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Super Yang–Mills action from WZW-like open superstring field theory including the Ramond sector

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In the framework of WZW-like open superstring field theory (SSFT) including the Ramond (R) sector whose action was constructed by Kunitomo and Okawa, we truncate the string fields in both the Neveu–Schwarz (NS) and R sectors up to the lowest massless level and obtain the ten-dimensional super Yang–Mills (SYM) action with bosonic extra term by explicit calculation of the SSFT action. Furthermore, we compute a contribution from the massive part up to the lowest order and find that the bosonic extra term is canceled and instead a fermionic extra term appears, which can be interpreted as a string correction to the SYM action. This calculation is an extension to the R sector of the earlier work by Berkovits and Schnabl in the NS sector. We also study gauge transformation, equation of motion, and spacetime supersymmetry transformation of the massless component fields induced from those of string fields.

Subject Index B16, B28

1. Introduction

It is expected that open superstring theory describes super Yang–Mills theory at low energy. Some time ago, Berkovits and Schnabl [1] showed that the Yang–Mills action is derived from Berkovits' Wess–Zumino–Witten (WZW)-like action for open superstring field theory [2] in the Neveu–Schwarz (NS) sector. Recently, Kunitomo and Okawa have constructed a complete action for open superstring field theory including the Ramond (R) sector as an extension of the WZW-like action [3]. Therefore, it is natural to expect that the ten-dimensional super Yang–Mills (SYM) action can be derived from it. Furthermore, an explicit formula for spacetime supersymmetry transformation in terms of string fields has been proposed in Refs. [4,5] and hence we can compare the induced transformation of the component fields and the conventional supersymmetry transformation of SYM.

As a first step toward the above issue, we adopt the level truncation method at the lowest level, which corresponds to the massless component fields due to the GSO projection. We perform explicit calculations in the superstring field theory in both NS and R sectors and obtain the SYM action with extra bosonic term, $O(A_\mu^4)$, in terms of the component fields [6]. We compute a contribution to the effective action of massless fields from the massive part of string fields in the zero-momentum sector, up to lowest order with respect to the coupling constant, and find that the extra $O(A_\mu^4)$ is canceled[1] and, instead, an extra fermionic term, $O(\lambda_{\dot\alpha}^4)$, appears in the order of $(\alpha')^1$, which can be interpreted as a string correction to SYM.

[1] This result has already been obtained in Ref. [1] in computation in the NS sector.

We also derive the induced gauge and supersymmetry transformations up to nonzero lowest order and find that the resulting formulas are consistent with those of the conventional SYM.

The organization of this paper is as follows. In the next section, we briefly review the action of open superstring field theory in terms of Kunitomo and Okawa's formulation. In Sect. 3, we derive the explicit form of the level-truncated action at the lowest level. In Sect. 4, we evaluate the contribution from the massive part. In Sect. 5, we find the induced transformations from those of the superstring field theory. In Sect. 6, we give some concluding remarks. In the appendix we summarize our convention on spin fields, which is necessary for explicit computations.

2. A brief review of a WZW-like action for open superstring field theory including the R sector

Let us first review Kunitomo and Okawa's action [3] for open superstring field theory (SSFT).

As an extension of Berkovits' WZW-like action in the NS sector, Kunitomo and Okawa proposed a complete action for open superstring field theory including the R sector as follows:

$$S[\Phi,\Psi] = S_{\rm NS}[\Phi] + S_{\rm R}[\Phi,\Psi], \tag{2.1}$$

$$S_{\rm NS}[\Phi] = -\int_0^1 dt\langle A_t(t), QA_\eta(t)\rangle, \tag{2.2}$$

$$S_{\rm R}[\Phi,\Psi] = -\frac{1}{2}\langle\!\langle \Psi, YQ\Psi\rangle\!\rangle - \int_0^1 dt\langle A_t(t), (F(t)\Psi)^2\rangle. \tag{2.3}$$

The action is a functional of string fields: Φ and Ψ. $\langle\ ,\ \rangle$ and $\langle\!\langle\ ,\ \rangle\!\rangle$ are the BPZ inner product in the large and small Hilbert space, respectively. Q is the BRST operator and η is the zeromode of $\eta(z) : \eta = \oint \frac{dz}{2\pi i}\eta(z)$. We use the relation of the superconformal ghosts between (β,γ) and (ξ,η,ϕ) as $\beta(z) = \partial\xi e^{-\phi}(z)$ and $\gamma(z) = e^{\phi}\eta(z)$. Φ is a Grassmann even string field in the NS sector, whose ghost number ($n_{\rm gh}$) and picture number ($n_{\rm pic}$) are both zero, and expanded by the states in the large Hilbert space. $A_t(t)$, $A_\eta(t)$, and $F(t)$ are defined by

$$A_t(t) = (\partial_t e^{t\Phi})e^{-t\Phi} = \Phi\,, \qquad A_\eta(t) = (\eta e^{t\Phi})e^{-t\Phi}, \tag{2.4}$$

$$F(t)\Psi = \Psi + \Xi\left\{A_\eta(t),\Psi\right\} + \Xi\left\{A_\eta(t), \Xi\left\{A_\eta(t),\Psi\right\}\right\} + \cdots \tag{2.5}$$

$$= \sum_{k=0}^{\infty} \underbrace{\Xi\{A_\eta(t), \Xi\{A_\eta(t),\ldots,\Xi\{A_\eta(t),}_{k}\Psi\}\ldots\}\}, \tag{2.6}$$

where $\Xi = \Theta(\beta_0)$, the Heaviside step function of β_0.[2] Ψ is a Grassmann odd string field in the R sector with $(n_{\rm gh}, n_{\rm pic}) = (1, -1/2)$, and it is expanded by the states in the *restricted* small Hilbert space. Namely, the conditions for the R string field are

$$\eta\Psi = 0, \qquad XY\Psi = \Psi. \tag{2.7}$$

X and Y are kinds of picture-changing operators with picture number 1 and -1, respectively:

$$X = -\delta(\beta_0)G_0 + \delta'(\beta_0)b_0 = \{Q, \Theta(\beta_0)\}, \tag{2.8}$$

$$Y = -c_0\delta'(\gamma_0), \tag{2.9}$$

where $XYX = X$ and $YXY = Y$ hold and therefore XY gives a projector: $(XY)^2 = XY$.

[2] In Ref. [7], a better expression for Ξ has been proposed.

In the following, we consider SSFT on the N BPS D9-branes in the flat ten-dimensional spacetime, and hence the string fields have the Chan–Paton factors implicitly and the GSO projection is imposed on the string fields.

Gauge transformation The action in Eq. (2.1) is invariant under the following infinitesimal gauge transformations:

$$A_{\delta_{g(\Lambda,\lambda,\Omega)}} = Q\Lambda + D_\eta\Omega + \{F\Psi, F\Xi(\{F\Psi,\Lambda\} - \lambda)\}, \qquad A_\delta \equiv (\delta e^{\Phi})e^{-\Phi}, \tag{2.10}$$

$$\delta_{g(\Lambda,\lambda)}\Psi = Q\lambda + X\eta F\Xi D_\eta(\{F\Psi,\Lambda\} - \lambda), \tag{2.11}$$

where $F = F(t=1)$,

$$D_\eta B = \eta B - \left[A_\eta, B\right\}, \qquad A_\eta = A_\eta(t=1), \tag{2.12}$$

and the gauge parameter λ in the R sector is in the restricted small Hilbert space, namely,

$$\eta\lambda = 0, \qquad XY\lambda = \lambda. \tag{2.13}$$

Λ and Ω are gauge parameters in the NS sector in the large Hilbert space.

Equation of motion Taking the variation of the action in Eq. (2.1) with respect to the string fields, Φ and Ψ, we have the equations of motion as follows:

$$QA_\eta + (F\Psi)^2 = 0, \qquad Q\Psi + X\eta F\Psi = 0. \tag{2.14}$$

The second equation is consistent with the condition for Ψ, which is in the restricted small Hilbert space.

Spacetime supersymmetry In the ten-dimensional Minkowski spacetime, the SSFT action in Eq. (2.1) is invariant under the spacetime supersymmetry transformation, which is given by [4]:[3]

$$A_{\delta_\mathcal{S}} = e^{\Phi}\,\mathcal{S}\Xi(e^{-\Phi}F\Psi e^{\Phi})\,e^{-\Phi} + \{F\Psi, F\Xi A_\mathcal{S}\}, \qquad A_\mathcal{S} = (\mathcal{S}e^{\Phi})e^{-\Phi}, \tag{2.15}$$

$$\delta_\mathcal{S}\Psi = X\eta F\Xi\mathcal{S}A_\eta, \tag{2.16}$$

where $\mathcal{S}$ is a derivation with respect to the star product of string fields and is given by a constant Weyl spinor $\epsilon_{\dot\alpha}$ and the spin field with $n_{\rm pic} = -1/2$:

$$\mathcal{S} = \epsilon_{\dot\alpha}\oint\frac{dz}{2\pi i}S^{\dot\alpha}_{(-1/2)}(z). \tag{2.17}$$

We use a bosonized formulation of spin fields as in the appendix for the following explicit computation.

3. Level truncation of string fields in the NS and R sectors

Here, we evaluate the SSFT action in Eq. (2.1) using the level truncation method explicitly. We define the level of string fields as the eigenvalue of $L_0 = \{Q, b_0\}$, except for a contribution from momentum. We expand the string fields with component fields up to the lowest level in both NS and R sectors.

[3] Another form of supersymmetry transformation has been proposed in Ref. [5]. The difference does not matter in our paper because we compute only the linearized one in Eq. (5.20).

3.1. Level-truncated action in the NS sector

In the NS sector, we expand a string field Φ with $(n_{\rm gh}, n_{\rm pic}) = (0, 0)$ in the large Hilbert space. The lowest-level state is

$$c\xi e^{-\phi} e^{ik\cdot X}(0)|0\rangle, \tag{3.1}$$

which corresponds to the tachyon, but it is excluded by the GSO projection of Eq. (A.5). The lowest-level states on the GSO projected space are

$$e^{ik\cdot X}(0)|0\rangle, \qquad c\xi\psi^{\mu}e^{-\phi}e^{ik\cdot X}(0)|0\rangle, \qquad c\partial c\xi\partial\xi e^{-2\phi}e^{ik\cdot X}(0)|0\rangle, \tag{3.2}$$

which correspond to the massless level. The first one can be eliminated by the Ω-gauge transformation in Eq. (2.10), which can be rewritten as $e^{-\Phi}\delta_{g(\Omega)}e^{\Phi} = \eta(e^{-\Phi}\Omega e^{\Phi})$ and has an expression for finite transformation: $e^{\Phi'} = e^{\Phi}g$ with $\eta g = 0$, because it can be rewritten as $e^{ik\cdot X}(0)|0\rangle = \eta\left(\xi e^{ik\cdot X}(0)|0\rangle\right)$. In the following, we take into account only the other two states and their component fields as a level-truncated string field Φ_0 in the NS sector. Namely, we use a partial gauge-fixing condition, $\xi_0\Phi = 0$. After Ref. [1], we use the notation

$$\Phi_A = \int \frac{d^{10}k}{(2\pi)^{10}} A_{\mu}(k)\mathcal{V}_A^{\mu}(k)(0)|0\rangle, \quad \mathcal{V}_A^{\mu}(k) = c\xi e^{-\phi}\psi^{\mu}(z)\, e^{ik\cdot X(z,\bar{z})}, \tag{3.3}$$

$$\Phi_B = \int \frac{d^{10}k}{(2\pi)^{10}} B(k)\mathcal{V}_B(k)(0)|0\rangle, \quad \mathcal{V}_B(k) = c\partial c\xi\partial\xi e^{-2\phi}(z)\, e^{ik\cdot X(z,\bar{z})}, \tag{3.4}$$

where $A_{\mu}(k)$ and $B(k)$ are the Fourier modes of component bosonic fields in the ten-dimensional spacetime.

The NS action in Eq. (2.2), which is the same as Berkovits' WZW-like action, can be expanded as

$$\begin{aligned} S_{\rm NS}[\Phi] &= \sum_{M,N=0}^{\infty} \frac{(-1)^N(M+N)!}{(M+N+2)!M!N!}\langle Q\Phi, \Phi^M(\eta\Phi)\Phi^N\rangle \\ &= \frac{1}{2}\langle Q\Phi, \eta\Phi\rangle + \frac{1}{3!}(\langle Q\Phi, \Phi\,\eta\Phi\rangle - \langle Q\Phi, \eta\Phi\,\Phi\rangle) \\ &\quad + \frac{1}{4!}(\langle Q\Phi, \Phi^2\,\eta\Phi\rangle - 2\langle Q\Phi, \Phi\,\eta\Phi\,\Phi\rangle + \langle Q\Phi, \eta\Phi\,\Phi^2\rangle) + O(\Phi^5), \end{aligned} \tag{3.5}$$

and we truncate the string field Φ in the NS sector to the sum of Eqs. (3.3) and (3.4): $\Phi_0 = \Phi_A + \Phi_B$.

Using the BRST transformations

$$\begin{aligned} [Q, \mathcal{V}_A^{\mu}(k)] &= -\alpha' k^2 c\partial c\xi e^{-\phi}\psi^{\mu}e^{ik\cdot X} \\ &\quad - \sqrt{2\alpha'}c(:k\cdot\psi\psi^{\mu}: + \frac{i}{\alpha'}\partial X^{\mu} + (:\eta\xi: + \partial\phi)k^{\mu})e^{ik\cdot X} + \eta e^{\phi}\psi^{\mu}e^{ik\cdot X}, \end{aligned} \tag{3.6}$$

$$[Q, \mathcal{V}_B(k)] = -\sqrt{2\alpha'}c\partial c\xi k\cdot\psi e^{-\phi}e^{ik\cdot X} + (-\partial c + 2c(:\eta\xi: + \partial\phi))e^{ik\cdot X} \tag{3.7}$$

on the real axis, where $Q = \oint \frac{dz}{2\pi i}(c(T^{\rm m} + T^{\phi} + T^{\xi\eta}) + bc\partial c + e^{\phi}\eta G^{\rm m} - \eta\partial\eta e^{2\phi}b)$, $T^{\rm m} = -\frac{1}{\alpha'}\partial X^{\mu}\partial X_{\mu} - \frac{1}{2}\psi^{\mu}\partial\psi_{\mu}$, and $G^{\rm m} = i\sqrt{\frac{2}{\alpha'}}\psi^{\mu}\partial X_{\mu}$, we can evaluate the kinetic term $\frac{1}{2}\langle Q\Phi_0, \eta\Phi_0\rangle$. For the interaction terms, we use the explicit form of the conformal maps, which defines the n-string term. Namely, for $A_k = A_k(0)|0\rangle$ $(k = 1, 2, \ldots, n)$, we have

$$\langle A_1, A_2 \cdots A_n\rangle = \langle g_1^{(n)} \circ A_1(0)\, g_2^{(n)} \circ A_2(0) \cdots g_n^{(n)} \circ A_n(0)\rangle_{\mathrm{UHP}}, \tag{3.8}$$

$$g_k^{(n)}(z) = h^{-1}\left(e^{i\pi\frac{2k-1-n}{n}}(h(z))^{\frac{2}{n}}\right) = \tan\left(\frac{2}{n}\arctan z + \frac{\pi}{2n}(2k-1-n)\right), \tag{3.9}$$

with the map from the upper half-plane to the unit disk: $h(z) = \frac{1+iz}{1-iz}$. The normalization of the large Hilbert space is given by

$$\langle \xi(y)\frac{1}{2}c\partial c\partial^2 c(z)e^{-2\phi(w)}e^{ik\cdot X(x,\bar{x})}\rangle_{\mathrm{UHP}} = (2\pi)^{10}\delta^{10}(k). \tag{3.10}$$

In particular, with respect to the ϕ-charge, the terms such as $\Phi^{n-2}\eta\Phi\, Q\Phi$ are linear combinations of $e^{q\phi}$ with $q \leq 2-n$ because $\Phi_A \sim e^{-\phi}$; $\eta\Phi_A \sim e^{-\phi}$; $\Phi_B \sim e^{-2\phi}$; $\eta\Phi_B \sim e^{-2\phi}$; $Q\Phi_A \sim e^{-\phi}, 1, e^{\phi}$ and $Q\Phi_B \sim e^{-\phi}, 1$, which imply that the higher-order interaction terms, $O(\Phi^5)$ in Eq. (3.5), vanish for $\Phi_0 = \Phi_A + \Phi_B$. Then, we obtain

$$\begin{aligned} S_{\mathrm{NS}}[\Phi_0] = &\int d^{10}x\,\mathrm{Tr}\left[\frac{\alpha'}{2}A_\mu\partial^2 A^\mu + i\sqrt{2\alpha'}B\partial^\mu A_\mu + B^2 + \frac{i\sqrt{2\alpha'}}{2}\partial_\mu\tilde{A}_\nu[\tilde{A}^\mu,\tilde{A}^\nu]\right] \\ &+\int d^{10}x 2^{\frac{\alpha'}{2}((\partial_1-\partial_3)^2+(\partial_2-\partial_4)^2)}\mathrm{Tr} \\ &\left[\frac{1}{8}A_\mu(x_1)A_\nu(x_2)A^\mu(x_3)A^\nu(x_4) - \frac{1}{2}A_\mu(x_1)A_\nu(x_2)A^\nu(x_3)A^\mu(x_4)\right]_{x_i=x}, \end{aligned} \tag{3.11}$$

where the trace is taken over the indices of the Chan–Paton factors, T_a; $A_\mu(x)$ and $B(x)$ are given by

$$A_\mu(x) = A_\mu^a(x)T_a = \int\frac{d^{10}k}{(2\pi)^{10}}A_\mu(k)e^{ik\cdot x}, \qquad B(x) = B^a(x)T_a = \int\frac{d^{10}k}{(2\pi)^{10}}B(k)e^{ik\cdot x}; \tag{3.12}$$

and $\tilde{A}_\mu$ is defined as

$$\tilde{A}_\mu(x) = K^{-\alpha'\partial^2}A_\mu(x), \qquad K \equiv \frac{4}{3\sqrt{3}}. \tag{3.13}$$

We note that the reality condition for the NS string field, $\mathrm{bpz}^{-1}\circ\Phi^\dagger = -\Phi$ [8], implies the (anti-)Hermiticity of the component fields: $A_\mu(x)^\dagger = A_\mu(x)$ and $B(x)^\dagger = -B(x)$.

Integrating out the scalar component field $B(x)$ in Eq. (3.11),[4] or using the equation of motion for B, $B = -i\sqrt{\alpha'/2}\,\partial^\mu A_\mu$, and taking the small-momentum limit: $K^{-\alpha'\partial^2} \sim 1$ and $2^{\frac{\alpha'}{2}((\partial_1-\partial_3)^2+(\partial_2-\partial_4)^2)} \sim 1$, we have

$$\begin{aligned} S_{\mathrm{NS}}[\Phi_0] = &-\frac{\alpha'}{4}\int d^{10}x\,\mathrm{Tr}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - \frac{i}{\sqrt{2\alpha'}}[A_\mu,A_\nu]\right)^2 \\ &-\frac{1}{4}\int d^{10}x\,\mathrm{Tr}[\frac{1}{2}A_\mu A_\nu A^\mu A^\nu + A_\mu A_\nu A^\nu A^\mu]. \end{aligned} \tag{3.14}$$

The first line on the right-hand side corresponds to the ordinary Yang–Mills action and the second line is the difference from it. Actually, it is known that the difference is canceled by the contribution from massive component fields [1].

[4] We note that Φ_B does not contribute to the terms including the R string fields, up to the lowest level, as we see in Sect. 3.2.

3.2. Level-truncated action including the R string field

In the R sector, the string field Ψ, which has $(n_{\rm gh}, n_{\rm pic}) = (1, -1/2)$, is in the restricted small Hilbert space. The lowest-level states are given by spin fields with $(-1/2)$-picture:

$$cS^{\alpha}_{(-1/2)}e^{ik\cdot X}(0)|0\rangle, \qquad cS^{\dot\alpha}_{(-1/2)}e^{ik\cdot X}(0)|0\rangle, \tag{3.15}$$

where α and $\dot\alpha$ are spinor indices with 16 components, and they correspond to the massless level. By the GSO projection in Eq. (A.5), only the state with the dotted spinor remains in our convention. Hence, we expand the level-truncated string field Ψ_0 as

$$\Psi_0 = \int \frac{d^{10}k}{(2\pi)^{10}}\lambda_{\dot\alpha}(k)\mathcal{V}^{\dot\alpha}_{\lambda}(k)(0)|0\rangle, \quad \mathcal{V}^{\dot\alpha}_{\lambda}(k) = cS^{\dot\alpha}_{(-1/2)}(z)\,e^{ik\cdot X(z,\bar z)}, \tag{3.16}$$

where $\lambda_{\dot\alpha}(k)$ is the Fourier mode of a fermionic component field, which is the Weyl spinor in the ten-dimensional spacetime. This string field Ψ_0 is Grassmann odd, and we find that it is indeed in the restricted small Hilbert space: $\eta\Psi_0 = 0$ and $XY\Psi_0 = \Psi_0$.

Similarly to the NS sector, we derive an explicit expression of the action including the R string field in terms of the component fields. The action in Eq. (2.3) is expanded as

$$S_{\rm R}[\Phi,\Psi] = -\frac{1}{2}\langle\!\langle \Psi, YQ\Psi\rangle\!\rangle - \langle\Phi,\Psi^2\rangle + O(\Phi^2\Psi^2), \tag{3.17}$$

and we truncate the string fields up to the lowest level: the NS string field Φ to $\Phi_0 = \Phi_A + \Phi_B$ and the R string field Ψ to Ψ_0 given in Eq. (3.16).

The kinetic term in the R sector is evaluated using the normalization of the small Hilbert space:

$$\langle\!\langle \frac{1}{2}c\partial c\partial^2 c(z)e^{-2\phi(w)}e^{ik\cdot X(x,\bar x)}\rangle\!\rangle_{\rm UHP} = (2\pi)^{10}\delta^{10}(k). \tag{3.18}$$

It is convenient to use $\langle\!\langle \Psi, YQ\Psi\rangle\!\rangle = \langle\!\langle \Psi, Y_{\rm mid}Q\Psi\rangle\!\rangle$ [3] for calculation, where $Y_{\rm mid} = Y(i)$ is the midpoint insertion of the conventional inverse picture-changing operator $Y(z) = c\partial\xi e^{-2\phi}(z)$. With the BRST transformation

$$[Q, \mathcal{V}^{\dot\alpha}_{\lambda}(k)] = -\alpha' k^2 c\partial cS^{\dot\alpha}_{(-1/2)}e^{ik\cdot X} - i\sqrt{\alpha'}k_\mu(\Gamma^\mu)^{\dot\alpha}{}_{\beta}\eta cS^{\beta}_{(1/2)}e^{ik\cdot X} \tag{3.19}$$

on the real axis, we have

$$-\frac{1}{2}\langle\!\langle \Psi_0, Y_{\rm mid}Q\Psi_0\rangle\!\rangle = \frac{\sqrt{\alpha'}}{2}\int d^{10}x\,{\rm Tr}\left[\lambda_{\dot\alpha}(\Gamma^\mu C)^{\dot\alpha\dot\beta}\partial_\mu\lambda_{\dot\beta}\right], \tag{3.20}$$

where

$$\lambda_{\dot\alpha}(x) = \lambda^a_{\dot\alpha}(x)T_a = \int \frac{d^{10}k}{(2\pi)^{10}}\lambda_{\dot\alpha}(k)e^{ik\cdot x}. \tag{3.21}$$

For the cubic interaction term, we find $\langle\Phi_B,\Psi_0^2\rangle = 0$ by counting the number of c ghosts. With a similar manipulation to the NS sector, the remaining term is evaluated as

$$-\langle\Phi_A,\Psi_0^2\rangle = \frac{i}{\sqrt{2}}\int d^{10}x\,{\rm Tr}\left[\tilde A_\mu\tilde\lambda_{\dot\alpha}(\Gamma^\mu C)^{\dot\alpha\dot\beta}\tilde\lambda_{\dot\beta}\right], \tag{3.22}$$

where

$$\tilde\lambda_{\dot\alpha}(x) = K^{-\alpha'\partial^2}\lambda_{\dot\alpha}(x) \tag{3.23}$$

as in Eq. (3.13). For higher interaction terms, we find that $O(\Phi^2\Psi^2)$ in Eq. (3.17) with $\Phi = \Phi_0$ and $\Psi = \Psi_0$ vanishes thanks to the normalization in Eq. (3.10) because $\Phi_A \sim c$, $\Phi_B \sim cc$, and $\Psi_0 \sim c$, with respect to the bc-ghost sector. From Eqs. (3.20), (3.22), and (3.17), for the small-momentum limit $K^{-\alpha'\partial^2} \sim 1$, we have obtained

$$S_{\rm R}[\Phi_0, \Psi_0] = -\frac{\sqrt{\alpha'}}{2}\int d^{10}x\, {\rm Tr}\left[i\hat{\lambda}^\alpha (C\Gamma^\mu)_{\alpha\beta} D_\mu \hat{\lambda}^\beta\right], \tag{3.24}$$

where

$$\hat{\lambda}^\alpha = C^{\alpha\dot{\beta}}\lambda_{\dot{\beta}}, \tag{3.25}$$

$$D_\mu\hat{\lambda} = \partial_\mu\hat{\lambda} - \frac{i}{\sqrt{2\alpha'}}[A_\mu, \hat{\lambda}]. \tag{3.26}$$

The above form of $S_{\rm R}[\Phi_0, \Psi_0]$ just corresponds to the gaugino term of the ten-dimensional SYM action.

4. Contribution from the massive part

Here, we consider a contribution to the effective action of massless fields, A_μ and $\hat{\lambda}^\alpha$, obtained in the previous section, from the massive part of string fields, Φ and Ψ, in the same way as Ref. [1]. The SSFT action with the coupling constant g, which is obtained by replacing $g^{-2}S[g\Phi, g\Psi]$ using Eq. (2.1) with $S[\Phi, \Psi]$, can be expanded as

$$\begin{aligned} S[\Phi, \Psi] = &-\frac{1}{2}\langle \eta\Phi, Q\Phi\rangle + \frac{g}{6}\langle \eta\Phi, [\Phi, Q\Phi]\rangle \\ &-\frac{1}{2}\langle\!\langle \Psi, YQ\Psi\rangle\!\rangle - g\langle \Phi, \Psi^2\rangle + O(g^2). \end{aligned} \tag{4.1}$$

In this section we concentrate on the zero-momentum sector, and then the level-truncated string fields, Φ_0, Ψ_0, satisfy $Q\eta\Phi_0 = 0$, $Q\Psi_0 = 0$ because of Eqs. (5.17) and (5.18) with $B = -i\sqrt{\alpha'/2}\,\partial^\mu A_\mu$. We also note that Φ_0 satisfies $\xi_0\eta\Phi_0 = \Phi_0$, namely, the partial gauge-fixing condition. Around the massless part of the string fields, Φ_0, Ψ_0, we expand the string fields Φ, Ψ as $\Phi_0 + R$, $\Psi_0 + S$, where R and S are the massive part in the NS and R sector respectively, and we have

$$\begin{aligned} S[\Phi_0 + R, \Psi_0 + S] - S[\Phi_0, \Psi_0] = &-\frac{1}{2}\langle \eta R, QR\rangle + \frac{g}{2}\langle \eta R, [\Phi_0, Q\Phi_0]\rangle - g\langle R, (\Psi_0)^2\rangle \\ &-\frac{1}{2}\langle\!\langle S, YQS\rangle\!\rangle - g\langle\!\langle S, \{\eta\Phi_0, \Psi_0\}\rangle\!\rangle + \cdots \end{aligned} \tag{4.2}$$

from Eq. (4.1). Here, $(+\cdots)$ denotes the higher-order terms in g when R and S are assumed to be $O(g)$. Varying the above with respect to the massive part, we have equations of motion

$$Q\eta R = g\left(-\frac{1}{2}\{\eta\Phi_0, Q\Phi_0\} - (\Psi_0)^2\right) + \cdots, \qquad QS = g\left(-X\{\eta\Phi_0, \Psi_0\}\right) + \cdots, \tag{4.3}$$

and these can be solved by using the propagators in Ref. [9] as

$$R_{\rm s} = -g\frac{\xi_0 b_0}{L_0}\left(\frac{1}{2}\eta[\Phi_0, Q\Phi_0] + (\Psi_0)^2\right), \qquad S_{\rm s} = -g\frac{b_0 X\eta}{L_0}[\Phi_0, \Psi_0], \tag{4.4}$$

up to lowest order in g, where $R_{\rm s}$ satisfies the partial gauge-fixing condition: $\xi_0 R_{\rm s} = 0$.

Expanding the massive part of string fields around Eq. (4.4) as $R = R' + R_{\rm s},\ S = S' + S_{\rm s}$, we have

$$\begin{aligned} &S[\Phi_0 + R' + R_{\rm s}, \Psi_0 + S' + S_{\rm s}] - S[\Phi_0, \Psi_0] \\ &= \frac{1}{2}\langle QR', \eta R'\rangle - \frac{1}{2}\langle\!\langle S', YQS'\rangle\!\rangle \\ &\quad - \frac{1}{2}\langle QR_{\rm s}, \eta R_{\rm s}\rangle + \frac{1}{2}\langle\!\langle S_{\rm s}, YQS_{\rm s}\rangle\!\rangle + \cdots , \end{aligned} \tag{4.5}$$

where linear terms with respect to R' and S' vanish and therefore the massive part decouples from the massless part. The second line on the right-hand side gives a contribution to the effective action of the massless fields, which can be computed as

$$\begin{aligned} -\frac{1}{2}\langle QR_{\rm s}, \eta R_{\rm s}\rangle + \frac{1}{2}\langle\!\langle S_{\rm s}, YQS_{\rm s}\rangle\!\rangle &= \frac{g^2}{8}\left\langle [\Phi_0, Q\Phi_0], \frac{b_0}{L_0}\{\eta\Phi_0, Q\Phi_0\}\right\rangle + \frac{g^2}{2}\left\langle\!\!\left\langle (\Psi_0)^2, \frac{b_0}{L_0}(\Psi_0)^2\right\rangle\!\!\right\rangle \\ &\quad - \frac{g^2}{2}\left\langle (\Psi_0)^2, \frac{b_0}{L_0}[\Phi_0, Q\Phi_0]\right\rangle - \frac{g^2}{2}\left\langle \{Q\Phi_0, \Psi_0\}, \frac{b_0}{L_0}[\Phi_0, \Psi_0]\right\rangle. \end{aligned} \tag{4.6}$$

In the computation for the term including $S_{\rm s}$, we manipulated X as in Ref. [9]. Namely, we rewrote it as

$$\begin{aligned} \frac{1}{2}\langle\!\langle S_{\rm s}, YQS_{\rm s}\rangle\!\rangle &= \frac{g^2}{2}\left\langle\!\!\left\langle \{\eta\Phi_0, \Psi_0\}, \frac{b_0X}{L_0}\{\eta\Phi_0, \Psi_0\}\right\rangle\!\!\right\rangle \\ &= \frac{g^2}{2}\left\langle \xi_0\eta\Phi_0, \Psi_0\frac{b_0X}{L_0}\{\eta\Phi_0, \Psi_0\}\right\rangle + \frac{g^2}{2}\left\langle \xi_0\eta\Phi_0, \frac{b_0X}{L_0}\{\eta\Phi_0, \Psi_0\}\Psi_0\right\rangle \\ &= \frac{g^2}{2}\left\langle [\Phi_0, \Psi_0], \frac{b_0X}{L_0}\{\eta\Phi_0, \Psi_0\}\right\rangle \\ &= \frac{g^2}{2}\left\langle [\Phi_0, \Psi_0], \frac{b_0}{L_0}Q\Xi\{\eta\Phi_0, \Psi_0\}\right\rangle = \frac{g^2}{2}\left\langle [\Phi_0, \Psi_0], \left(1 - Q\frac{b_0}{L_0}\right)\Xi\{\eta\Phi_0, \Psi_0\}\right\rangle \\ &= \frac{g^2}{2}\langle [\Phi_0, \Psi_0], \Xi\{\eta\Phi_0, \Psi_0\}\rangle - \frac{g^2}{2}\left\langle \{Q\Phi_0, \Psi_0\}, \frac{b_0}{L_0}[\Phi_0, \Psi_0]\right\rangle, \end{aligned} \tag{4.7}$$

where the first term of the last expression vanishes due to the number of c-ghosts and the normalization in Eq. (3.10).

The first term of the right-hand side of Eq. (4.6) has been evaluated in Ref. [1]:

$$\begin{aligned} &\frac{1}{8}\langle [\Phi_0, Q\Phi_0], \frac{b_0}{L_0}\{\eta\Phi_0, Q\Phi_0\}\rangle \\ &= -\int d^{10}x \mathrm{Tr}[A_\mu A_\nu A_\rho A_\sigma]\int_0^\infty dt\, e^{-t}\left(e^{-2t}a^2 - \frac{1}{a^2}\right)\left(\frac{\eta^{\mu\rho}\eta^{\nu\sigma}}{(a^{-1}+e^{-t}a)^4} + \frac{\eta^{\mu\sigma}\eta^{\nu\rho}}{(a^{-1}-e^{-t}a)^4}\right) \\ &= \frac{1}{4}\int d^{10}x\, \mathrm{Tr}\left[\frac{1}{2}A_\mu A_\nu A^\mu A^\nu + A_\mu A_\nu A^\nu A^\mu\right], \end{aligned} \tag{4.8}$$

where

$$a = \tan\frac{\pi}{8} = \sqrt{2} - 1. \tag{4.9}$$

Equation (4.8) cancels the extra term of Eq. (3.14) in the NS sector. We note that the component field B in Φ_0 does not contribute to the above thanks to the equation of motion, $B = -i\sqrt{\alpha'/2}\,\partial^\mu A_\mu$ in the lowest order in g, which is negligible in the zero-momentum sector. The remaining terms of the right-hand side of Eq. (4.6), which includes the R sector, do not shift the coefficient of the $O(A_\mu \hat{\lambda}\hat{\lambda})$ term of Eq. (3.24). Furthermore, we can neglect the third and fourth terms in Eq. (4.6), namely

$$-\frac{1}{2}\left\langle (\Psi_0)^2, \frac{b_0}{L_0}[\Phi_0, Q\Phi_0]\right\rangle = 0, \qquad -\frac{1}{2}\left\langle \{Q\Phi_0, \Psi_0\}, \frac{b_0}{L_0}[\Phi_0, \Psi_0]\right\rangle = 0, \tag{4.10}$$

because of Eq. (3.10), noting that $\mathcal{V}_A^\mu(k=0) = c\xi e^{-\phi}\psi^\mu$, $[Q, \mathcal{V}_A^\mu(k=0)] = -i\sqrt{2/\alpha'}\, c\partial X^\mu + \eta e^\phi \psi^\mu$, $\mathcal{V}_\lambda^{\dot{\alpha}}(k=0) = cS^{\dot{\alpha}}_{(-1/2)}$. The second term of Eq. (4.6) can be evaluated, in a similar way to Eq. (4.8), as

$$\begin{aligned}
&\frac{1}{2}\left\langle\!\!\left\langle (\Psi_0)^2, \frac{b_0}{L_0}(\Psi_0)^2 \right\rangle\!\!\right\rangle = \frac{1}{2}\int d^{10}x \mathrm{Tr}[\lambda_{\dot{\alpha}}\lambda_{\dot{\beta}}\lambda_{\dot{\gamma}}\lambda_{\dot{\delta}}]\left\langle\!\!\left\langle cS^{\dot{\alpha}}_{(-1/2)} * cS^{\dot{\beta}}_{(-1/2)}, \frac{b_0}{L_0}(cS^{\dot{\gamma}}_{(-1/2)} * cS^{\dot{\delta}}_{(-1/2)}) \right\rangle\!\!\right\rangle \\
&= \frac{1}{2}\int d^{10}x \mathrm{Tr}[\lambda_{\dot{\alpha}}\lambda_{\dot{\beta}}\lambda_{\dot{\gamma}}\lambda_{\dot{\delta}}]\Big\langle 0|cS^{\dot{\alpha}}_{(-1/2)}(-\sqrt{3})cS^{\dot{\beta}}_{(-1/2)}(\sqrt{3}) \\
&\quad U_3 \frac{b_0}{L_0} U_3^\dagger cS^{\dot{\gamma}}_{(-1/2)}\left(\frac{1}{\sqrt{3}}\right) cS^{\dot{\delta}}_{(-1/2)}\left(\frac{-1}{\sqrt{3}}\right)|0\Big\rangle \\
&= \frac{1}{2}\int d^{10}x \mathrm{Tr}[\lambda_{\dot{\alpha}}\lambda_{\dot{\beta}}\lambda_{\dot{\gamma}}\lambda_{\dot{\delta}}]\int_0^\infty dt \langle S^{\dot{\alpha}}_{(-1/2)}\left(-\frac{1}{a}\right) S^{\dot{\beta}}_{(-1/2)}\left(\frac{1}{a}\right) S^{\dot{\gamma}}_{(-1/2)}(e^{-t}a) S^{\dot{\delta}}_{(-1/2)}(-e^{-t}a)\rangle \\
&\quad \times \left\langle 0|c\left(-\frac{1}{a}\right)c\left(\frac{1}{a}\right)b_0 c(e^{-t}a)c(-e^{-t}a)|0\right\rangle,
\end{aligned} \tag{4.11}$$

where a is given in Eq. (4.9) and U_3 is given in Refs. [10,11], which corresponds to the conformal map $\tan\left(\frac{2}{3}\arctan z\right)$. For the bc-ghost sector, we have

$$\left\langle 0|c\left(-\frac{1}{a}\right)c\left(\frac{1}{a}\right)b_0 c(e^{-t}a)c(-e^{-t}a)|0\right\rangle = -4e^{-t}\left(e^{-2t}a^2 - \frac{1}{a^2}\right) \tag{4.12}$$

with the normalization $\langle 0|c_{-1}c_0c_1|0\rangle = 1$. As for the ϕ-ghost and the spin field sector, we note that[5]

$$\begin{aligned}
&\langle S^{\dot{\alpha}_1}_{(-1/2)}(z_1) S^{\dot{\alpha}_2}_{(-1/2)}(z_2) S^{\dot{\alpha}_3}_{(-1/2)}(z_3) S^{\dot{\alpha}_4}_{(-1/2)}(z_4)\rangle \\
&= \delta_{\dot{A}_1+\dot{A}_2+\dot{A}_3+\dot{A}_4,0}\left(\prod_{p<q}(z_{pq})^{\sum_{i=1}^5 \dot{A}_p^i \dot{A}_q^i - \frac{1}{4}}\right)\exp\left[i\pi\left(\sum_{p<q}\sum_{i,j=1}^5 \dot{A}_p^i M_{ij}\dot{A}_q^j + \frac{1}{2}\sum_{j=1}^5 M_{6j}(\dot{A}_2^j + \dot{A}_4^j)\right)\right] \\
&= \frac{1}{2z_{12}z_{13}z_{24}z_{34}}(\Gamma^\mu C)^{\dot{\alpha}_1\dot{\alpha}_2}(\Gamma_\mu C)^{\dot{\alpha}_3\dot{\alpha}_4} - \frac{1}{2z_{13}z_{14}z_{23}z_{24}}(\Gamma^\mu C)^{\dot{\alpha}_4\dot{\alpha}_1}(\Gamma_\mu C)^{\dot{\alpha}_2\dot{\alpha}_3},
\end{aligned} \tag{4.13}$$

where $z_{pq} = z_p - z_q$, and it leads to

$$\begin{aligned}
&\langle S^{\dot{\alpha}}_{(-1/2)}\left(-\frac{1}{a}\right) S^{\dot{\beta}}_{(-1/2)}\left(\frac{1}{a}\right) S^{\dot{\gamma}}_{(-1/2)}(e^{-t}a) S^{\dot{\delta}}_{(-1/2)}(-e^{-t}a)\rangle \\
&\quad = \frac{e^t}{8(a^{-1}+e^{-t}a)^2}(\Gamma^\mu C)^{\dot{\alpha}\dot{\beta}}(\Gamma_\mu C)^{\dot{\gamma}\dot{\delta}} - \frac{1}{2(a^{-2}-e^{-2t}a^2)^2}(\Gamma^\mu C)^{\dot{\delta}\dot{\alpha}}(\Gamma_\mu C)^{\dot{\beta}\dot{\gamma}}.
\end{aligned} \tag{4.14}$$

[5] The last expression is consistent with the formula in Ref. [12] using the Fierz identity:

$$(\Gamma^\mu C)^{\dot{\alpha}\dot{\beta}}(\Gamma_\mu C)^{\dot{\gamma}\dot{\delta}} + (\Gamma^\mu C)^{\dot{\alpha}\dot{\gamma}}(\Gamma_\mu C)^{\dot{\delta}\dot{\beta}} + (\Gamma^\mu C)^{\dot{\alpha}\dot{\delta}}(\Gamma_\mu C)^{\dot{\beta}\dot{\gamma}} = 0.$$

With the above and Eq. (4.12), Eq. (4.11) is rewritten as

$$\frac{1}{2}\left\langle\!\!\left\langle (\Psi_0)^2, \frac{b_0}{L_0}(\Psi_0)^2 \right\rangle\!\!\right\rangle = \frac{1}{4}\int d^{10}x \mathrm{Tr}[\lambda_{\dot{\alpha}}\lambda_{\dot{\beta}}\lambda_{\dot{\gamma}}\lambda_{\dot{\delta}}]\int_0^\infty dt$$
$$\left(\frac{a^{-1}-e^{-t}a}{a^{-1}+e^{-t}a}(\Gamma^\mu C)^{\dot{\alpha}\dot{\beta}}(\Gamma_\mu C)^{\dot{\gamma}\dot{\delta}} - \frac{4e^{-t}}{a^{-2}-e^{-2t}a^2}(\Gamma^\mu C)^{\dot{\delta}\dot{\alpha}}(\Gamma_\mu C)^{\dot{\beta}\dot{\gamma}}\right)$$
$$= \frac{1}{4}\int d^{10}x \mathrm{Tr}\left[(\lambda^T\Gamma^\mu C\lambda)(\lambda^T\Gamma_\mu C\lambda)\right]\int_0^\infty dt\left(1+\frac{2ae^{-t}}{a^{-1}-e^{-t}a}\right). \quad (4.15)$$

However, the last expression includes the divergent integration $\int_0^\infty dt\ 1$, which can be interpreted as a contribution from the massless part ($L_0 = 0$) in the integration $\frac{1}{L_0} = \int_0^\infty dt\ e^{-tL_0}$. In deriving the equations of motion for the massive part, Eq. (4.3), we should multiply a projection $\mathcal{P}$, which subtracts the massless part, on the right-hand side, and this implies that we should replace $\frac{1}{L_0} \to \frac{1}{L_0}\mathcal{P}$ in the above. Namely, inserting the projection, we can subtract the divergent term as

$$\frac{1}{2}\left\langle\!\!\left\langle (\Psi_0)^2, \frac{b_0}{L_0}\mathcal{P}(\Psi_0)^2 \right\rangle\!\!\right\rangle = \frac{1}{4}\int d^{10}x \mathrm{Tr}\left[(\lambda^T\Gamma^\mu C\lambda)(\lambda^T\Gamma_\mu C\lambda)\right]\int_0^\infty dt\frac{2ae^{-t}}{a^{-1}-e^{-t}a}$$
$$= \frac{1}{2}\log\left(2(\sqrt{2}-1)\right)\int d^{10}x \mathrm{Tr}\left[(\hat{\lambda}^T C\Gamma^\mu\hat{\lambda})(\hat{\lambda}^T C\Gamma_\mu\hat{\lambda})\right], \quad (4.16)$$

where we have used Eqs. (4.9) and (3.25).

Eventually, including a contribution from the massive part to the massless effective action in both NS and R sectors, Eqs. (3.14) and (3.24), we have

$$S = \int d^{10}x\ \mathrm{Tr}\Bigg[-\frac{\alpha'}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - \frac{ig}{\sqrt{2\alpha'}}[A_\mu,A_\nu]\right)^2 - \frac{\sqrt{\alpha'}}{2}i\hat{\lambda}^T C\Gamma^\mu\left(\partial_\mu\hat{\lambda} - \frac{ig}{\sqrt{2\alpha'}}[A_\mu,\hat{\lambda}]\right)$$
$$+\frac{1}{2}\log\left(2(\sqrt{2}-1)\right)g^2(\hat{\lambda}^T C\Gamma^\mu\hat{\lambda})(\hat{\lambda}^T C\Gamma_\mu\hat{\lambda})\Bigg]. \quad (4.17)$$

Rewriting $\sqrt{\alpha'}A_\mu \to A_\mu,\ (\alpha')^{1/4}\hat{\lambda} \to \hat{\lambda}$, and $\frac{1}{\sqrt{2\alpha'}}g \to g$ for the canonical form, we obtain

$$S = \int d^{10}x\ \mathrm{Tr}\Bigg[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu,A_\nu]\right)^2 - \frac{1}{2}i\hat{\lambda}^T C\Gamma^\mu\left(\partial_\mu\hat{\lambda} - ig[A_\mu,\hat{\lambda}]\right)$$
$$+\log\left(2(\sqrt{2}-1)\right)\alpha' g^2(\hat{\lambda}^T C\Gamma^\mu\hat{\lambda})(\hat{\lambda}^T C\Gamma_\mu\hat{\lambda})\Bigg], \quad (4.18)$$

where the first line is the same as the action of ten-dimensional SYM and the second line can be regarded as an α'-correction due to a superstring.

Concerning the α'-correction, we comment on the term of the form $\alpha' g\mathrm{Tr}\left[F_{\mu\nu}F^{\nu\sigma}F_\sigma{}^\mu\right]$ for the field strength, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu,A_\nu]$, which was investigated in Refs. [13,14]. In this section, we have included only the zero-momentum sector for contributions from the massive part, and then if the above term exists, it should appear in the form $\alpha' g^4\mathrm{Tr}\left[i[A_\mu,A_\nu][A^\nu,A^\sigma][A_\sigma,A^\mu]\right]$, which is a higher order in g than Eq. (4.18). In this sense, it is necessary to perform further computations including the nonzero-momentum sector or higher-order terms in g in order to compare the action from SSFT with the non-Abelian Born–Infeld action [15].

5. Induced transformations at the lowest order

Here, we derive the gauge and spacetime supersymmetry transformations for massless component fields from those of string fields given in Refs. [3–5]. We perform explicit computations up to the lowest order for simplicity.

5.1. *Gauge transformation*

The linearized version of Eqs. (2.10) and (2.11) is given by

$$\delta_g^{(0)}\Phi = Q\Lambda + \eta\Omega, \qquad \delta_g^{(0)}\Psi = Q\lambda. \tag{5.1}$$

At the massless level, we have

$$\Lambda_\varepsilon = \int \frac{d^{10}k}{(2\pi)^{10}}\varepsilon(k)c\xi\partial\xi e^{-2\phi}e^{ik\cdot X}(0)|0\rangle, \qquad \Omega_\omega = \int \frac{d^{10}k}{(2\pi)^{10}}\omega(k)\xi e^{ik\cdot X}(0)|0\rangle \tag{5.2}$$

for the gauge transformation parameter string fields in the NS sector, Λ and Ω, and we have no states for λ-gauge transformation in the R sector at this level. In order to respect the partial gauge-fixing condition $\xi_0\Phi = 0$, we take the gauge parameter as $\omega(k) = \varepsilon(k) = \varepsilon^a(k)T_a$ and we have

$$Q\Lambda_\varepsilon + \eta\Omega_\varepsilon = \int \frac{d^{10}k}{(2\pi)^{10}}\left(-\alpha' k^2\varepsilon(k)\mathcal{V}_B(0) - \sqrt{2\alpha'}k_\mu\varepsilon(k)\mathcal{V}_A^\mu(0)\right)|0\rangle. \tag{5.3}$$

Then, from the linearized gauge transformation at the massless level,

$$\delta_\varepsilon^{(0)}(\Phi_A + \Phi_B) \equiv Q\Lambda_\varepsilon + \eta\Omega_\varepsilon, \tag{5.4}$$

we have obtained the induced transformation for component fields:

$$\delta_\varepsilon^{(0)}B(x) = \alpha'\partial^2\varepsilon(x), \qquad \delta_\varepsilon^{(0)}A_\mu(x) = i\sqrt{2\alpha'}\partial_\mu\varepsilon(x), \qquad \varepsilon(x) = \int \frac{d^{10}k}{(2\pi)^{10}}\varepsilon(k)e^{ik\cdot x}. \tag{5.5}$$

For the gaugino field $\hat{\lambda}^\alpha(x)$, the transformation is trivial: $\delta_\varepsilon^{(0)}\hat{\lambda}^\alpha = 0$ at the linearized level. In order to get a nontrivial transformation, we should include the interaction term of SSFT in the gauge transformation. Expanding Eq. (2.11) as

$$\delta_{g(\Lambda)}\Psi = X\eta F\Xi D_\eta\{F\Psi, \Lambda\} = X\eta\{\Psi, \Lambda\} + O(\Phi\Psi), \tag{5.6}$$

we define

$$\delta_\varepsilon^{(1)}\Psi_0 = X\eta\{\Psi_0, \Lambda_\varepsilon\}. \tag{5.7}$$

Since the star product of Ψ_0 and Λ_ε has $(n_{\rm gh}, n_{\rm pic}) = (0, -1/2)$ and its ϕ-charge is $-5/2$, we expand as

$$\{\Psi_0, \Lambda_\varepsilon\} = \int \frac{d^{10}k}{(2\pi)^{10}}|\varphi^{\dot\alpha}(k)\rangle(C^{-1})_{\dot\alpha\alpha}\langle\varphi^{c\alpha}(-k), \{\Psi_0, \Lambda_\varepsilon\}\rangle + \cdots \tag{5.8}$$

at the massless level, where

$$\varphi^{\dot\alpha}(k) = c\partial c\,\xi\partial\xi\, S^{\dot\alpha}_{(-5/2)}e^{ik\cdot X}(0)|0\rangle, \qquad \varphi^{c\alpha}(k) = c\eta S^\alpha_{(1/2)}e^{ik\cdot X}(0)|0\rangle, \tag{5.9}$$

which satisfy the normalization

$$\langle \varphi^{c\alpha}(k_1), \varphi^{\dot\alpha}(k_2)\rangle = C^{\alpha\dot\alpha}(2\pi)^{10}\delta^{10}(k_1+k_2). \tag{5.10}$$

Using the relations

$$\begin{aligned}\langle \varphi^{c\alpha}(k_1), \{\Psi_0, \Lambda_\varepsilon\}\rangle &= \int \frac{d^{10}k_2}{(2\pi)^{10}}\int \frac{d^{10}k_3}{(2\pi)^{10}}(-1)\,[\lambda_{\dot\alpha}(k_2), \varepsilon(k_3)] \\ &\quad \times C^{\alpha\dot\alpha}K^{\alpha'(k_1^2+k_2^2+k_3^2)}(2\pi)^d\delta^d(k_1+k_2+k_3), \end{aligned}\tag{5.11}$$

$$X\eta|\varphi^{\dot\alpha}(k)\rangle = -\mathcal{V}_\lambda^{\dot\alpha}(k)(0)|0\rangle, \tag{5.12}$$

we have obtained

$$\delta_\varepsilon^{(1)}\Psi_0 = \int \frac{d^{10}k}{(2\pi)^{10}}\delta_\varepsilon^{(1)}\lambda_{\dot\alpha}(k)\mathcal{V}_\lambda^{\dot\alpha}(k)(0)|0\rangle + \cdots, \tag{5.13}$$

$$\delta_\varepsilon^{(1)}\lambda_{\dot\alpha}(k) = \int \frac{d^{10}k_2}{(2\pi)^{10}}\int \frac{d^{10}k_3}{(2\pi)^{10}}\,[\lambda_{\dot\alpha}(k_2), \varepsilon(k_3)]\,K^{\alpha'(k^2+k_2^2+k_3^2)}(2\pi)^d\delta^d(k_2+k_3-k), \tag{5.14}$$

and hence the induced gauge transformation of the component field is

$$\delta_\varepsilon^{(1)}\hat\lambda^\alpha(x) = -\left[\varepsilon(x), \hat\lambda^\alpha(x)\right] \tag{5.15}$$

for small momentum: $K^{\alpha'(k^2+k_2^2+k_3^2)} \sim 1$. Equations (5.5) and (5.15) are consistent with the ordinary gauge transformation of the SYM.

5.2. *Supersymmetry transformation at the linearized level*

First, we derive explicit expressions for equations of motion for component fields from those for string fields. The equations of motion in Eq. (2.14) are linearized as

$$Q\eta\Phi = 0, \qquad Q\Psi = 0. \tag{5.16}$$

At the massless level, we have

$$\begin{aligned} Q\eta\Phi_0 = \int \frac{d^{10}k}{(2\pi)^{10}}\Big(&-2\big(B(k) - \sqrt{\frac{\alpha'}{2}}k^\mu A_\mu(k)\big)c\eta e^{ik\cdot X}(0)|0\rangle \\ &- \sqrt{2\alpha'}\big(k_\mu B(k) - \sqrt{\frac{\alpha'}{2}}k^2 A_\mu(k)\big)c\partial c e^{-\phi}\psi^\mu e^{ik\cdot X}(0)|0\rangle\Big), \end{aligned}\tag{5.17}$$

$$Q\Psi_0 = \int \frac{d^{10}k}{(2\pi)^{10}}\lambda_{\dot\alpha}\left(\alpha' k^2 c\partial c S^{\dot\alpha}_{(-1/2)} + i\sqrt{\alpha'}k_\mu(\Gamma^\mu)^{\dot\alpha}{}_\alpha \eta c S^\alpha_{(1/2)}\right)e^{ik\cdot X}(0)|0\rangle, \tag{5.18}$$

which imply that the induced linearized equations of motion are

$$B + i\sqrt{\frac{\alpha'}{2}}\partial^\mu A_\mu = 0, \qquad i\partial_\mu B - \sqrt{\frac{\alpha'}{2}}\partial^2 A_\mu = 0, \qquad \Gamma^\mu\partial_\mu\hat\lambda = 0, \tag{5.19}$$

in terms of component fields. The first two equations correspond to the Maxwell equation for the gauge field A_μ and the last equation corresponds to the Dirac equation, and hence they are consistent with SYM at the linearized level.

The spacetime supersymmetry transformations in Eqs. (2.15) and (2.16) are linearized as

$$\delta_{\mathcal{S}}^{(0)}\Phi = \mathcal{S}\Xi\Psi, \qquad \delta_{\mathcal{S}}^{(0)}\Psi = X\mathcal{S}\eta\Phi. \tag{5.20}$$

At the massless level, the first transformation is computed as

$$\begin{aligned}\delta_{\mathcal{S}}^{(0)}\Phi_0 &= \mathcal{S}\Xi\Psi_0\\ &= \int \frac{d^{10}k}{(2\pi)^{10}}\frac{i}{\sqrt{2}}(\epsilon^T\Gamma_\mu C\lambda)\mathcal{V}_A^\mu(k)(0)|0\rangle + \eta\Omega_0,\end{aligned} \tag{5.21}$$

where $\Omega_0 \equiv \xi_0\mathcal{S}(\Xi - \xi_0)\Psi_0$ is a kind of Ω-gauge transformation in Eq. (2.10). Then, up to Ω-gauge transformation, the induced linearized transformations of bosonic component fields are obtained:

$$\delta_{\mathcal{S}}^{(0)}A_\mu = \frac{1}{\sqrt{2}}\bar{\hat{\epsilon}}\Gamma_\mu\hat{\lambda}, \qquad \delta_{\mathcal{S}}^{(0)}B = 0, \tag{5.22}$$

where $\hat{\epsilon} = C\epsilon$ and $\bar{\hat{\epsilon}} = \hat{\epsilon}^T C$. For the second equation of Eq. (5.20), we have calculated as follows:

$$\begin{aligned}\delta_{\mathcal{S}}^{(0)}\Psi_0 &= X\mathcal{S}\eta\Phi_0\\ &= \int \frac{d^{10}k}{(2\pi)^{10}}\Bigg(\sqrt{\frac{\alpha'}{2}}\frac{1}{2}(k_\mu A_\nu(k) - k_\nu A_\mu(k))(\epsilon^T\Gamma^{\mu\nu})_{\dot{\alpha}}\\ &\quad + \big(B(k) + \sqrt{\frac{\alpha'}{2}}k^\mu A_\mu(k)\big)\epsilon_{\dot{\alpha}}\Bigg)\mathcal{V}_\lambda^{\dot{\alpha}}(k)(0)|0\rangle.\end{aligned} \tag{5.23}$$

This implies that the induced linearized transformation of the fermionic component field is given by

$$\delta_{\mathcal{S}}^{(0)}\hat{\lambda} = i\sqrt{\frac{\alpha'}{2}}\frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\Gamma^{\mu\nu}\hat{\epsilon} + (B + i\sqrt{\frac{\alpha'}{2}}\partial^\mu A_\mu)\hat{\epsilon}. \tag{5.24}$$

We find that the induced transformations of Eqs. (5.22) and (5.24) are consistent with the conventional supersymmetry transformation in the ten-dimensional SYM, up to the equations of motion in Eq. (5.19), at the linearized level.

6. Concluding remarks

In this paper, we have truncated the string fields in both NS and R sectors in the framework of Kunitomo and Okawa's SSFT up to the lowest level (massless level) and, by evaluating the action explicitly in terms of the component fields, we have obtained the ten-dimensional SYM action plus an extra $O(A_\mu^4)$ term. We have also investigated a contribution from the massive part in the lowest order in g in the zero-momentum sector and observed that the extra $O(A_\mu^4)$ cancels in the NS sector and instead an extra $O(\lambda_{\dot{\alpha}}^4)$ appears from the R sector, which can be interpreted as α'-correction.[6] We have derived the gauge transformation and the spacetime supersymmetry transformation for the massless component fields induced from those of string fields at the lowest order. Our explicit calculation implies that the lowest-level truncation of Kunitomo and Okawa's SSFT action is consistent with the ten-dimensional SYM theory.

We have some remaining issues. At the present stage, we have no explicit formula for the reality condition of the string fields including the R sector. We expect that the Majorana condition for the

[6] As noted at the end of Sect. 4, we should include the nonzero-momentum sector in order to discuss $O(\alpha')$-correction completely.

massless component field in the R sector, $\hat{\lambda}^\dagger \Gamma^0 = \hat{\lambda}^T C$, should be imposed by a consistent reality condition for string fields. It would be interesting to perform similar calculations in other SSFTs such as A_∞-type SSFT and (modified) cubic SSFT. Our concrete computations in terms of component fields might be useful to find new methods to construct solutions of the equation of motion such as SSFT version of the BPS condition.

We hope that our work becomes one of the steps toward physical applications of SSFT.

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Appendix A. Convention of the spin fields

Here, we summarize our convention for explicit computations including the R sector, which is based on the method developed in Ref. [12]. The worldsheet fermion ψ^μ ($\mu = 0, 1, \ldots, 9$) can be bosonized using ϕ^a ($a = 1, 2, \ldots, 5$) as

$$i\,2^{-1/2}(\psi^0 \mp \psi^1) \simeq e^{\pm\phi^1} c_{\pm e_1}, \tag{A.1}$$

$$2^{-1/2}(\psi^{2a-2} \mp i\psi^{2a-1}) \simeq e^{\pm\phi^a} c_{\pm e_a}, \qquad a = 2, 3, 4, 5, \tag{A.2}$$

where the operator product expansion (OPE) among ϕ^a is $\phi^a(z)\phi^b(w) \sim \delta^{a,b}\log(z-w)$, $a, b = 1, \ldots, 5$. Involving the bosonized ghost $\phi \equiv \phi^6$, such as $\phi^6(z)\phi^6(w) \sim -\log(z-w)$, the cocycle factor c_λ ($\lambda = \sum_{i=1}^6 \lambda^i e_i$) is defined by

$$c_\lambda = e^{i\pi \sum_{i,j=1}^6 \lambda^i M_{ij}[\partial\phi^j]_0}, \qquad [\partial\phi^i]_0 = \oint \frac{dz}{2\pi i}\,\partial\phi^i, \tag{A.3}$$

where M_{ij} ($i, j = 1, 2, \ldots, 6$) is given by the matrix

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & -1 & -1 & 1 & 0 \end{pmatrix}. \tag{A.4}$$

The GSO (+) projection can be expressed as

$$\frac{1 + (-1)^G}{2}, \qquad G = \sum_{i=1}^6 [\partial\phi^i]_0. \tag{A.5}$$

The spin fields with $n_{\rm pic} = \pm 1/2$ are expressed as

$$S^{\alpha}_{(\pm 1/2)} = e^{\sum_{i=1}^{5} A^i\phi^i \pm \frac{1}{2}\phi} c_{\sum_{i=1}^{5} A^i e_i \pm \frac{1}{2} e_6}, \qquad A^i = \pm\frac{1}{2}, \quad (i = 1, \ldots, 5), \quad \prod_{i=1}^{5} A^i > 0, \tag{A.6}$$

$$S^{\dot\alpha}_{(\pm 1/2)} = e^{\sum_{i=1}^{5} \dot A^i\phi^i \pm \frac{1}{2}\phi} c_{\sum_{i=1}^{5} \dot A^i e_i \pm \frac{1}{2} e_6}, \qquad \dot A^i = \pm\frac{1}{2}, \quad (i = 1, \ldots, 5), \quad \prod_{i=1}^{5} \dot A^i < 0. \tag{A.7}$$

In general, for $S_\lambda \equiv e^{\sum_{i=1}^{6} \lambda^i \phi^i} c_\lambda$ with $\lambda = \sum_{i=1}^{6} \lambda^i e_i$, the OPE is

$$S_\lambda(y) S_{\lambda'}(z) = (y-z)^{\lambda^i \eta_{ij} \lambda'^j} e^{\lambda^i \phi^i(y) + \lambda'^i \phi^i(z)} e^{i\pi \lambda^i M_{ij} \lambda'^j} c_{\lambda+\lambda'}, \tag{A.8}$$

where $\eta_{ij} = {\rm diag}(1,1,1,1,1,-1)$. Corresponding to the above convention, we define the Γ-matrix as

$$\Gamma^0 = -i\sigma_1 \otimes 1_2 \otimes 1_2 \otimes 1_2 \otimes 1_2, \qquad \Gamma^1 = \sigma_2 \otimes 1_2 \otimes 1_2 \otimes 1_2 \otimes 1_2, \tag{A.9}$$

$$\Gamma^2 = \sigma_3 \otimes \sigma_2 \otimes 1_2 \otimes 1_2 \otimes 1_2, \qquad \Gamma^3 = -\sigma_3 \otimes \sigma_1 \otimes 1_2 \otimes 1_2 \otimes 1_2, \tag{A.10}$$

$$\Gamma^4 = -\sigma_3 \otimes \sigma_3 \otimes \sigma_1 \otimes 1_2 \otimes 1_2, \qquad \Gamma^5 = -\sigma_3 \otimes \sigma_3 \otimes \sigma_2 \otimes 1_2 \otimes 1_2, \tag{A.11}$$

$$\Gamma^6 = -\sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_2 \otimes 1_2, \qquad \Gamma^7 = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_1 \otimes 1_2, \tag{A.12}$$

$$\Gamma^8 = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_1, \qquad \Gamma^9 = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_2, \tag{A.13}$$

and $\Gamma^{11} = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\Gamma^6\Gamma^7\Gamma^8\Gamma^9 = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3$, where σ_i $(i = 1, 2, 3)$ is the Pauli matrix, and we take

$$C = e^{\frac{3}{4}\pi i} \sigma_2 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_2, \qquad C^T = -C, \quad C^\dagger = C^{-1}. \tag{A.14}$$

Then, we have

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}, \qquad C^{-1}\Gamma^\mu C = -\Gamma^{\mu T}, \qquad (\Gamma^\mu)^\dagger = \Gamma^0 \Gamma^\mu \Gamma^0. \tag{A.15}$$

In the same way as the linear combination of the bosonization, Eqs. (A.1) and (A.2), we define

$$\Gamma^{\pm e_1} = \frac{i}{\sqrt{2}}(\Gamma^0 \mp \Gamma^1) = \sqrt{2}\sigma_\mp \otimes 1_2 \otimes 1_2 \otimes 1_2 \otimes 1_2, \tag{A.16}$$

$$\Gamma^{\pm e_2} = \frac{1}{\sqrt{2}}(\Gamma^2 \mp i\Gamma^3) = \pm\sqrt{2} i\sigma_3 \otimes \sigma_\mp \otimes 1_2 \otimes 1_2 \otimes 1_2, \tag{A.17}$$

$$\Gamma^{\pm e_3} = \frac{1}{\sqrt{2}}(\Gamma^4 \mp i\Gamma^5) = -\sqrt{2}\sigma_3 \otimes \sigma_3 \otimes \sigma_\mp \otimes 1_2 \otimes 1_2, \tag{A.18}$$

$$\Gamma^{\pm e_4} = \frac{1}{\sqrt{2}}(\Gamma^6 \mp i\Gamma^7) = \mp\sqrt{2} i\sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_\mp \otimes 1_2, \tag{A.19}$$

$$\Gamma^{\pm e_5} = \frac{1}{\sqrt{2}}(\Gamma^8 \mp i\Gamma^9) = \sqrt{2}\sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_\mp, \tag{A.20}$$

where $\sigma_+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$, $\sigma_- = \frac{1}{2}(\sigma_1 - i\sigma_2)$, and they can be rewritten as

$$\Gamma^{\pm e_j} = (\pm i)^{j-1} \sqrt{2} (\sigma_3 \otimes)^{j-1} \sigma_\mp (\otimes 1_2)^{5-j} \qquad j = 1, 2, 3, 4, 5. \tag{A.21}$$

Using the above equations, we find

$$\Gamma^{\pm e_j} = \begin{pmatrix} 0 & (\Gamma^{\pm e_j})^{\alpha}{}_{\dot\beta} \\ (\Gamma^{\pm e_j})^{\dot\alpha}{}_{\beta} & 0 \end{pmatrix}, \quad j = 1,2,3,4,5, \tag{A.22}$$

$$(\Gamma^{\pm e_j})^{\alpha}{}_{\dot\beta} = \delta_{\pm e_j + A, \dot B}\sqrt{2}\, e^{\pm i\pi \sum_{k=1}^{5} M_{jk}\dot B^k}, \qquad (\Gamma^{\pm e_j})^{\dot\alpha}{}_{\beta} = \delta_{\pm e_j + \dot A, B}\sqrt{2}\, e^{\pm i\pi \sum_{k=1}^{5} M_{jk} B^k}, \tag{A.23}$$

$$C = \begin{pmatrix} 0 & C^{\alpha\dot\beta} \\ C^{\dot\alpha\beta} & 0 \end{pmatrix}, \tag{A.24}$$

$$C^{\alpha\dot\beta} = \delta_{A+\dot B,0}\, e^{-i\pi \sum_{i,j=1}^{6} A_+^i M_{ij} A_+^j}, \qquad A_+ \equiv (A^i, 1/2), \tag{A.25}$$

$$C^{\dot\alpha\beta} = -\delta_{\dot A+B,0}\, e^{-i\pi \sum_{i,j=1}^{6} \dot A_-^i M_{ij} \dot A_-^j}, \qquad \dot A_- \equiv (\dot A^i, -1/2), \tag{A.26}$$

$$e^{i\pi \sum_{j=1}^{5} M_{6j} A^j} = i, \quad e^{i\pi \sum_{j=1}^{5} M_{6j} \dot A^j} = -i, \quad (\Gamma^\mu C)^{\alpha\beta} = (\Gamma^\mu C)^{\beta\alpha}, \quad (\Gamma^\mu C)^{\dot\alpha\dot\beta} = (\Gamma^\mu C)^{\dot\beta\dot\alpha}, \tag{A.27}$$

and, furthermore, for $C^{-1} = \begin{pmatrix} 0 & C^{-1}_{\alpha\dot\beta} \\ C^{-1}_{\dot\alpha\beta} & 0 \end{pmatrix}$,

$$C^{-1}_{\alpha\dot\beta} = -\delta_{A+\dot B,0}\, e^{i\pi \sum_{i,j=1}^{6} A_+^i M_{ij} A_+^j}, \qquad C^{-1}_{\dot\alpha\beta} = \delta_{\dot A+B,0}\, e^{i\pi \sum_{i,j=1}^{6} \dot A_-^i M_{ij} \dot A_-^j}, \tag{A.28}$$

$$C^{\alpha\dot\beta} C^{-1}_{\dot\beta\gamma} = \delta^\alpha_\gamma, \quad C^{\dot\alpha\beta} C^{-1}_{\beta\dot\gamma} = \delta^{\dot\alpha}_{\dot\gamma}, \quad (C^{-1}\Gamma^\mu)_{\alpha\beta} = (C^{-1}\Gamma^\mu)_{\beta\alpha}, \quad (C^{-1}\Gamma^\mu)_{\dot\alpha\dot\beta} = (C^{-1}\Gamma^\mu)_{\dot\beta\dot\alpha}. \tag{A.29}$$

Here, we should note that the correspondence of the spinor index is

$$\alpha \leftrightarrow A = (\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2) = \sum_{i=1}^{5} A^i e_i, \quad \prod_{i=1}^{5} A^i > 0, \tag{A.30}$$

$$\dot\alpha \leftrightarrow \dot A = (\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2) = \sum_{i=1}^{5} \dot A^i e_i, \quad \prod_{i=1}^{5} \dot A^i < 0, \tag{A.31}$$

for undotted and dotted spinors.

For the spin field with $n_{\text{pic}} = r$,

$$S^{\hat\alpha}_{(r)} = e^{\sum_{i=1}^{5} \hat A^i \phi^i + r\phi} c_{\sum_{i=1}^{5} \hat A^i e_i + r e_6}, \qquad \alpha = (\alpha, \dot\alpha) \leftrightarrow \hat A, \tag{A.32}$$

we have the OPE:

$$\psi^\mu(y) S^{\hat\alpha}_{(r)}(z) \sim (y-z)^{-\frac{1}{2}} \frac{1}{\sqrt{2}} (\Gamma^\mu)^{\hat\alpha}{}_{\hat\beta} S^{\hat\beta}_{(r)}(z).$$

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