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Holographic conductivity in the massive gravity with power-law Maxwell field

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ABSTRACT

We obtain a new class of topological black hole solutions in $(n+1)$-dimensional massive gravity in the presence of the power-Maxwell electrodynamics. We calculate the conserved and thermodynamic quantities of the system and show that the first law of thermodynamics is satisfied on the horizon. Then, we investigate the holographic conductivity for the four and five dimensional black brane solutions. For completeness, we study the holographic conductivity for both massless $(m=0)$ and massive $(m \neq 0)$ gravities with power-Maxwell field. The massless gravity enjoys translational symmetry whereas the massive gravity violates it. For massless gravity, we observe that the real part of conductivity, Re$[\sigma]$, decreases as charge q increases when frequency ω tends to zero, while the imaginary part of conductivity, Im$[\sigma]$, diverges as $\omega \to 0$. For the massive gravity, we find that Im$[\sigma]$ is zero at $\omega = 0$ and becomes larger as q increases (temperature decreases), which is in contrast to the massless gravity. It also has a maximum value for $\omega \neq 0$ which increases with increasing q (with fixed p) or increasing p (with fixed q) for $(2+1)$-dimensional dual system, where p is the power parameter of the power-law Maxwell field. Interestingly, we observe that in contrast to the massless case, Re$[\sigma]$ has a maximum value at $\omega = 0$ (known as the Drude peak) for $p = (n+1)/4$ (conformally invariant electrodynamics) and this maximum increases with increasing q. In this case $(m \neq 0)$ and for different values of p, the real and imaginary parts of the conductivity has a relative extremum for $\omega \neq 0$. Finally, we show that for high frequencies, the real part of the holographic conductivity have the power law behavior in terms of frequency, ω^a where $a \propto (n+1-4p)$. Some similar behaviors for high frequencies in possible dual CFT systems have been reported in experimental observations.

1. Introduction

A century after Einstein's discovery namely general relativity, the domain of its applications has become as vast as it covers even condensed matter physics which seemed at the opposite end of physics building compared to gravity [1]. This strange topic which connects gravity to almost all fields of physics (see [2]) is called gauge/gravity duality (GGD); the extended version of AdS/CFT correspondence [3]. GGD has attracted increasing interests during recent years and become one of the most promising fields of physics which is hoped to be able to solve many of unsolved problems in different fields of physics including condensed matter physics.

Real materials in condensed matter physics do not respect the translational symmetry i.e. there is a dissipation in momentum. The momentum dissipation may come from the existence of a lattice or impurities. Although this dissipation has no important influence on the values of some observable, it affects the behavior of some others for instance conductivity. The DC conductivity in the presence of translational symmetry diverges, whereas in the absence of this symmetry (when momentum is dissipating) it has a finite value. In the context of GGD, it is important to study a gravity model which includes holographic momentum dissipation. There are some attempts to construct such gravity model [4]. One of these models proposed by D. Vegh [5], provides an effective bulk description of a theory in which momentum is no longer

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conserved. The conservation of momentum is due to the diffeomorphism invariance of stress–energy tensor in dual theory. In [5], the proposal is to break this symmetry holographically by giving a mass to graviton state. The resulting gravity is therefore *massive gravity*. One of the advantages of this theory is that the black hole solutions of it are solvable analytically and therefore it is an excellent toy model to study holographically the properties of materials without momentum conservation.

Thermal behaviors of black hole solutions in the context of massive gravity was explored extensively in recent years [5–8]. Thermodynamics of linearly charged massive black branes has been investigated in [5]. In [6], a class of higher-dimensional linearly charged solutions with positive, negative and zero constant curvature of horizon in the context of massive gravity accompanied by a negative cosmological constant has been presented and thermodynamics and phase structure of these black solutions have been studied in both canonical and grand canonical ensembles. In [7], van der Waals phase transitions of linearly charged black holes in massive gravity have been investigated and it has been shown that the massive gravity can present substantially different thermodynamic behavior in comparison with Einstein gravity. Also it has been shown that the graviton mass can cause a range of new phase transitions for topological black holes which are forbidden for other cases. The properties of massive solutions have been studied in different scenarios [9]. From holographic point of view, the behaviors of different holographic quantities have been studied in [5,10–22]. The behavior of holographic conductivity for systems dual to linearly charged massive black branes has been explored in [5]. In [11], a holographic superconductor has been constructed in the massive gravity background. [13] studies holographic superconductor-normal metal-superconductor Josephon junction in the massive gravity. Also the holographic thermalization process has been investigated in this context [14]. Analytic DC thermo-electric conductivities in the context of massive gravity have been calculated in [12]. In massive Einstein–Maxwell-dilaton gravity, DC and Hall conductivities have been computed in [15]. [16] presents a holographic model for insulator/metal phase transition and colossal magnetoresistance within massive gravity. Inspired by the recent action/complexity duality conjecture, it has been shown in [22] that the holographic complexity grows linearly with time in the context of massive gravity.

As we mentioned above, one of the quantities which is affected by momentum dissipation is conductivity. On the other hand, the choice of electrodynamics model has a direct influence on the behavior of conductivity. So, it is worthy to consider the effects of nonlinearity as well as massive gravity on the conductivity of the black hole solutions. It is well-known that the nonlinear electrodynamics brings reach physics compared to the linear Maxwell electrodynamics. For example, Maxwell theory is conformally invariant only in four dimensions and thus the corresponding energy–momentum tensor is only traceless in four dimensions. A natural question then arises: Is there an extension of Maxwell action in arbitrary dimensions that is traceless and hence possesses the conformal invariance? The answer is positive and the invariant Maxwell action under conformal transformation $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $A_\mu \to A_\mu$ in $(n+1)$-dimensions is given by [23],

$$S_m = \int d^{n+1}x\sqrt{-g}(-\mathcal{F})^p,$$

where $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ is the Maxwell invariant, provided $p = (n+1)/4$. The associated energy–momentum tensor of the above Maxwell action is given by

$$T_{\mu\nu} = 2\left(pF_{\mu\eta}F_\nu^{\ \eta}\mathcal{F}^{p-1} - \frac{1}{4}g_{\mu\nu}\mathcal{F}^p\right). \tag{1}$$

One can easily check that the above energy–momentum tensor is traceless for $p = (n+1)/4$. Also, quantum electrodynamics predicts that the electrodynamic field behaves nonlinearly through the presence of virtual charged particles that is reported by Heisenberg and Euler [24]. Hence, nonlinear electrodynamics has been subject of much researches [25–27]. This motivates us to extend the linearly charged black hole solutions of massive gravity [5,6] to nonlinearly charged ones in the presence of power-law Maxwell electrodynamics and investigate the thermodynamics of them as well as the behavior of conductivity corresponding to the dual system. In addition to power-law Maxwell electrodynamics, other types of nonlinear electrodynamics have been introduced in [28–30]. In spite of the special property for $p = (n+1)/4$, different aspects of various solutions have been investigated for different p's [31–33]. In the context of AdS/CFT correspondence, the power-law Maxwell field has been considered as electrodynamics source in [34–39].

The layout of this letter is as follows. In section 2, we present the action of the massive gravity in the presence of power-Maxwell electrodynamics and then by varying the action we obtain the field equations. We also derive a class of topological black hole solutions of the field equations in higher dimensions. In section 3, we study thermodynamics of the solutions and examine the first law of thermodynamics for massive black holes with power-law Maxwell field. In section 4, we investigate the holographic conductivity of black brane solutions in the presence of a power-law Maxwell gauge field. In particular, we shall disclose the effects of the power-law Maxwell electrodynamics as well as massive gravity on the holographic conductivity of dual systems. We finish with closing remarks in section 5.

2. Action and massive gravity solutions

The $(n+1)$-dimensional $(n \geq 3)$ action describing Einstein-massive gravity accompanied by a negative cosmological constant Λ in the presence of power-law Maxwell electrodynamics is

$$\mathcal{S} = \int d^{n+1}x\mathcal{L}, \tag{2}$$

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}\left[\mathcal{R} - 2\Lambda + (-\mathcal{F})^p + m^2\sum_i^4 c_i\mathcal{U}_i(g,\Gamma)\right], \tag{3}$$

where g and $\mathcal{R}$ are respectively the determinant of the metric and the Ricci scalar and $\Lambda = -n(n-1)/2l^2$ is the negative cosmological constant where l is the AdS radius. $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$ is electrodynamic tensor where A_ν is vector potential. p determines the nonlinearity of the electrodynamic field. For $p = 1$, the linear Maxwell gauge field will be recovered. In action (2), Γ is the reference metric, c_i's are constants and $\mathcal{U}_i$'s are symmetric polynomials of eigenvalues of the $(n+1)\times(n+1)$ matrix $\mathcal{K}^\mu_\nu \equiv \sqrt{g^{\mu\alpha}\Gamma_{\alpha\nu}}$ so that

$$\mathcal{U}_1 = [\mathcal{K}], \tag{4}$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - \left[\mathcal{K}^2\right], \tag{5}$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}]\left[\mathcal{K}^2\right] + 2\left[\mathcal{K}^3\right], \tag{6}$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6\left[\mathcal{K}^2\right][\mathcal{K}]^2 + 8\left[\mathcal{K}^3\right][\mathcal{K}] + 3\left[\mathcal{K}^2\right]^2 - 6\left[\mathcal{K}^4\right], \tag{7}$$

where the square root in $\mathcal{K}$ is related to mean matrix square root i.e. $\left(\sqrt{\mathcal{K}}\right)^\mu_{\ \nu}\left(\sqrt{\mathcal{K}}\right)^\nu_{\ \lambda} = \mathcal{K}^\mu_\lambda$ and rectangular brackets mean trace $[\mathcal{K}] \equiv \mathcal{K}^\mu_\mu$. Here m is the massive gravity parameter so that in limit $m \to 0$, one recovers the diffeomorphism invariant Einstein–Hilbert action with a gauge field and a negative cosmological constant. The equations of motion for gravitation and gauge field are

$$R_{\mu\nu}-\frac{1}{2}\mathcal{R}g_{\mu\nu}+\Lambda g_{\mu\nu}-2pF_{\mu\lambda}F_{\nu}{}^{\lambda}\left(-\mathcal{F}\right)^{p-1}-\frac{1}{2}\left(-\mathcal{F}\right)^{p}g_{\mu\nu}+m^{2}\chi_{\mu\nu}=0, \tag{8}$$

$$\nabla_{\mu}\left(\mathcal{F}^{p-1}F^{\mu\nu}\right)=0, \tag{9}$$

which are obtained by varying the action (2) with respect to the metric tensor $g_{\mu\nu}$ and gauge field A_μ respectively. In Eq. (8), we have

$$\begin{aligned}\chi_{\mu\nu}=&-\frac{c_1}{2}\left(\mathcal{U}_1 g_{\mu\nu}-\mathcal{K}_{\mu\nu}\right)-\frac{c_2}{2}\left(\mathcal{U}_2 g_{\mu\nu}-2\mathcal{U}_1\mathcal{K}_{\mu\nu}+2\mathcal{K}_{\mu\nu}^2\right)\\&-\frac{c_3}{2}(\mathcal{U}_3 g_{\mu\nu}-3\mathcal{U}_2\mathcal{K}_{\mu\nu}+6\mathcal{U}_1\mathcal{K}_{\mu\nu}^2-6\mathcal{K}_{\mu\nu}^3)\\&-\frac{c_4}{2}(\mathcal{U}_4 g_{\mu\nu}-4\mathcal{U}_3\mathcal{K}_{\mu\nu}+12\mathcal{U}_2\mathcal{K}_{\mu\nu}^2\\&-24\mathcal{U}_1\mathcal{K}_{\mu\nu}^3+24\mathcal{K}_{\mu\nu}^4).\end{aligned} \tag{10}$$

The static spacetime line element takes the usual form

$$ds^2=-f(r)dt^2+f^{-1}(r)dr^2+r^2h_{ij}dx^idx^j, \tag{11}$$

where $f(r)$ is the metric function and h_{ij} is a function of coordinates x_i which spanned an $(n-1)$-dimensional hypersurface with constant scalar curvature $(n-1)(n-2)k$ and volume ω_{n-1}. Without loss of generality, one can take $k=0,1,-1$, such that the black hole horizon or cosmological horizon in (11) can be a zero (flat), positive (elliptic) or negative (hyperbolic) constant curvature hypersurface. The reference metric (fixed symmetric tensor) $\Gamma_{\mu\nu}$ can be considered as [5,6]

$$\Gamma_{\mu\nu}=\mathrm{diag}(0,0,c_0^2h_{ij}), \tag{12}$$

where c_0 is a positive constant. Using (11) and (12), one can easily calculates $\mathcal{U}_i$'s as

$$\begin{aligned}\mathcal{U}_1&=\frac{(n-1)c_0}{r},\\ \mathcal{U}_2&=\frac{(n-1)(n-2)c_0^2}{r^2},\\ \mathcal{U}_3&=\frac{(n-1)(n-2)(n-3)c_0^3}{r^3},\\ \mathcal{U}_4&=\frac{(n-1)(n-2)(n-3)(n-4)c_0^4}{r^4}.\end{aligned} \tag{13}$$

Notice that $\mathcal{U}_3$ and $\mathcal{U}_4$ vanish for $(3+1)$-dimensional spacetime while $\mathcal{U}_4=0$ for $(4+1)$-dimensional spacetime. Using the metric (11), the electrodynamic field can be immediately found as

$$F_{tr}=-F_{rt}=\frac{q}{r^{(n-1)/(2p-1)}}, \tag{14}$$

where q is a constant parameter related to the total charge of black hole. Inserting Eqs. (12), (13) and (14) into field equations (8), one receives

$$\begin{aligned}&\frac{f'}{r}+\frac{(n-2)f}{r^2}-\frac{(n-2)k}{r^2}+\frac{2\Lambda}{n-1}+\frac{2p-1}{n-1}\left(2q^2r^{-\frac{2n-2}{2p-1}}\right)^p\\&-\frac{c_0m^2}{r}\left(c_1+\frac{(n-2)c_0c_2}{r}+\frac{(n-2)(n-3)c_0^2c_3}{r^2}+\frac{(n-2)(n-3)(n-4)c_0^3c_4}{r^3}\right)=0,\end{aligned} \tag{15}$$

$$\begin{aligned}&f''+\frac{2(n-2)f'}{r}+\frac{(n-2)(n-3)f}{r^2}-\frac{(n-2)(n-3)k}{r^2}\\&+2\Lambda-\left(2q^2r^{-\frac{2n-2}{2p-1}}\right)^p-\frac{(n-2)c_0m^2}{r}\left(c_1+\frac{(n-3)c_0c_2}{r}+\frac{(n-3)(n-4)c_0^2c_3}{r^2}+\frac{(n-3)(n-4)(n-5)c_0^3c_4}{r^3}\right)=0,\end{aligned} \tag{16}$$

where prime denotes the derivative with respect to r. Solving above equations, $f(r)$ can be obtained as

$$\begin{aligned}f(r)=&k-\frac{m_0}{r^{n-2}}-\frac{2\Lambda r^2}{n(n-1)}+\frac{2^pq^{2p}(2p-1)^2}{(n-1)(n-2p)r^{2(np-3p+1)/(2p-1)}}\\&+\frac{c_0m^2r}{n-1}\left(c_1+\frac{(n-1)c_0c_2}{r}+\frac{(n-1)(n-2)c_0^2c_3}{r^2}+\frac{(n-1)(n-2)(n-3)c_0^3c_4}{r^3}\right),\end{aligned} \tag{17}$$

where m_0 is an integration constant which is related to total mass of black hole as we see later. One may note that the metric function (17) reduces to those of Refs. [5,6] in the case $p=1$. Also the solution (17), in the absent of massive parameter ($m=0$), leads to

$$f_0(r)=k-\frac{m_0}{r^{n-2}}-\frac{2\Lambda r^2}{n(n-1)}+\frac{2^pq^{2p}(2p-1)^2}{(n-1)(n-2p)r^{2(np-3p+1)/(2p-1)}}, \tag{18}$$

which was presented in [32]. The mass parameter (m_0) in Eq. (17) can be found as

$$\begin{aligned}m_0=&kr_+^{n-2}-\frac{2\Lambda r_+^n}{n(n-1)}+\frac{2^pq^{2p}(2p-1)^2}{(n-1)(n-2p)r_+^{(n-2p)/(2p-1)}}\\&+\frac{c_0m^2r_+^{n-1}}{n-1}\left(c_1+\frac{(n-1)c_0c_2}{r_+}+\frac{(n-1)(n-2)c_0^2c_3}{r_+^2}+\frac{(n-1)(n-2)(n-3)c_0^3c_4}{r_+^3}\right),\end{aligned} \tag{19}$$

where r_+ is the radius of the event horizon given by the largest root of $f(r_+)=0$. According to Eq. (14) and regarding $A_t(r)=\int F_{rt}dr$, the gauge potential A_t can be calculated as

$$A_t\left(r\right)=\mu+\frac{q(2p-1)}{(n-2p)r^{(n-2p)/(2p-1)}}. \tag{20}$$

In (20), μ is the chemical potential of the quantum field theory locates on boundary which can be found by demanding the regularity condition on the horizon i.e. $A_t\left(r_+\right)=0$ as

$$\mu=\frac{q(2p-1)}{(2p-n)r_+^{(n-2p)/(2p-1)}}. \tag{21}$$

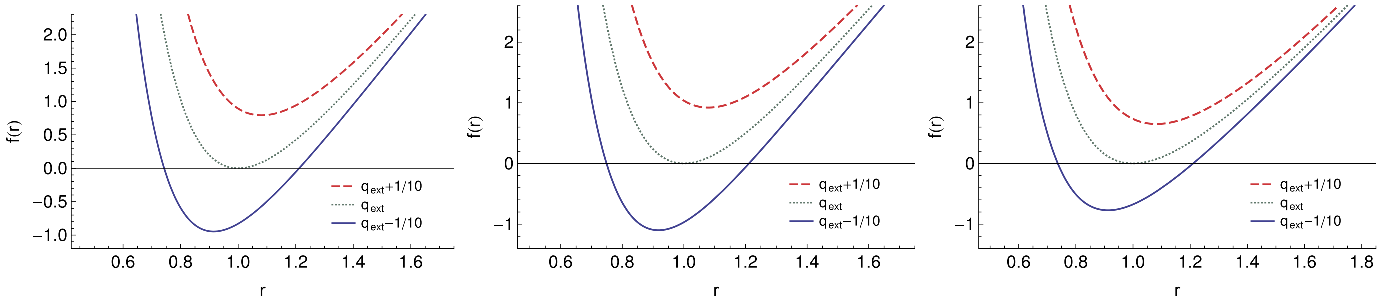
One should note that the electric potential $A_t\left(r\right)$ has a finite value at infinity ($r\to\infty$) provided the parameter p is restricted as

$$\frac{1}{2}<p<\frac{n}{2}, \tag{22}$$

obtained from $(n-2p)/(2p-1)>0$. One can also obtain the electric potential as

$$U=A_\nu\chi^\nu\left|_{r\to ref}-A_\nu\chi^\nu\right|_{r=r_+}, \tag{23}$$

where $\chi=C\partial_t$ is the null generator of the horizon and C is a constant. When one applies the power-law Maxwell electrodynamics,



(a) $k=0,\ q_{\mathrm{ext}}=2.03$ (b) $k=1,\ q_{\mathrm{ext}}=2.25$ (c) $k=-1,\ q_{\mathrm{ext}}=1.78$

Fig. 1. The behavior of $f(r)$ versus r for $n=4$, $l=1$, $p=5/4$, $m=1$, $r_+=1$, $c_0=1$, $c_1=1$, $c_2=3/2$, $c_3=-1/2$ and $c_4=1$.

it is common to use a general Killing vector with a constant C [40,41]. This is due to the fact that every linear combination of Killing vectors is also a Killing vector. Then, C is fixed so that the first law of thermodynamics is satisfied [40,41]. For linear Maxwell case ($p=1$), the constant C reduces to 1. Choosing infinity as the reference point, one can calculate the electric potential energy

$$U=C\mu. \tag{24}$$

One can obtain the Hawking temperature of the black hole on the event horizon as

$$\begin{aligned} T &= \frac{f'(r_+)}{4\pi} \\ &= \frac{(n-2)k}{4\pi r_+} - \frac{2\Lambda r_+}{4\pi(n-1)} + \frac{2^p q^{2p}(1-2p)}{4\pi(n-1)r_+^{(2p[n-2]+1)/(2p-1)}} \\ &\quad + \frac{c_0 m^2}{4\pi}\left(c_1 + \frac{(n-2)c_0c_2}{r_+} + \frac{(n-2)(n-3)c_0^2c_3}{r_+^2} \right. \\ &\quad \left. + \frac{(n-2)(n-3)(n-4)c_0^3c_4}{r_+^3}\right). \end{aligned} \tag{25}$$

The extremal black hole, whose temperature vanishes, can be also determined by an extremal charge,

$$\begin{aligned} q_{\mathrm{ext}}^{2p} &= \frac{(n-1)(n-2)r_{\mathrm{ext}}^{2[p(n-3)+1]/(2p-1)}}{(2p-1)2^p} - \frac{\Lambda r_{\mathrm{ext}}^{2p(n-1)/(2p-1)}}{(2p-1)2^{p-1}} \\ &\quad + \frac{c_0 m^2 (n-1) r_{\mathrm{ext}}^{[2p(n-2)+1]/(2p-1)}}{(2p-1)2^p}\left(c_1 + \frac{(n-2)c_0c_2}{r_{\mathrm{ext}}} \right. \\ &\quad \left. + \frac{(n-2)(n-3)c_0^2c_3}{r_{\mathrm{ext}}^2} + \frac{(n-2)(n-3)(n-4)c_0^3c_4}{r_{\mathrm{ext}}^3}\right). \end{aligned} \tag{26}$$

For $q>q_{\mathrm{ext}}$, there is a naked singularity in spacetime while $q<q_{\mathrm{ext}}$ describes solutions with two inner and outer horizons (r_+ and r_-). These two horizons degenerate for $q=q_{\mathrm{ext}}$. The behaviors of the metric function $f(r)$ versus r for different topologies of horizon are depicted in Fig. 1.

Up to now, we have obtained the higher-dimensional black hole solutions in the context of massive gravity and in the presence of power-law Maxwell gauge field. In the next section, we will study the thermodynamics of the obtained solutions. To do that, we shall obtain the Smarr-type formula and check the satisfaction of the first law of black holes thermodynamics.

3. Thermodynamics of massive gravity

The main purpose of this section is to examine the first law of thermodynamics for massive black holes with power-law Maxwell field. It was shown that the entropy of black holes in massive gravity still obeys the area law [6]. It is easy to show that the entropy of black hole per unit volume ω_{n-1} as an extensive quantity of thermodynamics is given by [6]

$$S=\frac{r_+^{n-1}}{4}, \tag{27}$$

which is a quarter of the event horizon area [6,42]. The electric charge of black hole per unit volume ω_{n-1} can be calculated through the use of Gauss law

$$Q=\frac{1}{4\pi}\int r^{n-1}(-\mathcal{F})^{p-1}F_{\mu\nu}n^{\mu}u^{\nu}dr, \tag{28}$$

where n^{μ} and u^{ν} are respectively the unit spacelike and timelike normals to the hypersurface of radius r defined by

$$n^{\mu}=\frac{1}{\sqrt{-g_{tt}}}dt=\frac{1}{\sqrt{f(r)}}dt,\quad u^{\nu}=\frac{1}{\sqrt{g_{rr}}}dr=\sqrt{f(r)}dr. \tag{29}$$

Thus, one can obtain

$$Q=\frac{2^{p-1}q^{2p-1}}{4\pi}. \tag{30}$$

In order to obtain the mass of black holes in massive gravity one can apply the Hamiltonian approach presented in Ref. [6]. The total mass (M) of massive black hole per unit volume ω_{n-1} can be calculated as [6]

$$M=\frac{(n-1)m_0}{16\pi}, \tag{31}$$

where m_0 as a function of the horizon radius r_+ was given in Eq. (19). In order to check the first law of thermodynamic, we need to compute Smarr-type formula for mass M as a function of extensive quantities entropy and electric charge. Using relations (27), (30) and (31), one can obtain the Smarr-type formula for mass as

$$\begin{aligned} M(S,Q) &= \frac{k(n-1)(4S)^{(n-2)/(n-1)}}{16\pi} - \frac{\Lambda(4S)^{n/(n-1)}}{8\pi n} \\ &\quad + \frac{Q^{2p/(2p-1)}(2p-1)^2}{2(n-2p)(4S)^{\frac{n-2p}{(n-1)(2p-1)}}}\left(\frac{\pi}{2^{p-3}}\right)^{1/(2p-1)} \\ &\quad + \frac{c_0 m^2 S}{4\pi}\left(c_1 + \frac{(n-1)c_0c_2}{(4S)^{1/(n-1)}} + \frac{(n-1)(n-2)c_0^2c_3}{(4S)^{2/(n-1)}} \right. \\ &\quad \left. + \frac{(n-1)(n-2)(n-3)c_0^3c_4}{(4S)^{3/(n-1)}}\right). \end{aligned} \tag{32}$$

Now, one can show that the thermodynamic quantities satisfy the first law of thermodynamic as

$$dM = TdS + UdQ, \tag{33}$$

in which

$$T = \left(\frac{\partial M}{\partial S}\right)_Q \quad \text{and} \quad U = \left(\frac{\partial M}{\partial Q}\right)_S, \tag{34}$$

provided $C = p$ in (24). As it is clear, for linear Maxwell case ($p = 1$), the constant C is reduced to 1. In the remainder of this work, we study the effect of power-law Maxwell electrodynamics on the holographic conductivity of dual systems with and without translational symmetry.

4. Holographic conductivity

In this section, we will obtain the electrical transport behavior of the dual field theory in the presence of a power-law Maxwell gauge field. In order to do this, one should use the solution of the black brane ($k = 0$) found in the previous section. First, we investigate the effects of the power-law Maxwell electrodynamics on the holographic conductivity of dual systems in which momentum is conserved ($m = 0$). Next, we consider the solutions dual to the systems which no longer possess momentum conservation ($m \neq 0$).

4.1. Vanishing m

The planer $(n+1)$-dimensional metric can be rewritten as

$$ds^2 = -\mathcal{F}(u)dt^2 + l^2\mathcal{F}(u)^{-1}u^{-4}du^2 + l^2u^{-2}\sum_{i=1}^{n-1}dx_i^2, \tag{35}$$

which is given by defining $u = lr^{-1}$ in the metric (11). Accordingly, the event horizon of black brane is at $u_+ = lr_+^{-1}$ and the n-dimensional thermal field theory lives at $u = 0$. The metric function of spacetime in absence of massive parameter is

$$\mathcal{F}(u) = -m_0 l^{2-n}u^{n-2} + u^{-2} + 2^p q^{2p}(2p-1)^2(n-1)^{-1} \times (n-2p)^{-1}\left[l^{-1}u\right]^{2(np-3p+1)/(2p-1)}, \tag{36}$$

obtained by substituting $r = lu^{-1}$ and $k = 0$ in Eq. (18). Perturbing the vector potential component A_x and the metric component g_{tx} by turning on $a_x(u)e^{-i\omega t}$ and $g_{tx}(u)e^{-i\omega t}$ respectively, we can easily derive two linear equations of motion for electrodynamics

$$a_x'' + \left((8p-n-3)(2p-1)^{-1}u^{-1} + \mathcal{F}'\mathcal{F}^{-1}\right)a_x' + l^2\omega^2u^{-4}\mathcal{F}^{-2}a_x + h'\mathcal{F}^{-1}\left(g_{tx}' + 2u^{-1}g_{tx}\right) = 0, \tag{37}$$

and for gravity

$$g_{tx}' + 2u^{-1}g_{tx} + 2^{p+1}ph'\left(u^4l^{-2}h'^2\right)^{p-1}a_x = 0, \tag{38}$$

where now the prime means derivative with respect to u and $h(u)$ is electric potential in the form

$$h(u) = \mu + \frac{q(2p-1)u^{(n-2p)/(2p-1)}}{(n-2p)l^{(n-2p)/(2p-1)}}, \tag{39}$$

which is obtained by transforming $r \to lu^{-1}$ in Eq. (20). By eliminating g_{tx} between Eqs. (37) and (38), the differential equation for a_x is

$$a_x'' + \left((8p-n-3)(2p-1)^{-1}u^{-1} + \mathcal{F}'\mathcal{F}^{-1}\right)a_x' + a_x\mathcal{F}^{-1}\left(l^2\omega^2u^{-4}\mathcal{F}^{-1} - 2^{p+1}ph'^2\left(u^4l^{-2}h'^2\right)^{p-1}\right) = 0. \tag{40}$$

The behavior of above relation near the boundary ($u \to 0$) is

$$a_x'' + (4p-n-1)(2p-1)^{-1}u^{-1}a_x' + \cdots = 0, \tag{41}$$

which has the following solution

$$a_x(u) = a_1 + a_2u^{(n-2p)/(2p-1)} + \cdots, \tag{42}$$

where a_1 and a_2 are two constant parameters. To calculate the expectation value of current for boundary theory, we can use the following formula [43,44]

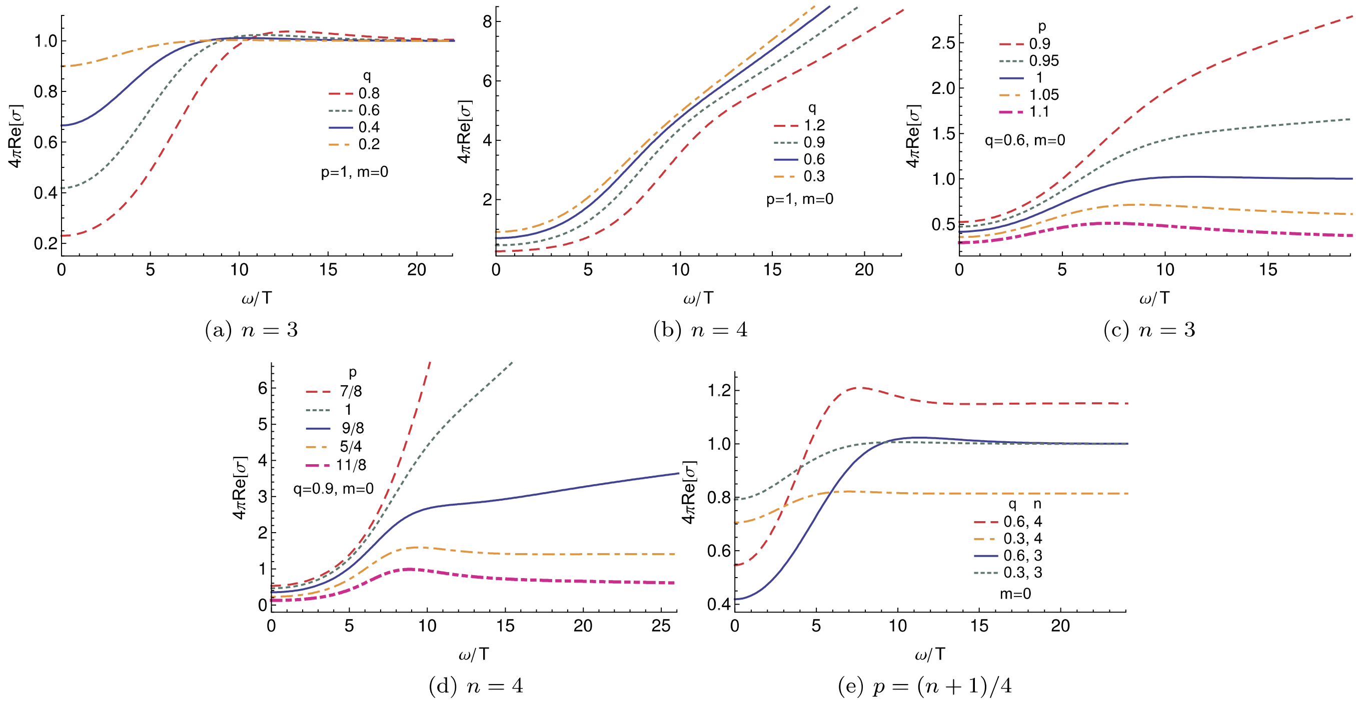
$$\langle J_x\rangle = \left.\frac{\partial\mathcal{L}}{\partial(\partial_u\delta a_x)}\right|_{u=0}, \tag{43}$$

where $\delta a_x = a_x(u)e^{-i\omega t}$ and $\mathcal{L}$ was given in Eq. (3). So, it is obvious that the holographic conductivity can be obtained as

$$\sigma = \frac{\langle J_x\rangle}{E_x} = -\frac{\langle J_x\rangle}{\partial_t\delta a_x} = -\frac{i\langle J_x\rangle}{\omega\delta a_x} = \frac{2^{p-3}p(n-2p)q^{2(p-1)}a_2}{(2p-1)\pi i\omega a_1}. \tag{44}$$

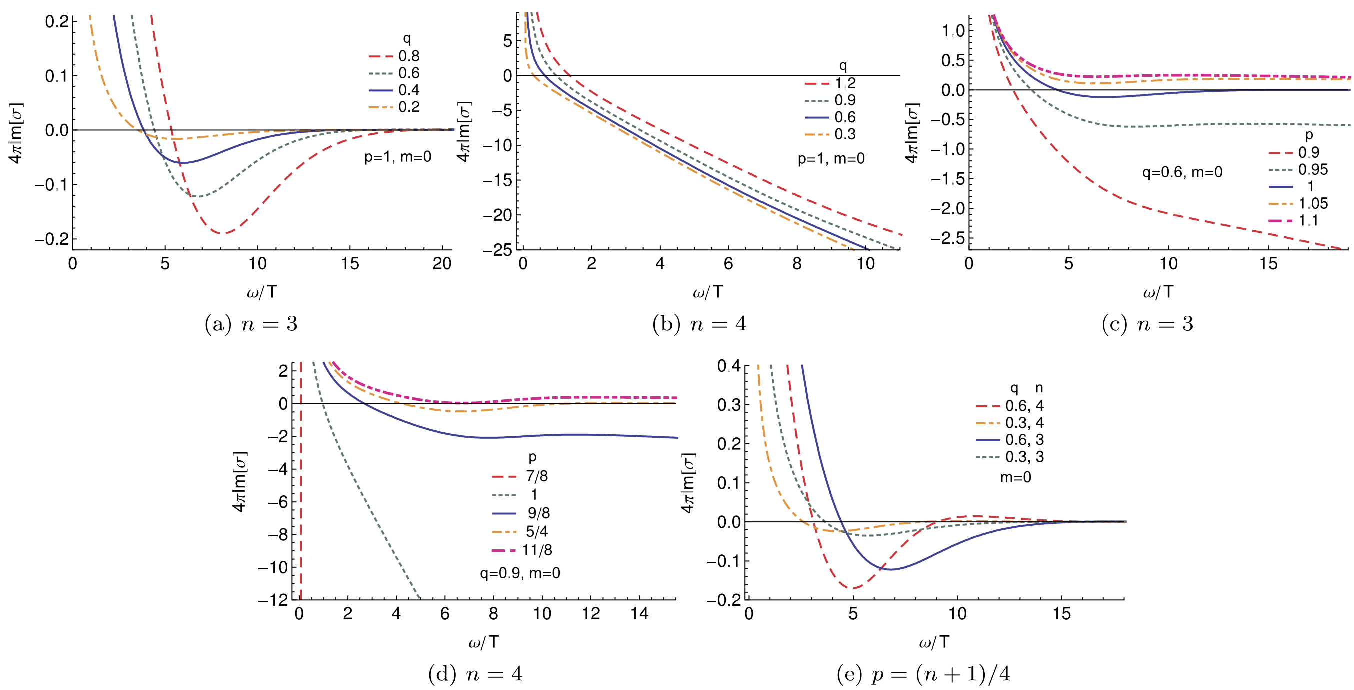
It is easy to show that the holographic conductivity (44) reduces to $\sigma = a_2/(4\pi i\omega a_1)$ for $n = 3$ and $p = 1$ [5,43]. In Figs. 2(a) and 3(a), the behaviors of real and imaginary parts of holographic conductivity for linear Maxwell case ($p = 1$) are illustrated as a function of ω/T and for various values of the charges of black brane q for $n = 3$. This figure shows that the real part of conductivity Re[σ] decreases as q increases (temperature decreases) for $\omega \to 0$ (Fig. 2(a)). Our numerical computations show that Re[σ] diverges at $\omega = 0$ independent of the value of the charge parameter q. Also, the maximum value of Re[σ] is greater for greater q's. We observe that Re[σ] tends to a constant for high frequencies independent of the value of the charge parameter. Next, we turn to study imaginary part of the conductivity Im[σ] plotted in Fig. 3(a). Imaginary part of conductivity includes a minimum for different charges. This minimum is deeper for larger charges (lower temperatures). At $\omega = 0$, imaginary part of conductivity Im[σ] diverges (Fig. 3(a)). This fact supports our numerical computation which shows that real part of conductivity blows up at zero frequency, according to Kramers–Kronig relation. For high frequencies, the imaginary part of conductivity vanishes independent of the value of charge. In Figs. 2(b) and 3(b), the behaviors of real and imaginary parts of holographic conductivity for linear Maxwell in terms of frequency for different values of black brane's charge q for $n = 4$ are depicted. For low frequencies the behavior of holographic conductivity is the same as the case $n = 3$. However, for high frequencies the behaviors are different. In $n = 3$ case, the real (imaginary) part of conductivity tends to a constant for high frequencies whereas for $n = 4$ case it increases (decreases) as ω increases.

Now, we intend to study the effect of nonlinearity of the electrodynamics (power parameter p of the power-law Maxwell field) on holographic conductivity. Figs. 2(c), 2(d), 3(c) and 3(d) show the behavior of Re[σ] and Im[σ] as a function of ω/T for different values of p (restricted by $1/2 < p < n/2$) for $n = 3$ and 4. In the $\omega \to 0$ limit, increasing p leads to the smaller Re[σ]. For high frequencies, Re[σ] increases (decreases) as linear function of ω/T and its slope increases (decreases) as p decreases (increases) for $p < (n+1)/4$ ($p > (n+1)/4$). For $p = (n+1)/4$, Re[σ] and Im[σ] tend to a constant for high frequencies as one can see in Figs. 2(e) and 3(e). Above behaviors show that for high frequencies Re[σ] $\propto \omega^a$ where $a \propto n + 1 - 4p$. This result is important from holographic point of view since similar results can be found in experimental observations [45,46]. In [45], for a $(2+1)$-dimensional graphene system, it was reported that the value of Re[σ] tends to a constant for large frequencies. We observed such a behavior in the conformally invariant case, $p = (n+1)/4$. For conductivity of a $(2+1)$-dimensional single-layer graphene induced by mild oxygen



(a) $n = 3$ (b) $n = 4$ (c) $n = 3$ (d) $n = 4$ (e) $p = (n+1)/4$

Fig. 2. The behaviors of real parts of conductivity σ versus ω/T for $m = 0$ with $l = r_+ = 1$.



(a) $n = 3$ (b) $n = 4$ (c) $n = 3$ (d) $n = 4$ (e) $p = (n+1)/4$

Fig. 3. The behaviors of imaginary parts of conductivity σ versus ω/T for $m = 0$ with $l = r_+ = 1$.

plasma exposure, a positive slope with respect to frequency for high frequencies has been reported in [46]. We observed similar behavior for conductivity in case of $p < (n+1)/4$. For all values of p, we see that Im[σ] blows up at zero frequency (Figs. 3(c) and 3(d)). For high frequencies, imaginary part of conductivity decreases for low values of p, whereas it flattens for bigger p's.

4.2. Nonvanishing m

Now, we intend to demonstrate the influence of power-law Maxwell parameter p on the holographic conductivity in massive gravity theory. Employing again $r \to lu^{-1}$ and setting $k = 0$ in (17), we obtain

$$\begin{aligned}\mathcal{F}(u) = &-m_0 l^{2-n} u^{n-2} + u^{-2} + 2^p q^{2p} (2p-1)^2 (n-1)^{-1} \\ &\times (n-2p)^{-1} \left[l^{-1}u\right]^{2(np-3p+1)/(2p-1)} \\ &+ (n-1)^{-1} c_0 m^2 l u^{-1} \Big(c_1 + (n-1) l^{-1} c_0 c_2 u \\ &+ (n-1)(n-2) l^{-2} c_0^2 c_3 u^2 \\ &+ (n-1)(n-2)(n-3) l^{-3} c_0^3 c_4 u^3 \Big). \end{aligned} \tag{45}$$

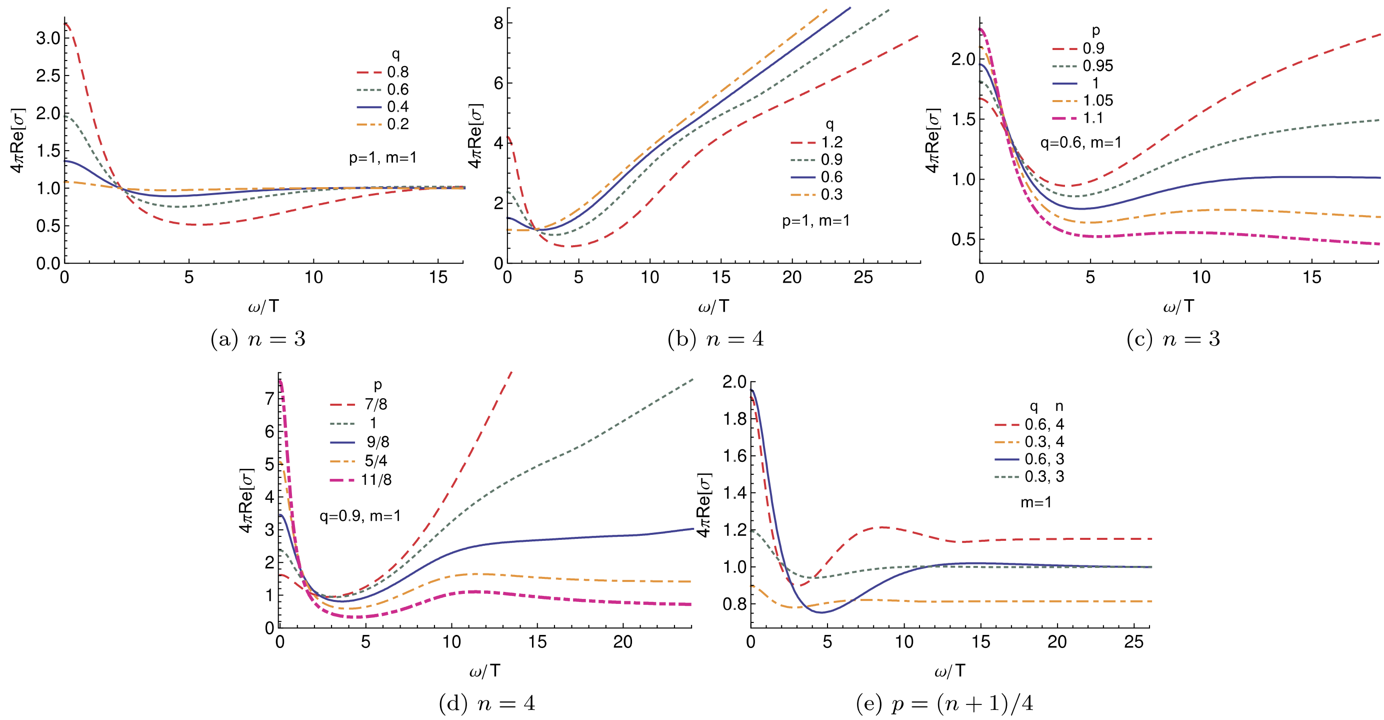


Fig. 4. The behaviors of real parts of conductivity σ versus ω/T for $m=1$ with $l=r_+=1$, $c_0=1$, $c_1=-1$ and $c_2=0$.

Hereon, we should perturb the gauge field and the metric by turning on $a_x(u)e^{-i\omega t}$, $g_{tx}(u)e^{-i\omega t}$ and $g_{ux}(u)e^{-i\omega t}$. At the linear regime, we have three independent differential equations for gauge field

$$\left(\mathcal{F}a_x'\right)' + (8p-n-3)\,(2p-1)^{-1}\,u^{-1}\mathcal{F}a_x' + l^2\omega^2u^{-4}\mathcal{F}^{-1}a_x + h'\left(g_{tx}' + 2u^{-1}g_{tx} + i\omega g_{ux}\right) = 0, \tag{46}$$

and for massive gravity

$$g_{tx}' + 2u^{-1}g_{tx} + i\omega g_{ux} + 2^{p+1}ph'\left(u^4l^{-2}h'^2\right)^{p-1}a_x + ic_0l^{-2}\omega^{-1}u^2\,\Xi\mathcal{F}g_{ux} = 0, \tag{47}$$

$$g_{tx}'' + (5-n)u^{-1}g_{tx}' - 2(n-2)u^{-2}g_{tx} + i\omega g_{ux}' + 2^{p+1}ph'\left(u^4l^{-2}h'^2\right)^{p-1}a_x' - i(n-3)\omega u^{-1}g_{ux} + c_0\,\Xi u^{-2}\mathcal{F}^{-1}g_{tx} = 0, \tag{48}$$

in which

$$\Xi = m^2\left(c_1lu^{-1} + 2(n-2)c_0c_2 + 3(n-3)(n-2)c_0^2c_3l^{-1}u + 4(n-4)(n-3)(n-2)c_0^3c_4l^{-2}u^2\right). \tag{49}$$

Eliminating g_{tx} between Eqs. (46), (47) and (48), one arrives at the two following second-order differential equations

$$\left(\mathcal{F}a_x'\right)' + (8p-n-3)\,(2p-1)^{-1}\,u^{-1}\mathcal{F}a_x' + \left[l^2\omega^2u^{-4}\mathcal{F}^{-1} - 2^{p+1}ph'^2\left(u^4l^{-2}h'^2\right)^{p-1}\right]a_x - ic_0l^{-2}\omega^{-1}\mathcal{F}h'\,\Xi u^2g_{ux} = 0, \tag{50}$$

$$l^{-2}u^{-2}\left(u^4\Xi^{-1}\mathcal{F}\left(u^2\Xi\mathcal{F}g_{ux}\right)'\right)' - i2^{p+1}p\omega c_0^{-1}u^{-2}\left[\Xi^{-1}u^4\mathcal{F}a_x\left(h'\left(u^4l^{-2}h'^2\right)^{p-1}\right)'\right]' + i(n-3)2^{p+1}p\omega c_0^{-1}u^{-2}\left[\Xi^{-1}u^3\mathcal{F}a_xh'\left(u^4l^{-2}h'^2\right)^{p-1}\right]' - (n-3)l^{-2}u^{-2}(u^5\mathcal{F}^2g_{ux})' + \omega^2g_{ux} - i2^{p+1}p\omega h'\left(u^4l^{-2}h'^2\right)^{p-1}a_x + c_0u^2l^{-2}\,\Xi\mathcal{F}g_{ux} = 0. \tag{51}$$

One can show that the solution of differential equation (50) near boundary ($u\to 0$) is

$$a_x'' + (4p-n-1)\,(2p-1)^{-1}\,u^{-1}a_x' + \cdots = 0, \tag{52}$$

which is the same as (41) and also the holographic conductivity has the same form as (44). To solve above differential equations numerically, we impose incoming boundary conditions at the horizon

$$a_x(u),\ g_{ux}(u) \propto (u_+-u)^{-i\omega/4\pi T}, \tag{53}$$

where T is the Hawking temperature.

In Figs. 4 and 5, we depict the holographic conductivity for $(2+1)$- and $(3+1)$-dimensional dual systems including momentum dissipation in the presence of linear Maxwell and nonlinear electrodynamics. Fig. 5 shows that the imaginary part of conductivity near zero frequency does not have diverging behavior in the presence of momentum dissipation. Consequently, according to Kramers–Kronig relation, the real part of conductivity does not diverge at $\omega=0$ and includes a Drude peak (in contrast with the case of previous subsection with no momentum dissipation where imaginary part of conductivity blows up at zero frequency and accordingly real part diverges there). Also, real part of DC conductivity becomes larger as q (p) increases. For high frequencies, the behaviors of real and imaginary parts of conductivity for $n=3$ and 4 in terms of black brane charge q and nonlinear parameter p are similar to the case of previous subsection with no momentum dissipation.

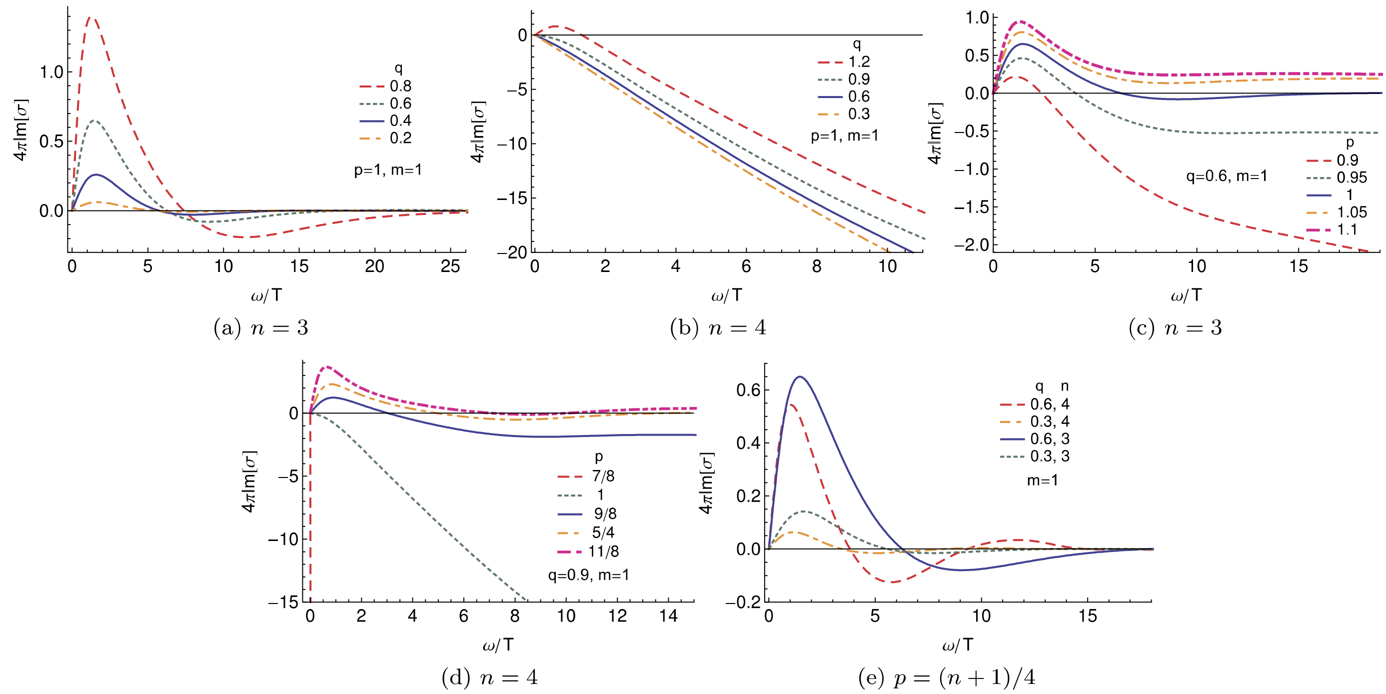


Fig. 5. The behaviors of imaginary parts of conductivity σ versus ω/T for $m = 1$ with $l = r_+ = 1$, $c_0 = 1$, $c_1 = -1$ and $c_2 = 0$.

5. Closing remarks

A gravity theory called massive gravity [5] was proposed in order to describe a class of strongly interacting quantum field theories with broken translational symmetry via a holographic principle. In this letter, we consider the massive gravity theory when the gauge field is in the form of the power-Maxwell electrodynamics. First, we derive a class of higher dimensional topological black hole solutions of this theory. Then, we calculate the conserved and thermodynamic quantities of the system and check that these quantities satisfy the first law of black holes thermodynamics on the horizon.

The main purpose of this letter is to investigate the electrical transport behavior of the dual field theory in the presence of a power-law Maxwell gauge field for the obtained solutions. In order to clarify the effects of the massive gravity on the holographic conductivity, we have first considered the holographic conductivity of the dual systems in which momentum is conserved ($m = 0$). Then, we have extended our study to the case where translational symmetry is broken and consequently the system no longer possess momentum conservation ($m \neq 0$). For both cases, we have plotted the behavior of the real and imaginary parts of the holographic conductivity in terms of the frequency per temperature (ω/T) for $(2+1)$- and $(3+1)$-dimensional dual systems. In the former case ($m = 0$), we observed that the real part of conductivity Re[σ] for $n = 3$ decreases as q increases (temperature decreases) for $\omega \to 0$. Besides, Re[σ] has a maximum which is greater for greater charges. Also, Re[σ] tends to a constant for high frequencies independent of the value of charge. In addition, the imaginary part of conductivity Im[σ] diverges as $\omega \to 0$. For high frequencies, the imaginary part of conductivity vanishes independent of the value of charge. The low frequencies behavior of holographic conductivity for $n = 4$ is the same as the case of $n = 3$. For high frequencies, in contrast with $n = 3$, the real (imaginary) part of conductivity increases (decreases) as Re[σ] increases for $n = 4$. Next, we explored the effect of the power-law Maxwell field on holographic electrical transport. We observed that increasing p leads to the smaller Re[σ] for $\omega \to 0$ while for high frequencies Re[σ] $\propto \omega^a$ where $a \propto (n + 1 - 4p)$. Similar results for high frequencies can be found in experimental observations on $(2+1)$-dimensional graphene systems [45,46]. This is important from holographic point of view.

In the latter case ($m \neq 0$), we find out that the imaginary part of the DC conductivity, Im[σ], is zero at $\omega = 0$ and becomes larger as q increases (temperature decreases). This is in contrast to the case without momentum dissipation. It also has a maximum value for $\omega \neq 0$ which increases with increasing q (with fixed p) or increasing p (with fixed q) for $n = 3$. For the real part of the conductivity, Re[σ], we see that in case of $p = 1$ the maximum value (Drude peak) achieves at $\omega = 0$. Again this is in contrast to the former case ($m = 0$) in which the minimum value of Re[σ], occurs for $\omega \to 0$. For different values of the power parameter, p, the real and imaginary part of the conductivity has relative minimum and maximum, respectively. Finally, we observed that both real and imaginary parts of the holographic conductivity are similar to the previous case for high frequencies.

In this work, we obtained the conductivity by applying the linear response theory where the electric field is treated as a probe. This may restrict the study from fully explaining the effects of nonlinearity of electrodynamics model. Therefore, it is an interesting issue for future researches to consider the case where the properties of the system are functions of electric field. In such case, nonlinear response happens. Some examples of such studies can be found in literature in Refs. [47–52].

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