduli, the authors of [11] have found that non-BPS, extremal black holes correspond to *local*, but not global, volume minimizers of the corresponding curve classes, as there is always a disconnected, piecewise-calibrated representative (union of holomorphic and anti-holomorphic curves) which corresponds to the BPS/anti-BPS black hole constituents, and whose smaller volume implies that the aforementioned WGC is satisfied, yielding to the decay of non-BPS black holes into widely-separated BPS and anti-BPS particles.

On the other hand, within some of the same few moduli models, in [11] it was discovered that non-BPS extremal black strings correspond to *global* minima of the corresponding effective magnetic potential, and thus the existence of a phenomenon called *"recombination"* was established [12]: holomorphic and anti-holomorphic constituents of the same homology class fuse together to make a smaller cycle, and by the WGC this yields to the prediction that there should be *stable*, non-BPS black strings (with small charge) in the spectrum of the resulting supergravity theory. It is here worth emphasizing that these black strings are only kinematically stable against decay into constituent BPS/anti-BPS pairs. Moreover, in black strings still persists the usual Gregory-Laflamme instability [13–15] against metric perturbation. Further, we should recall that, for a given supporting electric or magnetic charge configuration, in the whole treatment of [11] all extremal black hole resp. black string solutions have been found to be *unique*.

In this context, the investigation of the possible *non-uniqueness* of attractors with different entropy or tension is of considerable importance, as it may provide evidence for the existence of volume-minimizing representatives in the same homology class having different values of their local minimal volume; this fact can actually be traced back to the homological structure and topological data of the Calabi-Yau threefolds under consideration. In the present paper, developing and extending the analysis of [11], we plan to carry out an in-depth investigation of extremal (non-BPS) black hole and black string attractors in five-dimensional, minimal supergravity theory arising from Calabi-Yau compactification of M-theory, especially for what concerns their possible non-uniqueness, as well as the stability against the decay into constituent elements with small charges.

Our main result will be two-fold: on one hand, we will confirm the existence of kinematically stable, non-BPS, extremal black strings (which, when combined with the WGC, hints for the fact that such strings should be microscopic with small charges); on the other hand, for a given supporting magnetic charge configuration, we will also find evidence for the non-uniqueness of such (non-BPS) stable extremal black string attractors, thus hinting for the

aforementioned existence of volume-minimizing representatives in the same homology class. For our purposes, one-modulus Calabi-Yau manifolds provide the simplest class of models in which these issues can be explored; classification of one-modulus Calabi-Yau manifolds as complete intersections of product of projective spaces (CICY) as well as hypersurfaces in toric varieties (THCY) has been carried out in various studies [16, 17]; for instance, the relevant cohomology data along with the respective Kähler cones have been reported for these manifolds in [18], hence providing the needed ingredients for our investigation.

The plan of the present paper is as follows.

We will start and recall basic facts on extremal (electric) black hole attractors in five dimensional, minimal supergravity in section 2, then focussing on one-modulus Calabi-Yau threefolds in section 3. After a general treatment in section 3.1, in section 3.2 we analyze the uniqueness of black hole attractor solutions in a variety of one-modulus Calabi-Yau threefolds: in models with $c = d = 0$ and $a = b = 0$ in sections 3.2.1 and 3.2.2, respectively, and in complete intersection Calabi-Yau (CICY) and in Calabi-Yau manifolds arising as a hypersurface in a toric variety (THCY), respectively in sections 3.2.3 resp. 3.2.4. We will then compute the recombination factor for non-BPS extremal black hole attractors in one-modulus THCY models in section 3.2.5, obtaining instability of such solutions in all cases. Then, we proceed and recall basic facts on extremal (magnetic) black string attractors in five dimensional, minimal supergravity in section 4, then focussing again on one-modulus Calabi-Yau threefolds in section 5. Then, in section 5.1 we analyze the uniqueness of black string attractor solutions in a variety of one-modulus Calabi-Yau threefolds: in models with $c = d = 0$ and $a = b = 0$ in sections 5.1.1 and 5.1.2, respectively, and in CICY one-modulus models in section 5.1.3. Then, in section 5.2 we present evidence for multiple non-BPS extremal black strings in one-modulus THCY models. After that, we will compute the recombination factor for non-BPS extremal black strings in the same class of models in section 5.3, obtaining stability of such solutions in most of the models under consideration. We make some conclusive remarks and comments on further possible developments in the final section 6. Various appendices conclude and complete the paper. In Appendices A and B we respectively recall the electric attractor equations and present the magnetic attractor equations in minimal $D = 5$ supergravity. Finally, in Appendices C–D and E–F we report (in various tables) explicit results on extremal black hole resp. black string attractors in the one-modulus CICY and THCY models.

2 5D black hole attractors

In this paper we will focus on extremal solutions arising from the compactification of M-theory on a Calabi-Yau manifold [19, 20], whose low-energy, field theory limit results into minimal, $N = 2$ supergravity theory in five space-time dimensions [21]. The Kähler moduli of the Calabi-Yau manifold gives rise to vector multiplets in the resulting five dimensional theory. The moduli space exhibits the structure of a very special geometry [22]. Critical points in this theory has been studied extensively [23]. In the following we will first outline the basic formalism in order to obtain the stabilization equation as well as to set up the notations and conventions. Here we will mostly use the conventions of [23]. In the case of

coupling to n vector multiplets, the black hole effective potential in five dimensions is given by ($I, J = 1, \ldots, n+1$, and $i, j = 1, \ldots, n$)

$$V = G^{IJ}q_I q_J = Z^2 + \frac{3}{2}g^{ij}\partial_i Z \partial_j Z, \tag{2.1}$$

where the central charge reads as[1]

$$Z_e = t^I q_I, \tag{2.2}$$

in terms of the electric charge q_I and the (pull-back of the) Kähler moduli t^I. The metric g_{ij} of the real special moduli space $\mathcal{M}$ (namely, the scalar manifold of the corresponding supergravity theory, with $\dim_{\mathbb{R}}\mathcal{M} = n$) is given as

$$g_{ij} = \frac{3}{2}\partial_i t^I \partial_j t^J G_{IJ}, \quad g^{ij}g_{jk} = \delta^i_k, \tag{2.3}$$

where G_{IJ} is its pull-back onto the $(n+1)$-dimensional "ambient space", which is the canonical metric associated to the cubic form $C_{IJK}t^I t^J t^K$,

$$G_{IJ} := -\frac{1}{3}\left.\frac{\partial^2 \log C_{LMN}t^L t^M t^N}{\partial t^I \partial t^J}\right|_* = \left(3C_{ILM}C_{JNP}t^L t^M t^N t^P - 2C_{IJM}t^M\right)_*, \tag{2.4}$$

where "$*$" denotes the evaluation at

$$C_{IJK}t^I t^J t^K = 1. \tag{2.5}$$

Here C_{IJK} are the triple intersection numbers associated with the Calabi-Yau manifold, which ultimately fix the whole bosonic sector of the Lagrangian density of $N = 2$, $D = 5$ Maxwell-Einstein supergravity [24]:

$$\frac{\mathcal{L}}{\sqrt{-\mathbf{g}}} = -\frac{1}{2}R - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{J|\mu\nu} - \frac{1}{2}g_{ij}\partial_\mu \varphi^i \partial^\mu \varphi^j + \frac{1}{6^{3/2}\sqrt{-\mathbf{g}}}C_{IJK}\varepsilon^{\lambda\mu\nu\rho\sigma}F^I_{\lambda\mu}F^J_{\nu\rho}A^K_\sigma, \tag{2.6}$$

where $\mathbf{g} := \det \mathbf{g}_{\mu\nu}$, and $\mathbf{g}_{\mu\nu}$ is the space-time metric Introducing

$$\Pi^{IJ} := g^{ij}\partial_i t^I \partial_j t^J, \tag{2.7}$$

we can write the effective potential as

$$V = Z^2 + \frac{3}{2}\Pi^{IJ}q_I q_J. \tag{2.8}$$

It has been shown in [25] that

$$\Pi^{IJ} = -\frac{1}{3}(C^{IJ} - t^I t^J), \tag{2.9}$$

where C^{IJ} is the inverse of the matrix[2] $C_{IJ} = C_{IJK}t^K$. Substituting this, we find

$$V = \frac{3}{2}Z^2 - \frac{1}{2}C^{IJ}q_I q_J \tag{2.10}$$

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[1] In the following treatment, the subscript "e" (for "*electric*") will be understood for brevity.

[2] In homogeneous scalar manifolds, since $C^{IJK}C_{J(LM|}C_{K|N)P} = \frac{1}{108}\left(\delta^K_P C_{LMN} + 3\delta^K_{(L}C_{MN)P}\right)$, it holds that $C^{IJ} = 108\left(C^{IJK}C_{KMN}t^M t^N - \frac{1}{36}t^I t^J\right)$.

We will use the method of Lagrange multiplier to extremize this potential subjected to the constraint $C_{IJK}t^I t^J t^K = 1$. Extremizing

$$\tilde{V} = \frac{3}{2}Z^2 - \frac{1}{2}C^{IJ}q_I q_J + \lambda(C_{IJK}t^I t^J t^K - 1) \tag{2.11}$$

with respect to t^K and λ we find

$$0 = C_{IJK}t^I t^J t^K - 1; \tag{2.12}$$

$$0 = 3Zq_K - \frac{1}{2}\partial_K C^{IJ}q_I q_J + 3\lambda C_{KIJ}t^I t^J. \tag{2.13}$$

Multiplying by t^K in the second of the above and using[3]

$$\partial_K C^{IJ} = -C^{IL}C^{JM}C_{KLM}, \tag{2.14}$$

we find

$$3\lambda = -3Z^2 - \frac{1}{2}C^{IJ}q_I q_J. \tag{2.15}$$

Using this value of λ we find the equations of motion

$$3Z\left(q_K - ZC_{KIJ}t^I t^J\right) + \frac{1}{2}(C^{IL}C^{JM}C_{KLM} - C^{IJ}C_{KLM}t^L t^M)q_I q_J = 0 \tag{2.16}$$

along with the constraint (2.5). The equation of motion can be rewritten in a compact form as

$$C_{KLM}(C^{IL}C^{JM} - C^{IJ}t^L t^M)\left(q_J + 6ZC_{JN}t^N\right)q_I = 0. \tag{2.17}$$

The supersymmetric critical points correspond to[4]

$$q_K - ZC_{KIJ}t^I t^J = 0. \tag{2.18}$$

The non-BPS black holes can be found upon solving (2.16) such that $\left(q_K - ZC_{KIJ}t^I t^J\right) \neq 0$. It is worth exploring whether we can obtain an equation analogous to (2.18) for the non-BPS critical points. A naïve analysis of (2.17) might suggest the non-BPS solutions to the equation of motion given in terms of $q_J + 6ZC_{JN}t^N = 0$. However, for such a solution $C_{IJK}t^I t^J t^K = -1/6$ and hence it is not consistent with the constraint (2.5).

In order to obtain the algebraic equation corresponding to non-BPS critical points, we set

$$X_I := q_I - ZC_{IJK}t^J t^K \tag{2.19}$$

Note that the constraint (2.5) implies

$$t^I X_I = 0 \tag{2.20}$$

Substituting $q_I = X_I + ZC_{IJK}t^J t^K$ in (2.16) we find

$$8ZX_K + C_{KLM}(C^{IL}C^{JM} - C^{IJ}t^L t^M)X_I X_J = 0 \tag{2.21}$$

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[3]In homogeneous scalar manifolds, one can compute that $\partial_K C^{IJ} = 216C^{IJR}C_{RKM}t^M - 3\left(t^I\delta_K^J + t^J\delta_K^I\right)$.

[4]Note that (2.18) perfectly matches the relation (3.14) of [26], obtained in the so-called "new attractor" approach to the 5D attractor equations (cf. section 3.1.4 of [26]).

Solving the above along with the constraint $X_I t^I = 0$ we can obtain the expression for X_I as a function of t^I and q_I. It might be easier to solve $X_I t^I = 0$ first. This will give a possible solution for X_I up to an overall multiplicative factor. We can determine it as follows. Multiply both sides of (2.21) with $C^{KN}X_N$ to obtain

$$8ZC^{IJ}X_I X_J + C_{KLM}C^{KI}C^{LJ}C^{MN}X_I X_J X_N = 0.\tag{2.22}$$

This will determine the overall multiplicative factor in X_I. We can substitute the resulting expression back in (2.21) to verify whether it holds. The trivial solution $X_I = 0$ corresponding to BPS critical points whereas any non-zero solution for X_I will correspond to non-BPS black holes.

Note that, from the "new attractor" approach to 5D attractors treated in [26] (see appendix A), (2.19) can equivalently be rewritten as

$$X_I = \frac{3^{3/2}}{2^{5/2}}\frac{1}{Z}T^{ijk}\partial_j Z \partial_k Z \partial_i t_I,\tag{2.23}$$

where $t_I = C_{IJK}t^J t^K$ [26, 27], and

$$T^{ijk} = g^{il}g^{jm}g^{kn}T_{lmn};\tag{2.24}$$
$$T_{ijk} = \partial_i t^I \partial_j t^J \partial_k t^K C_{IJK}.\tag{2.25}$$

The condition (2.20) is consistent with (2.23), because

$$t^I \partial_i t_I = 0, \ \forall i,\tag{2.26}$$

as a consequence of the normalization condition

$$t^I t_I = 1.\tag{2.27}$$

3 One-modulus models

3.1 General treatment

We will now focus our attention to one-modulus models, by setting $n = 1$; thus, $I, J = 1, 2$. In order to avoid cluttering of indices, we will use the notation[5] $t^1 = x, t^2 = y$. By defining $a := C_{111}$, $b := C_{112}$, $c := C_{122}$, $d := C_{222}$, we find

$$C_{IJ} = \begin{pmatrix} ax + by & bx + cy \\ bx + cy & cx + dy \end{pmatrix},\tag{3.1}$$

and its inverse

$$C^{IJ} = \frac{1}{Lxy - Nx^2 - My^2}\begin{pmatrix} cx + dy & -(bx + cy) \\ -(bx + cy) & ax + by \end{pmatrix},\tag{3.2}$$

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[5]This differs from the notations of [9] where they introduced $\bar{t}^I = Z^{1/2}t^I$ and $x = \bar{t}^1, y = \bar{t}^2$.

where $L := ad - bc$, $M := c^2 - bd$, $N := b^2 - ac$. Further[6]

$$C_{IJ}t^J = \begin{pmatrix} ax^2 + 2bxy + cy^2 \\ bx^2 + 2cxy + dy^2 \end{pmatrix} \tag{3.3}$$

and

$$C^{IJ}q_J = \frac{1}{Lxy - Nx^2 - My^2} \begin{pmatrix} q_1(cx + dy) - q_2(bx + cy) \\ -q_1(bx + cy) + q_2(ax + by) \end{pmatrix}. \tag{3.4}$$

The equations of motion (2.16) have the following lengthy expressions:

$$6Z(q_1 - ZA_1) + \frac{(aQ_1^2 + 2bQ_1Q_2 + cQ_2^2)}{(Lxy - Nx^2 - My^2)^2} - \frac{A_1(q_1Q_1 + q_2Q_2)}{(Lxy - Nx^2 - My^2)} = 0; \tag{3.5}$$

$$6Z(q_2 - ZA_2) + \frac{(bQ_1^2 + 2cQ_1Q_2 + dQ_2^2)}{(Lxy - Nx^2 - My^2)^2} - \frac{A_2(q_1Q_1 + q_2Q_2)}{(Lxy - Nx^2 - My^2)} = 0, \tag{3.6}$$

where, for easy reading, we have introduced the notations

$$A_1 := ax^2 + 2bxy + cy^2, \ A_2 := bx^2 + 2cxy + dy^2 \tag{3.7}$$

and[7]

$$Q_1 := q_1(cx + dy) - q_2(bx + cy), \ Q_2 := -q_1(bx + cy) + q_2(ax + by). \tag{3.8}$$

These give rise to two coupled degree seven equations, which in general cannot be solved to obtain exact analytic expression for the moduli t^I in terms of the "charges" Q_I. The supersymmetric critical points, corresponding to BPS black hole attractors, are obtained from (2.18),

$$q_1 - Z(ax^2 + 2bxy + cy^2) = 0; \tag{3.9}$$

$$q_2 - Z(bx^2 + 2cxy + dy^2) = 0. \tag{3.10}$$

The general solution for these equations have been obtained in [9]. The analogous equations for the non-BPS critical points, corresponding to non-BPS black hole attractors, can be obtained by solving (2.21) for X_I along with $X_I t^I = 0$. The later condition can easily be solved to find $X_I = \hat{X}\tilde{X}_I$ with $\tilde{X}_1 = -y, \tilde{X}_2 = x$. The overall multiplicative factor $\hat{X}$ can be obtained from (2.22) upon substituting the above form of X_I in it. We find

$$\hat{X} = -\frac{8ZC^{IJ}\tilde{X}_I\tilde{X}_J}{C_{KLM}C^{KP}C^{LQ}C^{MN}\tilde{X}_P\tilde{X}_Q\tilde{X}_N}. \tag{3.11}$$

This can be further simplified using

$$C^{IJ}\tilde{X}_J = \frac{1}{\det C} \begin{pmatrix} -A_2 \\ A_1 \end{pmatrix} \text{ and } C^{IJ}\tilde{X}_I\tilde{X}_J = \frac{1}{\det C}, \tag{3.12}$$

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[6]$C_{IJ}t^J = C_{IJK}t^It^K$ is the (un-normalized) 'Jordan dual' [28] of t^I.

[7]Note that $Q^I = (Lxy - Nx^2 - My^2)C^{IJ}q_J$ should have a contravariant I-index; we choose covariant indices, but this is irrelevant in our treatment.

where $C := (C_{IJ})$. This gives rise to

$$\hat{X} = -\frac{8Z \det^2 C}{C_{IJK} A^I A^J A^K},\tag{3.13}$$

where $A^1 = -A_2, A^2 = A_1$. Upon using $C_{IJK} t^I t^J t^K = 1$, the degree six polynomial $C_{IJK} A^I A^J A^K$ can be shown to reduce to the cubic

$$(2b^3 - 3abc + a^2 d)x^3 + 3(b^2 c - 2ac^2 + abd)x^2 y - 3(bc^2 - 2b^2 d + acd)xy^2 - (2c^2 - 3bcd + ad^2)y^3$$

To summarise, the equation of motion corresponding to the non-BPS critical point for an arbitrary one-modulus model is obtained by rewriting (2.19) with the positions written below (3.10) and using the result (3.13):

$$q_I - Z C_{IJK} t^J t^K + \frac{8Z \det^2 C}{C_{IJK} A^I A^J A^K} \tilde{X}_I = 0 .\tag{3.14}$$

For a given value of C_{IJK} and for a given set of charges, this equation can be solved numerically to obtain the values of the moduli t^I corresponding to a non-BPS critical point.

The effective black hole potential (2.1) (or, equivalently, (2.8) or (2.10)) has the expression

$$V = \frac{1}{2} \left[3(q_1 x + q_2 y)^2 - \frac{q_1^2(cx + dy) - 2q_1 q_2 (bx + cy) + q_2^2(ax + by)}{Lxy - Nx^2 - My^2} \right] .\tag{3.15}$$

By adopting the normalization of [11], the (Bekenstein-Hawking) black hole entropy S can be determined from the critical value of the effective black hole potential as[8]

$$S = 2\pi \left(\frac{V}{9} \right)^{3/4} .\tag{3.16}$$

For BPS solution:

$$V = Z^2 = (q_1 x + q_2 y)^2 \implies S = \frac{2}{3\sqrt{3}} \pi |Z|^{3/2} = \frac{2}{3\sqrt{3}} \pi |q_1 x + q_2 y|^{3/2} .\tag{3.17}$$

We anticipate here that in *all* $36 + 48 = 84$ one-modulus models (of CICY and THCY type) we have considered in the present paper, we will find that *all* (BPS and non-BPS) black hole (electric) attractors are *unique*, confirming and generalizing the results of [11]. Moreover, by analising the so-called *recombination factor*, we will also find that *all* non-BPS black holes are actually unstable against the decay into their BPS/anti-BPS constituent pairs, again confirming and generalizing the results of [11].

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[8]Actually, the effective black hole potential discussed in [11] differs from ours by a factor of 2/3 and hence we have $(V/9)^{3/4}$ instead of $(V/6)^{3/4}$.

3.2 Uniqueness of attractors

3.2.1 $c = d = 0$

Before analysing the class of one-modulus models in detail, we observe that the corresponding equations of motion (3.5)–(3.6) take a particularly simple form if we set $c = d = 0$. A number of Calabi-Yau models do possess intersection numbers satisfying either of these conditions; for instance, the K3 fibration considered in [11] is one such example. Upon setting $c = 0 = d$ and introducing $t = x/y$ and $q = q_1/q_2$, the constraint (2.5) reduces to

$$(at^3 + 3bt^2)y^3 = 1 \, . \tag{3.18}$$

Using the above, the BPS equation reduces to the simple form

$$t(a - bq) + 2b = 0 \, , \tag{3.19}$$

which gives rise to the *unique* solution

$$t = \frac{2b}{bq - a} \, . \tag{3.20}$$

The critical values for the moduli x, y are given by

$$x = \frac{2^{1/3}}{(3bq - a)^{1/3}}, \quad y = \frac{bq - a}{2^{2/3}b(3bq - a)^{1/3}} . \tag{3.21}$$

The solution will lie inside the Kähler cone (or, in other words, the conditions of positivity of critical values of the moduli x and y are satisfied) provided $q > a/b$ and $3bq > a$. The effective potential for the above critical point is

$$V = \frac{q_2^2 \, (3bq - a)^{4/3}}{2^{4/3}b^2} \, , \tag{3.22}$$

and hence the entropy

$$S = 2\pi \left(\frac{V}{9} \right)^{3/4} = \frac{\pi(3bq - a)}{3\sqrt{3}} \left| \frac{q_2}{b} \right|^{3/2} \, . \tag{3.23}$$

A similar analysis for the non-BPS case leads to the linear equation

$$6b + t(a + 3bq) = 0 \, , \tag{3.24}$$

resulting the *unique* solution

$$t = -\frac{6b}{a + 3bq} \, . \tag{3.25}$$

The expression for the moduli are given as

$$x = \frac{2^{1/3}}{(a - 3bq)^{1/3}} \, , \ y = \frac{-(a + 3bq)}{3 \, 2^{2/3}b(a - 3bq)^{1/3}} \, . \tag{3.26}$$

The solution lies within the Kähler cone, provided $a/b + 3q < 0$ and $a > 3bq$. The entropy of the black hole is given by

$$S = \frac{\pi(a - 3bq)}{3\sqrt{3}} \left| \frac{q_2}{b} \right|^{3/2} \, . \tag{3.27}$$

3.2.2 $a = b = 0$

A similar analysis can be done for the case $a = b = 0$. For the BPS solution, we find

$$t = \frac{c - dq}{2cq} \,. \tag{3.28}$$

This gives rise to

$$x = \frac{c - dq}{c(2q)^{2/3}(3c - dq)^{1/3}} \,, \ y = \left(\frac{2q}{3c - dq}\right)^{1/3} \,. \tag{3.29}$$

The solution lies within the Kähler cone for $q < c/d$ and $d < 3c/q$. The entropy of the corresponding solution is

$$S = \frac{\pi(3c - dq)}{3\sqrt{3}q} \left|\frac{qq_2}{c}\right|^{3/2} \,. \tag{3.30}$$

For the non-BPS case we find

$$t = -\frac{3c + dq}{6cq} \,. \tag{3.31}$$

Thus, we have

$$x = \frac{3c + dq}{3c(2q)^{2/3}(3c - dq)^{1/3}} \,, \ y = \left(\frac{2q}{dq - 3c}\right)^{1/3} \,, \tag{3.32}$$

with entropy

$$S = \frac{\pi(dq - 3c)}{3\sqrt{3}q} \left|\frac{qq_2}{c}\right|^{3/2} \,. \tag{3.33}$$

The Kähler cone conditions are $d > 3c/q$ and $3/q + d/c < 0$.

3.2.3 CICY

We will now systematically analyse BPS as well as non-BPS black hole attractors in one-modulus Calabi-Yau models arising as complete intersection of hypersurfaces in product of projective spaces. A few of these complete intersection Calabi-Yau (CICY) manifolds were already treated in [11]. Intersection numbers and other relevant cohomology data for all 36 such one-modulus CICY manifolds have been recently computed and reported in [18] (cfr. appendix A therein). We intend to examine extremal black holes in all these CICY models in order to investigate the issue of *non-uniqueness* of such solutions. In the following treatment, we will first workout one model in full detail, and then summarise our results for the remaining models in various tables in appendix C.

We consider the CICY model with configuration matrix [18]

$$\begin{pmatrix} 0 & 0 & 2 & 1 \\ 2 & 2 & 1 & 1 \end{pmatrix} \,. \tag{3.34}$$

The Calabi-Yau manifold constitutes of the intersection of four hypersurfaces, which are given by the zero locus of polynomials with bi-degrees $(0, 2), (0, 2), (2, 1)$ and $(1, 1)$ respectively in the product space $\mathbb{P}^2 \times \mathbb{P}^5$. Each of the columns of the configuration matrix represents the bi-degree of the respective polynomial. The Kähler cone consists of the positive quadrant

in the xy-plane. The intersection numbers of the Calabi-Yau manifold are given by[9] $a = 0, b = 2/3, c = 2$ and $d = 4/3$. The volume of the Calabi-Yau manifold is given by

$$\mathcal{V} = 2x^2y + 6xy^2 + \frac{4}{3}y^3 \,. \tag{3.35}$$

We will first consider the BPS equations:

$$3q_1 - 2y(2x + 3y)(q_1x + q_2y) = 0 \,, \tag{3.36}$$

$$3q_2 - 2(x^2 + 6xy + 2y^2)(q_1x + q_2y) = 0 \,. \tag{3.37}$$

In addition, we need to impose the constraing $\mathcal{V} = 1$, which turns out to be

$$2x^2y + 6xy^2 + \frac{4}{3}y^3 = 1 \,. \tag{3.38}$$

To analyse the above equations we will do the rescaling $t = x/y$ and $q = q_1/q_2$. Solving (3.38) for y as a function of t, we find

$$y = \left(\frac{3}{2(3t^2 + 9t + 2)} \right)^{1/3} \tag{3.39}$$

Substituting the above in (3.36) we find

$$qt^2 + 2(3q - 1)t + (2q - 3) = 0 \,. \tag{3.40}$$

Solving the above for t we find the critical value

$$t_\pm = \frac{1}{q} \left(1 - 3q \pm \sqrt{7q^2 - 3q + 1} \right) \,. \tag{3.41}$$

We need to make sure that the solution lies in the Kähler cone, i.e., the critical value of t must be positive. It can be observed that t_- is negative for all values of q, whereas t_+ becomes positive for $0 < q < 3/2$. Thus, the equations of motion admit a *unique* BPS attractor for $0 < q < 3/2$.

Though it is sufficient for our purpose to have the expression for t as a function of q, for the sake completeness, we reproduce in the following the form of the moduli x, y in terms of the ratio the electric charges $q = q_1/q_2$:

$$x = \left(\frac{3}{2q} \right)^{1/3} \frac{1 - 3q + \sqrt{7q^2 - 3q + 1}}{\left((23q^2 - 18q + 6) - (9q - 6)\sqrt{7q^2 - 3q + 1} \right)^{1/3}} \,, \tag{3.42}$$

$$y = \left(\frac{3q^2}{2} \right)^{1/3} \frac{1}{\left((23q^2 - 18q + 6) - (9q - 6)\sqrt{7q^2 - 3q + 1} \right)^{1/3}} \,. \tag{3.43}$$

We will now compute the black hole entropy for the above configuration. For the BPS solution, the effective potential:

$$V = (q_1x + q_2y)^2 = \left(\frac{3}{2} \right)^{2/3} q_2^2 \frac{(1 + qt)^2}{(3t^2 + 9t + 2)^{2/3}} \,. \tag{3.44}$$

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[9]Our convention for the overall normalization of the triple intersection numbers differs from [11, 18] by a factor of 6.

Substituting $t = t_+$ in this result, we then find the entropy of the black hole to be

$$S = \frac{\pi q}{3} \sqrt{\frac{2q_2^3 \left(2 - 3q + \sqrt{7q^2 - 3q + 1}\right)^3}{(23q^2 - 18q + 6) - (9q - 6)\sqrt{7q^2 - 3q + 1}}} \, . \tag{3.45}$$

We will now turn our attention to the non-BPS attractors. The equations of motion are given as

$$q_1 - \frac{2}{3}y(q_1 x + q_2 y)\left((2x + 3y) + \frac{8(x^2 + 3xy + 7y^2)^2}{(2x + 3y)(x^2 + 3xy - 12y^2)}\right) = 0, \tag{3.46}$$

$$q_2 - \frac{2}{3}(q_1 x + q_2 y)\left((x^2 + 6xy + 9y^2) - \frac{8x(x^2 + 3xy + 7y^2)^2}{(2x + 3y)(x^2 + 3xy - 12y^2)}\right) = 0 \, . \tag{3.47}$$

To find the non-BPS black holes, we need to solve the above equations along with the volume constraint (3.38). In order to simplify (3.46) we will once more introduce the variable $t = x/y$ and the charge ratio $q = q_1/q_2$. In terms of these quantities, the equations of motion become

$$q - \frac{2y^3(qt + 1)(12t^4 + 72t^3 + 181t^2 + 219t + 284)}{3(2t + 3)(t^2 + 3t - 12)} = 0 \, , \tag{3.48}$$

$$1 + \frac{2y^3(qt + 1)(6t^5 + 27t^4 + 141t^3 + 444t^2 + 683t + 72)}{3(2t + 3)(t^2 + 3t - 12)} = 0 \, . \tag{3.49}$$

Substituting the expression for y from (3.39) in the above we find one linearly independent equation involving t and q. For any given q it can be numerically solved to obtain the corresponding value of t. We need the verify whether this corresponds to a physical solution with t lying within the Kähler cone. It is however much more instructive to deal with the inverse problem, i.e., express q as a function of t. Upon simplification, we find

$$q = -\frac{12t^4 + 72t^3 + 181t^2 + 219t + 284}{6t^5 + 27t^4 + 141t^3 + 444t^2 + 638t + 72} \, . \tag{3.50}$$

We find that the r.h.s. of the above equation is a monotonically increasing function of t. For $t = 0$ the charge ratio q takes the value $-71/18$, and it vanishes as $t \to \infty$. Since the function is monotonic, we have a *unique* non-BPS black hole for every value of q in the range $-71/18 < q < 0$. This is in contrast to the BPS case, where we have a *unique* solution for $0 < q < 3/2$. Thus, both the solutions are mutually exclusive.

The black hole effective potential V takes the form

$$\frac{V}{q_2^2} = \left(\frac{3}{2}\right)^{2/3} \left(3t^2 + 9t + 2\right)^{4/3} \frac{36t^6 + 324t^5 + 1557t^4 + 4482t^3 + 10329t^2 + 15192t + 12272}{\left(6t^5 + 27t^4 + 141t^3 + 444t^2 + 638t + 72\right)^2} \tag{3.51}$$

where t is the critical value in the above equation.

As an example, consider the value $q = -48/83$. This gives the solution $t = 1$. Thus, we have

$$x = y = \left(\frac{3}{28}\right)^{1/3} \, . \tag{3.52}$$

The entropy for this configuration is

$$S = \frac{7\pi}{249\sqrt{83}} \left(\frac{1381 q_2^2}{2}\right)^{3/4}. \tag{3.53}$$

A similar analysis can be carried out for all other 35 one-modulus CICY models. In appendix C we summarise both BPS as well as non-BPS solutions along with their respective ranges of validity for all such 35 one-modulus CICY models. In *all* such models, both BPS and non-BPS black hole attractors are *unique*.

3.2.4 THCY

In this subsection we will consider one-modulus Calabi-Yau manifolds arising as a hyper-surface in a toric variety. A toric variety is specified in terms of a reflexive polytope[10] with a specific triangulation of its faces [29]. In the case of Calabi-Yau threefolds we need to consider reflexive polytopes in four dimensions. There is a one-to-one map from the faces of the reflexive polytope to the vertices of the dual polytope. Typically, the four dimensional vectors corresponding the these vertices are not linearly independent. A generic dual polytope with n vertices $\vec{v}_i$ will have $(n-4)$ linear relationships like $(r = 1, \ldots, n-4)$

$$\sum_{i=1}^{n} q_i^r \vec{v}_i = 0. \tag{3.54}$$

The coefficients q_i^r constitute a $(n-4) \times n$ matrix, which is called as the weight matrix or charge matrix of the toric variety. Each of these vertices is associated with a homogenenous coordinate $z_i \in \mathbb{C}^n$ and the rows of the weight matrix give equivalence relations among these homogeneous coordinates. Removing a fixed point set F which is determined by the given triangulation, and taking quotient with the above mentioned equivalence relation gives rise to the four dimentional toric variety. The number of rows of the weight matrix gives the Picard number of the toric variety. Since we are interested in toric varieties with Picard number two, we need to consider dual polytopes with six vertices.

A complete classification of all such reflexive polytopes in four dimensions has been carried out [17]. Hypersurfaces with vanishing first Chern class in these toric varieties give rise to Calabi-Yau manifolds (THCY). The intersection numbers as well as the Kähler cone and other relevant cohomology data for the one-modulus THCY models have also been recently computed [18] (cfr. appendix B therein), and the corresponding cubic forms are available in the database [30] (see also [31]). As done for the CICY's, in the following treatment we will consider one specific model in detail, and then summarize the results for the remaining THCY models in various tables in appendix D.

The toric variety of our interest is described by the weight matrix

$$\begin{pmatrix} -1 & 1 & 1 & 0 & 2 & 3 \\ 1 & 0 & 0 & 1 & 0 & -2 \end{pmatrix}. \tag{3.55}$$

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[10]An integral polytope is called reflexive if its dual polytope is also integral.

The triangulation is specified in terms of the Stanley-Reisner ideal

$$\langle z_0 z_3, z_1 z_2 z_4 z_5 \rangle \, . \tag{3.56}$$

The triple intersection numbers of the corresponding THCY model in a basis where the Kähler cone coincides with the first quadrant of the Argand-Gauss plane are $a = 1/3, b = 1/2, c = 1/2$ and $d = 1/2$. The volume of the Calabi-Yau manifold is

$$\mathcal{V} = \frac{x^3}{3} + \frac{3x^2 y}{2} + \frac{3xy^2}{2} + \frac{y^3}{2} \, . \tag{3.57}$$

The BPS equations are

$$q_1 - \frac{1}{6}(q_1 x + q_2 y)(2x^2 + 3y^2 + 6xy) = 0, \tag{3.58}$$

$$q_2 - \frac{1}{2}(q_1 x + q_2 y)(x + y)^2 = 0 \, . \tag{3.59}$$

Once again we will use the rescaled coordinate $t = x/y$ and the charge ratio $q = q_1/q_2$. The constraint $\mathcal{V} = 1$ gives

$$y = \left(\frac{6}{2t^3 + 9t^2 + 9t + 3} \right)^{1/3} \, . \tag{3.60}$$

Substituting the above expression for y in (3.58), we find

$$3q(t + 1)^2 - (2t^2 + 6t + 3) = 0 \, . \tag{3.61}$$

It is straightforward to write down the solutions to the above equation. We find

$$t_{\pm} = \frac{3(1 - q) \pm \sqrt{3(1 - q)}}{3q - 2} \, . \tag{3.62}$$

Here t_+ corresponds to the physical solution lying inside the Kähler cone for q taking values in the range $2/3 < q < 1$.

We will now compute the entropy for this configuration. The effective potential for the BPS black hole is

$$V = (q_1 x + q_2 y)^2 = 6^{2/3} q_2^2 \frac{(1 + qt)^2}{(2t^3 + 9t^2 + 9t + 3)^{2/3}}, \tag{3.63}$$

and thus the entropy reads

$$S = \left(\frac{2q_2}{3} \right)^{3/2} \pi \sqrt{\frac{\left(3q^2 - 6q + 2 - q\sqrt{3(1 - q)}\right)^3}{\left(- 9q^3 + 36q^2 - 36q + 8\right) + \sqrt{3(1 - q)}(9q^2 - 16q + 4)}}. \tag{3.64}$$

We will now consider the non-BPS equations of motion. Substituting the values of the intersection numbers in (3.14), we find

$$q_1 - (q_1 x + q_2 y) \left(\frac{1}{6}(2x^2 + 6xy + 3y^2) + \frac{4x^2 y(x + y)^2}{4x^3 + 9x^2 y + 9xy^2 + 3y^3} \right) = 0, \tag{3.65}$$

$$q_2 - (q_1 x + q_2 y) \left(\frac{1}{2}(x + y)^2 - \frac{4x^3(x + y)^2}{4x^3 + 9x^2 y + 9xy^2 + 3y^3} \right) = 0 \, . \tag{3.66}$$

Substituting $x = ty, q_1 = qq_2$ and using the constraint (3.60) in the above equations, we obtain

$$3q(t+1)^2\left(4t^3 - 9t^2 - 9t - 3\right) + 8t^5 + 66t^4 + 132t^3 + 111t^2 + 45t + 9 = 0\,. \qquad (3.67)$$

For a given q, we can numerically solve the above equation to obtain the value of t. To obtain a qualitative behaviour, we will instead solve the above equation for q as a function of t. We find

$$q = -\frac{8t^5 + 66t^4 + 132t^3 + 111t^2 + 45t + 9}{3(t+1)^2\left(4t^3 - 9t^2 - 9t - 3\right)} \qquad (3.68)$$

Note that, in the physical $t > 0$ region the denominator of the r.h.s. in the above equation vanishes for $t = t_* \simeq 3.06$. At the $x = 0$ boundary of the moduli space q takes the value $q = 1$. For all values of q in the range $1 < q < \infty$ the solution for t lies in the region $0 < t < t_*$. Similarly, at the $y = 0$ boundary of the moduli space $q = -2/3$. Thus, for $-\infty < q < -2/3$ the value of t lies in the range $t_* < t < \infty$. For $-2/3 < q < 1$, the equation (3.67) does not admit any solution with $t > 0$. Beyond this region, there is a *unique* non-BPS black hole solution for any given value of q.

As an example, consider the value $q = 371/204$. This gives rise to the critical value $t = 1$. The corresponding values for the moduli x and y are given by

$$x = \left(\frac{6}{23}\right)^{1/3} = y\,. \qquad (3.69)$$

The black hole effective potential corresponding to the non-BPS black hole as a function of t at the critical point is given by

$$\frac{V}{q_2^2} = \frac{2^{2/3}\left(2t^3 + 9t^2 + 9t + 3\right)^{4/3}\left(112t^6 + 360t^5 + 441t^4 + 282t^3 + 135t^2 + 54t + 9\right)}{3\sqrt[3]{3}(t+1)^4\left(4t^3 - 9t^2 - 9t - 3\right)^2} \qquad (3.70)$$

We can use the above expression to compute the entropy. For example, for the case of $q = 371/204$, the black hole entropy is given by

$$S = 23\,\pi\,\frac{(1393\,q_2^2)^{3/4}}{306\sqrt{102}}\,. \qquad (3.71)$$

A similar analysis can be carried out for all other 47 one-modulus THCY models. In appendix D we summarise both BPS as well as non-BPS solutions along with their respective ranges of validity for all such 47 one-modulus THCY models. In *all* such models, both BPS and non-BPS black hole attractors are *unique*.

Thus, in *all* $36 + 48 = 84$ one-modulus models (of CICY and THCY type) we have considered, we have found that *all* (*both* BPS and non-BPS) black hole (electric) attractors are *unique*, confirming and generalizing the results of [11].

3.2.5 Non-BPS black holes: recombination factor and instability

It is important to analyse the issue of stability for the non-BPS black holes. In this context an important quantity, namely the *recombination factor R* has been introduced in [12].

It is given by the ratio of the mass of the non-BPS black holes to that of the minimal piecewise calibrated representative corresponding to the same homology class. For $R > 1$ the non-BPS black hole is unstable, and it decays into the corresponding BPS-anti-BPS constituent which form the piecewise calibrated representative. On the other hand, the value $R < 1$ indicates that the constituent BPS-anti-BPS pairs recombine, in order to give rise to a stable non-BPS black hole in the spectrum [11]. In the following treatment, we compute the recombination factor for non-BPS black holes in the one-modulus THCY model treated above.

We consider an $M2$-brane of charge q_I wrapped on the curve $C = \alpha C_1 + \beta C_2$. Thus, we have

$$q_1 = J_1 \cdot C = \alpha J_1 \cdot C_1 + \beta J_1 \cdot C_2 \text{ and } q_2 = J_2 \cdot C = \alpha J_2 \cdot C_1 + \beta J_2 \cdot C_2. \tag{3.72}$$

For the basis where the Kähler cone coincides with the first quadrant, we have

$$J_1 = \frac{1}{2} D_4 \text{ and } J_2 = \frac{3}{4} D_4 - \frac{1}{2} D_5. \tag{3.73}$$

By considering the one-modulus THCY model treated above, from (3.55) we find the intersection numbers

$$\begin{aligned}
C_1 \cdot D_0 &= -1, & C_1 \cdot D_1 &= C_1 \cdot D_2 = 1, & C_1 \cdot D_3 &= 0, & C_1 \cdot D_4 &= 2, & C_1 \cdot D_5 &= 3, \\
C_2 \cdot D_0 &= 1, & C_2 \cdot D_1 &= C_2 \cdot D_2 = 0, & C_2 \cdot D_3 &= 1, & C_2 \cdot D_4 &= 0, & C_2 \cdot D_5 &= -2.
\end{aligned} \tag{3.74}$$

Thus, we have $q_1 = \alpha$ and $q_2 = \beta$.

The $M2$ brane wrapping the curve C will give rise to a non-BPS black hole. For simplicity's sake, we will here confine ourselves to deal with a *doubly-extremal* black hole of charge q_I, in which thus the moduli are fixed to the respective attractor value.[11] The mass M_C of the black hole is given by the square root of the critical value of the black hole effective potential:

$$M_C = \sqrt{V}. \tag{3.75}$$

Let $C^\cup$ be the minimum volume piecewise calibrated representative of the class $[C]$ and denote $M_{C^\cup}$ to be the mass of the $M2$ brane wrapping $C^\cup$. We find

$$M_{C^\cup} = \int_{C^\cup} J = t^1 |\alpha| + t^2 |\beta| = t^1 |q_1| + t^2 |q_2|. \tag{3.76}$$

We can rewrite the above as

$$M_{C^\cup} = y |q_2| (1 + |q|t). \tag{3.77}$$

Thus, the recombination factor in the present case is given by

$$R = \frac{M_C}{M_{C^\cup}} = \left. \frac{\sqrt{V}}{y |q_2| (1 + |q|t)} \right|_{t = t_c}, \tag{3.78}$$

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[11]The generalization to non-doubly extremal but extremal non-BPS black holes may be discussed by exploiting the so-called *first order formalism*, as recently treated in [32] (see also refs. therein).
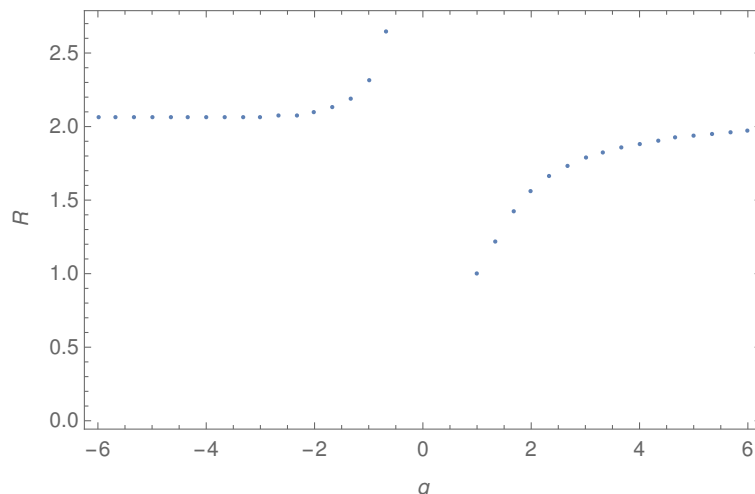
Figure 1. Recombination factor for the foregoing non-BPS black hole for different values of q.

where t_c denotes the critical value of t. Using the expression of the effective potential in (3.70) we find

$$R = \frac{2^{1/6}\left(2t^3+9t^2+9t+3\right)^{5/6}\sqrt{112t^6+360t^5+441t^4+282t^3+135t^2+54t+9}}{3^{5/6}(t+1)^2\left(4t^3-9t^2-9t-3\right)(|q|t+1)}\Bigg|_{t=t_c}. \quad (3.79)$$

Notice that the q_2 dependence on $V_{\rm cr}$ drops out in the ratio, and hence the recombination factor R only depends upon the value of q.

From the discussion below (3.67) we observe that the non-BPS solution does not exist for $-2/3 < q < 1$. There are two branches of solutions for $q > 1$ and for $q < -2/3$. We can numerically evaluate R in both the branches. The value $q = 1$ corresponds to $t = 0$. This gives rise to the value $R = 1$. We find that R increases monotonically in this branch. For the second branch, $R = \sqrt{7}$ for $q = -2/3$. As we decrease the value of q further, R continues to decrease till $q \simeq -4.01$ where it takes the minimum value $R \simeq 2.06$. It increases beyond this value and rises to $R \simeq 2.12$ as $q \to -\infty$. In both the branches the value of R remains greater than 1 throughout.

In figure 1 we plot R as a function of q in the two branches $q < -2/3$ and $q > 1$. As we can see, R increases monotonically in $q > 1$ region. In the $q < -2/3$ region, it decreases rapidly till $q \simeq -4.01$ and then increases very slowly. To understand these results better, we also plot R as a function of the critical value t_c in figure 2. The cusp at $t_c = t_\star \simeq 3.06$ separates the two branches of solutions. From the graph we can clearly see that $R > 1$ throughout, and thus we can conclude that the non-BPS solution remains unstable for all values of q.

Thus, we can conclude that such black holes does *not* enjoy recombination, and they are unstable, decaying into BPS and anti-BPS constituents; this confirms the results of [11], in which *all* non-BPS black holes were found to be unstable.

– 18 –
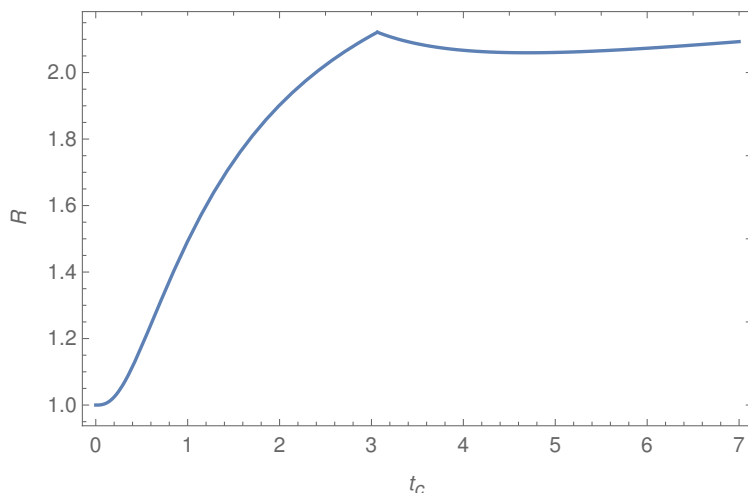
Figure 2. Recombination factor for the same non-BPS black hole as a function of the critical value t_c.

4   5D black string attractors

We will now turn our attention to BPS as well as non-BPS black string configurations in five dimensions. We will first consider general analysis for arbitrary number of moduli. The BPS condition in this case gives rise to a *unique* solution. We will then develope formalism to study the non-BPS configurations. Here we will use the conventions of [23]. The black string effective potential in five dimensions is given by (2.1), with the central charge reading

$$Z = C_{IJK} p^I t^J t^K. \tag{4.1}$$

This implies that

$$\partial_i Z = 2 C_{IJK} p^I t^J \partial_i t^K, \tag{4.2}$$

and thus, by recalling the definition (2.7), the effective potential can be written as

$$V = Z^2 + 6 C_{IJK} p^I t^J C_{LMN} p^L t^M g^{ij} \partial_i t^K \partial_j t^N = Z^2 + 6 \Pi^{KN} C_{IJK} C_{LMN} p^I p^L t^J t^M. \tag{4.3}$$

Then, by recalling (2.9), one can write

$$V = G_{IJ} p^I p^J = Z^2 - 2 \left(C^{KN} - t^K t^N \right) C_{IJK} C_{LMN} p^I p^L t^J t^M$$
$$= 3 Z^2 - 2 C_{LMN} p^N p^L t^M = 3 Z^2 - 2 C_{IJ} p^I p^J. \tag{4.4}$$

We will use the method of Lagrange multiplier to extremize this potential subjected to the constraint $C_{IJK} t^I t^J t^K = 1$. Extremizing

$$\check{V} = 3 Z^2 - 2 C_{IJ} p^I p^J + \lambda (C_{IJK} t^I t^J t^K - 1) \tag{4.5}$$

with respect to t^I and λ we find

$$0 = C_{IJK} t^I t^J t^K - 1; \tag{4.6}$$

$$0 = 12 Z C_{IJK} p^J t^K - 2 C_{IJK} p^J p^K + 3 \lambda C_{IJK} t^J t^K$$
$$= C_{IJK} \left(12 Z p^J t^K - 2 p^J p^K + 3 \lambda t^J t^K \right). \tag{4.7}$$

Multiplying by t^I in the second of the above, we find

$$3\lambda = -12Z^2 + 2C_{IJK}t^I p^J p^K = -12Z^2 + C_{IJ}p^I p^J. \tag{4.8}$$

Using this value of λ we find the equation of motion

$$0 = 12ZC_{IJK}p^J t^K - 2C_{IJK}p^J p^K - 12Z^2 C_{IJK}t^J t^K + 2C_{LM}p^L p^M C_{IJK}t^J t^K$$
$$= 12ZC_{IJK}t^K \left(p^J - Zt^J\right) - 2p^J p^K \left(C_{IJK} - C_{MJK}t^M C_{ILN}t^L t^N\right), \tag{4.9}$$

along with the constraint (2.5). Multiplying C^{IJ} we find

$$6Z(p^J - Zt^J) - C^{JK}C_{KLM}p^L p^M + C_{LM}p^L p^M t^J = 0. \tag{4.10}$$

The equation of motion can also be rewritten in a compact form as

$$C_{ILM}p^K \left(6Zt^J - p^J\right) \left(\frac{1}{2}\delta_J^L \delta_K^M + \frac{1}{2}\delta_K^L \delta_J^M - t^L t^M C_{JKN}t^N\right) = 0. \tag{4.11}$$

The supersymmetric critical points correspond to

$$t^I = \frac{p^I}{Z}. \tag{4.12}$$

This equation can be rewritten as

$$t^I = \frac{p^I}{\left(C_{JKL}p^J p^K p^L\right)^{1/3}}. \tag{4.13}$$

Thus, for a given set of (supporting, magnetic) charges, there *always* is a *unique* BPS black string solution.

The non-BPS black holes can be found upon solving (4.11) such that $Zt^I - p^I \neq 0$. It is worth exploring whether we can obtain an equation analogous to eq. (4.12) for the non-BPS critical points. A naïve analysis of (4.11) might suggest the non-BPS solutions to the equation of motion given in terms of $6Zt^I = p^I$. However, such for such a solution $C_{IJK}t^I t^J t^K = 1/6$ and hence it is not consistent with the constraint (2.5).

In order to obtain the algebraic equation corresponding non-BPS critical points, we set

$$X^I \equiv p^I - Zt^I \tag{4.14}$$

Note that the constraint (2.5) implies

$$C_{IJK}t^I t^J X^K = 0 \tag{4.15}$$

Substituting $p^I = X^I + Zt^I$ in (4.9) we find

$$0 = 12ZC_{IJK}X^J t^K - 2\left(X^J + Zt^J\right)\left(X^K + Zt^K\right)\left(C_{IJK} - C_{MJK}t^M C_{INP}t^N t^P\right)$$
$$= 12ZC_{IJK}X^J t^K$$
$$\quad - 2\left(X^J X^K + ZX^J t^K + Zt^J X^K + Z^2 t^J t^K\right)\left(C_{IJK} - C_{MJK}t^M C_{INP}t^N t^P\right)$$
$$= 12ZC_{IJK}X^J t^K - 2X^J X^K C_{IJK} - 4ZX^J t^K C_{IJK} - 2Z^2 t^J t^K C_{IJK}$$
$$\quad + 2X^J X^K C_{MJK}t^M C_{INP}t^N t^P$$
$$\quad + 4ZX^J t^K C_{MJK}t^M C_{INP}t^N t^P + 2Z^2 t^J t^K C_{MJK}t^M C_{INP}t^N t^P$$
$$= 8ZC_{IJ}X^J - 2C_{IJK}X^J X^K + 2C_{MN}X^M X^N C_{IJ}t^J. \tag{4.16}$$

This equation can be simplified a bit upon multiplying both sides of (4.16) with C^{KI} to obtain

$$
\begin{aligned}
0 &= 8ZC^{KI}C_{IJ}X^J - 2C^{KI}C_{IJM}X^JX^M + 2C_{MN}X^MX^NC^{KI}C_{IJ}t^J \\
&= 8ZX^K - 2C^{KI}C_{IJM}X^JX^M + 2C_{LM}X^LX^Mt^K,
\end{aligned}
\tag{4.17}
$$

which can be rewritten as

$$
4ZX^I + X^JX^KC_{JKL}\left(t^Lt^I - C^{LI}\right) = 0.
\tag{4.18}
$$

Solving the above along with the constraint $C_{IJK}t^It^JX^K = 0$ we can obtain the expression for X^I as a function of t^I and p^I. It might be easier to solve $C_{IJK}t^It^JX^K = 0$ first. This will give a possible solution for X^I up to an overall multiplicative factor. We can determine it as follows. Multiply both sides of (4.16) with X^I and use the constraint (4.15) to obtain

$$
4ZC_{IJ}X^IX^J - C_{IJK}X^IX^JX^K = 0.
\tag{4.19}
$$

This will determine the overall multiplicative factor in X^I. We can substitute the resulting expression back in (4.18) to verify that it holds. The trivial solution $X^I = 0$ corresponding to BPS critical points whereas any non-zero solution for X^I in the above equation will correspond to a non-BPS black string.

Note that, from the "magnetic" version of the "new attractor" approach to 5D attractors treated in [26, 27] (see appendix A), (4.14) can equivalently be rewritten as

$$
X^I = \frac{3^{3/2}}{2^{5/2}}\frac{1}{Z}T^{ijk}\partial_jZ\partial_kZ\partial_it^I.
\tag{4.20}
$$

5 One-modulus models

We will now focus our attention to one-modulus models. As before, we will use the notation $t^1 = x, t^2 = y$. From section 3, we here report

$$
C_{IJ} = \begin{pmatrix} ax + by & bx + cy \\ bx + cy & cx + dy \end{pmatrix},
\tag{5.1}
$$

and its inverse

$$
C^{IJ} = \frac{1}{Lxy - Nx^2 - My^2}\begin{pmatrix} cx + dy & -(bx + cy) \\ -(bx + cy) & ax + by \end{pmatrix}.
\tag{5.2}
$$

Furthermore,[12] $C_{IJK}p^Jp^K =: (P_1, P_2)^T$, where we have introduced the notation

$$
P_1 = a(p^1)^2 + 2bp^1p^2 + c(p^2)^2, \ \ P_2 = b(p^1)^2 + 2cp^1p^2 + d(p^2)^2.
\tag{5.3}
$$

Thus, we find

$$
C^{JK}C_{KLM}p^Lp^M = \frac{1}{(Lxy - Nx^2 - My^2)}\begin{pmatrix} P_1(cx + dy) - P_2(bx + cy) \\ -P_1(bx + cy) + P_2(ax + by) \end{pmatrix}.
\tag{5.4}
$$

---

[12]The P's are the (un-normalized) "Jordan duals" [28] of the magnetic charges p's.

The equations of motion can now be expressed as

$$6Z(p^1 - Zx) + x(xP_1 + yP_2) - \frac{P_1(cx + dy) - P_2(bx + cy)}{(Lxy - Nx^2 - My^2)} = 0; \qquad (5.5)$$

$$6Z(p^2 - Zy) + y(xP_1 + yP_2) + \frac{P_1(bx + cy) - P_2(ax + by)}{(Lxy - Nx^2 - My^2)} = 0. \qquad (5.6)$$

These give rise to two coupled degree seven equations in variables x and y. They give rise to both BPS as well as non-BPS critical points. The BPS critical points are obtained upon solving $(p^1 - Zx) = 0 = (p^2 - Zy)$, and have the exact expression

$$x = \frac{p^1}{(p^1 P_1 + p^2 P_2)^{1/3}}; \qquad (5.7)$$

$$y = \frac{p^2}{(p^1 P_1 + p^2 P_2)^{1/3}}. \qquad (5.8)$$

This result can be regarded as the generalization of the treatment of [9] (done for the *electric* black holes) to *magnetic* black strings.

It is however in general not possible to obtain the non-BPS critical points. The formulation discussed in the previous section gives rise to a somewhat simpler set of equations for the non-BPS critical points. We need to solve (4.18) along with the constraint (4.15). The constraint (4.15) can be solved for X^I in terms of the moduli t^I up to an overall multiplicative factor. To find its value, let $\tilde{X}^I$ be a solution of (4.15). Then, substitute $X^I = \check{X}\tilde{X}^I$ in (4.19) to obtain the value of the multiplicative factor $\check{X}$ as

$$\check{X} = \frac{4Z C_{MN}\tilde{X}^M \tilde{X}^N}{C_{IJK}\tilde{X}^I \tilde{X}^J \tilde{X}^K}. \qquad (5.9)$$

For one-modulus models, it is easy to solve the constraint (4.15). Recall from (3.3) we have $C_{IJ}t^J = (A_1, A_2)^T$, with A_1, A_2 given in (3.7). Thus, (4.15) can be solved to obtain $X^I = \check{X}\tilde{X}^I$ with

$$\tilde{X}^1 = -A_2 = A^1, \text{ and } \tilde{X}^2 = A_1 = A^2. \qquad (5.10)$$

Upon using $C_{IJK}t^I t^J t^K = 1$ we find that $C_{IJ}A^I A^J = \det C$ and hence

$$\check{X} = \frac{4Z \det C}{C_{IJK}A^I A^J A^K}. \qquad (5.11)$$

Thus, the non-BPS critical point corresponding to black strings for an arbitrary one-modulus model is given by

$$p^I - Zt^I - \frac{4Z \det C}{C_{IJK}A^I A^J A^K}A^I = 0. \qquad (5.12)$$

For a given value of C_{IJK} and for a given set of charges, this equation can be solved numerically to obtain the values of the moduli t^I corresponding to a non-BPS critical point.

The effective black hole potential has the expression

$$V = 3\left[p^1(ax^2 + 2bxy + cy^2) + p^2(bx^2 + 2cxy + dy^2)\right]^2$$
$$- 2\left[(p^1)^2(ax + by) + 2p^1p^2(bx + cy) + (p^2)^2(cx + dy)\right]. \qquad (5.13)$$

By adopting the normalization of [11], the black string tension T can be determined from the critical value of the effective black hole potential as[13]

$$T = \sqrt{V}. \qquad (5.14)$$

For BPS solution:

$$V = Z^2 = \left[p^1(ax^2 + 2bxy + cy^2) + p^2(bx^2 + 2cxy + dy^2) \right]^2 ; \qquad (5.15)$$

$$\Downarrow$$

$$T = |Z| = \left| p^1(ax^2 + 2bxy + cy^2) + p^2(bx^2 + 2cxy + dy^2) \right| . \qquad (5.16)$$

We anticipate here that in *all* 36 one-modulus models of CICY type we will find that *all* non-BPS black string (magnetic) attractors are *unique*, confirming and generalizing the results of [11]. On the other hand, in most of the 48 one-modulus models of THCY type we will find that there exist *multiple* non-BPS black string (magnetic) attractors, a phenomenon which was not observed in [11] Moreover, by analising the so-called *recombination factor*, we will also find evidence for the existence of non-BPS black strings which enjoy *recombination*, and thus that are actually stable against the decay into their BPS/anti-BPS constituent pairs, again confirming and generalizing the results of [11].

5.1 Uniqueness of attractors

5.1.1 $c = d = 0$

Before considering specific models in detail, we would like to note that also for black string we have *unique* solution for the special case of $c = d = 0$. Upon setting $c = 0 = d$ and introducing $t = x/y$ and $p = p^1/p^2$, rescaling and solving the non-BPS equation we find

$$t = -\frac{3bp}{3b + 2ap} . \qquad (5.17)$$

Thus, the black string solution in this case is given by

$$x = -\frac{3b + 2ap}{3bp^{1/3}(3b + ap)^{2/3}} , \qquad (5.18)$$

$$y = \frac{3b + 2ap}{3bp^{2/3}(3b + ap)^{1/3}} . \qquad (5.19)$$

The Kähler cone condition is given by $3/p + 2a/b < 0$ and $a + 3b/p < 0$. The tension of the black string is

$$T = \left| p^2 p^{2/3}(3b + ap)^{1/3} \right| . \qquad (5.20)$$

---

[13]Actually, the effective black string potential discussed in [11] differs from ours by a factor of 3/2 and hence we have $\sqrt{V}$ instead of $\sqrt{\frac{3}{2}V}$.

5.1.2 $a = b = 0$

A similar analysis can be done for the case $a = b = 0$. We find

$$t = -\frac{2d + 3cp}{3c} \tag{5.21}$$

and hence

$$x = \frac{2d + 3cp}{3c(d + 3cp)^{1/3}}, \ y = -\frac{1}{(d + 3cp)^{1/3}}. \tag{5.22}$$

The string tension is

$$T = \left| p_2(d + 3cp)^{1/3} \right|. \tag{5.23}$$

The Kähler cone condition is $d + 3cp < 0$ and $(2d + 3cp)/c < 0$.

5.1.3 CICY

We will now consider black string attractor solutions in the 36 one-modulus CICY models, already treated in section 3.2.3. Since BPS black string attractors are always *unique*, we will henceforth only analyse the corresponding non-BPS equations. Once again, we will workout in some detail the one-modulus CICY model considered in section 3.2.3. Substituting the values of the intersection numbers for the hypersurface (3.34) in the non-BPS equations (5.12) we find

$$0 = p^1 - \frac{2\left(6x^4 + 45x^3y + 93x^2y^2 + 156xy^3 + 56y^4\right)\left(p^1y(2x+3y) + p^2\left(x^2 + 6xy + 2y^2\right)\right)}{3(2x+3y)\left(x^2 + 3xy - 12y^2\right)}, \tag{5.24}$$

$$0 = 2p^1y^2(2x+3y)\left(3x^2 + 9xy + 40y^2\right)$$
$$+ p^2\left[6x^4y + 54x^3y^2 + 4y^2\left(-9 + 40y^3\right) + x^2\left(3 + 200y^3\right) + x\left(9y + 516y^4\right)\right]. \tag{5.25}$$

Substituting $x = yt$, $p^1 = p^2p$ and then using the constraint $\mathcal{V} = 1$ from (3.60), we find that the above equations take the simple form

$$\left(6t^4 + 45t^3 + 93t^2 + 156t + 56\right) + p\left(6t^3 + 27t^2 + 107t + 120\right) = 0. \tag{5.26}$$

This is a quartic equation and hence we can write down the exact solution for t in terms of p. However, before doing so we will analyse the above equation qualitatively. Solving the above for p as a function of t we find

$$p = -\frac{6t^4 + 45t^3 + 93t^2 + 156t + 56}{6t^3 + 27t^2 + 107t + 120} \tag{5.27}$$

As $t \to 0$ the r.h.s. goes to $-7/15$, and it goes to $-\infty$ in the limit $t \to \infty$. Thus, for positive t the value of p must be less than $-7/15$. The r.h.s. is a monotonic function, and hence we have a *unique* solution with $t > 0$ for $p < -7/15$. The string tension $T = \sqrt{V}$ is given by

$$T = \left(\frac{3}{2}\right)^{-1/3} |p^2| \frac{\sqrt{p^2\left(6t^2 + 18t + 23\right) + 2p\left(9t^2 + 8t + 6\right) + 3t^4 + 18t^3 + 54t^2 + 24t + 4}}{\left(3t^2 + 9t + 2\right)^{2/3}}. \tag{5.28}$$

As an example, let us consider the value $t = 1$. It corresponds to $p = -89/65$. The moduli are same as (3.52):

$$x = y = \left(\frac{3}{28}\right)^{1/3}.$$ (5.29)

The corresponding tension of the black string $T = \sqrt{V}$ is given by

$$T = |p^2| \, \frac{2^{7/6}\sqrt{1381}}{65} \, \left(\frac{7}{3}\right)^{1/3} \simeq 1.7 \, |p^2|.$$ (5.30)

We will now consider the exact solution. We find

$$t = \sqrt{\frac{133\left(p^2 + 3p + 7\right)}{4\mathcal{D}_2(p)} - \frac{\left(24p^3 + 108p^2 + 2746p + 3957\right)}{32\sqrt{3\mathcal{D}_3(p)}} - \frac{\mathcal{D}_2(p)}{16} + \frac{\mathcal{D}_3(p)}{96}}$$
$$+ \frac{1}{8}\sqrt{\frac{\mathcal{D}_3(p)}{3}} - \frac{p}{4} - \frac{15}{8},$$ (5.31)

where we have used the notation

$$\mathcal{D}_1(p) := 19\left(1372p^6 + 12348p^5 + 172731p^4 + 851166p^3 + 1565367p^2 + 1032552p + 518096\right),$$

$$\mathcal{D}_2(p) := 76^{1/3}\left(1425p^2 + 4275p + 950 + \sqrt{\mathcal{D}_1(p)}\right)^{1/3},$$

$$\mathcal{D}_3(p) := 12p^2 + 36p + 179 + 4\mathcal{D}_2(p) - \frac{2128\left(p^2 + 3p + 7\right)}{\mathcal{D}_2(p)}.$$ (5.32)

A similar analysis can be carried out for all other 35 one-modulus CICY models. In appendix E we report the non-BPS solutions along with their respective ranges of validity for all such 35 one-modulus CICY models. In *all* such models, non-BPS black string attractors are *unique*.

5.2 THCY *Multiple* black strings

In this section we will consider non-BPS black string attractors in the various one-modulus THCY models, already introduced in section 3.2.4. Once again, we will workout in some detail the one-modulus THCY model considered in section 3.2.4. Setting $x = ty$ and $p^1 = pp^2$, we obtain the quartic equation

$$p(8t^4 + 28t^3 + 27t^2 + 3t - 3) + t\left(16t^3 + 45t^2 + 45t + 15\right) = 0.$$ (5.33)

To understand the qualitative features of the equation we solve the above for p:

$$p = -\frac{t\left(16t^3 + 45t^2 + 45t + 15\right)}{8t^4 + 28t^3 + 27t^2 + 3t - 3}.$$ (5.34)

Note that the coefficients in the denominator change sign once. Thus, according to Descartes' rule, it must admit *at least* one positive root where the rational polynomial function at the r.h.s. diverges. Numerically solving the equation

$$8t^4 + 28t^3 + 27t^2 + 3t - 3 = 0,$$ (5.35)

we find that it admits only one positive root $t = t_\star \simeq 0.25$. The rational function is monotonic in the region $0 < t < t_\star$. It vanishes at $t = 0$, diverges at $t = t_\star$ and takes positive values in the interval $(0, t_\star)$. Thus, for all $p > 0$, we have a *unique* solution for $t \in (0, t_\star)$. On the other hand, the rational function is no longer monotonic for $t > t_\star$. Differentiating it with respect to t, we find that the extrema occurs for

$$88t^6 + 144t^5 - 261t^4 - 762t^3 - 675t^2 - 270t - 45 = 0 \,. \tag{5.36}$$

Using Descartes' rule, we find that this also has *at least* one positive root. Solving numerically, we find the only positive root of the above equation is at $t = t_0 \simeq 2.24$. This corresponds to a local maximum of the rational function. Its value at $t = t_0$ is given by $p_0 \simeq -1.78$. On the other hand, p takes the value -2 as $t \to \infty$. Numerically solving (5.33) with $p = -2$, we find $t = t_m \simeq 0.86$. Thus, for all $p < -2$, we have *unique* black string solutions in the narrow window $t_\star < t < t_m$. However, for all $-2 < p < p_0$ we have double roots for the equation (5.33) in the region $t > t_m$, thereby leading to *multiple critical points for the black string attractor*. The values of the moduli x, y in terms of t are given by (3.60):

$$x = \left(\frac{6t^3}{2t^3 + 9t^2 + 9t + 3} \right)^{1/3} \,, \quad y = \left(\frac{6}{2t^3 + 9t^2 + 9t + 3} \right)^{1/3} \,. \tag{5.37}$$

The tension of the black string given by

$$T = \frac{|p^2|}{6^{1/3} \left(2t^3 + 9t^2 + 9t + 3\right)^{2/3}} \left(\begin{array}{c} p^2 \left(4t^4 + 24t^3 + 54t^2 + 42t + 9\right) \\ +6p \left(2t^4 + 8t^3 + 15t^2 + 12t + 3\right) \\ +15t^4 + 42t^3 + 54t^2 + 36t + 9 \end{array} \right)^{1/2} \,. \tag{5.38}$$

Here t takes the critical value for a given p.

As an example, consider the value $p = -121/63$. It admits two solutions for t, namely $t_1 = 1$ and $t_2 \simeq 14.5$. The corresponding values of (x, y) are

$$(x_1, y_1) = \left(\left(\frac{6}{23}\right)^{1/3}, \left(\frac{6}{23}\right)^{1/3} \right), \quad (x_2, y_2) \simeq (1.31, 0.09) \,. \tag{5.39}$$

The tension for these two solutions are

$$T_1 = \sqrt{\frac{199}{567}} \left(\frac{23}{6}\right)^{1/3} |p^2| \simeq 0.93 \, |p^2|, \tag{5.40}$$

and

$$T_2 \simeq 0.89 \, |p^2| \,. \tag{5.41}$$

We will now write down the exact expression for both these solutions. They are given by

$$2t_\pm = \sqrt{\mathcal{D}_3(p)} - \frac{28p + 45}{16(p+2)}$$
$$\pm \sqrt{ \frac{3 \left(208p^2 + 408p + 105\right)}{256(p+2)^2} + \frac{11 \left(64p^3 + 720p^2 + 1476p + 705\right)}{2048(p+2)^3 \sqrt{\mathcal{D}_3(p)}} - \mathcal{D}_3(p) }, \tag{5.42}$$

where we have introduced the notation

$$\mathcal{D}_1(p) := -1616p^6 + 10152p^5 + 60777p^4 + 104634p^3 + 84375p^2 + 33750p + 5625,$$

$$\mathcal{D}_2(p) := \left(\frac{3}{2}\right)^{1/3} \left(50p^3 + 225p^2 + 225p + 75 + \sqrt{\mathcal{D}_1(p)}\right)^{1/3},$$

$$\mathcal{D}_3(p) := \frac{21p(p+1)}{8(p+2)\mathcal{D}_2(p)} + \frac{\mathcal{D}_2(p)}{8(p+2)} + \frac{208p^2 + 408p + 105}{256(p+2)^2}. \tag{5.43}$$

A similar analysis can be carried out for all other 37 one-modulus THCY models. The results are summarized in appendix F, and they provide evidence for a remarkable difference with respect to the results of [11]; indeed, interestingly, *we find multiple black string non-BPS attractor solutions for most of the models*. Intriguingly, we also find that the (local) minimum value of the effective black string potential (and thus, the tension of the non-BPS black string) is *different* for *different, multiple* attractors; again, this fact highlights a new phenomenon with respect to the findings of [11]: at a geometrical level, this should correspond to connected locally volume-minimizing representatives of the (non-BPS) homology class having different values of their (local) minimal volumes as a function of the moduli.

5.3 Non-BPS black strings: recombination factor and stability

As done in section 3.2.5 for black hole attractors, here also we can introduce the recombination factor in order to study the stability of non-BPS black string doubly-extremal. Analogously, the recombinaton factor R is defined by the ratio of the black string tension to that of the minimum piecewise calibrated representative in the same homology class as the black string. For $R > 1$ the non-BPS black string decays into constituent BPS-anti-BPS pairs, whereas for $R < 1$ we have a kinematically stable black string as a result of recombination.

A black string of charge $p^I = (p^1, p^2)$ is obtained upon wrapping an $M5$ brane on the divisor $D = p^1 J_1 + p^2 J_2$. For a double extremal solution, the string tension T is given by the square root of the corresponding effective potential

$$T = \sqrt{V}|_{t=t_c}. \tag{5.44}$$

Let $D^\cup$ be the minimum volume piecewise calibrated representative of the class $[D]$ with volume $V_{D^\cup}$. Then,

$$V_{D^\cup} = |p^1| \int_{J_1} J \wedge J + |p^2| \int_{J_2} J \wedge J, \tag{5.45}$$

which gives rise to

$$V_{D^\cup} = |p^2|(A_2 + |p|A_1)|_{t=t_c}, \tag{5.46}$$

where A_1 and A_2 are defined in (3.7). The recombination factor is given by the ratio

$$R = \frac{\sqrt{V}}{V_{D^\cup}}\bigg|_{t=t_c}. \tag{5.47}$$
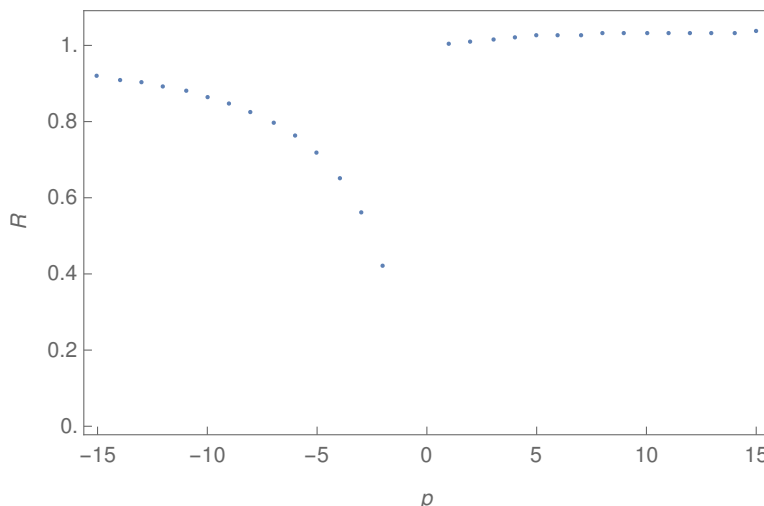
Figure 3. The recombination factor of the foregoing non-BPS black string for various values of p.

In the following treatment, we compute the recombination factor for non-BPS black strings in the one-modulus THCY model treated above. We start and recall here eq. (5.38):

$$T = \frac{|p^2|}{6^{1/3}\left(2t^3 + 9t^2 + 9t + 3\right)^{2/3}} \cdot \begin{pmatrix} p^2\left(4t^4 + 24t^3 + 54t^2 + 42t + 9\right) \\ +6p\left(2t^4 + 8t^3 + 15t^2 + 12t + 3\right) \\ +15t^4 + 42t^3 + 54t^2 + 36t + 9 \end{pmatrix}^{1/2}. \tag{5.48}$$

On the other hand, substituting the value of the intersection numbers in (5.46) we find

$$V_{D^\cup} = \frac{|p^2|\left(\left(2t^2 + 6t + 3\right)|p| + 3(t+1)^2\right)}{6^{1/3}\left(2t^3 + 9t^2 + 9t + 3\right)^{2/3}}. \tag{5.49}$$

Taking the ratio, we find the expression for the recombination factor

$$R = \frac{1}{\left(2t^2 + 6t + 3\right)|p| + 3(t+1)^2} \begin{pmatrix} p^2\left(4t^4 + 24t^3 + 54t^2 + 42t + 9\right) \\ +6p\left(2t^4 + 8t^3 + 15t^2 + 12t + 3\right) \\ +15t^4 + 42t^3 + 54t^2 + 36t + 9 \end{pmatrix}^{1/2}. \tag{5.50}$$

Note that in eqs. (5.48)–(5.50) t takes its critical value. The plot for R in the range $p < -2$ and $p > 0$ are shown below in figures 3–5.

For $p > 0$ and for large negative p (i.e. $p = p_m \lesssim -44.856$) the value of R is greater than 1 and the solution remains unstable. Whereas for p in the range $p_m < p < p_0$ we have stable non-BPS attractor. Thus, in some, suitable ranges of the supporting magnetic charges, black strings do enjoy *recombination*, and they are thus stable against the decaying into their BPS/anti-BPS constituents.

It is here worth remarking that there exist *at least* some subsector of the supporting magnetic charge ratio p for which the *multiple* non-BPS black string attractors (if any) are kinematically *stable*. This adds interest and physical relevance to the discovery of *multiple* non-BPS black string attractors, which is a result of the present paper (and which, for
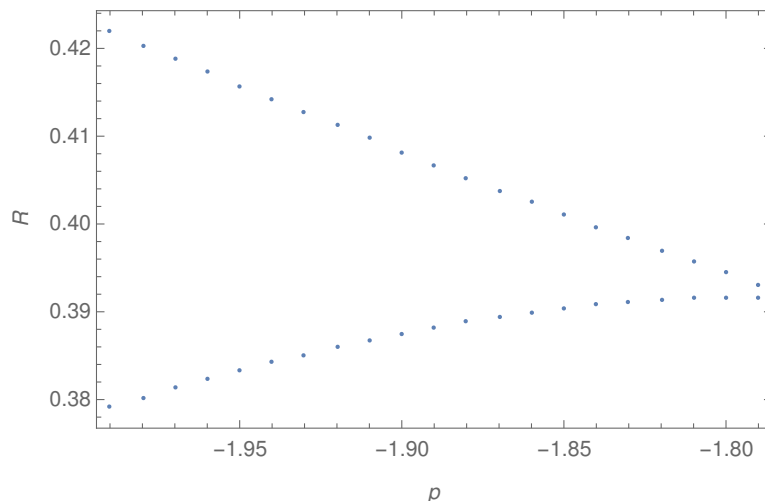
Figure 4. The recombination factor for various values of p supporting multiple non-BPS black strings.
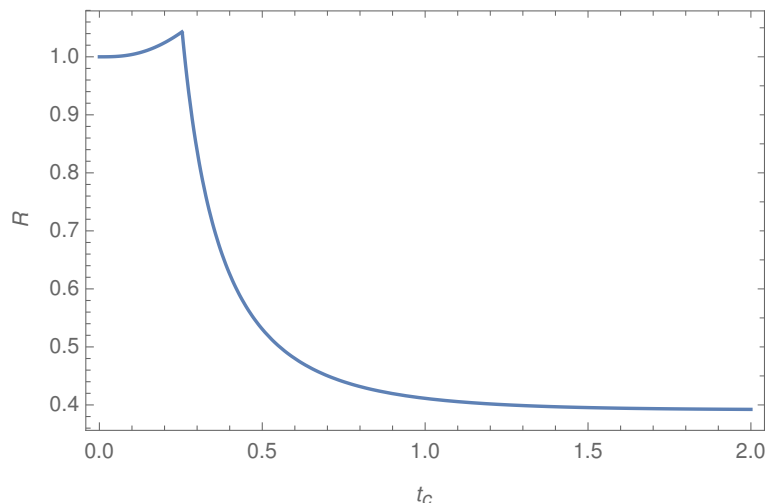
Figure 5. The recombination factor of the same non-BPS black string(s) as a function of the critical value t_c.

instance, was not observed in [11]). Indeed, in [11] stable non-BPS (doubly-extremal) black strings were observed, but they were unique. Here, we have discovered multiple (and, in the doubly-extremal case, *stable*) non-BPS black strings in most of one-modulus THCY models.

6 Conclusions

Motivated by the relevance of extremal non-BPS black holes and black strings for the Weak Gravity Conjecture (WGC) [2] and by the recently established [11] evidence for stable non-BPS remnants of black strings in minimal, $N = 2$ five-dimensional supergravity, we have investigated non-BPS attractors in the low-energy limit of compactifications of M-theory on Calabi-Yau threefolds with $h_{1,1} = 2$ moduli.

On one hand, by computing the so-called *recombination factor* for doubly-extremal non-BPS black holes, we have confirmed the results of [11] in the whole set of CICY and THCY one-modulus models (respectively made of 36 and 48 models): for a given, supporting electric charge configuration, non-BPS extremal black holes are always *unstable* and *unique*. This means that such solutions correspond to local, but not global, volume minimizers of the corresponding curve classes, as there is always a disconnected, piecewise-calibrated representative (union of holomorphic and anti-holomorphic curves) which corresponds to the BPS/anti-BPS black hole constituents, and whose smaller volume implies that the WGC is satisfied, yielding to the decay of non-BPS black holes into widely-separated BPS and anti-BPS particles. In other words, non-BPS black holes are bound to decay to BPS and anti-BPS constituents, and, by sweeping all one-modulus CICY and THCY models, we have found, as in [11], no examples of macroscopic black holes whose mass predicts a stable remnant microscopic black hole coming from Calabi-Yau threefolds.

On the other hand, we have extended the results of [11] concerning the stability of some non-BPS (doubly-extremal) black string solutions, confirming the existence of a phenomenon called *"recombination"* for a large fraction of the whole set of CICY and THCY one-modulus models: in such a phenomenon, holomorphic and anti-holomorphic constituents of the same homology class fuse together to make a smaller cycle, and by the WGC this yields to the prediction that there should be microscopic and *stable*, non-BPS black strings (with small charge) in the spectrum of the resulting supergravity theory. Thus, extremal non-BPS configurations, at least for large charges, may have robust features similar to what one sees for supersymmetric, BPS states. However, it should be here remarked that, for a given supporting magnetic charge configuration, in all CICY one-modulus models all black string solutions have been found to be *unique*.

A new evidence, which constitute the novel contribution of the present investigation to the study of non-BPS attractors within the WGC, is the *non-uniqueness* of non-BPS, extremal (*stable*) black strings in most of one-modulus THCY models: for a given, supporting magnetic charge configurations, in many models there exist multiple non-BPS black string attractors with different tensions, which are stable: in these models, recombination occurs in presence of connected, locally volume-minimizing, representatives of the same (non-BPS) homology class having different values of their (local) minimal volumes as a function of the moduli. All this begs for a clearer mathematical explanation. From a physical perspective, by the WGC, a given (small) magnetic charge configuration may support multiple non-BPS, extremal black string solutions which are stable against the decay into constituent BPS/anti-BPS black string constituents.

As speculated in [11], a possible explanation for such a difference between non-BPS black holes and non-BPS black strings may lie in the fact that black holes correspond to (M2 branes wrapping) 2-cycles, which are thus less than half of the dimension of the Calabi-Yau threefold, whereas black strings correspond to (M5 branes wrapping) 4-cycles, with dimension bigger than half of the dimension of the Calabi-Yau threefold. This might hint, *at least* for the class of one-modulus THCY models which has been investigated here, for an intersection for higher dimensional cycles due to local instability modes localized where holomorphic and anti-holomorphic cycles intersect.

Many different developments may be considered, starting from the present paper. For instance, it would be interesting to compute the recombination factor for non-BPS black holes and black strings which are extremal but not doubly-extremal; as pointed out along the treatment, this can be done by exploiting the so-called first order formalism for extremal solutions, as recently done in [32]. Also, one could consider to carry out an extensive analysis over other classes of $h_{1,1} \geqslant 3$-moduli Calabi-Yau threefolds, starting from the topological data provided by existing classifications (see e.g. [33] and [34] for recent studies). Concerning non-uniqueness of attractors (in the same basin of attraction of the moduli space), by exploiting a Kaluza-Klein compactification from 5 to 4 space-time dimensions, it would be interesting to relate the multiple non-BPS, extremal and kinematically stable black strings found in the present work with the multiple four-dimensional extremal black hole attractors related to non-trivial involutory matrices, as found in [35, 36].

Acknowledgments

A   5D electric "new attractor" approach

In this appendix we will recall the so-called "new attractor" approach to the attractor equations of extremal (electric) black holes in $N = 2$, $D = s + t = 4 + 1$ Maxwell-Einstein supergravity coupled to n Abelian vector multiplets [26, 27].

We start and observe that the metric G_{IJ} of the $(n + 1)$-dimensional "ambient space", which is the pull-back of the metric g_{ij} of the scalar manifold $\mathcal{M}$ as well as the canonical metric associated to the cubic form $C_{IJK} t^I t^J t^K$, reads

$$G_{IJ} = t_I t_J + \frac{3}{2} g^{ij} \partial_i t_I \partial_j t_J, \tag{A.1}$$

and its inverse reads

$$G^{IJ} = t^I t^J + \frac{3}{2} g^{ij} \partial_i t^I \partial_j t^J, \tag{A.2}$$

where

$$t_I = C_{IJK} t^J t^K. \tag{A.3}$$

Thus, the following identity holds in projective special real geometry:

$$\delta_J^I = t_J t^I + \frac{3}{2} g^{ij} \partial_i t^I \partial_j t_J. \tag{A.4}$$

By contracting such an identity with q_I and defining the (electric) central charge function as

$$Z := t^I q_I \Rightarrow \partial_i Z = \partial_i t^I q_I, \tag{A.5}$$

one obtains the following identity:[14]

$$q_I = t_I Z + \frac{3}{2} g^{ij} \partial_i t_I \partial_j Z \,, \tag{A.6}$$

holding in the background of an extremal black hole.

For BPS black hole attractors,

$$\partial_j Z = 0 \Rightarrow \partial_j V = 0, \tag{A.7}$$

and hence, from the identity (A.6) one finds

$$q_I = t_I Z, \tag{A.8}$$

which is an algebraic, equivalent re-writing of the electric BPS attractor equations (A.7) (cfr. e.g. (3.14) of [26]).

On the other hand, for non-BPS black hole attractors,

$$\left. \begin{array}{c} \partial_j V = 0 \\ \partial_j Z \neq 0 \text{ for some } j \end{array} \right\} \Rightarrow \partial_j Z = \frac{1}{2Z} \sqrt{\frac{3}{2}} T_{jkl} g^{km} g^{lp} \partial_m Z \partial_p Z, \tag{A.9}$$

where

$$T_{ijk} = - \left(\frac{3}{2} \right)^{3/2} \partial_i t^I \partial_j t^J \partial_k t^K C_{IJK} \,. \tag{A.10}$$

Thus, from the identity (A.6) one finds[15]

$$q_I = t_I Z + \frac{1}{2} \left(\frac{3}{2} \right)^{3/2} \frac{1}{Z} T^{imp} \partial_i t_I \partial_m Z \partial_p Z \,, \tag{A.11}$$

where $T^{imp} := g^{ij} g^{km} g^{lp} T_{jkl}$. Clearly, (A.11) is an equivalent re-writing of the non-BPS electric attractor equations (A.9).

B 5D magnetic "new attractor" approach

In this appendix we will present recall the so-called "new attractor" approach to the attractor equations of extremal (magnetic) black strings in $N = 2$, $D = s + t = 4 + 1$ Maxwell-Einstein supergravity coupled to n Abelian vector multiplets, which has not been considered e.g. in [26, 27], and which for non-BPS attractors, as far as we know, has never been presented in the literature.

By contracting the identity (A.4) with p^J and defining the (magnetic) central charge function as

$$Z := t_I p^I \Rightarrow \partial_i Z = \partial_i t_I p^I, \tag{B.1}$$

one obtains the following identity:

$$p^I = t^I Z + \frac{3}{2} g^{ij} \partial_i t^I \partial_j Z \,, \tag{B.2}$$

---

[14]The "+" in front of the second term of the r.h.s. of (A.6) corrects a typo e.g. in (3.4) of [26].

[15]The "+" in front of the second term of the r.h.s. of (A.11) corrects a typo e.g. in (3.15) of [26].

holding in the background of an extremal black string solution.

For BPS black string attractors,

$$\partial_j Z = 0 \Rightarrow \partial_j V = 0, \tag{B.3}$$

and hence, from the identity (B.2) one finds

$$p^I = t^I Z, \tag{B.4}$$

which is an algebraic, equivalent re-writing of the magnetic BPS attractor equations (B.3).

On the other hand, for non-BPS black string attractors,

$$\left.\begin{array}{r} \partial_j V = 0 \\ \partial_j Z \neq 0 \text{ for some } j \end{array}\right\} \Rightarrow \partial_j Z = \frac{1}{2Z}\sqrt{\frac{3}{2}} T_{jkl} g^{km} g^{lp} \partial_m Z \partial_p Z. \tag{B.5}$$

Thus, from the identity (B.2) one finds

$$p^I = t^I Z + \frac{1}{2}\left(\frac{3}{2}\right)^{3/2} \frac{1}{Z} T^{imp} \partial_i t^I \partial_m Z \partial_p Z\,, \tag{B.6}$$

which is an equivalent re-writing of the non-BPS magnetic attractor equations (B.5).

C CICY black holes

In this appendix we will report the results of the study of extremal black hole attractors in one-modulus complete intersection Calabi-Yau (CICY) models, recently discussed in [18] (cfr. appendix A therein). For one modulis, CICY's are given by intersections of hypersurfaces in an ambient space of the form $\mathcal{A} = \mathbb{P}^{n_1} \times \mathbb{P}^{n_2}$. As we have mentioned at the start of section 3.2.3, the Calabi-Yau manifold is specified by a configuration matrix. Each column of the configuration matrix represents the bi-degree of a polynomial whose zero *locus* defines a hypersurface in $\mathcal{A}$. The common zero *locus* of all these polynomials becomes a Calabi-Yau manifold provided the sum of the ith row elements of the configuration matrix takes the value $n_i + 1$.

In tables 1–3, we report the extremal black hole attractor solutions for 20 one-modulus CICY models. In particular, the first model of table 1 is the one explicitly treated in section 3.2.3. The first column of tables 1–3 indicates the CICY label as well as its configuration matrix, as from [18]. The superscripts in the configuration matrix are the Hodge numbers $h_{1,1} = 2$ and $h_{2,1}$, respectively, whereas the subscript is the Euler number $\chi := 2(h_{1,1} - h_{2,1})$ of the CICY model. On the other hand, as indicated in the first row of tables 1–3, in the various partitions of their second column we respectively specify: the intersection numbers of the CICY model, the critical values of $t = x/y$ as a function of the charge ratio $q = q_1/q_2$ for the BPS black hole attractors, along with the range of q for which the BPS moduli lie inside the Kähler cone, the expression for q as a rational polynomial function of t for the non-BPS black hole attractors, along with the range of q for which the non-BPS moduli lie within the Kähler cone, and finally the expression for the recombination factor. Note that in all such tables t takes its critical value.

CICY label, Configuration Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	BPS solution	Non-BPS solution
		Range of validity	Range of validity
		Recombination factor	
7643 $\begin{pmatrix} 0\ 0\ 2\ 1 \\ 2\ 2\ 1\ 1 \end{pmatrix}^{2,46}_{-88}$	$\frac{2}{3}\begin{pmatrix} 3\ 2 \\ 1\ 0 \end{pmatrix}$	$\frac{1-3q+\sqrt{7q^2-3q+1}}{q}$	$-\frac{12t^4+72t^3+181t^2+219t+284}{6t^5+27t^4+141t^3+444t^2+638t+72}$
		$0 < q < \frac{3}{2}$	$-\frac{71}{18} < q < 0$
		$\frac{(3t^2+9t+2)\sqrt{36t^6+324t^5+1557t^4+4482t^3+10329t^2+15192t+12272}}{(6t^5+27t^4+141t^3+444t^2+638t+72)(1-qt)}$	
7644 $\begin{pmatrix} 2\ 0\ 1\ 1\ 1 \\ 0\ 2\ 1\ 1\ 1 \end{pmatrix}^{2,46}_{-88}$	$\frac{2}{3}\begin{pmatrix} 3\ 1 \\ 3\ 1 \end{pmatrix}$	$\frac{3(1-q)+\sqrt{6q^2-8q+6}}{3q-1}$	$-\frac{7t^5+123t^4+258t^3+238t^2+123t+51}{51t^5+123t^4+258t^3+238t^2+123t+7}$
		$\frac{1}{3} < q < 3$	$-\frac{51}{7} < q < -\frac{7}{51}$
		$\frac{(t^3+9t^2+9t+1)\sqrt{481t^6+1854t^5+3555t^4+4220t^3+3555t^2+1854t+481}}{(51t^5+123t^4+238t^3+258t^2+123t+7(1-qt)}$	
7668 $\begin{pmatrix} 0\ 2\ 1 \\ 3\ 1\ 1 \end{pmatrix}^{2,47}_{-90}$	$\frac{1}{2}\begin{pmatrix} 3\ 2 \\ 1\ 0 \end{pmatrix}$	$\frac{1-3q+\sqrt{7q^2-3q+1}}{q}$	$-\frac{12t^4+72t^3+181t^2+219t+284}{6t^5+27t^4+141t^3+444t^2+638t+72}$
		$0 < q < \frac{3}{2}$	$-\frac{71}{18} < q < 0$
		$\frac{(3t^2+9t+2)\sqrt{36t^6+324t^5+1557t^4+4482t^3+10329t^2+15192t+12272}}{(6t^5+27t^4+141t^3+444t^2+638t+72)(1-qt)}$	
7725 $\begin{pmatrix} 0\ 0\ 1\ 1\ 1 \\ 2\ 2\ 1\ 1\ 1 \end{pmatrix}^{2,50}_{-96}$	$\frac{2}{3}\begin{pmatrix} 3\ 3 \\ 1\ 0 \end{pmatrix}$	$\frac{1-3q+\sqrt{6q^2-3q+1}}{q}$	$-\frac{4t^4+24t^3+59t^2+69t+69}{2t^5+9t^4+39t^3+114t^2+159t+27}$
		$0 < q < 1$	$-\frac{23}{9} < q < 0$
		$\frac{3(t^2+3t+1)\sqrt{4t^6+36t^5+165t^4+450t^3+915t^2+1206t+849}}{(2t^5+9t^4+39t^3+114t^2+159t+27)(1-qt)}$	
7726 $\begin{pmatrix} 0\ 1\ 1\ 1\ 1 \\ 2\ 1\ 1\ 1\ 1 \end{pmatrix}^{2,50}_{-96}$	$\frac{1}{3}\begin{pmatrix} 6\ 4 \\ 4\ 1 \end{pmatrix}$	$\frac{4-6q+\sqrt{10(2q^2-2q+1)}}{4q-1}$	$-\frac{3t^5+70t^4+220t^3+300t^2+220t+112}{4(7t^5+25t^4+65t^3+100t^2+70t+8)}$
		$\frac{1}{4} < q < \frac{3}{2}$	$-\frac{7}{2} < q < -\frac{3}{28}$
		$\frac{(t^3+12t^2+18t+4)\sqrt{89t^6+516t^5+1440t^4+2440t^3+2820t^2+2016t+704}}{4(7t^5+25t^4+65t^3+100t^2+70t+8)(1-qt)}$	
7758 $\begin{pmatrix} 0\ 2\ 1 \\ 2\ 1\ 2 \end{pmatrix}^{2,52}_{-100}$	$\frac{1}{3}\begin{pmatrix} 5\ 2 \\ 2\ 0 \end{pmatrix}$	$\frac{2-5q+\sqrt{21q^2-10q+4}}{2q}$	$-\frac{96t^4+480t^3+1018t^2+1045t+1289}{48t^5+180t^4+858t^3+2320t^2+2816t+190}$
		$0 < q < \frac{5}{2}$	$-\frac{1289}{190} < q < 0$
		$\frac{(6t^2+15t+2)\sqrt{576t^6+4320t^5+17748t^4+43740t^3+89637t^2+115530t+83113}}{2(24t^5+90t^4+429t^3+1160t^2+1408t+95)(1-qt)}$	
7759 $\begin{pmatrix} 0\ 2\ 1\ 1 \\ 2\ 1\ 1\ 1 \end{pmatrix}^{2,52}_{-100}$	$\frac{1}{3}\begin{pmatrix} 5\ 2 \\ 4\ 1 \end{pmatrix}$	$\frac{4-5q+\sqrt{17q^2-18q+11}}{4q-1}$	$-\frac{35t^5+821t^4+2146t^3+2458t^2+1567t+821}{2(172t^5+503t^4+1214t^3+1646t^2+982t+67)}$
		$\frac{1}{4} < q < \frac{5}{2}$	$-\frac{821}{134} < q < -\frac{35}{344}$
		$\frac{(t^3+12t^2+15t+2)\sqrt{11873t^6+56262t^5+133323t^4+195668t^3+205611t^2+134094t+43793}}{2(172t^5+503t^4+1214t^3+1646t^2+982t+67)(1-qt)}$	

Table 1. BPS and non-BPS extremal black holes in one-modulus CICY models, 1/3.

CICY label Configuration Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	BPS solution	Non-BPS solution
		Range of validity	Range of validity
		Recombination Factor	
7761 $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}^{2,52}_{-100}$	$\frac{5}{6}\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$	$\frac{2(1-q)+\sqrt{2q^2-3q+2}}{2q-1}$	$-\frac{5t^5+58t^4+124t^3+119t^2+64t+22}{22t^5+64t^4+119t^3+124t^2+58t+5}$
		$\frac{1}{2}<q<2$	$-\frac{22}{5}<q<-\frac{5}{22}$
		$\frac{(t^3+6t^2+6t+1)\sqrt{281t^6+1212t^5+2472t^4+3046t^3+2472t^2+1212t+281}}{(22t^5+64t^4+119t^3+124t^2+58t+5)(1-qt)}$	
7799 $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}^{2,55}_{-106}$	$\frac{1}{6}\begin{pmatrix} 7 & 2 \\ 7 & 2 \end{pmatrix}$	$\frac{7(1-q)+\sqrt{5(7q^2-9q+7)}}{7q-2}$	$-\frac{32t^5+658t^4+1372t^3+1253t^2+637t+280}{280t^5+637t^4+1253t^3+1372t^2+658t+32}$
		$\frac{2}{7}<q<\frac{7}{2}$	$-\frac{35}{4}<q<-\frac{4}{35}$
		$\frac{(2t^3+21t^2+21t+2)\sqrt{12256t^6+45696t^5+86205t^4+101030t^3+86205t^2+45696t+12256}}{\sqrt{5}(280t^5+637t^4+1253t^3+1372t^2+658t+32)(1-qt)}$	
7807 $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \end{pmatrix}^{2,56}_{-108}$	$\frac{1}{3}\begin{pmatrix} 5 & 4 \\ 2 & 0 \end{pmatrix}$	$\frac{2-5q+\sqrt{17q^2-10q+4}}{2q}$	$-\frac{96t^4+480t^3+986t^2+965t+831}{2(24t^5+90t^4+333t^3+820t^2+957t+130)}$
		$0<q<\frac{5}{4}$	$-\frac{831}{260}<q<0$
		$\frac{(6t^2+15t+4)\sqrt{576t^6+4320t^5+16596t^4+37980t^3+65373t^2+72870t+43529}}{2(24t^5+90t^4+333t^3+820t^2+957t+130)(1-qt)}$	
7808 $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 \end{pmatrix}^{2,56}_{-108}$	$\frac{1}{2}\begin{pmatrix} 3 & 3 \\ 1 & 0 \end{pmatrix}$	$\frac{1-3q+\sqrt{6q^2-3q+1}}{q}$	$-\frac{4t^4+24t^3+59t^2+69t+69}{2t^5+9t^4+39t^3+114t^2+159t+27}$
		$0<q<1$	$-\frac{23}{9}<q<0$
		$\frac{3(t^2+3t+1)\sqrt{4t^6+36t^5+165t^4+450t^3+915t^2+1206t+849}}{(2t^5+9t^4+39t^3+114t^2+159t+27)(1-qt)}$	
7809 $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{pmatrix}^{2,56}_{-108}$	$\frac{1}{6}\begin{pmatrix} 9 & 5 \\ 7 & 2 \end{pmatrix}$	$\frac{7-9q+\sqrt{46q^2-53q+31}}{7q-2}$	$-\frac{656t^5+13402t^4+36092t^3+42197t^2+26617t+11861}{5392t^5+16457t^4+37057t^3+49162t^2+29567t+2815}$
		$\frac{2}{7}<q<\frac{9}{5}$	$-\frac{11861}{2815}<q<-\frac{41}{337}$
		$\frac{(2t^3+21t^2+27t+5)\sqrt{3(353632t^6+1752528t^5+4196475t^4+6114530t^3+6107025t^2+3776778t+1143907)}}{(5392t^5+16457t^4+37057t^3+49162t^2+29567t+2815)(1-qt)}$	
7821 $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix}^{2,58}_{-112}$	$\frac{1}{6}\begin{pmatrix} 8 & 5 \\ 4 & 0 \end{pmatrix}$	$\frac{2(1-2q)+\sqrt{11q^2-8q+4}}{2q}$	$-\frac{4(24t^4+96t^3+158t^2+124t+87)}{48t^5+144t^4+432t^3+856t^2+801t+85}$
		$0<q<\frac{8}{5}$	$-\frac{348}{85}<q<0$
		$\frac{(12t^2+24t+5)\sqrt{144t^6+864t^5+2664t^4+4896t^3+6801t^2+6114t+2951}}{(48t^5+144t^4+432t^3+856t^2+801t+85)(1-qt)}$	
7833 $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}^{2,59}_{-114}$	$\frac{1}{6}\begin{pmatrix} 7 & 2 \\ 3 & 0 \end{pmatrix}$	$\frac{3-7q+\sqrt{43q^2-21q+9}}{3q}$	$-\frac{9(108t^4+504t^3+1005t^2+973t+1208)}{486t^5+1701t^4+7965t^3+20412t^2+23298t+1120}$
		$0<q<\frac{7}{2}$	$-\frac{1359}{140}<q<0$
		$\frac{3(9t^2+21t+2)\sqrt{2916t^6+20412t^5+79461t^4+185598t^3+368577t^2+455616t+317536}}{(486t^5+1701t^4+7965t^3+20412t^2+23298t+1120)(1-qt)}$	

Table 2. BPS and non-BPS extremal black holes in one-modulus CICY models, 2/3.

CICY label, Configuration Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	BPS solution	Non-BPS solution
		Range of validity	Range of validity
		Recombination Factor	
7844 $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}^{2,62}_{-120}$	$\frac{1}{3}\begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$	$\frac{2-3q+\sqrt{7q^2-6q+4}}{2q}$	$-\frac{96t^4+288t^3+362t^2+219t+142}{48t^5+108t^4+282t^3+444t^2+319t+18}$
		$0 < q < 3$	$-\frac{71}{9} < q < 0$
		$\frac{(6t^2+9t+1)\sqrt{576t^6+2592t^5+6228t^4+8964t^3+10329t^2+7596t+3068}}{(48t^5+108t^4+282t^3+444t^2+319t+18)(1-qt)}$	
7853 $\begin{pmatrix} 0 & 2 & 1 \\ 2 & 2 & 1 \end{pmatrix}^{2,64}_{-124}$	$\frac{2}{3}\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$	$\frac{1-2q+\sqrt{3q^2-2q+1}}{q}$	$-\frac{6t^4+24t^3+40t^2+32t+26}{3t^5+9t^4+30t^3+62t^2+59t+5}$
		$0 < q < 2$	$-\frac{26}{5} < q < 0$
		$\frac{(3t^2+6t+1)\sqrt{9t^6+54t^5+171t^4+324t^3+483t^2+462t+241}}{(3t^5+9t^4+30t^3+62t^2+59t+5)(1-qt)}$	
7863 $\begin{pmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}^{2,66}_{-128}$	$\frac{1}{3}\begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix}$	$\frac{3(1-q)+\sqrt{6q^2-8q+6}}{3q-1}$	$-\frac{7t^5+123t^4+258t^3+238t^2+123t+51}{51t^5+123t^4+258t^3+238t^2+123t+7}$
		$\frac{1}{3} < q < 3$	$-\frac{51}{7} < q < -\frac{7}{51}$
		$\frac{(t^3+9t^2+9t+1)\sqrt{481t^6+1854t^5+3555t^4+4220t^3+3555t^2+1854t+481}}{(51t^5+123t^4+238t^3+258t^2+123t+7)(1-qt)}$	
7868 $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}^{2,68}_{-132}$	$\frac{1}{6}\begin{pmatrix} 7 & 5 \\ 3 & 0 \end{pmatrix}$	$\frac{3-7q+\sqrt{34q^2-21q+9}}{3q}$	$-\frac{9(108t^4+504t^3+969t^2+889t+739)}{486t^5+1701t^4+6021t^3+13986t^2+15297t+1855}$
		$0 < q < \frac{7}{5}$	$-\frac{6651}{1855} < q < 0$
		$\frac{3(9t^2+21t+5)\sqrt{2916t^6+20412t^5+73629t^4+158382t^3+258579t^2+273042t+155041}}{(486t^5+1701t^4+6021t^3+13986t^2+15297t+1855)(1-qt)}$	
7883 $\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}^{2,77}_{-150}$	$\frac{1}{6}\begin{pmatrix} 5 & 2 \\ 3 & 0 \end{pmatrix}$	$\frac{3-5q+\sqrt{19q^2-15q+9}}{3q}$	$-\frac{9(108t^4+360t^3+501t^2+335t+232)}{486t^5+1215t^4+3429t^3+5940t^2+4722t+320}$
		$0 < q < \frac{5}{2}$	$-\frac{261}{40} < q < 0$
		$\frac{3(9t^2+15t+2)\sqrt{2916t^6+14580t^5+38637t^4+61290t^3+76977t^2+61920t+27232}}{(486t^5+1215t^4+3429t^3+5940t^2+4722t+320)(1-qt)}$	
7884 $\begin{pmatrix} 3 \\ 3 \end{pmatrix}^{2,83}_{-162}$	$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	$\frac{1-q+\sqrt{q^2-q+1}}{q}$	$-\frac{4t^4+8t^3+7t^2+3t+2}{2t^5+3t^4+7t^3+8t^2+4t}$
		$q > 0$	$q < 0$
		$\frac{3(t+1)\sqrt{4t^6+12t^5+21t^4+22t^3+21t^2+12t+4}}{(2t^4+3t^3+7t^2+8t+4)(1-qt)}$	

Table 3. BPS and non-BPS extremal black holes in one-modulus CICY models, 3/3.

Our findings show that *there are no multiple extremal BPS black holes* for a given value of (supporting electric charge ratio) q. Analogously, for extremal non-BPS attractors, for any given q within the specified range there is a *unique t* inside the Kähler cone; thus, all 20 one-modulus CICY models under consideration give *unique* non-BPS black hole attractors. Moreover, we should remark that all BPS and non-BPS solutions are *mutually exclusive*. Except for the bi-cubic model (which is the last model treated in table 3), in which any given q supports either a BPS or a non-BPS black hole attractor, the range of allowed values of q is finite.

We have numerically evaluated the recombination factor for the non-BPS black hole attractors in the entire moduli space for all allowed values of q; we have found that the recombination factor is always greater than 1 for all the 20 one-modulus CICY models under consideration: therefore, all non-BPS black holes in the 20 one-modulus CICY models listed in tables 1–3 are *unstable*.

Since the one-modulus CICY models have been classified in a set of 36 models (cfr. e.g. [18]), a natural question arises: what about the remaining 16 one-modulus CICY models, not reported in tables 1–3? All such unlisted models have either $c = d = 0$ or $a = b = 0$, and thus the uniqueness of their BPS and non-BPS black hole attractors has been discussed in sections 3.2.1 and 3.2.2, respectively. Moreover, in such models the recombination factor of non-BPS black holes can be computed exactly. For $a = b = 0$, we find

$$R = 3 \left(\theta(-q) \left| \frac{3c - dq}{9c + dq} \right| + \theta(q) \right) , \qquad (C.1)$$

where $\theta(q)$ is the Heavyside step function. As we have noticed in section 3.2.2, for the attractor solution to lie within the Kähler cone, one must have $d > 3c/q$ and $3/q + d/c < 0$. Thus, for $c, d > 0$, which is the case in all models, the first condition $d > 3c/q$ is automatically satisfied, whereas the second condition implies that q must be negative. In addition, we must have $-3 < dq/c < 0$. Using this, we find that $1 < R < 3$, and hence *all* non-BPS black hole attractor solutions are *unstable*.

Similarly, for $c = d = 0$, we find

$$R = 3 \left(\theta(-q) \left| \frac{a - 3bq}{a + 9bq} \right| + \theta(q) \right) . \qquad (C.2)$$

Once again, by imposing the Kähler cone conditions $a/b + 3q < 0$ and $a > 3bq$, we obtain that $R > 1$, and hence *all* non-BPS black hole attractor solutions are *unstable*.

Let us observe that:

- The one-modulus CICY models labelled by 7643 and 7668, i.e. the first and the third model of table 1, have the matrices of intersection numbers reciprocally proportional, and they also share the same entries in all partitions of the second column, such as the same expressions for t as a function of q for BPS attractors, as well as the same expressions of q as a rational polynomial function of t for non-BPS attractors. However, the fact that their intersection numbers are different (notwithstanding being proportional) implies that the cubic constraint defining the 5D scalar manifold $\mathcal{M}$ (i.e. $C_{IJK} t^I t^J t^K = 1$) yields different expressions for the moduli, and the similarity of the

various expressions observed here is a mere coincidence. Furthermore, such models also have different $h_{2,1}$ (and thus different χ), as well as different configuration matrices.

D   THCY black holes

In this appendix we will report the results of the study of extremal black hole attractors in the 48 one-modulus toric hypersurface Calabi-Yau (THCY), recently discussed in [18] (cfr. appendix B therein), and whose cubic forms (and thus intersection numbers) can be obtained e.g. from the database [30].

In tables 4–8, we report the extremal black hole attractor solutions for 37 one-modulus THCY models, and we do not include the model[16] already treated in detail in section 3.2.4. The first column of tables 4–8 indicates the polytope label as well as the charge matrix of the ambient toric variety, as from [18]. The superscripts in the configuration matrix are the Hodge numbers $h_{1,1} = 2$ and $h_{2,1}$, respectively, whereas the subscript is the Euler number χ of the THCY model. On the other hand, as indicated in the first row of tables 4–8 and as reported in tables 1–3 for one-modulus CICY models, in the various partitions of their second column we respectively specify: the intersection numbers of the THCY model, the critical values of t as a function of the charge ratio q for the BPS black hole attractors, along with the range of q for which the BPS moduli lie inside the Kähler cone, the expression for q as a rational polynomial function of t for the non-BPS black hole attractors, along with the range of q for which the non-BPS moduli lie within the Kähler cone, and finally the expression for the recombination factor. Again, in all such tables t takes its critical value.

Again, as for one-modulus CICY models, our findings for one-modulus THCY models show that *there are no multiple extremal BPS black holes* for a given value of (supporting electric charge ratio) q. Analogously, for extremal non-BPS attractors, for any given q within the specified range there is a *unique t* inside the Kähler cone; thus, all 37 one-modulus THCY models under consideration give *unique* non-BPS black hole attractors. Moreover, we should remark that all BPS and non-BPS solutions are *mutually exclusive*.

We have numerically evaluated the recombination factor for the non-BPS black hole attractors in the entire moduli space for all allowed values of q; we have found that the recombination factor is always greater than 1 for all the 37 one-modulus THCY models under consideration: therefore, all non-BPS black holes in the 37 one-modulus THCY models listed in tables 4–8 are *unstable*.

Since the Calabi-Yau threefolds with $h_{1,1} = 2$ constructed as hypersurfaces in toric varieties (THCYs) associated with the 36 reflexive four-dimensional polytopes with six rays, and their various triangulations, have been classified in a set of 48 models (cfr. appendix B of [18]), a natural question arises: what about the remaining 10 one-modulus THCY models, not reported in tables 4–8? Some of such unlisted models have either $c = d = 0$ or $a = b = 0$, and thus the uniqueness of their BPS and non-BPS black hole attractors has been discussed in sections 3.2.1 and 3.2.2, respectively. Apart from these, we have also not included a

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[16]Such a one-modulus THCY model corresponds to the polytope label $(3,1)^{2,74}_{-144}$, and the corresponding charge matrix is given by (3.55); note that this model has the same set of intersection numbers as in the models $(4,1)^{2,74}_{-144}$ and $(4,2)^{2,74}_{-144}$, i.e., the 3rd and 4th models of table 4.

few models for which the triple intersection numbers are identical to some of the models already discussed here; they are related to each other by a flop (for more detail, see e.g. the discussion in [18]).

Some remarks are in order:

- The model $(2,1)^{2,38}_{-72}$, i.e. the second model of table 4 is marked red, because for such a model one of the two eigenvalues of G_{IJ} is always negative in the ranges of q supporting either the BPS or the non-BPS black hole attractors. Thus, this model does not give rise to physically consistent black hole attractors.

- There are models (like the models $(1,1)^{2,29}_{-54}$ and $(5,1)^{2,83}_{-162}$, i.e. the first and the fifth of table 4) with the same entries in the second column and intersection numbers related to one another by an overall rescaling; they have different polytope labels, $h_{2,1}$, χ and charge matrices. This can be explained by observing that the equations of motion (2.16) is invariant under the rescaling $C_{IJK} \to \alpha C_{IJN}, t^I \to \alpha^{-1/3}t^I$, $\alpha \in \mathbb{R}$; thus, the attractor moduli will differ by an overall factor, but the ratio $t = t^1/t^2$ will remain invariant. Thus, the solution listed in table 4 which relate the charge ratio with t remain the same for both the models. The recombination factor too remains the same upon an overall rescaling of the intersection numbers.

- There are models (like the models $(4,1)^{2,74}_{-144}$ and $(4,2)^{2,74}_{-144}$, i.e. the third and the fourth of table 4) which share the intersection numbers of the model explicitly treated in section 3.2.4, labelled as $(3,1)^{2,74}_{-144}$, but with different charge matrices. Such models share the same entries in the second column as well as the same intersection numbers and polytope labels; they seem to differ only for the charge matrices. This can be explained by noticing that the polytope corresponding to such models admits more than one triangulation, such that different triangulations correspond to different charge matrices. However all such models are related to one another by a flop.

- There are models (like the models $(20,1)^{2,106}_{-208}$ and $(21,1)^{2,106}_{-208}$, i.e. the first and the second of table 6, or like the models $(25,1)^{2,122}_{-240}$ and $(26,1)^{2,122}_{-240}$, i.e. the eighth of table 6 and the first of table 7) which share the same entries in the second column and the same intersection numbers; they seem to differ only for the charge matrices and the polytope labels. This is somewhat a surprising result, because the corresponding two-dimensional ambient spaces are not related by any symmetry. As far as we know, it looks like a mere coincidence that such pairs exist.

- There are models (like the models $(26,1)^{2,122}_{-240}$ and $(27,1)^{2,122}_{-240}$, i.e. the 1first and the second of table 7) with the same $h_{2,1}$, χ, and intersecting numbers related by $(a,b,c,d) \leftrightarrow (d,c,b,a)$. They have different entries in the second column and different charge matrices, as well. These models correspond to different polytopes; though the Hodge numbers coincide, they give rise to different Calabi-Yau threefolds. As far as we know, there is not any deeper reason why the intersection numbers are related by $(a,b,c,d) \leftrightarrow (d,c,b,a)$; e.g., no explanation has been given for such pairs in [18].

Polytope Label, Charge Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	BPS Solution	non-BPS Solution
		Range of validity	Range of validity
		Recombination Factor	
$(1,1)^{2,29}_{-54}$ $\begin{pmatrix} 1\ 0\ 0\ 0\ 1\ 1 \\ 0\ 1\ 1\ 1\ 0\ 0 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 1\ 0 \\ 1\ 0 \end{pmatrix}$	$\frac{\sqrt{q^2-q+1}-q+1}{q}$	$-\frac{4t^4+8t^3+7t^2+3t+2}{2t^5+3t^4+7t^3+8t^2+4t}$
		$q>0$	$q<0$
		$\frac{3\sqrt{(t+1)\left(4t^7+16t^6+33t^5+43t^4+43t^3+33t^2+16t+4\right)}}{6t^4+11t^3+14t^2+11t+6}$	
$(2,1)^{2,38}_{-72}$ $\begin{pmatrix} 0\ 0\ 1\ 1\ 0\ \ 1 \\ 1\ 1\ 0\ 0\ 1\ -3 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 1\ 9 \\ 3\ 9 \end{pmatrix}$	$\frac{3-q-\sqrt{26q(3-q)}}{3(q-3)}$	$\frac{(3t+1)^2\left(9t^3+9t^2+3t+61\right)}{27t^5+45t^4-516t^3-354t^2-59t-75}$
		$\frac{1}{9}<q<3$	$q<-\frac{61}{75}$ & $q>3$
		$\frac{3\left(3t^3+3t^2+t+3\right)\sqrt{81t^6+162t^5+135t^4-2592t^3-2637t^2-882t-23}}{\left(-27t^5-45t^4+516t^3+354t^2+59t+75\right)(1+|q|t)}$	
$(4,1)^{2,74}_{-144}$ $\begin{pmatrix} -1\ \ 2\ \ 1\ 1\ \ 3\ \ 0 \\ \ 1\ -1\ 0\ 0\ -1\ 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3\ 3 \\ 3\ 2 \end{pmatrix}$	$\frac{3(1-q)+\sqrt{3(1-q)}}{3q-2}$	$-\frac{8t^5+66t^4+132t^3+111t^2+45t+9}{3(t+1)^2\left(4t^3-9t^2-9t-3\right)}$
		$\frac{2}{3}<q<1$	$q<-\frac{2}{3}$ & $q>1$
		$\frac{2^{1/6}\left(2t^3+9t^2+9t+3\right)^{5/6}\sqrt{112t^6+360t^5+441t^4+282t^3+135t^2+54t+9}}{3^{5/6}(t+1)^2\left(4t^3-9t^2-9t-3\right)(|q|t+1)}$	
$(4,2)^{2,74}_{-144}$ $\begin{pmatrix} \ 2\ -1\ 1\ 1\ 0\ \ 3 \\ -1\ \ 1\ \ 0\ 0\ 1\ -1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3\ 3 \\ 3\ 2 \end{pmatrix}$	$\frac{3(1-q)+\sqrt{3(1-q)}}{3q-2}$	$-\frac{8t^5+66t^4+132t^3+111t^2+45t+9}{3(t+1)^2\left(4t^3-9t^2-9t-3\right)}$
		$\frac{2}{3}<q<1$	$q<-\frac{2}{3}$ & $q>1$
		$\frac{2^{1/6}\left(2t^3+9t^2+9t+3\right)^{5/6}\sqrt{112t^6+360t^5+441t^4+282t^3+135t^2+54t+9}}{3^{5/6}(t+1)^2\left(4t^3-9t^2-9t-3\right)(|q|t+1)}$	
$(5,1)^{2,83}_{-162}$ $\begin{pmatrix} 1\ 0\ 1\ 0\ 0\ 1 \\ 0\ 1\ 0\ 1\ 1\ 0 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1\ 0 \\ 1\ 0 \end{pmatrix}$	$\frac{\sqrt{q^2-q+1}-q+1}{q}$	$-\frac{4t^4+8t^3+7t^2+3t+2}{2t^5+3t^4+7t^3+8t^2+4t}$
		$q>0$	$q<0$
		$\frac{3\sqrt{(t+1)\left(4t^7+16t^6+33t^5+43t^4+43t^3+33t^2+16t+4\right)}}{6t^4+11t^3+14t^2+11t+6}$	
$(6,1)^{2,84}_{-164}$ $\begin{pmatrix} 1\ -1\ 1\ 1\ \ 2\ \ 0 \\ 0\ \ 1\ \ 0\ 0\ -1\ 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 5\ 5 \\ 5\ 3 \end{pmatrix}$	$\frac{5(1-q)+\sqrt{10(1-q)}}{5q-3}$	$-\frac{21t^5+195t^4+390t^3+320t^2+125t+25}{5(t+1)^2\left(9t^3-15t^2-15t-5\right)}$
		$\frac{3}{5}<q<1$	$q<-\frac{7}{15}$ & $q>1$
		$\frac{\left(3t^3+15t^2+15t+5\right)\sqrt{369t^6+1170t^5+1395t^4+840t^3+375t^2+150t+25}}{5(t+1)^2\left(9t^3-15t^2-15t-5\right)(1+|q|t)}$	
$(10,1)^{2,86}_{-168}$ $\begin{pmatrix} 1\ 0\ 0\ 1\ 0\ \ 1 \\ 0\ 1\ 1\ 0\ 1\ -1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3\ 0 \\ 5\ 5 \end{pmatrix}$	$\frac{5-3q+\sqrt{9q^2-15q+10}}{5(q-1)}$	$-\frac{125t^5+825t^4+1350t^3+1485t^2+1215t+486}{9t\left(75t^4+275t^3+465t^2+360t+108\right)}$
		$q>1$	$q<-\frac{5}{27}$
		$\frac{\left(5t^2+15t+9\right)\sqrt{3625t^6+15750t^5+33075t^4+41850t^3+32805t^2+14580t+2916}}{3\left(75t^4+275t^3+465t^2+360t+108\right)(1-qt)}$	
$(11,1)^{2,86}_{-168}$ $\begin{pmatrix} 1\ 0\ 0\ \ 1\ \ 1\ 0 \\ 0\ 1\ 1\ -1\ -1\ 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3\ \ 0 \\ 7\ 11 \end{pmatrix}$	$\frac{7-3q+\sqrt{9q^2-21q+16}}{7q-11}$	$\frac{77t^5-267t^4-1134t^3-2133t^2-1701t-486}{9t\left(233t^4+721t^3+897t^2+504t+108\right)}$
		$q>\frac{11}{7}$	$q<\frac{77}{2097}$
		$\frac{\left(11t^2+21t+9\right)\sqrt{14569t^6+57582t^5+102735t^4+103950t^3+61965t^2+20412t+2916}}{3\left(233t^4+721t^3+897t^2+504t+108\right)(1+|q|t)}$	

Table 4. BPS and non-BPS extremal black holes in one-modulus THCY models, 1/5.

Polytope Label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	BPS Solution	non-BPS Solution
		Range of validity	Range of validity
		Recombination Factor	
$(12,1)^{2,86}_{-168}$ $\begin{pmatrix} 1 & -1 & -1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 & -1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 10 & 14 \\ 6 & 3 \end{pmatrix}$	$\frac{6-10q+\sqrt{2(8q^2-9q+3)}}{6q-3}$	$-\frac{3(9t^5+102t^4+372t^3+642t^2+572t+228)}{2(45t^5+288t^4+843t^3+1308t^2+978t+238)}$
		$\frac{1}{2} < q < \frac{5}{7}$	$-\frac{171}{119} < q < -\frac{3}{10}$
		$\frac{(3t^3+18t^2+30t+14)\sqrt{3(603t^6+5292t^5+20160t^4+42780t^3+53460t^2+37272t+11308)}}{2(45t^5+288t^4+843t^3+1308t^2+978t+238)(1-qt)}$	
$(14,1)^{2,86}_{-168}$ $\begin{pmatrix} 1 & -1 & -1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 & -2 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 13 & 17 \\ 9 & 6 \end{pmatrix}$	$\frac{9-13q+\sqrt{16q^2-15q+3}}{9q-6}$	$\frac{3(72t^5+534t^4+1500t^3+1881t^2+895t+15)}{396t^5+3195t^4+10137t^3+15450t^2+10977t+2737}$
		$\frac{2}{3} < q < \frac{13}{17}$	$\frac{45}{2737} < q < \frac{6}{11}$
		$\frac{(6t^3+27t^2+39t+17)\sqrt{3(720t^6+8424t^5+42651t^4+117750t^3+184077t^2+152826t+52331)}}{(396t^5+3195t^4+10137t^3+15450t^2+10977t+2737)(1+qt)}$	
$(15,1)^{2,86}_{-168}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 & 1 & -1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3 & 1 \\ 3 & 3 \end{pmatrix}$	$\frac{3(1-q)+\sqrt{6q(q-1)}}{3(q-1)}$	$\frac{3(t+1)^2(3t^3+9t^2+9t-11)}{9t^5+45t^4+132t^3+174t^2+87t+5}$
		$1 < q < 3$	$-\frac{33}{5} < q < 1$
		$\frac{(3t^3+9t^2+9t+1)\sqrt{9t^6+54t^5+135t^4+384t^3+747t^2+666t+217}}{(9t^5+45t^4+132t^3+174t^2+87t+5)(1+|q|t)}$	
$(16,1)^{2,90}_{-176}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & -1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 5 & 2 \\ 5 & 5 \end{pmatrix}$	$\frac{5(1-q)+\sqrt{15q(q-1)}}{5(q-1)}$	$\frac{5(t+1)^2(5t^3+15t^2+15t-16)}{25t^5+125t^4+355t^3+460t^2+230t+16}$
		$1 < q < \frac{5}{2}$	$-5 < q < 1$
		$\frac{(5t^3+15t^2+15t+2)\sqrt{25t^6+150t^5+375t^4+1010t^3+1905t^2+1680t+544}}{(25t^5+125t^4+355t^3+460t^2+230t+16)(1+|q|t)}$	
$(17,1)^{2,92}_{-180}$ $\begin{pmatrix} 1 & -1 & -1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 4 & 7 \\ 2 & 1 \end{pmatrix}$	$\frac{2-4q+\sqrt{q(2q-1)}}{2q-1}$	$\frac{(t+2)^2(t^3+6t^2+12t+1)}{2t^5+20t^4+87t^3+188t^2+188t+63}$
		$\frac{1}{2} < q < \frac{4}{7}$	$\frac{4}{63} < q < \frac{1}{2}$
		$\frac{(t^3+6t^2+12t+7)\sqrt{t^6+12t^5+60t^4+194t^3+444t^2+600t+337}}{(2t^5+20t^4+87t^3+188t^2+188t+63)(1+qt)}$	
$(17,2)^{2,92}_{-180}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 1 & 1 & -2 & 0 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix}$	$\frac{3-q+\sqrt{q^2-3q+2}}{3q-7}$	$\frac{63t^5+127t^4+66t^3-21t^2-27t-6}{t(59t^4+159t^3+161t^2+72t+12)}$
		$q > \frac{7}{3}$	$q < \frac{63}{59}$
		$\frac{(7t^2+9t+3)\sqrt{337t^6+1422t^5+2499t^4+2322t^3+1197t^2+324t+36}}{(59t^4+159t^3+161t^2+72t+12)(1+|q|t)}$	
$(18,1)^{2,95}_{-186}$ $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -2 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3 & 0 \\ 7 & 14 \end{pmatrix}$	$\frac{7-3q+\sqrt{9q^2-21q+7}}{7(q-2)}$	$\frac{2744t^5+6762t^4+5292t^3-189t^2-1701t-486}{9t(196t^4+637t^3+861t^2+504t+108)}$
		$q > 2$	$q < \frac{14}{9}$
		$\frac{(14t^2+21t+9)\sqrt{5488t^6+24696t^5+56889t^4+76734t^3+56133t^2+20412t+2916}}{3(196t^4+637t^3+861t^2+504t+108)(1+|q|t)}$	
$(19,1)^{2,102}_{-200}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & -2 & 1 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 3 & 1 \\ 6 & 12 \end{pmatrix}$	$\frac{3(2-q)+\sqrt{3q(q-2)}}{6(q-2)}$	$\frac{3(2t+1)^2(12t^3+18t^2+9t-2)}{72t^5+180t^4+222t^3+132t^2+33t+2}$
		$2 < q < 3$	$-3 < q < 2$
		$\frac{(12t^3+18t^2+9t+1)\sqrt{144t^6+432t^5+540t^4+564t^3+441t^2+180t+28}}{(72t^5+180t^4+222t^3+132t^2+33t+2)(1+|q|t)}$	

Table 5. BPS and non-BPS extremal black holes in one-modulus THCY models, 2/5.

Polytope Label, Charge Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	BPS Solution	non-BPS Solution
		Range of validity	Range of validity
		Recombination Factor		
$(20,1)^{2,106}_{-208}$ $\begin{pmatrix} 1\ 1\ 1\ 0\ -3\ 8 \\ 0\ 0\ 0\ 1\ 1\ -2 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 12\ 36 \\ 4\ 1 \end{pmatrix}$	$\frac{2\left(2-6q+\sqrt{1-3q}\right)}{4q-1}$	$-\frac{5t^5+108t^4+648t^3+1692t^2+2160t+1296}{12(t+3)^2\left(t^3-12t^2-36t-36\right)}$
		$\frac{1}{4}<q<\frac{1}{3}$	$q<-\frac{5}{12}$ & $q>\frac{1}{3}$
		$\frac{\left(t^3+12t^2+36t+36\right)\sqrt{17t^6+168t^5+648t^4+1368t^3+2160t^2+2592t+1296}}{4(t+3)^2\left(t^3-12t^2-36t-36\right)(1+|q|t)}$		
$(21,1)^{2,106}_{-208}$ $\begin{pmatrix} 1\ -3\ 1\ 1\ 5\ 0 \\ 0\ 1\ 0\ 0\ -1\ 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 12\ 36 \\ 4\ 1 \end{pmatrix}$	$\frac{2\left(2-6q+\sqrt{1-3q}\right)}{4q-1}$	$-\frac{5t^5+108t^4+648t^3+1692t^2+2160t+1296}{12(t+3)^2\left(t^3-12t^2-36t-36\right)}$
		$\frac{1}{4}<q<\frac{1}{3}$	$q<-\frac{5}{12}$ & $q>\frac{1}{3}$
		$\frac{\left(t^3+12t^2+36t+36\right)\sqrt{17t^6+168t^5+648t^4+1368t^3+2160t^2+2592t+1296}}{4(t+3)^2\left(t^3-12t^2-36t-36\right)(1+|q|t)}$		
$(22,1)^{2,106}_{-208}$ $\begin{pmatrix} 1\ 0\ 0\ 0\ 1\ 0 \\ 0\ 1\ 1\ 2\ -3\ 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 4\ 1 \\ 12\ 36 \end{pmatrix}$	$\frac{2(3-q)+\sqrt{q(q-3)}}{6(q-3)}$	$\frac{12(3t+1)^2\left(36t^3+36t^2+12t-1\right)}{1296t^5+2160t^4+1692t^3+648t^2+108t+5}$
		$3<q<4$	$-\frac{12}{5}<q<3$
		$\frac{3\left(36t^3+36t^2+12t+1\right)\sqrt{1296t^6+2592t^5+2160t^4+1368t^3+648t^2+168t+17}}{\left(1296t^5+2160t^4+1692t^3+648t^2+108t+5\right)(1+|q|t)}$		
$(23,1)^{2,116}_{-228}$ $\begin{pmatrix} -2\ 3\ -2\ -4\ 5\ 0 \\ 1\ 0\ 1\ 2\ -1\ 3 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 25\ 98 \\ 5\ 1 \end{pmatrix}$	$\frac{5(1-5q)+3\sqrt{3q(5q-1)}}{5q-1}$	$\frac{(t+5)^2\left(t^3+15t^2+75t-64\right)}{5t^5+125t^4+1439t^3+8140t^2+20350t+14896}$
		$\frac{1}{5}<q<\frac{25}{98}$	$-\frac{100}{931}<q<\frac{1}{5}$
		$\frac{\left(t^3+15t^2+75t+98\right)\sqrt{t^6+30t^5+375t^4+3418t^3+23145t^2+87600t+131104}}{\left(5t^5+125t^4+1439t^3+8140t^2+20350t+14896\right)(1+|q|t)}$		
$(23,2)^{2,116}_{-228}$ $\begin{pmatrix} 0\ 1\ 0\ 0\ 1\ 2 \\ 2\ -3\ 2\ 4\ -5\ 0 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1\ 0 \\ 8\ 49 \end{pmatrix}$	$\frac{8-q+\sqrt{q^2-8q+15}}{8q-49}$	$\frac{3724t^5+2815t^4+520t^3-84t^2-36t-3}{2t\left(754t^4+760t^3+287t^2+48t+3\right)}$
		$q>\frac{49}{8}$	$q<\frac{931}{377}$
		$\frac{\left(49t^2+24t+3\right)\sqrt{32776t^6+50952t^5+32865t^4+11232t^3+2142t^2+216t+9}}{2\left(754t^4+760t^3+287t^2+48t+3\right)(1+|q|t)}$		
$(24,1)^{2,120}_{-236}$ $\begin{pmatrix} 1\ -1\ 1\ 2\ 0\ 3 \\ -1\ 4\ -1\ -5\ 3\ 0 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 8\ 2 \\ 32\ 101 \end{pmatrix}$	$\frac{8(4-q)+3\sqrt{6(4-q)}}{32q-101}$	$-\frac{15655t^5+26528t^4+13264t^3+2938t^2+320t+16}{2(4t+1)^2\left(61t^3-96t^2-24t-2\right)}$
		$\frac{101}{32}<q<4$	$q<-\frac{15655}{1952}$ & $q>4$
		$\frac{\left(101t^3+96t^2+24t+2\right)\sqrt{134617t^6+112704t^5+37392t^4+6956t^3+960t^2+96t+4}}{2(4t+1)^2\left(61t^3-96t^2-24t-2\right)(1+|q|t)}$		
$(24,2)^{2,120}_{-236}$ $\begin{pmatrix} 1\ -4\ 1\ 5\ -3\ 0 \\ 0\ 1\ 0\ -1\ 1\ 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 23\ 101 \\ 5\ 1 \end{pmatrix}$	$\frac{5-23q+\sqrt{24q^2-14q+2}}{5q-1}$	$\frac{3t^5+79t^4+746t^3+3312t^2+7067t+5869}{-t^5+79t^4+1268t^3+7078t^2+17249t+15655}$
		$\frac{1}{5}<q<\frac{23}{101}$	$q<-3$ & $q>\frac{5869}{15655}$
		$\frac{\left(t^3+15t^2+69t+101\right)\sqrt{73t^6+1542t^5+13539t^4+63256t^3+165903t^2+231666t+134617}}{\left(t^5-79t^4-1268t^3-7078t^2-17249t-15655\right)(1+|q|t)}$		
$(25,1)^{2,122}_{-240}$ $\begin{pmatrix} -3\ 1\ 1\ 1\ 0\ 7 \\ 1\ 0\ 0\ 0\ 1\ -2 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 21\ 63 \\ 7\ 2 \end{pmatrix}$	$\frac{7(1-3q)+\sqrt{7(1-3q)}}{7q-2}$	$\frac{16t^5+294t^4+1764t^3+4851t^2+6615t+3969}{441(t+3)^2\left(t^2+3t+3\right)}$
		$\frac{2}{7}<q<\frac{1}{3}$	$q>\frac{1}{3}$
		$\frac{\left(2t^3+21t^2+63t+63\right)\sqrt{32t^6+336t^5+1449t^4+3654t^3+6615t^2+7938t+3969}}{147(t+3)^2\left(t^2+3t+3\right)(1+qt)}$		

Table 6. BPS and non-BPS extremal black holes in one-modulus THCY models, 3/5.

Polytope Label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	BPS Solution	non-BPS Solution
		Range of validity	Range of validity
		Recombination Factor	
$(26,1)^{2,122}_{-240}$ $\begin{pmatrix} 1 & -3 & 1 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 21 & 63 \\ 7 & 2 \end{pmatrix}$	$\frac{7(1-3q)+\sqrt{7(1-3q)}}{7q-2}$	$\frac{16t^5+294t^4+1764t^3+4851t^2+6615t+3969}{441(t+3)^2(t^2+3t+3)}$
		$\frac{2}{7} < q < \frac{1}{3}$	$q > \frac{1}{3}$
		$\frac{(2t^3+21t^2+63t+63)\sqrt{32t^6+336t^5+1449t^4+3654t^3+6615t^2+7938t+3969}}{147(t+3)^2(t^2+3t+3)(1+qt)}$	
$(27,1)^{2,122}_{-240}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & -3 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 7 & 2 \\ 21 & 63 \end{pmatrix}$	$\frac{7(3-q)+\sqrt{7q(q-3)}}{21(q-3)}$	$\frac{441t(3t+1)^2(3t^2+3t+1)}{3969t^5+6615t^4+4851t^3+1764t^2+294t+16}$
		$3 < q < \frac{7}{2}$	$0 < q < 3$
		$\frac{3(63t^3+63t^2+21t+2)\sqrt{3969t^6+7938t^5+6615t^4+3654t^3+1449t^2+336t+32}}{(3969t^5+6615t^4+4851t^3+1764t^2+294t+16)(1+qt)}$	
$(30,2)^{2,128}_{-252}$ $\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 2 \\ 2 & -3 & 2 & 2 & -3 & 0 \end{pmatrix}$	$\frac{2}{3}\begin{pmatrix} 1 & 0 \\ 6 & 27 \end{pmatrix}$	$\frac{1}{2q-9}$	$\frac{3t-1}{2t}$
		$q > \frac{9}{2}$	$q < \frac{3}{2}$
		$\frac{3(3-q)}{3-2q+|q|}$	
$(31,1)^{2,128}_{-252}$ $\begin{pmatrix} 1 & -1 & -1 & 2 & 0 & 1 \\ -2 & 3 & 3 & -5 & 1 & 0 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 16 & 6 \\ 42 & 109 \end{pmatrix}$	$\frac{42-16q+\sqrt{4q^2-18q+20}}{42q-109}$	$\frac{15369t^5+29210t^4+21840t^3+7986t^2+1416t+96}{7522t^5+14700t^4+11374t^3+4344t^2+816t+60}$
		$\frac{109}{42} < q < \frac{8}{3}$	$\frac{8}{5} < q < \frac{15369}{7522}$
		$\frac{(109t^3+126t^2+48t+6)\sqrt{83881t^6+221868t^5+244500t^4+143652t^3+47448t^2+8352t+612}}{2(3761t^5+7350t^4+5687t^3+2172t^2+408t+30)(1+qt)}$	
$(31,2)^{2,128}_{-252}$ $\begin{pmatrix} 2 & -3 & -3 & 5 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 2 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 25 & 109 \\ 5 & 1 \end{pmatrix}$	$\frac{5(1-5q)+4\sqrt{q(5q-1)}}{5q-1}$	$\frac{(t+5)^2(t^3+15t^2+75t+13)}{5t^5+125t^4+1362t^3+7370t^2+18425t+15369}$
		$\frac{1}{5} < q < \frac{25}{109}$	$\frac{325}{15369} < q < \frac{1}{5}$
		$\frac{(t^3+15t^2+75t+109)\sqrt{t^6+30t^5+375t^4+3044t^3+17535t^2+59550t+83881}}{(5t^5+125t^4+1362t^3+7370t^2+18425t+15369)(1+qt)}$	
$(32,1)^{2,128}_{-252}$ $\begin{pmatrix} 1 & -3 & -3 & 1 & 4 & 0 \\ 0 & 1 & 1 & 0 & -1 & 1 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 15 & 54 \\ 4 & 1 \end{pmatrix}$	$\frac{5-18q}{4q-1}$	$\frac{t+9}{18}$
		$\frac{1}{4} < q < \frac{5}{18}$	$q > \frac{1}{2}$
		3	
$(33,1)^{2,132}_{-260}$ $\begin{pmatrix} 1 & -1 & -1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 2 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 4 & 7 \\ 2 & 1 \end{pmatrix}$	$\frac{2(1-2q)+\sqrt{q(2q-1)}}{2q-1}$	$\frac{(t+2)^2(t^3+6t^2+12t+1)}{2t^5+20t^4+87t^3+188t^2+188t+63}$
		$\frac{1}{2} < q < \frac{4}{7}$	$\frac{4}{63} < q < \frac{1}{2}$
		$\frac{(t^3+6t^2+12t+7)\sqrt{t^6+12t^5+60t^4+194t^3+444t^2+600t+337}}{(2t^5+20t^4+87t^3+188t^2+188t+63)(1+qt)}$	
$(33,2)^{2,132}_{-260}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 2 \\ -1 & 1 & 1 & 1 & -2 & 0 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix}$	$\frac{3-q+\sqrt{q^2-3q+2}}{3q-7}$	$\frac{63t^5+127t^4+66t^3-21t^2-27t-6}{t(59t^4+159t^3+161t^2+72t+12)}$
		$q > \frac{7}{3}$	$q < \frac{63}{59}$
		$\frac{(7t^2+9t+3)\sqrt{337t^6+1422t^5+2499t^4+2322t^3+1197t^2+324t+36}}{(59t^4+159t^3+161t^2+72t+12)(1+|q|t)}$	

Table 7. BPS and non-BPS extremal black holes in one-modulus THCY models, 4/5.

Polytope Label, Charge Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	BPS Solution	non-BPS Solution
		Range of validity	Range of validity
		Recombination Factor	
$(34,1)^{2,132}_{-260}$ $\begin{pmatrix} 1 & -2 & -2 & -4 & 7 & 0 \\ 0 & 1 & 1 & 2 & -3 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 49 & 144 \\ 21 & 9 \end{pmatrix}$	$\frac{7(3-7q)+\sqrt{q(7q-3)}}{3(7q-3)}$	$\frac{(3t+7)^2\left(9t^3+63t^2+147t+112\right)}{189t^5+2205t^4+10311t^3+24108t^2+28126t+13072}$
		$\frac{3}{7}<q<\frac{49}{114}$	$\frac{343}{817}<q<\frac{3}{7}$
		$\frac{3\left(3t^3+21t^2+49t+38\right)\sqrt{81t^6+1134t^5+6615t^4+20682t^3+36729t^2+35280t+14368}}{\left(189t^5+2205t^4+10311t^3+24108t^2+28126t+13072\right)(1+qt)}$	
$(34,2)^{2,132}_{-260}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 2 \\ -1 & 2 & 2 & 4 & -7 & 0 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1 & 0 \\ 8 & 57 \end{pmatrix}$	$\frac{8-q+\sqrt{q^2-8q+7}}{8q-57}$	$\frac{3268t^5+2277t^4+536t^3+4t^2-12t-1}{2t\left(262t^4+232t^3+93t^2+16t+1\right)}$
		$q>\frac{57}{8}$	$q<\frac{817}{131}$
		$\frac{3\left(19t^2+8t+1\right)\sqrt{3592t^6+3912t^5+2409t^4+992t^3+222t^2+24t+1}}{\left(524t^4+464t^3+186t^2+32t+2\right)(1+|q|t)}$	
$(35,1)^{2,144}_{-284}$ $\begin{pmatrix} 1 & -2 & -2 & -2 & 5 & 0 \\ 0 & 1 & 1 & 1 & -2 & 1 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 25 & 62 \\ 10 & 4 \end{pmatrix}$	$\frac{5(2-5q)+\sqrt{q(5q-2)}}{2(5q-2)}$	$\frac{(2t+5)^2\left(4t^3+30t^2+75t+59\right)}{40t^5+500t^4+2514t^3+6320t^2+7900t+3906}$
		$\frac{2}{5}<q<\frac{25}{62}$	$\frac{1475}{3906}<q<\frac{2}{5}$
		$\frac{\left(4t^3+30t^2+75t+62\right)\sqrt{16t^6+240t^5+1500t^4+5068t^3+9885t^2+10650t+4969}}{\left(40t^5+500t^4+2514t^3+6320t^2+7900t+3906\right)(1+qt)}$	
$(35,2)^{2,144}_{-284}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 2 \\ -1 & 2 & 2 & 2 & -5 & 0 \end{pmatrix}$	$\frac{2}{3}\begin{pmatrix} 1 & 0 \\ 6 & 31 \end{pmatrix}$	$\frac{6-q+\sqrt{q^2-6q+5}}{6q-31}$	$-\frac{-1953t^5-1865t^4-570t^3+6t^2+27t+3}{478t^5+600t^4+316t^3+72t^2+6t}$
		$q>\frac{31}{6}$	$q<\frac{1953}{478}$
		$\frac{\left(31t^2+18t+3\right)\sqrt{4969t^6+8514t^5+7575t^4+3996t^3+1143t^2+162t+9}}{\left(478t^4+600t^3+316t^2+72t+6\right)(1+|q|t)}$	
$(36,1)^{2,272}_{-540}$ $\begin{pmatrix} 0 & 0 & 0 & 2 & 3 & 1 \\ 1 & 1 & 1 & 0 & 0 & -3 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 1 & 0 \\ 3 & 9 \end{pmatrix}$	$\frac{3-q+\sqrt{(q-3)q}}{3(q-3)}$	$\frac{(3t+1)^2\left(9t^3+9t^2+3t-2\right)}{t\left(27t^4+45t^3+51t^2+24t+4\right)}$
		$q>3$	$q<3$
		$\frac{3\left(3t^2+3t+1\right)\sqrt{81t^6+162t^5+135t^4+162t^3+117t^2+36t+4}}{\left(27t^4+45t^3+51t^2+24t+4\right)(1+|q|t)}$	

Table 8. BPS and non-BPS extremal black holes in one-modulus THCY models, 5/5.

E CICY black strings

In this appendix we will report the results of the study of extremal black string attractors in one-modulus complete intersection Calabi-Yau (CICY) models. In tables 9–11, we report the extremal black string attractor solutions for 20 one-modulus CICY models. In particular, again, the first model of table 9 is the one explicitly treated in section 5.1.3. The first column of tables 9–11 indicates the CICY label as well as its configuration matrix, as from [18]. Again, the superscripts in the configuration matrix are the Hodge numbers $h_{1,1} = 2$ and $h_{2,1}$, respectively, whereas the subscript is the Euler number χ of the CICY model. On the other hand, as indicated in the first row of tables 9–11, for black strings only the non-BPS attractors need to be reported (the BPS ones are always *unique*): so, the second column reports the intersection numbers of the CICY model, the expression for p as

a rational polynomial function of t for the non-BPS black string attractors, along with the range of $p = p^1/p^2$ for which the non-BPS moduli lie within the Kähler cone, and finally the expression for the recombination factor. Note that in all such tables t takes its critical value.

Our findings show that *there are no multiple extremal non-BPS black strings* for a given value of (supporting magnetic charge ratio) p.

We have numerically evaluated the recombination factor for the non-BPS black string attractors in the entire moduli space for all allowed values of p; we have found that the recombination factor is always greater than 1 for all the 20 one-modulus CICY models under consideration: therefore, all non-BPS black strings in the 20 one-modulus CICY models listed in tables 9–11 are *stable* (for all the allowed values of p) against decay into their constituent BPS/anti-BPS black string pairs.

Since the one-modulus CICY models have been classified in a set of 36 models (cfr. e.g. [18]), a natural question arises: what about the remaining 16 one-modulus CICY models, not reported in tables 9–11? Again, all such unlisted models have either $c = d = 0$ or $a = b = 0$, and thus the uniqueness of their non-BPS black string attractors has been discussed in sections 5.1.1 and 5.1.2, respectively. Moreover, in such models the recombination factor of non-BPS black holes can be computed exactly. For $c = d = 0$, we find

$$R = 3\left(\theta(-p)\frac{3b + ap}{9b + ap} + \theta(p)\right). \tag{E.1}$$

Similarly, for $a = b = 0$, we find

$$R = 3\left(\theta(-p)\frac{d + 3cp}{d + 9cp} + \theta(p)\right). \tag{E.2}$$

Using the Kähler cone condition, we can see that $R < 1$ for all allowed values of p, and hence all these non-BPS string attractors are stable. Moreover we have numerically analysed the recombination factor for all the models listed in tables 9–11 and we found that the non-BPS attractors are stable for all the allowed values of p.

Let us observe that:

- As already noticed in remark 1 in appendix C, the first and the third model of table 9 have the matrices of intersection numbers reciprocally proportional, and they also share the same entries in all partitions of the second column, such as the same expressions of p as a rational polynomial function of t for non-BPS string attractors. However, the fact that their intersection numbers are different (notwithstanding being proportional) implies that the cubic constraint defining the 5D scalar manifold $\mathcal{M}$ (i.e. $C_{IJK}t^I t^J t^K = 1$) yields different expressions for the moduli, and the similarity of the various expressions observed here is a mere coincidence. Furthermore, such models also have different $h_{2,1}$ (and thus different χ), as well as different configuration matrices.

CICY label, Configuration Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	Non-BPS solution	Range of validity
		Recombination factor	
7643 $\begin{pmatrix} 0\ 0\ 2\ 1 \\ 2\ 2\ 1\ 1 \end{pmatrix}^{2,46}_{-88}$	$\frac{2}{3}\begin{pmatrix} 3\ 2 \\ 1\ 0 \end{pmatrix}$	$-\frac{6t^4+45t^3+93t^2+156t+56}{6t^3+27t^2+107t+120}$	$p < -\frac{7}{15}$
		$\frac{\sqrt{p^2(6t^2+18t+23)+2p(9t^2+8t+6)+3t^4+18t^3+54t^2+24t+4}}{-p(2t+3)+t^2+6t+2}$	
7644 $\begin{pmatrix} 2\ 0\ 1\ 1\ 1 \\ 0\ 2\ 1\ 1\ 1 \end{pmatrix}^{2,46}_{-88}$	$\frac{2}{3}\begin{pmatrix} 3\ 1 \\ 3\ 1 \end{pmatrix}$	$-\frac{25t^4+69t^3+63t^2+37t+6}{6t^4+37t^3+63t^2+69t+25}$	$-\frac{25}{6} < p < -\frac{6}{25}$
		$\frac{\sqrt{p^2(t^4+12t^3+54t^2+52t+21)+6p(t^4+4t^3+10t^2+4t+1)+21t^4+52t^3+54t^2+12t+1}}{p(t^2+6t+3)-3t^2-6t-1}$	
7668 $\begin{pmatrix} 0\ 2\ 1 \\ 3\ 1\ 1 \end{pmatrix}^{2,47}_{-90}$	$\frac{1}{2}\begin{pmatrix} 3\ 2 \\ 1\ 0 \end{pmatrix}$	$-\frac{6t^4+45t^3+93t^2+156t+56}{6t^3+27t^2+107t+120}$	$p < -\frac{7}{15}$
		$\frac{\sqrt{p^2(6t^2+18t+23)+2p(9t^2+8t+6)+3t^4+18t^3+54t^2+24t+4}}{-p(2t+3)+t^2+6t+2}$	
7725 $\begin{pmatrix} 0\ 0\ 1\ 1\ 1 \\ 2\ 2\ 1\ 1\ 1 \end{pmatrix}^{2,50}_{-96}$	$\frac{2}{3}\begin{pmatrix} 3\ 3 \\ 1\ 0 \end{pmatrix}$	$-\frac{2t^4+15t^3+33t^2+51t+24}{2t^3+9t^2+31t+33}$	$p < -\frac{8}{11}$
		$\frac{\sqrt{3}\sqrt{p^2(2t^2+6t+7)+p(6t^2+8t+6)+t^4+6t^3+18t^2+12t+3}}{-p(2t+3)+t^2+6t+3}$	
7726 $\begin{pmatrix} 0\ 1\ 1\ 1\ 1 \\ 2\ 1\ 1\ 1\ 1 \end{pmatrix}^{2,50}_{-96}$	$\frac{1}{3}\begin{pmatrix} 6\ 4 \\ 4\ 1 \end{pmatrix}$	$-\frac{11t^4+46t^3+66t^2+56t+16}{2t^4+17t^3+42t^2+62t+32}$	$-\frac{11}{2} < p < -\frac{1}{2}$
		$\frac{\sqrt{p^2(t^4+16t^3+96t^2+136t+76)+8p(t^4+6t^3+21t^2+16t+6)+4(9t^4+34t^3+54t^2+24t+4)}}{p(t^2+8t+6)-4(t^2+3t+1)}$	
7758 $\begin{pmatrix} 0\ 2\ 1 \\ 2\ 1\ 2 \end{pmatrix}^{2,52}_{-100}$	$\frac{1}{3}\begin{pmatrix} 5\ 2 \\ 2\ 0 \end{pmatrix}$	$-\frac{24t^4+150t^3+249t^2+365t+84}{24t^3+90t^2+319t+305}$	$p < -\frac{84}{305}$
		$\frac{\sqrt{p^2(24t^2+60t+67)+4p(15t^2+8t+5)+2(6t^4+30t^3+75t^2+20t+2)}}{p(4t+5)-2(t^2+5t+1)}$	
7759 $\begin{pmatrix} 0\ 2\ 1\ 1 \\ 2\ 1\ 1\ 1 \end{pmatrix}^{2,52}_{-100}$	$\frac{1}{3}\begin{pmatrix} 5\ 2 \\ 4\ 1 \end{pmatrix}$	$-\frac{123t^4+421t^3+471t^2+345t+68}{22t^4+177t^3+375t^2+521t+237}$	$-\frac{123}{22} < p < -\frac{68}{237}$
		$\frac{\sqrt{p^2(t^4+16t^3+96t^2+116t+59)+4p(2t^4+10t^3+33t^2+16t+5)+2(19t^4+58t^3+75t^2+20t+2)}}{p(t^2+8t+5)-2(2t^2+5t+1)}$	
7761 $\begin{pmatrix} 1\ 1\ 1\ 1\ 1 \\ 1\ 1\ 1\ 1\ 1 \end{pmatrix}^{2,52}_{-100}$	$\frac{5}{6}\begin{pmatrix} 2\ 1 \\ 2\ 1 \end{pmatrix}$	$-\frac{21t^4+62t^3+66t^2+39t+8}{8t^4+39t^3+66t^2+62t+21}$	$-\frac{21}{8} < p < -\frac{8}{21}$
		$\frac{\sqrt{p^2(t^4+8t^3+24t^2+22t+8)+2p(2t^4+8t^3+15t^2+8t+2)+8t^4+22t^3+24t^2+8t+1}}{p(t^2+4t+2)-2t^2-4t-1}$	

Table 9. Non-BPS extremal black strings in one-modulus CICY models, 1/3.

CICY label, Configuration Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	Non-BPS solution	Range of validity
		Recombination factor	
7799 $\begin{pmatrix} 2\ 1\ 1 \\ 1\ 2\ 1 \end{pmatrix}^{2,55}_{-106}$	$\frac{1}{6}\begin{pmatrix} 7\ 2 \\ 7\ 2 \end{pmatrix}$	$-\frac{276t^4+749t^3+651t^2+384t+56}{56t^4+384t^3+651t^2+749t+276}$	$-\frac{69}{14} < p < -\frac{14}{69}$
		$\frac{\sqrt{p^2\left(4t^4+56t^3+294t^2+286t+119\right)+2p\left(14t^4+56t^3+159t^2+56t+14\right)+119t^4+286t^3+294t^2+56t+4}}{p\left(2t^2+14t+7\right)-7t^2-14t-2}$	
7807 $\begin{pmatrix} 0\ 1\ 1\ 1 \\ 2\ 2\ 1\ 1 \end{pmatrix}^{2,56}_{-108}$	$\frac{1}{3}\begin{pmatrix} 5\ 4 \\ 2\ 0 \end{pmatrix}$	$-\frac{24t^4+150t^3+273t^2+355t+136}{24t^3+90t^2+263t+235}$	$p < -\frac{136}{235}$
		$\frac{\sqrt{p^2\left(24t^2+60t+59\right)+p\left(60t^2+64t+40\right)+2\left(6t^4+30t^3+75t^2+40t+8\right)}}{p(4t+5)-2\left(t^2+5t+2\right)}$	
7808 $\begin{pmatrix} 0\ 1\ 1\ 1 \\ 3\ 1\ 1\ 1 \end{pmatrix}^{2,56}_{-108}$	$\frac{1}{2}\begin{pmatrix} 3\ 3 \\ 1\ 0 \end{pmatrix}$	$-\frac{2t^4+15t^3+33t^2+51t+24}{2t^3+9t^2+31t+33}$	$p < -\frac{8}{11}$
		$\frac{\sqrt{3}\sqrt{p^2\left(2t^2+6t+7\right)+p\left(6t^2+8t+6\right)+t^4+6t^3+18t^2+12t+3}}{-p(2t+3)+t^2+6t+3}$	
7809 $\begin{pmatrix} 1\ 1\ 1\ 1 \\ 2\ 1\ 1\ 1 \end{pmatrix}^{2,56}_{-108}$	$\frac{1}{6}\begin{pmatrix} 9\ 5 \\ 7\ 2 \end{pmatrix}$	$-\frac{1196t^4+4277t^3+5223t^2+3809t+920}{248t^4+1832t^3+3891t^2+4985t+2219}$	$-\frac{299}{62} < p < -\frac{920}{2219}$
		$\frac{\sqrt{p^2\left(4t^4+56t^3+294t^2+358t+173\right)+2p\left(14t^4+72t^3+219t^2+140t+45\right)+111t^4+358t^3+486t^2+180t+25}}{p\left(2t^2+14t+9\right)-7t^2-18t-5}$	
7821 $\begin{pmatrix} 1\ 1\ 1 \\ 2\ 2\ 1 \end{pmatrix}^{2,58}_{-112}$	$\frac{1}{6}\begin{pmatrix} 8\ 5 \\ 4\ 0 \end{pmatrix}$	$-\frac{24t^4+120t^3+174t^2+182t+55}{24t^3+72t^2+170t+122}$	$p < -\frac{55}{122}$
		$\frac{\sqrt{8p^2\left(12t^2+24t+19\right)+16p\left(12t^2+10t+5\right)+48t^4+192t^3+384t^2+160t+25}}{-8p(t+1)+4t^2+16t+5}$	
7833 $\begin{pmatrix} 2\ 1 \\ 1\ 3 \end{pmatrix}^{2,59}_{-114}$	$\frac{1}{6}\begin{pmatrix} 7\ 2 \\ 3\ 0 \end{pmatrix}$	$-\frac{162t^4+945t^3+1431t^2+2016t+344}{9\left(18t^3+63t^2+217t+196\right)}$	$p < -\frac{86}{441}$
		$\frac{\sqrt{9p^2\left(6t^2+14t+15\right)+2p\left(63t^2+24t+14\right)+27t^4+126t^3+294t^2+56t+4}}{-p(6t+7)+3t^2+14t+2}$	
7844 $\begin{pmatrix} 2\ 1 \\ 2\ 2 \end{pmatrix}^{2,62}_{-120}$	$\frac{1}{3}\begin{pmatrix} 3\ 1 \\ 2\ 0 \end{pmatrix}$	$-\frac{24t^4+90t^3+93t^2+78t+14}{24t^3+54t^2+107t+60}$	$p < -\frac{7}{30}$
		$\frac{\sqrt{p^2\left(24t^2+36t+23\right)+2p\left(18t^2+8t+3\right)+12t^4+36t^3+54t^2+12t+1}}{-p(4t+3)+2t^2+6t+1}$	
7853 $\begin{pmatrix} 0\ 2\ 1 \\ 2\ 2\ 1 \end{pmatrix}^{2,64}_{-124}$	$\frac{2}{3}\begin{pmatrix} 2\ 1 \\ 1\ 0 \end{pmatrix}$	$-\frac{3t^4+15t^3+21t^2+23t+6}{3t^3+9t^2+23t+17}$	$p < -\frac{6}{17}$
		$\frac{\sqrt{2p^2\left(3t^2+6t+5\right)+4p\left(3t^2+2t+1\right)+3t^4+12t^3+24t^2+8t+1}}{-2p(t+1)+t^2+4t+1}$	

Table 10. Non-BPS extremal black strings in one-modulus CICY models, 2/3.

CICY label, Configuration Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	Non-BPS solution	Range of validity
		Recombination factor	
7863 $\begin{pmatrix} 2\ 1\ 1 \\ 2\ 1\ 1 \end{pmatrix}^{2,66}_{-128}$	$\frac{1}{3}\begin{pmatrix} 3\ 1 \\ 3\ 1 \end{pmatrix}$	$-\frac{25t^4+69t^3+63t^2+37t+6}{6t^4+37t^3+63t^2+69t+25}$	$-\frac{25}{6} < p < -\frac{6}{25}$
	$\frac{\sqrt{p^2(t^4+12t^3+54t^2+52t+21)+6p(t^4+4t^3+10t^2+4t+1)+21t^4+52t^3+54t^2+12t+1}}{p(t^2+6t+3)-3t^2-6t-1}$		
7868 $\begin{pmatrix} 1\ 1\ 1 \\ 3\ 1\ 1 \end{pmatrix}^{2,68}_{-132}$	$\frac{1}{6}\begin{pmatrix} 7\ 5 \\ 3\ 0 \end{pmatrix}$	$-\frac{162t^4+945t^3+1593t^2+1953t+680}{9(18t^3+63t^2+175t+147)}$	$p < -\frac{680}{1323}$
	$\frac{\sqrt{9p^2(6t^2+14t+13)+2p(63t^2+60t+35)+27t^4+126t^3+294t^2+140t+25}}{-p(6t+7)+3t^2+14t+5}$		
7883 $\begin{pmatrix} 2\ 1 \\ 3\ 1 \end{pmatrix}^{2,77}_{-150}$	$\frac{1}{6}\begin{pmatrix} 5\ 2 \\ 3\ 0 \end{pmatrix}$	$-\frac{162t^4+675t^3+783t^2+720t+152}{9(18t^3+45t^2+97t+60)}$	$p < -\frac{38}{135}$
	$\frac{\sqrt{9p^2(6t^2+10t+7)+p(90t^2+48t+20)+27t^4+90t^3+150t^2+40t+4}}{-p(6t+5)+3t^2+10t+2}$		
7884 $\begin{pmatrix} 3 \\ 3 \end{pmatrix}^{2,83}_{-162}$	$\frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$-\frac{t\left(2t^3+5t^2+3t+2\right)}{2t^3+3t^2+5t+2}$	$p < 0$
	$\frac{\sqrt{3}\sqrt{p^2(2t^2+2t+1)+2pt^2+t^2(t^2+2t+2)}}{2pt+p-t(t+2)}$		

Table 11. Non-BPS extremal black strings in one-modulus CICY models, 3/3.

F   THCY black strings

In this appendix we will report the most interesting results of our paper, namely the study of non-BPS extremal black string attractors in one-modulus toric hypersurface Calabi-Yau (THCY) models. Our findings show that *there are multiple extremal non-BPS black strings* in most of the one-modulus THCY models under consideration, and that they are also *stable*, *at least* in some subsector of the allowed range for the supporting magnetic charge ratio p. We should remark that the existence of *multiple* non-BPS black string attractors was not observed in the analysis of [11], and the fact that we also obtain that they can be stable adds interest and physical relevance to such a finding.

In tables 12–19, we report the non-BPS extremal black string attractor solutions for 37 one-modulus THCY models, and we do not include the model $(3,1)^{2,74}_{-144}$ already treated in detail in section 3.2.4. The first column of tables 12–19 indicates the polytope label as well as the charge matrix of the ambient toric variety, as from [18]. Again, the superscripts in the configuration matrix are the Hodge numbers $h_{1,1} = 2$ and $h_{2,1}$, respectively, whereas the subscript is the Euler number χ of the THCY model. On the other hand, in the various partitions of their second column we respectively specify: the triple intersection numbers, the non-BPS black string attractor solution, along with the constraints on p for the existence of the single solution, and of the *multiple* solution (if any), as well. Also, we report the analytical form of the recombination factor. As in previous tables, t takes the critical value in all these formulae. A numerical evaluation of the recombination factor allows us

Polytope label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of the non-BPS attractor		
$(1,1)^{2,29}_{-54}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	$-\frac{t\left(2t^3+5t^2+3t+2\right)}{2t^3+3t^2+5t+2}$	$p < 0$	NA
	$\dfrac{\sqrt{3}\sqrt{2pt^2+t^2\left(2+2t+t^2\right)+p^2\left(1+2t+2t^2\right)}}{t(2+t)+|\mathrm{p}|(1+2t)}$			
	All non-BPS attractors are stable			
$(2,1)^{2,38}_{-72}$ $\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & -3 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 1 & 9 \\ 3 & 9 \end{pmatrix}$	$\frac{9t^4-3t^3-9t^2-47t-12}{45t^3+45t^2+15t-7}$	$p < \frac{12}{7}$	$p > \frac{12}{7}$
	$\dfrac{\sqrt{3}\sqrt{27+12t+2t^2-48t^3+3t^4+6p\left(1+12t+28t^2+4t^3+3t^4\right)+p^2\left(-17-48t+18t^2+36t^3+27t^4\right)}}{9+2t+3t^2+|\mathrm{p}|(1+3t)^2}$			
$(4,1)^{2,74}_{-144}$ $\begin{pmatrix} -1 & 2 & 1 & 1 & 3 & 0 \\ 1 & -1 & 0 & 0 & -1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3 & 3 \\ 3 & 2 \end{pmatrix}$	$-\frac{t\left(16t^3+45t^2+45t+15\right)}{8t^4+28t^3+27t^2+3t-3}$	$p > 0$ & $p < -2$	$-2 < p < -1.78$
	$\dfrac{\sqrt{9+36t+54t^2+42t^3+15t^4+6p\left(3+12t+15t^2+8t^3+2t^4\right)+p^2\left(9+42t+54t^2+24t^3+4t^4\right)}}{3(1+t)^2+|\mathrm{p}|\left(3+6t+2t^2\right)}$			
	Stable solution for $-44.856 < p < -1.78$			
$(4,2)^{2,74}_{-144}$ $\begin{pmatrix} 2 & -1 & 1 & 1 & 0 & 3 \\ -1 & 1 & 0 & 0 & 1 & -1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3 & 3 \\ 3 & 2 \end{pmatrix}$	$-\frac{t\left(16t^3+45t^2+45t+15\right)}{8t^4+28t^3+27t^2+3t-3}$	$p > 0$ & $p < -2$	$-2 < p < -1.78$
	$\dfrac{\sqrt{9+36t+54t^2+42t^3+15t^4+6p\left(3+12t+15t^2+8t^3+2t^4\right)+p^2\left(9+42t+54t^2+24t^3+4t^4\right)}}{3(1+t)^2+|\mathrm{p}|\left(3+6t+2t^2\right)}$			
	Stable solution for $-44.856 < p < -1.78$			
$(5,1)^{2,83}_{-162}$ $\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	$-\frac{t\left(2t^3+5t^2+3t+2\right)}{2t^3+3t^2+5t+2}$	$p < 0$	NA
	$\dfrac{\sqrt{3}\sqrt{2pt^2+t^2\left(2+2t+t^2\right)+p^2\left(1+2t+2t^2\right)}}{t(2+t)+|\mathrm{p}|(1+2t)}$			
	All non-BPS attractors are stable			

Table 12. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 1/8.

to conclude that most of the non-BPS black string attractors, both in their single and *multiple* solution regimes, remain stable for certain range of p and then become unstable; in particular, the range of values of p which supports stable non-BPS black string attractors is specified in all models.

Again, since the Calabi-Yau threefolds with $h_{1,1} = 2$ constructed as hypersurfaces in toric varieties (THCYs) associated with the 36 reflexive four-dimensional polytopes with six rays, and their various triangulations, have been classified in a set of 48 models (cfr. appendix B of [18]), one might ask: what about the remaining 10 one-modulus THCY models, not reported in tables 12–19? Some of such unlisted models have either $c = d = 0$ or $a = b = 0$, and thus the uniqueness of their non-BPS black string attractors has been discussed in sections 5.1.1 and 5.1.2. Apart from these, we have again not included a few models for which the triple intersection numbers are identical to some of the models

Polytope label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of non-BPS attractors		
$(6,1)^{2,84}_{-164}$ $\begin{pmatrix} 1 & -1 & 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 5 & 5 \\ 5 & 3 \end{pmatrix}$	$-\frac{t\left(27t^3+75t^2+75t+25\right)}{12t^4+45t^3+45t^2+5t-5}$	$p>0\ \&\ p<-\frac{9}{4}$	$-\frac{9}{4}<p<-1.87$
		$\frac{\sqrt{10p\left(5+20t+24t^2+12t^3+3t^4\right)+5\left(5+20t+30t^2+24t^3+9t^4\right)+p^2\left(25+120t+150t^2+60t^3+9t^4\right)}}{5(1+t)^2+|\mathrm{p}|\left(5+10t+3t^2\right)}$		
		Stable solution for $-37.02<p<-1.87$		
$(10,1)^{2,86}_{-168}$ $\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & -1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3 & 0 \\ 5 & 5 \end{pmatrix}$	$-\frac{9t(25t^3+55t^2+45t+18)}{200t^4+675t^3+945t^2+675t+162}$	$-\frac{9}{8}<p<0$	NA
		$\frac{\sqrt{9t^2\left(6+10t+5t^2\right)+10pt^2\left(9+12t+5t^2\right)+p^2\left(27+90t+150t^2+100t^3+25t^4\right)}}{t(6+5t)+|\mathrm{p}|\left(3+10t+5t^2\right)}$		
		All non-BPS attractors are stable		
$(11,1)^{2,86}_{-168}$ $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & -1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 3 & 0 \\ 7 & 11 \end{pmatrix}$	$-\frac{9t(49t^3+91t^2+63t+18)}{704t^4+1827t^3+1917t^2+945t+162}$	$-\frac{441}{704}<p<0$	NA
		$\frac{\sqrt{9t^2\left(6+14t+9t^2\right)+2pt^2\left(63+132t+77t^2\right)+p^2\left(27+126t+294t^2+308t^3+121t^4\right)}}{t(6+7t)+|\mathrm{p}|\left(3+14t+11t^2\right)}$		
		All non-BPS attractors are stable		
$(12,1)^{2,86}_{-168}$ $\begin{pmatrix} 1 & -1 & -1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 & -1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 10 & 14 \\ 6 & 3 \end{pmatrix}$	$-\frac{81t^4+474t^3+1062t^2+1110t+448}{36t^4+243t^3+630t^2+762t+354}$	$-\frac{9}{4}<p<-\frac{224}{177}$	NA
		$\frac{\sqrt{3p^2\left(44+92t+72t^2+24t^3+3t^4\right)+4p\left(70+168t+153t^2+60t^3+9t^4\right)+4\left(49+140t+150t^2+69t^3+12t^4\right)}}{14+20t+6t^2+|\mathrm{p}|\left(10+12t+3t^2\right)}$		
		All non-BPS attractors are stable		
$(14,1)^{2,86}_{-168}$ $\begin{pmatrix} 1 & -1 & -1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 & -2 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 13 & 17 \\ 9 & 6 \end{pmatrix}$	$-\frac{72t^4+681t^3+2061t^2+2523t+1088}{72t^4+612t^3+1791t^2+2211t+993}$	$-\frac{1088}{993}<p<-1$	NA
		$\frac{\sqrt{289+884t+1014t^2+498t^3+87t^4+3p^2\left(67+166t+162t^2+72t^3+12t^4\right)+2p\left(221+612t+657t^2+312t^3+54t^4\right)}}{17+26t+9t^2+|\mathrm{p}|\left(13+18t+6t^2\right)}$		
		All non-BPS attractors are stable		

Table 13. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 2/8.

already discussed here; they are related to each other by a flop (for more detail, see e.g. the discussion in [18]).

Again, we remark that

- The second model of table 12 is marked red, because for such a model one of the two eigenvalues of G_{IJ} is always negative in the ranges of p supporting either the BPS or the non-BPS black string attractors. Thus, this model does not give rise to physically consistent black string attractors.

Polytope label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of non-BPS attractors		
$(15,1)^{2,86}_{-168}$	$\frac{1}{6}\begin{pmatrix} 3 & 1 \\ 3 & 3 \end{pmatrix}$	$\frac{3t^4-3t^3-27t^2-23t-4}{15t^3+45t^2+45t+17}$	$p > -\frac{4}{17}$	$-0.45 < p < -\frac{4}{17}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 & 1 & -1 \end{pmatrix}$		$\frac{\sqrt{1+12t+54t^2+48t^3+9t^4+6p\left(1+4t+12t^2+12t^3+3t^4\right)+3p^2\left(7+16t+18t^2+12t^3+3t^4\right)}}{1+6t+3t^2+3|\mathrm{p}|(1+t)^2}$		
		Stable attractors for $-0.45 < p < -0.05$		
$(16,1)^{2,90}_{-176}$	$\frac{1}{6}\begin{pmatrix} 5 & 2 \\ 5 & 5 \end{pmatrix}$	$\frac{5t^4-5t^3-45t^2-40t-8}{25t^3+75t^2+75t+28}$	$p > -\frac{2}{7}$	$-0.47 < p < -\frac{2}{7}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & -1 \end{pmatrix}$		$\frac{\sqrt{4+40t+150t^2+130t^3+25t^4+10p\left(2+8t+21t^2+20t^3+5t^4\right)+5p^2\left(11+26t+30t^2+20t^3+5t^4\right)}}{2+10t+5t^2+5|\mathrm{p}|(1+t)^2}$		
		Stable attractors for $-0.47 < p < -0.04$		
$(17,1)^{2,92}_{-180}$	$\frac{1}{2}\begin{pmatrix} 4 & 7 \\ 2 & 1 \end{pmatrix}$	$\frac{t^4-2t^3-36t^2-83t-56}{5t^3+30t^2+60t+41}$	$p > -\frac{56}{41}$	$-1.367 < p < -\frac{56}{41}$
$\begin{pmatrix} 1 & -1 & -1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 \end{pmatrix}$		$\frac{\sqrt{49+112t+96t^2+34t^3+4t^4+p^2\left(20+34t+24t^2+8t^3+t^4\right)+2p\left(28+56t+45t^2+16t^3+2t^4\right)}}{7+8t+2t^2+|\mathrm{p}|(2+t)^2}$		
		Stable attractors for $-1.367 < p < -0.016$		
$(17,2)^{2,92}_{-180}$	$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix}$	$-\frac{t(15t^3+37t^2+27t+6)}{56t^4+141t^3+123t^2+45t+6}$	$-\frac{15}{56} < p < 0$	NA
$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 1 & 1 & -2 & 0 \end{pmatrix}$		$\frac{\sqrt{t^2\left(6+18t+13t^2\right)+2pt^2\left(9+28t+21t^2\right)+p^2\left(3+18t+54t^2+84t^3+49t^4\right)}}{t(2+3t)+|\mathrm{p}|\left(1+6t+7t^2\right)}$		
		All non-BPS attractors are stable		
$(18,1)^{2,95}_{-186}$	$\frac{1}{6}\begin{pmatrix} 3 & 0 \\ 7 & 14 \end{pmatrix}$	$-\frac{9t(49t^2+63t+18)}{392t^4+1764t^3+2079t^2+945t+162}$	NA	$-0.22 < p < 0$
$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -2 & 1 \end{pmatrix}$		$\frac{\sqrt{9t^2\left(6+14t+7t^2\right)+14pt^2\left(9+24t+14t^2\right)+p^2\left(27+126t+294t^2+392t^3+196t^4\right)}}{t(6+7t)+|\mathrm{p}|\left(3+14t+14t^2\right)}$		
		All solutions for $-0.22 < p < -0.037$ and one branch of solutions with smaller t_c for the remaining allowed range $-0.037 < p < 0$ are stable		

Table 14. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 3/8.

Polytope label, Charge Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of non-BPS attractors		
$(19,1)^{2,102}_{-200}$	$\frac{1}{3}\begin{pmatrix} 3 & 1 \\ 6 & 12 \end{pmatrix}$	$\frac{12t^4-6t^3-27t^2-14t-2}{60t^3+90t^2+45t+8}$	$p>-\frac{1}{4}$	$-0.28<p<-\frac{1}{4}$
$\begin{pmatrix} 1\ 0\ 0\ 0\ \ 1\ \ 0 \\ 0\ 1\ 1\ 1\ -2\ 1 \end{pmatrix}$		$\frac{\sqrt{1+12t+54t^2+84t^3+36t^4+6p\left(1+8t+30t^2+48t^3+24t^4\right)+3p^2\left(5+28t+72t^2+96t^3+48t^4\right)}}{1+6t+6t^2+3|\text{p}|(1+2t)^2}$		
		All non-BPS attractors for $-0.28<p<-0.01$ are stable		
$(20,1)^{2,106}_{-208}$	$\frac{1}{6}\begin{pmatrix} 12 & 36 \\ 4 & 1 \end{pmatrix}$	$-\frac{3t(7t^3+60t^2+180t+180)}{4t^4+39t^3+108t^2+36t-108}$	$p>0\ \&\ p<-\frac{21}{4}$	$-\frac{21}{4}<p<-4.99$
$\begin{pmatrix} 1\ 1\ 1\ 0\ -3\ \ 8 \\ 0\ 0\ 0\ 1\ \ 1\ -2 \end{pmatrix}$		$\frac{\sqrt{24\left(54+72t+36t^2+9t^3+t^4\right)+8p\left(108+144t+63t^2+12t^3+t^4\right)+p^2\left(144+216t+96t^2+16t^3+t^4\right)}}{4(3+t)^2+|\text{p}|\left(12+8t+t^2\right)}$		
		All non-BPS attractors for $-181.52<p<-4.99$ are stable		
$(21,1)^{2,106}_{-208}$	$\frac{1}{6}\begin{pmatrix} 12 & 36 \\ 4 & 1 \end{pmatrix}$	$-\frac{3t(7t^3+60t^2+180t+180)}{4t^4+39t^3+108t^2+36t-108}$	$p>0\ \&\ p<-\frac{21}{4}$	$-\frac{21}{4}<p<-4.99$
$\begin{pmatrix} 1\ -3\ 1\ 1\ \ 5\ \ 0 \\ 0\ \ 1\ \ 0\ 0\ -1\ 1 \end{pmatrix}$		$\frac{\sqrt{24\left(54+72t+36t^2+9t^3+t^4\right)+8p\left(108+144t+63t^2+12t^3+t^4\right)+p^2\left(144+216t+96t^2+16t^3+t^4\right)}}{4(3+t)^2+|\text{p}|\left(12+8t+t^2\right)}$		
		All non-BPS attractors for $-181.52<p<-4.99$ are stable		
$(22,1)^{2,106}_{-208}$	$\frac{1}{6}\begin{pmatrix} 4 & 1 \\ 12 & 36 \end{pmatrix}$	$\frac{108t^4-36t^3-108t^2-39t-4}{540t^3+540t^2+180t+21}$	$p>-\frac{4}{21}$	$-0.2<p<-\frac{4}{21}$
$\begin{pmatrix} 1\ 0\ 0\ 0\ \ 1\ \ 0 \\ 0\ 1\ 1\ 2\ -3\ 1 \end{pmatrix}$		$\frac{\sqrt{1+16t+96t^2+216t^3+144t^4+24p^2\left(1+9t+36t^2+72t^3+54t^4\right)+8p\left(1+12t+63t^2+144t^3+108t^4\right)}}{1+8t+12t^2+4|\text{p}|(1+3t)^2}$		
		All non-BPS attractors for $-0.2<p<-0.005$ are stable		
$(23,1)^{2,116}_{-228}$	$\frac{1}{6}\begin{pmatrix} 25 & 98 \\ 5 & 1 \end{pmatrix}$	$\frac{t^4-5t^3-225t^2-1240t-1960}{5t^3+75t^2+375t+652}$	$p>-\frac{490}{163}$	$-3.1<p<-\frac{490}{163}$
$\begin{pmatrix} -2\ 3\ -2\ -4\ \ 5\ \ 0 \\ \ 1\ 0\ \ 1\ \ 2\ -1\ 3 \end{pmatrix}$		$\frac{\sqrt{9604+9800t+3750t^2+554t^3+25t^4+p^2\left(895+554t+150t^2+20t^3+t^4\right)+2p\left(2450+1960t+669t^2+100t^3+5t^4\right)}}{98+50t+5t^2+|\text{p}|(5+t)^2}$		
		All non-BPS attractors for $-3.1<p<-0.07$ are stable		

Table 15. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 4/8.

Polytope label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of non-BPS attractors		
$(23,2)^{2,116}_{-228}$ $\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 2 \\ 2 & -3 & 2 & 4 & -5 & 0 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1 & 0 \\ 8 & 49 \end{pmatrix}$	$-\frac{t(164t^3+137t^2+36t+3)}{1470t^4+1340t^3+435t^2+60t+3}$	$-\frac{82}{735} < p < 0$	NA
		$\frac{\sqrt{2t^2(3+24t+47t^2)+8pt^2(6+49t+98t^2)+p^2(3+48t+384t^2+1568t^3+2401t^4)}}{2t(1+4t)+|\mathrm{p}|(1+16t+49t^2)}$		
		All non-BPS attractors are stable		
$(24,1)^{2,120}_{-236}$ $\begin{pmatrix} 1 & -1 & 1 & 2 & 0 & 3 \\ -1 & 4 & -1 & -5 & 3 & 0 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 8 & 2 \\ 32 & 101 \end{pmatrix}$	$-\frac{t(667t^3+480t^2+120t+10)}{1616t^4+1273t^3+288t^2+8t-2}$	$p > 0 \text{ \& } p < -\frac{667}{1616}$	$-\frac{667}{1616} < p < -0.4$
		$\frac{\sqrt{4(1+16t+96t^2+283t^3+364t^4)+4p(8+128t+687t^2+1616t^3+1616t^4)+p^2(64+1132t+6144t^2+12928t^3+10201t^4)}}{2(1+4t)^2+|\mathrm{p}|(8+64t+101t^2)}$		
		All non-BPS attractors for $-18.02 < p < -0.4$ are stable		
$(24,2)^{2,120}_{-236}$ $\begin{pmatrix} 1 & -4 & 1 & 5 & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 23 & 101 \\ 5 & 1 \end{pmatrix}$	$-\frac{23t^4+357t^3+2055t^2+5191t+4848}{4t^4+65t^3+387t^2+1001t+949}$	$-\frac{23}{4} < p < -\frac{4848}{949}$	NA
		$\frac{\sqrt{10201+9292t+3174t^2+488t^3+29t^4+p^2(577+488t+150t^2+20t^3+t^4)+2p(2323+2020t+648t^2+92t^3+5t^4)}}{101+46t+5t^2+|\mathrm{p}|(23+10t+t^2)}$		
		All non-BPS attractors are stable		
$(25,1)^{2,122}_{-240}$ $\begin{pmatrix} -3 & 1 & 1 & 1 & 0 & 7 \\ 1 & 0 & 0 & 0 & 1 & -2 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 21 & 63 \\ 7 & 2 \end{pmatrix}$	$-\frac{9t(4t^3+35t^2+105t+105)}{8t^4+72t^3+189t^2+63t-189}$	$p > 0 \text{ \& } p < -\frac{9}{2}$	$-\frac{9}{2} < p < -4.48$
		$\frac{\sqrt{63(63+84t+42t^2+10t^3+t^4)+14p(189+252t+117t^2+24t^3+2t^4)+p^2(441+630t+294t^2+56t^3+4t^4)}}{7(3+t)+|\mathrm{p}|(21+14t+2t^2)}$		
		All non-BPS attractors for $-322.24 < p < -4.48$ are stable		
$(26,1)^{2,122}_{-240}$ $\begin{pmatrix} 1 & -3 & 1 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 21 & 63 \\ 7 & 2 \end{pmatrix}$	$-\frac{9t(4t^3+35t^2+105t+105)}{8t^4+72t^3+189t^2+63t-189}$	$p > 0 \text{ \& } p < -\frac{9}{2}$	$-\frac{9}{2} < p < -4.48$
		$\frac{\sqrt{63(63+84t+42t^2+10t^3+t^4)+14p(189+252t+117t^2+24t^3+2t^4)+p^2(441+630t+294t^2+56t^3+4t^4)}}{7(3+t)^2+|\mathrm{p}|(21+14t+2t^2)}$		
		All non-BPS attractors for $-322.24 < p < -4.48$ are stable		

Table 16. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 5/8.

Polytope label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of non-BPS attractors		
$(27,1)^{2,122}_{-240}$	$\frac{1}{6}\begin{pmatrix} 7 & 2 \\ 21 & 63 \end{pmatrix}$	$\frac{189t^4-63t^3-189t^2-72t-8}{945t^3+945t^2+315t+36}$	$p > -\frac{2}{9}$	$-0.223 < p < -\frac{2}{9}$
$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & -3 & 1 \end{pmatrix}$		$\frac{\sqrt{4+56t+294t^2+630t^3+441t^4+63p^2\left(1+10t+42t^2+84t^3+63t^4\right)+14p\left(2+24t+117t^2+252t^3+189t^4\right)}}{2+14t+21t^2+7|\mathrm{p}|(1+3t)^2}$		
		All non-BPS attractors for $-0.223 < p < -0.003$ are stable		
$(30,2)^{2,128}_{-252}$	$\frac{2}{3}\begin{pmatrix} 1 & 0 \\ 6 & 27 \end{pmatrix}$	$-\frac{t}{6t+1}$	$-\frac{1}{6} < p < 0$	NA
$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 2 \\ 2 & -3 & 2 & 2 & -3 & 0 \end{pmatrix}$		$\frac{1+3p}{1-p}$		
		All non-BPS attractors are stable		
$(31,1)^{2,128}_{-252}$	$\frac{1}{6}\begin{pmatrix} 16 & 6 \\ 42 & 109 \end{pmatrix}$	$-\frac{1539t^4+2618t^3+1656t^2+462t+48}{4360t^4+7425t^3+4710t^2+1320t+138}$	$-\frac{1539}{4360} < p < -\frac{8}{23}$	NA
$\begin{pmatrix} 1 & -1 & -1 & 2 & 0 & 1 \\ -2 & 3 & 3 & -5 & 1 & 0 \end{pmatrix}$		$\frac{\sqrt{4\left(9+96t+384t^2+681t^3+451t^4\right)+4p\left(48+504t+1989t^2+3488t^3+2289t^4\right)+p^2\left(264+2724t+10584t^2+18312t^3+11881t^4\right)}}{6+32t+42t^2+|\mathrm{p}|\left(16+84t+109t^2\right)}$		
		All non-BPS attractors are stable		
$(31,2)^{2,128}_{-252}$	$\frac{1}{6}\begin{pmatrix} 25 & 109 \\ 5 & 1 \end{pmatrix}$	$\frac{t^4-5t^3-225t^2-1295t-2180}{5t^3+75t^2+375t+641}$	$p > -\frac{2180}{641}$	$-3.405 < p < -\frac{2180}{641}$
$\begin{pmatrix} 2 & -3 & -3 & 5 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 2 \end{pmatrix}$		$\frac{\sqrt{11881+10900t+3750t^2+532t^3+25t^4+p^2\left(785+532t+150t^2+20t^3+t^4\right)+2p\left(2725+2180t+702t^2+100t^3+5t^4\right)}}{109+50t+5t^2+|\mathrm{p}|(5+t)^2}$		
		All non-BPS attractors for $-3.405 < p < -0.042$ are stable		
$(32,1)^{2,128}_{-252}$	$\frac{1}{3}\begin{pmatrix} 15 & 54 \\ 4 & 1 \end{pmatrix}$	$-\frac{9(t+4)}{2t+9}$	$-\frac{9}{2} < p < -4$	NA
$\begin{pmatrix} 1 & -3 & -3 & 1 & 4 & 0 \\ 0 & 1 & 1 & 0 & -1 & 1 \end{pmatrix}$		$3\left(\frac{p^2+9p+18}{p^2+9p-18}\right)$		
		All non-BPS attractors are stable		

Table 17. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 6/8.

Polytope label, Charge Matrix	$\begin{pmatrix} c\ d \\ b\ a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of non-BPS attractors		
$(33,1)^{2,132}_{-260}$ $\begin{pmatrix} 1 & -1 & -1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 2 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 4\ 7 \\ 2\ 1 \end{pmatrix}$	$\frac{t^4-2t^3-36t^2-83t-56}{5t^3+30t^2+60t+41}$	$p > -\frac{56}{41}$	$-1.37 < p < -\frac{56}{41}$
		$\frac{\sqrt{49+112t+96t^2+34t^3+4t^4+p^2\left(20+34t+24t^2+8t^3+t^4\right)+2p\left(28+56t+45t^2+16t^3+2t^4\right)}}{7+8t+2t^2+|p|(2+t)^2}$		
		All non-BPS attractors for $< -1.37 < p < -0.02$ are stable		
$(33,2)^{2,132}_{-260}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 2 \\ -1 & 1 & 1 & 1 & -2 & 0 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1\ 0 \\ 3\ 7 \end{pmatrix}$	$-\frac{t(15t^3+37t^2+27t+6)}{56t^4+141t^3+123t^2+45t+6}$	$-\frac{15}{56} < p < 0$	NA
		$\frac{\sqrt{t^2\left(6+18t+13t^2\right)+2pt^2\left(9+28t+21t^2\right)+p^2\left(3+18t+54t^2+84t^3+49t^4\right)}}{t(2+3t)+|p|\left(1+6t+7t^2\right)}$		
		All non-BPS attractors are stable		
$(34,1)^{2,132}_{-260}$ $\begin{pmatrix} 1 & -2 & -2 & -4 & 7 & 0 \\ 0 & 1 & 1 & 2 & -3 & 1 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 49\ 114 \\ 21\ 9 \end{pmatrix}$	$\frac{9t^4-21t^3-441t^2-1256t-1064}{45t^3+315t^2+735t+572}$	$p > -\frac{266}{143}$	NA
		$\frac{\sqrt{3}\sqrt{4332+7448t+4802t^2+1374t^3+147t^4+p^2\left(805+1374t+882t^2+252t^3+27t^4\right)+2p\left(1862+3192t+2055t^2+588t^3+63t^4\right)}}{114+98t+21t^2+|p|(7+3t)^2}$		
		All non-BPS attractors for $-\frac{266}{143} < p < 0$ are stable		
$(34,2)^{2,132}_{-260}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 2 \\ -1 & 2 & 2 & 4 & -7 & 0 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1\ 0 \\ 8\ 57 \end{pmatrix}$	$\frac{t(20t^3-27t^2-12t-1)}{266t^4+436t^3+153t^2+20t+1}$	$0 < p < \frac{10}{133}$	$-0.06 < p < 0$
		$\frac{\sqrt{3}\sqrt{2t^2\left(1+8t+13t^2\right)+8pt^2\left(2+19t+38t^2\right)+p^2\left(1+16t+128t^2+608t^3+1083t^4\right)}}{2t(1+4t)+|p|\left(1+16t+57t^2\right)}$		
		All non-BPS attractors for $-0.06 < p < -0.01$ are stable		
$(35,1)^{2,144}_{-284}$ $\begin{pmatrix} 1 & -2 & -2 & -2 & 5 & 0 \\ 0 & 1 & 1 & 1 & -2 & 1 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 25\ 62 \\ 10\ 4 \end{pmatrix}$	$\frac{4t^4-10t^3-225t^2-685t-620}{20t^3+150t^2+375t+313}$	$p > -\frac{620}{313}$	NA
		$\frac{\sqrt{p^2\left(635+1004t+600t^2+160t^3+16t^4\right)+4p\left(775+1240t+747t^2+200t^3+20t^4\right)+2\left(1922+3100t+1875t^2+502t^3+50t^4\right)}}{62+50t+10t^2+|p|(5+2t)^2}$		
		All non-BPS attractors for $-\frac{620}{313} < p < 0$ are stable		

Table 18. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 7/8.

Polytope label, Charge Matrix	$\begin{pmatrix} c & d \\ b & a \end{pmatrix}$	Non-BPS solution	Range of validity for Single solution	Range of validity for Multiple solutions
		Recombination factor		
		Stability of non-BPS attractors		
$(35,2)^{2,144}_{-284}$ $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 2 \\ -1 & 2 & 2 & 2 & -5 & 0 \end{pmatrix}$	$\frac{2}{3}\begin{pmatrix} 1 & 0 \\ 6 & 31 \end{pmatrix}$	$\frac{t(3t^3-53t^2-27t-3)}{310t^4+555t^3+255t^2+45t+3}$	$0 < p < \frac{3}{310}$	$-0.08 < p < 0$
	$\dfrac{\sqrt{2t^2\left(3+18t+23t^2\right)+4pt^2\left(9+62t+93t^2\right)+p^2\left(3+36t+216t^2+744t^3+961t^4\right)}}{2t(1+3t)+|\mathrm{p}|\left(1+12t+31t^2\right)}$			
	All non-BPS attractors for $-0.08 < p < -0.01$ are stable			
$(36,1)^{2,272}_{-540}$ $\begin{pmatrix} 0 & 0 & 0 & 2 & 3 & 1 \\ 1 & 1 & 1 & 0 & 0 & -3 \end{pmatrix}$	$\frac{1}{6}\begin{pmatrix} 1 & 0 \\ 3 & 9 \end{pmatrix}$	$\frac{t(9t^3-3t^2-9t-2)}{45t^3+45t^2+15t+2}$	$p > 0$	$-0.12 < p < 0$
	$\dfrac{\sqrt{3}\sqrt{6pt^2\left(1+4t+3t^2\right)+t^2\left(2+6t+3t^2\right)+p^2\left(1+6t+18t^2+36t^3+27t^4\right)}}{|\mathrm{p}|(1+3t)^2+t(2+3t)}$			
	All non-BPS attractors for $-0.12 < p < -0.025$ are stable			

Table 19. Non-BPS extremal (multiple) black strings in one-modulus THCY models, 8/8.

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