Masses and magnetic moments of hadrons with one and two open heavy quarks: Heavy baryons and tetraquarks

Wen-Xuan Zhang,[1,*] Hao Xu,[1,2,†] and Duojie Jia[1,2,‡]

[1] *Institute of Theoretical Physics, College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, China*

[2] *Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, China*

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In this paper we compute masses and magnetic moments of heavy baryons and tetraquarks with one and two open heavy flavors in a unified framework of the MIT bag model. Using the parameters of the MIT bag model, we confirm that an extra binding energy, which is supposed to exist between heavy quarks (c and b), and between heavy and strange quarks in literatures, is required to reconcile light hadrons with heavy hadrons. Numerical calculations are made for all light mesons and heavy hadrons (with one and two open heavy flavors) predicting the masses of doubly charmed baryons to be $M(\Xi_{cc}) = 3.604$ GeV, $M(\Xi^*_{cc}) = 3.714$ GeV, and that of the strange isosinglet tetraquark $ud\bar{s}\,\bar{c}$ with $J^P = 0^+$ to be $M(ud\bar{s}\,\bar{c}, 0^+) = 2.934$ GeV. The state mixing due to chromomagnetic interaction is shown to be sizable for the strange scalar tetraquark $nn\bar{s}\,\bar{c}$.

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I. INTRODUCTION

Four years ago, the LHCb Collaboration at CERN discovered the first doubly charmed baryon Ξ^{++}_{cc} with $J^P = 1/2^+$ and measured its mass to be 3621.40 ± 0.78 MeV [1]. Later, the Ξ^{++}_{cc} state was confirmed in the decay to $\Xi^{+}_{cc}\pi^+$ [2] and its lifetime, mass, and production cross section were subsequently measured [3,4]. Containing two charmed quarks, such a baryon provides a unique probe for quantum chromodynamics (QCD)—the gauge theory of strong interactions. In addition, the observation provides useful experimental information about the strength of the interaction between two heavy quarks, and enables us to further explore tetraquarks $QQ'\bar{q}\,\bar{q}$ containing two open heavy quarks, which is allowed by QCD. (See e.g., Refs. [5–7].) Recently, the LHCb Collaboration reported the first exotic state $X_0(2900)$ with open heavy flavors and mass 2866 ± 7 MeV[8], which is interpreted to be an isosinglet tetraquark $cs\bar{u}\,\bar{d}$ in Ref. [9]. More recently, the observation of a doubly charmed tetraquark T^{++}_{cc} is reported also by LHCb Collaboration [10]. These findings, among others, make it of interest to explore doubly heavy (DH) hadrons in details. There exists extensive studies of DH hadrons with various approaches, including the potential quark model and the bag model [11–21], AdS/QCD approach [22–26], and the relativistic quark model [27]. (See Refs [28,29] for recent reviews.)

In identifying and/or finding these DH hadrons experimentally, it is helpful to have a systematic estimate of masses and other properties of them within an unified framework. For instance, a mass prediction [14,16] of the doubly charmed baryon Ξ_{cc}, which are larger about 100 MeV than that measured in 2002 by the SELEX Collaboration at Fermilab [30] (awaiting confirmation), helps the LHCb Collaboration to search for the Ξ^{++}_{cc} [1] eventually.

In this work, we apply the MIT bag model [31,32] with chromomagnetic interaction and a strong coupling α_s running with the bag radius, to systematically study the open heavy baryons and tetraquarks with one and two heavy flavors and compute the masses and other static properties (magnetic moments, electric-charge radii) of them. It is confirmed that an extra binding energy between heavy quarks (c and b), and between heavy and strange quarks is required to reconcile light hadrons with heavy hadrons. Computed results are compared to other calculations and are consistent with the measured masses and other properties of light hadrons and singly heavy baryons in their ground states (except for π). For the $J^P = \frac{1}{2}^+$ states of heavy baryons Ξ_c, Ξ_b, Ξ_{bc}, Ω_{bc} and the heavy tetraquarks, the chromomagnetic mixing is taken into account and the respective mass splittings are computed variationally.

*zhangwx89@outlook.com
†xuh2020@nwnu.edu.cn
‡jiadj@nwnu.edu.cn

It is well known that the bag model [31] embodies two primary features of quantum chromodynamics (QCD); asymptotic freedom at short distance and confinement at long distance. The simple structure of the model enables us to describe mesons ($q\bar{q}$), baryons (qqq), and even hadrons made of multiquarks. In the past few decades, the bag model has been applied to describe the doubly heavy baryons [11–13] and multiquark hadrons, including light exotic baryons with five and seven nonstrange quarks [33]. In order to evaluate the masses of doubly heavy baryons, a large running strong coupling α_s was applied in Ref. [12]. A Coulomb-like interaction is derived between heavy quarks in a bag in Refs. [34,35].

This paper will be organized as follows. In Sec. II we review some basic relations of the MIT bag model, including chromomagnetic interaction (CMI) among the quarks in the bag. In Sec. III a systematic numerical calculation is performed for the established light and singly heavy (SH) baryons, with the optimal set of parameters obtained and the results for masses and other properties are reproduced. In Sec. IV we present detailed predictions for masses and other properties for doubly heavy baryons a nd the tetraquarks with one and two open heavy quarks. The paper ends with summary and conclusions in Sec. V.

II. METHOD FOR MIT BAG MODEL WITH CMI

A. Mass formula

Treating hadron as a spherical bag, the MIT bag model provides an approach to estimate masses and other properties of hadrons in their ground states [31,32], in which the chromomagnetic interaction is derived from the energy of a spherelike gluon field interacting with quark fields in bag [31]. The mass formula of hadron in MIT bag model is

$$M(R) = \sum_{i=n,s,c,b} n_i\omega_i + \frac{4}{3}\pi R^3 B - \frac{Z_0}{R} + \langle \Delta H \rangle, \quad (1)$$

$$\omega_i = \left(m_i^2 + \frac{x_i^2}{R^2}\right)^{1/2}, \quad (2)$$

where the first term is the kinematic energy of all quarks in the bag with radius R, the second is the volume energy of the bag with the bag constant B, the third is the zero point energy (ZPE) with coefficient Z_0, and $\langle \Delta H \rangle$ is the short-range interaction among quarks in the bag which we will address in this paper. In Eq. (1) n_i is the number of quarks or antiquarks in the bag with mass m_i and flavor i (where i can be the light nonstrange quarks $n = u, d$), the strange quark s, the charm quark c, and the bottom quark b. The value of R is to be determined variationally, and the dimensionless parameters $x_i = x_i(mR)$ are related to the bag radius R by an transcendental eigenequation

$$\tan x_i = \frac{x_i}{1 - m_i R - (m_i^2 R^2 + x_i^2)^{1/2}}. \quad (3)$$

The interaction energy $\langle \Delta H \rangle = B_{EB} + M_{\mathrm{CMI}}$ is composed of two energy terms:

(1) The spin-independent binding energy B_{EB}, due mainly to the short-range chromoelectric interaction between quarks and/or antiquarks. Owing to the smallness of the relativistic light quarks $n(= u, d)$, this energy scales mainly as $-\sum \alpha_s/r_{ij}$, and becomes sizable when both of the two quarks, i and j, are massive and are moving nonrelativistically. In the present work we treat this energy as the sum of the pair binding energies, $B_{QQ'}$ (B_{Qs}), between heavy quarks and between heavy quarks Q and strange quarks s [14,36,37]. The net effect for this chromoelectric interaction amounts to the introduction of five binding energies B_{cs}, B_{cc}, B_{bs}, B_{bb}, and B_{bc} for any quark pair in the color configuration $\bar{\mathbf{3}}_c$, which are extractable from heavy mesons and can be scaled to other color configurations.

(2) The chromomagnetic interaction energy, due to the perturbative gluon exchange between quarks (antiquarks) i and j,

$$M_{\mathrm{CMI}} = -\sum_{i<j} (\boldsymbol{\lambda_i} \cdot \boldsymbol{\lambda_j})(\boldsymbol{\sigma_i} \cdot \boldsymbol{\sigma_j}) C_{ij}, \quad (4)$$

with $\boldsymbol{\lambda}_i$ the Gell-Mann matrices, $\boldsymbol{\sigma}_i$ the Pauli matrices, and C_{ij} the CMI parameter. In the MIT bag model, the parameters C_{ij} are given by

$$C_{ij} = 3\frac{\alpha_s(R)}{R^3} \bar{\mu}_i \bar{\mu}_j I_{ij}, \quad (5)$$

$$\bar{\mu}_i = \frac{R}{6} \frac{4\alpha_i + 2\lambda_i - 3}{2\alpha_i(\alpha_i - 1) + \lambda_i}, \quad (6)$$

$$I_{ij} = 1 + 2\int_0^R \frac{dr}{r^4} \bar{\mu}_i \bar{\mu}_j = 1 + F(x_i, x_j), \quad (7)$$

where $\alpha_i \equiv \omega_i R$, $\lambda_i \equiv m_i R$, $\alpha_s(R)$ is the running strong coupling, $\bar{\mu}_i$ is the reduced magnetic moment without electric charge, and I_{ij} and $F(x_i, x_j)$ are rational functions of x_i and x_j, given explicitly by [31].

$$F(x_i,x_j) = \left(x_i\sin^2 x_i - \frac{3}{2}y_i\right)^{-1}\left(x_j\sin^2 x_j - \frac{3}{2}y_j\right)^{-1}\left\{-\frac{3}{2}y_iy_j - 2x_ix_j\sin^2 x_i\sin^2 x_j + \frac{1}{2}x_ix_j[2x_i\mathrm{Si}(2x_i) + 2x_j\mathrm{Si}(2x_j) - (x_i+x_j)\mathrm{Si}(2(x_i+x_j)) - (x_i-x_j)\mathrm{Si}(2(x_i-x_j))]\right\}, \tag{8}$$

where $y_i = x_i - \cos(x_i)\sin(x_i)$, x_i is the root of Eq. (3) for a given m_iR, and

$$\mathrm{Si}(x) = \int_0^x \frac{\sin(t)}{t}\mathrm{d}t. \tag{9}$$

In some applications of the bag model [11,12,34,35,38], the parameter α_s takes a logarithmic form

$$\alpha_s(R) = \frac{w}{\ln[\gamma + (R\Lambda_{\mathrm{QCD}})^{-n}]}, \tag{10}$$

where w and n ($= 1$ or 2) are the parameters, Λ_{QCD} is the QCD scale (0.2–0.5 GeV), and γ is prefactor used to avoid infrared divergence. Similar to Ref. [12], we take $w = 0.296$, $\Lambda_{\mathrm{QCD}} = 0.281$ GeV, $\gamma = 1$, and $n = 1$ to set

$$\alpha_s(R) = \frac{0.296}{\ln[1 + (0.281R)^{-1}]}, \tag{11}$$

which is plotted in Fig. 1. Among the four lines showing the running of α_s in the plot, the solid line shows notable variation and corresponds to a relatively lower value of α_s.

Given the parameter values of the quark mass m_i, the bag constant B, the ZPE coefficient Z_0, and the strong coupling constant $\alpha_s(R)$ depending on the bag radius R, one can apply a variational method to determine the respective bag radius R for each hadron and the respective x_i through Eq. (3). Then, it is straightforward to use Eqs. (1), (2), and (4) to compute the ground-state masses and other static properties (the magnetic moments and the charge radius) of the hadrons ranging from the light hadrons to heavy tetraquarks. The computed results for the light hadrons are listed and compared to that predicted by the original MIT bag model in Table I.

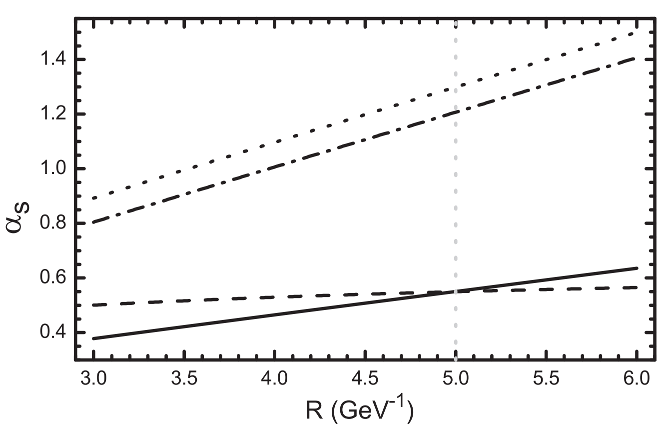

FIG. 1. Four running behaviors of strong coupling. The bag radius R ranges from 3 GeV^{-1} to 6 GeV^{-1}, and the standard radius is set to be 5 GeV^{-1} ($\approx$1 fm) for checking. The solid line represents our result (11) while the dashed line corresponds to Eq. (10) with $\gamma = 2.847$, $\Lambda_{\mathrm{QCD}} = 0.281$ GeV, and $w = 2\pi n/9$. The dotted line shows Eq. (10) with $\gamma = 1$, $\Lambda_{\mathrm{QCD}} = 0.281$ GeV and $w = 2\pi n/9$. The dotdashed line indicates that of Ref. [12]. All four behaviors adopt $n = 1$.

We stress that for a given hadronic state there is in principle a unique set of the solution x_i and R corresponding to the respective bag dynamics, as indicated by our computation. Owing to (x_i, R)-dependence of $\langle \Delta H \rangle$, a simple and analytic mass formula is lacking for the hadrons with chromomagnetic mixing since for those one has to first diagonalize the CMI matrices before the variational analysis, which amounts to higher-order algebraic equations.

B. Chromomagnetic interaction

In evaluating the spin-dependent mass due to the CMI (4), in which $\boldsymbol{\lambda}_i$ should be replaced by $-\boldsymbol{\lambda}_i^*$ for an antiquark, one has to diagonalize the CMI matrix for given hadron multiplets with certain spin parity J^P to give the respective mass splittings [7] within the multiplets. For this, we list all the flavor-spin-color wave functions of hadrons

TABLE I. Bag radius R (in GeV^{-1}) and mass prediction M_{bag} (in GeV) obtained from this work ($Z_0 = 1.83$) and the original MIT bag model for light hadrons, compared to the measured mass M_{exp} (in GeV) being isospin-averaged.

	MIT bag [32]		This work		
State	R	M_{bag}	R	M_{bag}	M_{exp} [39]
N	5.00	0.938	5.22	0.932	0.939
Δ	5.48	1.233	5.33	1.241	1.232
Λ	4.95	1.105	5.26	1.096	1.116
Σ	4.95	1.144	5.22	1.137	1.193
Ξ	4.91	1.289	5.27	1.282	1.318
Σ^*	5.43	1.382	5.38	1.383	1.385
Ξ^*	5.39	1.529	5.42	1.529	1.533
Ω	5.35	1.672	5.46	1.677	1.672
π	3.34	0.280	4.31	0.348	0.137
ω	4.71	0.783	4.55	0.776	0.783
K	3.26	0.497	4.34	0.561	0.496
K^*	4.65	0.928	4.63	0.918	0.894
ϕ	4.61	1.068	4.70	1.064	1.019

including the tetraquarks considered in this work, and present relevant formulas of the color and spin factors for them.

Mesons: The color wave function $\phi^M = |q_1\bar{q}_2\rangle$ can be one of two spin states (vectorlike and scalarlike)

$$\chi_1^M = |q_1\bar{q}_2\rangle_1, \qquad \chi_2^M = |q_1\bar{q}_2\rangle_0, \tag{12}$$

where subscript $J = 0$ or 1 outside the bracket denotes the total spin of the hadron. The spin-color wave functions with spin $J = 0$ and 1 are then

$$\phi^M\chi_1^M = |q_1\bar{q}_2\rangle_1, \qquad \phi^M\chi_2^M = |q_1\bar{q}_2\rangle_0. \tag{13}$$

Baryons: The color wave functions $\phi^B = |(q_1q_2)^{\bar{3}}q_3\rangle$ can be in one of three spin states

$$\begin{aligned}\chi_1^B &= |(q_1q_2)_1q_3\rangle_{3/2},\\ \chi_2^B &= |(q_1q_2)_1q_3\rangle_{1/2},\\ \chi_3^B &= |(q_1q_2)_0q_3\rangle_{1/2},\end{aligned} \tag{14}$$

where (q_1q_2) stands for a diquark with spin $J = 1$ or 0 in color configuration $\bar{\mathbf{3}}_c$.

To write the wave function for a hadron, the flavor symmetry has to be considered. For a flavor-symmetric wave function of (q_1q_2), with isospin $I = 1$ or identical flavors, we use a symbol $\delta_{12}^S = 1$. For a flavor-asymmetric wave function with $I = 0$, a symbol $\delta_{12}^A = 1$ will be used. For two quarks q_1 and q_2 with different flavors which go beyond isospin symmetry, one can use $\delta_{12}^S = \delta_{12}^A = 1$. With the help of the Pauli principle, one can write three flavor-spin-color wave functions for baryons

$$\begin{aligned}\phi^B\chi_1^B &= |(q_1q_2)_1^{\bar{3}}q_3\rangle_{3/2}\delta_{12}^S,\\ \phi^B\chi_2^B &= |(q_1q_2)_1^{\bar{3}}q_3\rangle_{1/2}\delta_{12}^S,\\ \phi^B\chi_3^B &= |(q_1q_2)_0^{\bar{3}}q_3\rangle_{1/2}\delta_{12}^A.\end{aligned} \tag{15}$$

Owing to the nondiagonal chromomagnetic interaction (4), some hadronic states with same J^P but different spin-color wave functions can mix (CMI mixing). For example, for the doubly heavy baryons Ξ_{bc} and Ξ'_{bc} with flavor structure $(bc)s$ the use of $\delta_{12}^S = \delta_{12}^A = 1$ is not enough to distinguish the two configurations $\phi^B\chi_2^B$ and $\phi^B\chi_3^B$ solely in terms of their J^P quantum numbers. Thus, the physical state must be one of the mixing states of them. See Sec. IV for the details of chromomagnetic mixing.

Tetraquarks: A tetraquark can have the color structure of either $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$ or $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$, with the respective color wave functions,

$$\phi_1^T = |(q_1q_2)^6(\bar{q}_3\bar{q}_4)^{\bar{6}}\rangle, \quad \phi_2^T = |(q_1q_2)^{\bar{3}}(\bar{q}_3\bar{q}_4)^3\rangle, \tag{16}$$

and it can take one of the following six states

$$\begin{aligned}&\chi_1^T = |(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1\rangle_2, \quad \chi_2^T = |(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1\rangle_1,\\ &\chi_3^T = |(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1\rangle_0, \quad \chi_4^T = |(q_1q_2)_1(\bar{q}_3\bar{q}_4)_0\rangle_1,\\ &\chi_5^T = |(q_1q_2)_0(\bar{q}_3\bar{q}_4)_1\rangle_1, \quad \chi_6^T = |(q_1q_2)_0(\bar{q}_3\bar{q}_4)_0\rangle_0,\end{aligned} \tag{17}$$

which lead to twelve basis wave functions

$$\begin{aligned}\phi_1^T\chi_1^T &= |(q_1q_2)_1^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_2\delta_{12}^A\delta_{34}^A,\\ \phi_2^T\chi_1^T &= |(q_1q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_2\delta_{12}^S\delta_{34}^S,\\ \phi_1^T\chi_2^T &= |(q_1q_2)_1^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_1\delta_{12}^A\delta_{34}^A,\\ \phi_2^T\chi_2^T &= |(q_1q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_1\delta_{12}^S\delta_{34}^S,\\ \phi_1^T\chi_3^T &= |(q_1q_2)_1^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_0\delta_{12}^A\delta_{34}^A,\\ \phi_2^T\chi_3^T &= |(q_1q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_0\delta_{12}^S\delta_{34}^S,\\ \phi_1^T\chi_4^T &= |(q_1q_2)_1^6(\bar{q}_3\bar{q}_4)_0^{\bar{6}}\rangle_1\delta_{12}^A\delta_{34}^S,\\ \phi_2^T\chi_4^T &= |(q_1q_2)_1^{\bar{3}}(\bar{q}_3\bar{q}_4)_0^3\rangle_1\delta_{12}^S\delta_{34}^A,\\ \phi_1^T\chi_5^T &= |(q_1q_2)_0^6(\bar{q}_3\bar{q}_4)_1^{\bar{6}}\rangle_1\delta_{12}^S\delta_{34}^A,\\ \phi_2^T\chi_5^T &= |(q_1q_2)_0^{\bar{3}}(\bar{q}_3\bar{q}_4)_1^3\rangle_1\delta_{12}^A\delta_{34}^S,\\ \phi_1^T\chi_6^T &= |(q_1q_2)_0^6(\bar{q}_3\bar{q}_4)_0^{\bar{6}}\rangle_0\delta_{12}^S\delta_{34}^S,\\ \phi_2^T\chi_6^T &= |(q_1q_2)_0^{\bar{3}}(\bar{q}_3\bar{q}_4)_0^3\rangle_0\delta_{12}^A\delta_{34}^A.\end{aligned} \tag{18}$$

We list all relevant color wave functions in Appendix A and spin wave functions in Appendix B. With them, one can evaluate the color and spin factors in Eq. (4) with the help of the following formulas,

$$\langle\boldsymbol{\lambda_i}\cdot\boldsymbol{\lambda_j}\rangle_{nm} = \sum_{\alpha=1}^{8}\mathrm{Tr}(c_{\mathrm{in}}^\dagger\lambda^\alpha c_{im})\mathrm{Tr}(c_{jn}^\dagger\lambda^\alpha c_{jm}), \tag{19}$$

$$\langle\boldsymbol{\sigma_i}\cdot\boldsymbol{\sigma_j}\rangle_{xy} = \sum_{\alpha=1}^{3}\mathrm{Tr}(\chi_{ix}^\dagger\sigma^\alpha\chi_{iy})\mathrm{Tr}(\chi_{jx}^\dagger\sigma^\alpha\chi_{jy}), \tag{20}$$

where n, m, and x, y, indicate the specific color and spin states respectively, i and j are the indexes of quarks (antiquarks), and the functions c and χ are the respective basis vectors in the color and spin spaces. Table II lists a set of nonmixed hadronic states with their respective CMIs.

Now, we are in the position to construct the matrix formula of the CMI energy (4) and diagonalize it so as to minimize the obtained mass formula. Adding the binding energy (for a heavy quark pair and for a pair of one heavy quark and one strange quark) to the bag energy, one can solve the dynamical parameters x_i and R, and thereby obtain the wave functions of a given hadron.

TABLE II. Chromomagnetic interactions for the nonmixing hadrons with respective wave functions. C_{ij} follows Eq. (5) with subscripts corresponding to quarks or antiquarks.

State	Wave function	CMI	State	Wave function	CMI
π	$\phi^M\chi_2^M$	$-16C_{nn}$	ω	$\phi^M\chi_1^M$	$\frac{16}{3}C_{nn}$
K	$\phi^M\chi_2^M$	$-16C_{sn}$	K^*	$\phi^M\chi_1^M$	$\frac{16}{3}C_{sn}$
			ϕ	$\phi^M\chi_1^M$	$\frac{16}{3}C_{ss}$
D	$\phi^M\chi_2^M$	$-16C_{cn}$	D^*	$\phi^M\chi_1^M$	$\frac{16}{3}C_{cn}$
D_s	$\phi^M\chi_2^M$	$-16C_{cs}$	D_s^*	$\phi^M\chi_1^M$	$\frac{16}{3}C_{cs}$
η_c	$\phi^M\chi_2^M$	$-16C_{cc}$	J/ψ	$\phi^M\chi_1^M$	$\frac{16}{3}C_{cc}$
N	$\phi^B\chi_3^B$	$-8C_{nn}$	Δ	$\phi^B\chi_1^B$	$8C_{nn}$
Λ	$\phi^B\chi_3^B$	$-8C_{nn}$	...	...	...
Σ	$\phi^B\chi_2^B$	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{sn}$	Σ^*	$\phi^B\chi_1^B$	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{sn}$
Λ_c	$\phi^B\chi_3^B$	$-8C_{nn}$	...	...	...
Σ_c	$\phi^B\chi_2^B$	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{cn}$	Σ_c^*	$\phi^B\chi_1^B$	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{cn}$
Ξ	$\phi^B\chi_2^B$	$\frac{8}{3}C_{ss}-\frac{32}{3}C_{sn}$	Ξ^*	$\phi^B\chi_1^B$	$\frac{8}{3}C_{ss}+\frac{16}{3}C_{sn}$
Ξ_c	$\phi^B\chi_3^B$	$-8C_{sn}$	...	...	...
Ξ_c'	$\phi^B\chi_2^B$	$\frac{8}{3}C_{sn}-\frac{16}{3}C_{cn}-\frac{16}{3}C_{cs}$	Ξ_c^*	$\phi^B\chi_1^B$	$\frac{8}{3}C_{sn}+\frac{8}{3}C_{cn}+\frac{8}{3}C_{cs}$
			Ω	$\phi^B\chi_1^B$	$8C_{ss}$
$ss\bar{c}\,\bar{c}$	$\phi_2^T\chi_2^T$	$\frac{8}{3}C_{ss}-\frac{16}{3}C_{cs}+\frac{8}{3}C_{cc}$	$ss\bar{c}\,\bar{c}$	$\phi_2^T\chi_1^T$	$\frac{8}{3}C_{ss}+\frac{16}{3}C_{cs}+\frac{8}{3}C_{cc}$
$(nn\bar{c}\,\bar{c})^{I=1}$	$\phi_2^T\chi_2^T$	$\frac{8}{3}C_{nn}-\frac{16}{3}C_{cn}+\frac{8}{3}C_{cc}$	$(nn\bar{c}\,\bar{c})^{I=1}$	$\phi_2^T\chi_1^T$	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{cn}+\frac{8}{3}C_{cc}$

C. Hadronic properties

Given the parameters x_i and R describing a hadronic state, mass and other properties (e.g., the charge radius and the magnetic moment) can be evaluated. Following the standard method, one can first calculate the contribution of a quark or an antiquark i with electric charge Q_i to the charge radius [31]

$$\langle r_E^2\rangle_i = Q_i R^2 \frac{\alpha_i[2x_i^2(\alpha_i-1)+4\alpha_i+2\lambda_i-3]}{3x_i^2[2\alpha_i(\alpha_i-1)+\lambda_i]} - Q_i R^2 \frac{\lambda_i[4\alpha_i+2\lambda_i-2x_i^2-3]}{2x_i^2[2\alpha_i(\alpha_i-1)+\lambda_i]}. \quad (21)$$

The sum of Eq. (21) then gives the charge radius of a hadronic state [40]

$$r_E = \left|\sum_i \langle r_E^2\rangle_i\right|^{1/2}. \quad (22)$$

We note that Eq. (22) also holds true for the chromomagnetic-mixing systems having the identical quark constituents.

For magnetic moment, the following equations [31,41], which are computed relative to the magnetic moment of the proton and has the unit of μ_p, are useful,

$$\mu_i = Q_i\bar{\mu}_i = Q_i\frac{R}{6}\frac{4\alpha_i+2\lambda_i-3}{2\alpha_i(\alpha_i-1)+\lambda_i}, \quad (23)$$

$$\mu = \left\langle \psi_{\rm spin}\left|\sum_i g_i\mu_i S_{iz}\right|\psi_{\rm spin}\right\rangle, \quad (24)$$

where $g_i = 2$, and S_{iz} is the third component of spin for an individual quark or antiquark. In all Tables for the results of the magnetic moments obtained from Eqs. (23) and (24), we transform them into that in the unit of the nuclear magneton μ_N, with the help of the measured data $\mu_p = 2.79285\mu_N$. If the chromomagnetic mixing enters, the total spin wave function becomes

$$|\psi_{\rm spin}\rangle = C_1\chi_1 + C_2\chi_2, \quad (25)$$

which by Eq. (24) gives

$$\mu = C_1^2\mu(\chi_1) + C_2^2\mu(\chi_2) + 2C_1C_2\mu^{\rm tr}(\chi_1,\chi_2), \quad (26)$$

with $\mu^{\rm tr}$ the cross term representing the transition moment [13] and (C_1, C_2) the eigenvector of the given mixing state. We list all the spin wave functions in Appendix B and derive magnetic moments for them and their possibly-mixed systems involved in this work. The results for the spin wave functions and the respective magnetic moments are listed in Table III collectively.

Note that the cross terms in CMI-mixing systems are not symmetric under the exchange between quarks q_1 and q_2 or between q_3 and q_4 in the flavor space. While the expression of the cross term for the diquark (ud) differs by a sign for (ud) and (du) within the symmetric or asymmetric flavor wave functions when $I_3 = 1$ or -1 in isospin space, respectively, the explicit computation via these wave functions can offset such a cross term. Similar conclusions also apply for hadrons as the hadron systems respect the $SU(2)$ isospin symmetry.

TABLE III. Sum rule for magnetic moments of spin states of mesons $(q_1\bar{q}_2)$, baryons $(q_1q_2)q_3$, and tetraquarks $(q_1q_2)(\bar{q}_3\bar{q}_4)$ and their spin-mixed systems.

$\psi_{\rm spin}$	μ
χ_1^M	$\mu_1+\mu_2$
χ_2^M	0
χ_1^B	$\mu_1+\mu_2+\mu_3$
χ_2^B	$\frac{1}{3}(2\mu_1+2\mu_2-\mu_3)$
χ_3^B	μ_3
$C_1\chi_2^B+C_2\chi_3^B$	$C_1^2\mu(\chi_2^B)+C_2^2\mu(\chi_3^B)+\frac{2}{\sqrt{3}}C_1C_2(\mu_2-\mu_1)$
χ_1^T	$\mu_1+\mu_2+\mu_3+\mu_4$
χ_2^T	$\frac{1}{2}(\mu_1+\mu_2+\mu_3+\mu_4)$
χ_3^T	0
χ_4^T	$\mu_1+\mu_2$
χ_5^T	$\mu_3+\mu_4$
χ_6^T	0
$C_1\chi_3^T+C_2\chi_6^T$	0
$C_1\chi_5^T+C_2\chi_4^T$	$C_1^2\mu(\chi_5^T)+C_2^2\mu(\chi_4^T)$
$C_1\chi_2^T+C_2\chi_4^T+C_3\chi_5^T$	$C_1^2\mu(\chi_2^T)+C_2^2\mu(\chi_4^T)+C_3^2\mu(\chi_5^T)+\sqrt{2}C_1C_2(\mu_3-\mu_4)+\sqrt{2}C_1C_3(\mu_2-\mu_1)$
$C_1\chi_2^T+C_2\chi_5^T+C_3\chi_4^T$	$C_1^2\mu(\chi_2^T)+C_2^2\mu(\chi_5^T)+C_3^2\mu(\chi_4^T)+\sqrt{2}C_1C_2(\mu_2-\mu_1)+\sqrt{2}C_1C_3(\mu_3-\mu_4)$

III. DETERMINATION OF PARAMETERS

In the MIT bag model, the parameters (nonstrange m_n and strange m_s quark masses, B, Z_0, and α_s) are determined based on the mass spectra of the light hadrons N, Δ, ω, and Ω in their ground states. The results read [31]

$$\left\{\begin{matrix} m_n=0, & m_s=0.279\ \text{GeV}, & \\ Z_0=1.83, & B^{1/4}=0.145\ \text{GeV}, & \alpha_s=0.55. \end{matrix}\right\} \tag{27}$$

We choose Eqs. (27) to be the parameters applying to both of the light and heavy hadrons, with one exception—that the strong coupling α_s changes with the size R of the hadron around 0.55, as given by Eq. (11). To fix the model parameters two tasks still remain.

The first task is to extract the heavy quark masses m_c and m_b. Given Eq. (27), one can apply Eqs. (1) and (4) to the heavy-light mesons D^* and B^* to fix numerically m_c and m_b, respectively. The results are

$$\{\, m_c=1.641\ \text{GeV}, \quad m_b=5.093\ \text{GeV}\,\} \tag{28}$$

The second is to fix the binding energy $B_{QQ'}$ (and $B_{Q\bar{Q}'}$, $Q,Q'=s,c,b$), which is proposed in Ref. [14] to occur in charmed-strange hadrons, bottom-strange hadrons, and heavy quadrennia. It can be due to nontrivial short-range interaction between two heavy quarks and between heavy and strange quarks [14,36,37]. Applied to the strange heavy mesons ($Q\bar{s}$ and $Q\overline{Q\prime}$), this binding enters the mass formula through

$$M(Q\bar{s})=\omega_Q+\omega_s+\frac{4}{3}\pi R^3B-\frac{Z_0}{R}+\langle H_{\rm CMI}\rangle+B_{Q\bar{s}},$$
$$M(Q\overline{Q\prime})=\omega_Q+\omega_{Q'}+\frac{4}{3}\pi R^3B-\frac{Z_0}{R}+\langle H_{\rm CMI}\rangle+B_{Q\overline{Q\prime}}. \tag{29}$$

Applying this to the case of the vector mesons $D_s^*=c\bar{s}$, this allows one to solve the binding term,

$$B_{c\bar{s}}=M(D_s^*)-\omega_c-\omega_s-\frac{4}{3}\pi R^3B+\frac{Z_0}{R}-\langle H_{\rm CMI}\rangle, \tag{30}$$

with the bag radius R solved variationally for the D_s^* mesons. Numerically, one finds, $B_{c\bar{s}}=-0.050$ GeV by Eq. (30). Note that the short-range color interaction of the quark pair $c\bar{s}$ in the color singlet (1_c) and in the heavy meson D_s^*, can be related to that of the cs pair in color antitriplet ($\bar{3}_c$) in a heavy baryon nsc. By the factor of $1/2$, one can reasonably assume, in the short range, that the strength of the cs interaction in $\bar{3}_c$ is half that of $c\bar{s}$ in 1_c. This follows from $B_{cs}=B_{c\bar{s}}/2=-0.025$ GeV. Here, the factor of one half can be extracted from the ratio of the color factor $-8/3$ in Eq. (A4) for 1_c and $-16/3$ Eq. (A5), evaluated in Appendix A; $1/2=(-8/3)/(-16/3)$. The same also holds true for the quark pair $Q\overline{Q\prime}$ in the heavy mesons B_s^*, J/ψ, Υ, and B_c^*. Thus, for the quark pair (Qs an $QQ\prime$) in the heavy baryon, we choose

$$B_{Qs}=B_{Q\bar{s}}/2, \qquad B_{QQ'}=B_{Q\overline{Q\prime}}/2, \tag{31}$$

for the quark pair in $\bar{\mathbf{3}}_c$ and the quark-antiquark pair in 1_c, and solve the model (1) for the heavy mesons B_s^*, J/ψ, Υ, and B_c^*, obtaining by Eqs. (29) and (31),

$$\left\{\begin{array}{ll} B_{cs} = B_{c\bar{s}}/2 = -0.025\ \text{GeV}, & B_{cc} = B_{c\bar{c}}/2 = -0.077\ \text{GeV}, \\ B_{bs} = B_{b\bar{s}}/2 = -0.032\ \text{GeV}, & B_{bb} = B_{b\bar{b}}/2 = -0.128\ \text{GeV}, \\ B_{bc} = B_{b\bar{c}}/2 = -0.101\ \text{GeV}, & \end{array}\right\}, \tag{32}$$

where the results for the $c\bar{s}$ pair are also included. Here in the computation, we have used $M(B_c^*) = 6.332$ GeV as the mass of the heavy meson B_c^* in Ref. [42] due to lacking of the measured B_c^*.

It can be seen from Eq. (32) that $B_{QQ'}$ depends monotonically on the reduced mass $\mu_{QQ'} = m_Q m_{Q'}/(m_Q + m_{Q'})$ of two involved quarks Q and Q'. The dependence (Fig. 2) can be approximated by

$$B_{QQ'}(\bar{3}_c) = 0.274\ \text{GeV} - 0.3604(\text{GeV}^{7/8})\mu_{QQ'}^{1/8}. \tag{33}$$

One can scale Eq. (32) for the pair in $\bar{\mathbf{3}}_c$ to the pair QQ' in other color configurations. This can be done by computing the explicit ratios of the color factors in Eq. (19) (evaluated in Appendix A). The scale factor $g([QQ']_R)$ is equal to the ratio of the color factor for representation R and the color factor for $\bar{3}_c$ for the pair $QQ'(= bb, cc, bc, bs, cs)$ and can be given explicitly by

$$\left\{\begin{array}{ll} g([QQ']_{\mathbf{1}_c}) = 2, & g([b\bar{s}]_{\mathbf{6}_c}) = g([c\bar{s}]_{\mathbf{6}_c}) = 5/4, \\ g([QQ']_{\mathbf{6}_c}) = -1/2, & g([b\bar{s}]_{\bar{\mathbf{3}}_c}) = g([c\bar{s}]_{\bar{\mathbf{3}}_c}) = 1/2, \end{array}\right\}. \tag{34}$$

The binding energy between the pair QQ' in R is then

$$B([QQ']_R) = g([QQ']_R) B_{QQ'}, \tag{35}$$

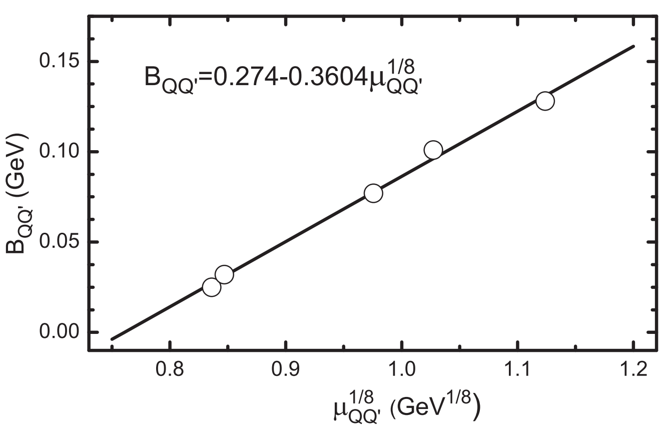

FIG. 2. Binding energy $B_{QQ'}$ (solid line) in Eq. (32) as a function of the reduced mass $\mu_{QQ'}$ of two quarks Q and Q'. Circles correspond to the pair data ($\mu_{QQ'}$,$B_{QQ'}$) with the respective $QQ' = sc, sb, cc, cb, bb$.

with $B_{QQ'} \equiv B([QQ']_{\bar{\mathbf{3}}_c})$ given in Eq. (32). For instance, for a pair QQ' in $R = \mathbf{1}_c$, the scaled result for the binding energy $B([QQ']_{\mathbf{1}_c}) = 2B_{QQ'}$. For a pair in $R = \mathbf{6}_c$, it is $-B_{QQ'}/2$. For $Q\bar{s}$ in a tetraquark $\bar{q}\,\bar{s}\,QQ'$, the scaled binding energy is $5B_{QQ'}/4$ for $Q\bar{s}$ in $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$, and is $B_{QQ'}/2$ for $Q\bar{s}$ in $\mathbf{3}_c \otimes \bar{\mathbf{3}}_c$. The total binding energy of the baryons and tetraquark systems are given by the sum of all pair binding energies and can be found in Eqs. (C8)–(C10) in Appendix C.

Given these values for the QQ' binding energies and the expressions for the CMI matrices in Eq. (4) with the coefficients derived in Appendixes A and B, one can numerically solve the MIT bag model (1) via the variational method for all established hadrons in their lowest-lying states and thereby compute masses, magnetic moments, and charge radii for them. The results are listed in Tables IV,V,VI,VII and VIII for the states without state mixing due to the CMI. The results for the DH baryons are also presented in Table VIII.

Some remarks are in order: (i) Some of the computed masses M_{bag} in Table I deviate from the measured masses by about 30–40 MeV; (ii) The antiparticles of mesons are not listed in Tables IV and V as they share the same masses, charge radii (but minus the magnetic moments) in comparison with the mesons in Tables. The antiparticles of the heavy tetraquarks are ignored as well; (iii) In Table VIII, our mass predictions 3.714 GeV (for Ξ_{cc}^*) are comparable to the quark-model prediction $M(\Xi_{cc}^*) = 3.727$ GeV [16], and also to 3706 ± 28 MeV and 3692 ± 28 MeV obtained using the lattice QCD [43,44] respectively. Table IX shows the comparison of our predictions with other works for DH baryons. The prediction 3.604 GeV for the Ξ_{cc} is consistent with the measured mass 3.621 GeV, considering the simplicity of the model; (iv) The predicted magnetic

TABLE IV. Computed masses (in GeV), magnetic moments (in μ_N), and charge radii (in fm), of light ground-states mesons, compared to the measured data. The blank cells indicate the values are the same as in the above.

State	R_0(GeV^{-1})	M_{bag}	M_{exp} [39]	μ_{bag}	r_E (fm)	r_E (fm) [39]
π^+	4.31	0.348	0.140	$\cdots$	0.62	0.66
ω	4.55	0.776	0.783	0	0	$\cdots$
K^+	4.34	0.561	0.494	$\cdots$	0.61	0.56
K^0			0.498	$\cdots$	0.13	0.28
K^{*+}	4.63	0.918	0.892	2.30	0.65	$\cdots$
K^{*0}			0.896	−0.18	0.15	$\cdots$
ϕ	4.70	1.064	1.019	0	0	$\cdots$

TABLE V. Computed masses (in GeV), magnetic moments (in μ_N), and charge radii (in fm), of heavy mesons in their ground states. We use 5.325 GeV and the blank cells indicate that the values are the same as above.

State	$R_0(\mathrm{GeV}^{-1})$	M_{bag}	M_{exp} [39]	μ_{bag}	r_E (fm)
D^+	3.63	1.835	1.870	$\cdots$	0.46
D^0			1.865	$\cdots$	0.25
D^{*+}	4.09	2.009[input]	2.010	1.21	0.51
D^{*0}			2.007	−0.98	0.29
B^+	3.14	5.248	5.279	$\cdots$	0.42
B^0			5.280	$\cdots$	0.17
B^{*+}	3.47	5.325[input]	5.325	1.32	0.46
B^{*0}				−0.53	0.19
D_s^+	3.77	1.961	1.968	$\cdots$	0.46
D_s^{*+}	4.17	2.112[input]	2.112	1.08	0.51
B_s^0	3.35	5.346	5.367	$\cdots$	0.16
B_s^{*0}	3.62	5.415[input]	5.415	1.01	0.17
η_c	3.15	3.002	2.984	$\cdots$	0
J/ψ	3.54	3.097[input]	3.097	0	0
B_c^+	2.53	6.273	6.274	$\cdots$	0.29
B_c^{*+}	2.81	6.332[input]	6.332	0.52	0.32
η_b	1.59	9.396	9.399	$\cdots$	0
Υ	1.80	9.460[input]	9.460	0	0

TABLE VI. Computed mass (in GeV) of ground-state light baryons. M_{exp} and μ_{exp} are the observed values of the mass and magnetic moments (in μ_N), respectively.

State	$R_0(\mathrm{GeV}^{-1})$	M_{bag}	M_{exp} [39]	μ_{bag}	μ_{exp} [39]	r_E (fm)
p	5.22	0.932	0.938	2.79	2.79	0.75
n			0.940	−1.86	−1.91	0
Δ^{++}	5.33	1.241	1.231	5.70	6.14	1.08
Δ^+			1.235	2.85	2.7	0.77
Δ^0			1.233	0	$\cdots$	0
Δ^-			$\cdots$	−2.85	$\cdots$	0.77
Λ	5.26	1.096	1.116	−0.71	−0.61	0.17
Σ^+	5.22	1.137	1.189	2.72	2.46	0.77
Σ^0			1.193	0.86	$\cdots$	0.17
Σ^-			1.197	−1.01	−1.16	0.73
Σ^{*+}	5.38	1.383	1.383	3.11	$\cdots$	0.79
Σ^{*0}			1.384	0.23	$\cdots$	0.18
Σ^{*-}			1.387	−2.64	$\cdots$	0.75
Ξ^0	5.27	1.282	1.315	−1.58	−1.25	0.25
Ξ^-			1.322	−0.64	−0.65	0.72
Ξ^{*0}	5.42	1.529	1.532	0.48	$\cdots$	0.26
Ξ^{*-}			1.535	−2.43	$\cdots$	0.74
Ω	5.46	1.677	1.672	−2.20	−2.02	0.72

moments in Table VI are in good agreement with the measured values, from which the magnetic moments for heavy baryons and tetraquarks are predicted; (v) Our prediction of 0.75 fm for the proton-charge radius is slightly lower than the newly-measured value 0.83 fm [45,46], but is similar to the original MIT bag model [31,32].

TABLE VII. Computed mass (in GeV), magnetic moments and charge radii of ground-state SH baryons (nonmixed). M_{exp} stands for the observed isospin-averaged mass [39].

State	$R_0(\mathrm{GeV}^{-1})$	M_{bag}	M_{exp} [39]	μ_{bag}	μ [27]	r_E (fm)
Λ_c	4.86	2.270	2.286	0.49	0.42	0.60
Σ_c^{++}	4.82	2.411	2.454	2.13	1.76	0.92
Σ_c^+			2.453	0.41	0.36	0.60
Σ_c^0			2.454	−1.31	−1.04	0.35
Σ_c^{*++}	5.01	2.512	2.518	4.07	$\cdots$	0.95
Σ_c^{*+}			2.518	1.39	$\cdots$	0.62
Σ_c^{*0}			2.518	−1.29	$\cdots$	0.37
Ω_c	4.93	2.680	2.695	−1.07	−0.85	0.28
Ω_c^*	5.10	2.764	2.766	−0.90	$\cdots$	0.29
Λ_b	4.60	5.648	5.620	−0.09	−0.06	0.25
Σ_b^+	4.64	5.835	5.811	2.23	2.07	0.71
Σ_b^0			$\cdots$	0.58	0.53	0.26
Σ_b^-			5.816	−1.07	−1.01	0.62
Σ_b^{*+}	4.73	5.872	5.830	3.29	$\cdots$	0.73
Σ_b^{*0}			$\cdots$	0.76	$\cdots$	0.26
Σ_b^{*-}			5.835	−1.77	$\cdots$	0.63
Ω_b	4.77	6.080	6.046	−0.86	−0.82	0.60
Ω_b^*	4.84	6.112	$\cdots$	−1.43	$\cdots$	0.60

IV. BARYONS AND TETRAQUARKS

A. Heavy baryons including the CMI mixing

Hadrons containing a diquark or an antiquark with different flavors, may not respect the flavor-symmetry of the wave function for involved light quark pairs. As such, the states with same J^P but different spin-color wave functions may mix due to the CMI (4), as mentioned in Sec. II (B). To begin with, we first consider the system of baryons with $J^P = 1/2^+$ in which two spin-color states $(\phi^B\chi_2^B, \phi^B\chi_3^B)$ can mix. The associated baryons are the Ξ_c, the Ξ_b, the Ξ_{bc}, and the Ω_{bc}.

In terms of the wave functions in color and spin space (see Appendixes A and B), one can compute the CMI matrices in the degenerate subspace of the spin-color basis $\phi^B\chi^B$ when the chromomagnetic mixing occurs (Appendix C). These CMI matrices depend upon C_{ij} with the subscripts (i,j) of C_{ij} denote the flavor constituents. One can diagonalize the CMI matrix, (C1), to write the mass formulas of the baryons using Eq. (1). This is done by solving the eigenvalues and eigenvectors of the matrix (C1) analytically and using the latter to identify (denote) the mixed states. Of course, the relevant binding energies (B_{cs}, B_{bs}, and B_{bc}) are included in the mass formulas.

In Table X we list our computed results of masses and other properties for the CMI-mixed systems of heavy baryons. The net effects of the state mixing (the second column of Table) are not so significant in general and they are somehow negligible in the case of singly heavy baryons. This can be due to the higher $SU(3)$ flavor symmetry and heavy-quark symmetry which suppress

TABLE VIII. Computed masses (in GeV), magnetic moments and charge radii of the ground-state doubly heavy baryons (nonmixed states). The magnetic moments by Ref. [27] are listed for comparison.

State	$R_0(\mathrm{GeV}^{-1})$	M_{bag}	μ_{bag}	μ [27]	r_E (fm)
Ξ_{cc}^{++}	4.42	3.604	0.12	0.13	0.78
Ξ_{cc}^{+}			0.91	0.72	0.45
Ξ_{cc}^{*++}	4.64	3.714	2.64	⋯	0.82
Ξ_{cc}^{*+}			0.16	⋯	0.47
Ξ_{bb}^{0}	3.71	10.311	−0.55	−0.53	0.29
Ξ_{bb}^{-}			0.11	0.18	0.45
Ξ_{bb}^{*0}	3.87	10.360	1.21	⋯	0.30
Ξ_{bb}^{*-}			−0.86	⋯	0.47
Ω_{cc}	4.49	3.726	0.86	0.67	0.48
Ω_{cc}^{*}	4.69	3.820	0.33	⋯	0.50
Ω_{bb}	3.83	10.408	0.07	0.04	0.45
Ω_{bb}^{*}	3.97	10.451	−0.75	⋯	0.47

TABLE IX. Computed mass and other calculations cited of doubly heavy baryons, all in GeV.

State	J	This work	[14]	[15]	[16]	[17]	[18]	[19]	[20]
Ξ_{cc}	$\frac{1}{2}$	3.604	3.627	⋯	3.620	3.676	3.612	3.547	3.557
Ξ_{cc}^{*}	$\frac{3}{2}$	3.714	3.690	3.72	3.727	3.753	3.706	3.719	3.661
Ω_{cc}	$\frac{1}{2}$	3.726	⋯	⋯	3.778	3.815	3.702	3.648	3.710
Ω_{cc}^{*}	$\frac{3}{2}$	3.820	⋯	3.78	3.872	3.876	3.783	3.770	3.800
Ξ_{bb}	$\frac{1}{2}$	10.311	10.162	⋯	10.202	10.340	10.197	10.185	10.062
Ξ_{bb}^{*}	$\frac{3}{2}$	10.360	10.184	10.3	10.237	10.367	10.236	10.216	10.101
Ω_{bb}	$\frac{1}{2}$	10.408	⋯	⋯	10.359	10.454	10.260	10.271	10.208
Ω_{bb}^{*}	$\frac{3}{2}$	10.451	⋯	10.4	10.389	10.486	10.297	10.289	10.244
Ξ_{bc}	$\frac{1}{2}$	6.953	6.914	⋯	6.933	7.011	6.919	6.904	6.846
Ξ_{bc}'	$\frac{1}{2}$	7.015	6.933	⋯	6.963	7.047	6.948	6.920	6.891
Ξ_{bc}^{*}	$\frac{3}{2}$	7.044	6.969	7.2	6.980	7.074	6.986	6.936	6.919
Ω_{bc}	$\frac{1}{2}$	7.064	⋯	⋯	7.088	7.136	6.986	6.994	6.999
Ω_{bc}'	$\frac{1}{2}$	7.116	⋯	⋯	7.116	7.165	7.009	7.005	7.036
Ω_{bc}^{*}	$\frac{3}{2}$	7.142	⋯	7.35	7.130	7.187	7.046	7.017	7.063

TABLE X. Predicted masses (in GeV), magnetic moments, and charge radii of heavy baryons. M_{exp} is the observed mass isospin-averaged [39]. Magnetic moments and charge radii are organized in the order of $I_3 = \frac{1}{2}, -\frac{1}{2}$ for $I = \frac{1}{2}$. The bag radius R_0 is in GeV^{-1}.

State	Eigenvector	R_0	M_{bag}	M_{exp} [39]	μ_{bag}	r_E (fm)
(Ξ_c', Ξ_c)	(0.05, 1.00)	4.89	2.436	2.469	0.37, 0.50	0.63, 0.32
	(−1.00, 0.05)	4.88	2.544	2.578	0.67, −1.20	0.63, 0.32
Ξ_c^{*}	1.00	5.06	2.636	2.646	1.61, −1.10	0.65, 0.33
(Ξ_b', Ξ_b)	(0.01, 1.00)	4.64	5.805	5.794	−0.12, −0.08	0.30, 0.60
	(−1.00, 0.01)	4.71	5.956	5.935	0.74, −0.97	0.30, 0.61
Ξ_b^{*}	1.00	4.79	5.991	5.954	0.96, −1.61	0.31, 0.62
(Ξ_{bc}, Ξ_{bc}')	(0.39, 0.92)	4.22	7.015	⋯	1.48, −0.33	0.58, 0.19
	(−0.92, 0.39)	4.09	6.953	⋯	−0.20, 0.09	0.56, 0.19
Ξ_{bc}^{*}	1.000	4.31	7.044	⋯	1.94, −0.37	0.59, 0.20
$(\Omega_{bc}, \Omega_{bc}')$	(0.40, 0.92)	4.29	7.117	⋯	−0.20	0.15
	(−0.92, 0.40)	4.18	7.064	⋯	0.06	0.14
Ω_{bc}^{*}	1.000	4.37	7.143	⋯	−0.22	0.15

the off-diagonal elements in matrix (C1). For this reason, we employ still the normal notations of the states for the SH baryons. For the computed masses of the SH baryons Ξ_c, the Ξ'_c, the Ξ_b, and the Ξ'_b are comparable with the measured data, as seen in the fifth column (with reasonable errors). The magnetic moments for Ξ_c^+, Ξ_c^0, Ξ_b^0, and Ξ_b^- are predicted to be $0.37\mu_N$, $0.50\mu_N$, $-0.12\mu_N$, and $-0.08\mu_N$ which are comparable to $0.35\mu_N$, $0.50\mu_N$, $-0.045\mu_N$, and $-0.08\mu_N$ in Ref. [47], respectively. The magnetic moments for Ξ_c^{*+}, Ξ_c^{*0}, Ξ_b^{*0}, and Ξ_b^{*-} are $1.61\mu_N$, $-1.10\mu_N$, $0.96\mu_N$, and $-1.61\mu_N$ comparable to $1.68\mu_N$, $-0.68\mu_N$, $0.50\mu_N$, and $-1.42\mu_N$ in Ref. [48], respectively.

B. Singly heavy tetraquarks

Let us consider the strange tetraquarks $nn\bar{s}\,\bar{c}$ and $nn\bar{s}\,\bar{b}$ containing one heavy quark, one strange quark, and two nonstrange light quarks. In such a case, the CMI mixing happens if $J \neq 2$. We use a combination of the spin-color basis functions $\phi^T\chi^T$ to denote the mixed states. For instance, the combination $(\phi_2^T\chi_3^T, \phi_1^T\chi_6^T)$ stands for a mixed state $c_1\phi_2^T\chi_3^T + c_2\phi_1^T\chi_6^T$ for $(J^P, I) = (0^+, 1)$. Similarly, other mixed states can be denoted as $(\phi_1^T\chi_3^T, \phi_2^T\chi_6^T)$ for $(J^P, I) = (0^+, 0)$, as $(\phi_2^T\chi_2^T, \phi_2^T\chi_4^T, \phi_1^T\chi_5^T)$ for $(J^P, I) = (1^+, 1)$ and $(\phi_1^T\chi_2^T, \phi_1^T\chi_4^T, \phi_2^T\chi_5^T)$ for $(J^P, I) = (1^+, 0)$. The binding energy matrices become diagonal in mass formula since the mixed states have two color configurations while the spin states are orthogonal. Note that diagonalization should be applied to the sum of the interaction matrices before evaluating the hadron mass.

Following the variational principle, we diagonalize the 2×2 matrix to solve two analytical eigenvalues and construct the mass formula as usual. Application of the same procedure to the 3×3 matrix is, however, not straightforward, for which the eigenvalues are some roots of a cubic equation. For this, we scan three sets of x_i and R to solve the cubic equation numerically so that one can obtain the minimized masses within three root eigenvalues.

Our numerical results are shown in Table XI, with a notable tetraquark of an isosinglet $nn\bar{s}\,\bar{c}$ with $J^P = 0^+$, which has two masses (2.934 GeV and 2.513 GeV) for its two mixed states. Comparing with the measured mass 2866 ± 7 MeV of $X_0(2900)$ reported by LHCb [8] and the quark model prediction of 2863.4 ± 12 MeV [9], our prediction 2.934 GeV is larger even if the model error of 40 MeV is subtracted. If we rather, as Karliner suggested for the color $\bar{\mathbf{3}} \otimes \mathbf{3}$ configuration, ignore the CMI mixing and evaluate directly the masses of the $\phi_1^T\chi_3^T$ and $\phi_2^T\chi_6^T$ states, the resulted masses lie around 2.7 GeV, away from the LHCb reported mass of the $X_0(2900)$. Our calculation suggests that chromomagnetic mixing is strong for the strange tetraquark $nn\bar{s}\,\bar{c}$ with $J^P = 0^+$ and yields a mass splitting as large as 420 MeV.

TABLE XI. Computed mass (in GeV), magnetic moments and charge radii of singly heavy tetraquarks $nn\bar{s}\bar{c}$ and $nn\bar{s}\bar{b}$. Magnetic moments and charge radii are organized in the order of $I_3 = 1, 0, -1$ for $I = 1$. The bag radius R_0 is in GeV^{-1}.

State	J^P	Eigenvector	R_0	M_{bag}	μ_{bag}	r_E (fm)
$(nn\bar{s}\,\bar{c})^{I=1}$	0^+	(0.54, 0.84)	5.73	3.218	⋯	0.91, 0.38, 0.73
		(−0.84, 0.55)	5.39	2.776	⋯	0.85, 0.35, 0.69
	1^+	(0.81, 0.58, 0.10)	5.46	3.001	3.48, 1.54, −0.40	0.86, 0.36, 0.70
		(0.25, −0.49, 0.84)	5.57	3.154	1.03, 0.23, −0.57	0.88, 0.37, 0.71
		(−0.54, 0.65, 0.54)	5.38	2.846	1.67, 0.04, −1.60	0.85, 0.35, 0.69
	2^+	1.00	5.64	3.075	4.27, 1.25, −1.77	0.89, 0.37, 0.72
$(nn\bar{s}\,\bar{c})^{I=0}$	0^+	(0.63, 0.77)	5.56	2.934	⋯	0.37
		(−0.78, 0.63)	5.19	2.513	⋯	0.34
	1^+	(0.77, 0.07, 0.64)	5.40	2.895	0.54	0.35
		(−0.23, −0.90, 0.36)	5.57	3.056	1.24	0.37
		(−0.60, 0.43, 0.68)	5.35	2.674	0.04	0.35
	2^+	1.00	5.66	3.063	1.26	0.37
$(nn\bar{s}\,\bar{b})^{I=1}$	0^+	(0.53, 0.85)	5.53	6.580	⋯	1.07, 0.71, 0.36
		(−0.85, 0.53)	5.28	6.202	⋯	1.02, 0.68, 0.34
	1^+	(0.66, 0.75, 0.06)	5.35	6.419	3.59, 1.37, −0.86	1.03, 0.69, 0.35
		(0.36, −0.38, 0.85)	5.43	6.554	1.33, 0.72, 0.12	1.05, 0.70, 0.35
		(−0.66, 0.55, 0.52)	5.22	6.228	1.99, 0.55, −0.89	1.01, 0.67, 0.34
	2^+	1.00	5.47	6.446	4.72, 1.79, −1.13	1.05, 0.70, 0.36
$(nn\bar{s}\,\bar{b})^{I=0}$	0^+	(0.66, 0.75)	5.41	6.327	⋯	0.69
		(−0.75, 0.66)	5.14	5.980	⋯	0.66
	1^+	(0.66, −0.23, 0.72)	5.33	6.322	0.71	0.68
		(−0.46, −0.88, 0.14)	5.38	6.454	1.31	0.69
		(−0.60, 0.43, 0.68)	5.14	6.038	0.61	0.66
	2^+	1.00	5.48	6.431	1.80	0.70

C. Doubly heavy tetraquarks

Now, let us consider the doubly heavy tetraquarks $qq\bar{Q}\bar{Q}$ with strangeness $S \leq 2$. In this case, the hadrons consist of the nonstrange tetraquarks $nn\bar{Q}\bar{Q}$ and the strange tetraquarks $ns\bar{Q}\bar{Q}$ and $ss\bar{Q}\bar{Q}$. They lie in a larger (compared to baryons) space spanned by more configurations (bases) in which the CMI mixing occurs variously. For the isotriplet tetraquarks with $J^P = 0^+$, the general ground state can be the mixed one, with the wave function $(\phi_2^T\chi_3^T, \phi_1^T\chi_6^T)$. For isosinglet tetraquark $nn\bar{Q}\bar{Q}$ with $J^P = 1^+$, the wave function has the form of $(\phi_2^T\chi_5^T, \phi_1^T\chi_4^T)$. In the case of the strange tetraquark $ns\bar{Q}\bar{Q}$ with $J^P = 1^+$, the wave function can be of $(\phi_2^T\chi_2^T, \phi_2^T\chi_5^T, \phi_1^T\chi_4^T)$ and the wave function of the tetraquark $nn\bar{c}\,\bar{b}$ is similar to that of $nn\bar{s}\bar{c}$. Note that the strange DH states $ns\bar{c}\,\bar{b}$ with mixing among six spin-color states are not considered for simplicity.

The computation of the mass and other properties of these DH tetraquarks is similar to that for the heavy baryons discussed in Sec. IV(A). Our numerical results for DH tetraquarks are listed in Table XII for the $nn\bar{Q}\,\overline{Q\prime}$, in Table XIII for the $ss\bar{Q}\,\overline{Q\prime}$ and Table XIV for the $ns\bar{Q}\,\overline{Q\prime}$. Our results for the mass predictions of the DH tetraquarks are summarized in Table XV and compared with some other calculations cited.

V. SUMMARY AND DISCUSSIONS

In this work, we have studied systematically masses and other properties of hadrons with one and two open heavy quarks within an unified framework of the MIT bag model with chromomagnetic interaction. Masses, magnetic moments, and charge radii of heavy baryons and heavy tetraquarks are computed systematically, including the predictions $M(\Xi_{cc}, 1/2^+) = 3.604$ GeV, $M(\Xi_{cc}^*, , 3/2^+) = 3.714$ GeV, and $M(ud\bar{s}\,\bar{c}, 0^+) = 2.934$ GeV for the strange isosinglet tetraquark $ud\bar{s}\,\bar{c}$. The state mixing due to chromomagnetic interaction is shown to be sizable for the strange scalar tetraquark $nn\bar{s}\,\bar{c}$, giving mass splitting as large as 420 MeV roughly, while it is small for other heavy hadrons.

We also confirm that a term of extra binding energy $B_{QQ'}$, proposed previously to exist among heavy quarks (c and b) and between heavy and strange quarks [14], is required to reconcile light hadron with heavy hadrons, with a useful formula provided for $B_{QQ'}$. This binding effect may rise from the enhanced short-range interaction between two relatively heavy quarks and makes the mass pattern and other properties of heavy hadrons differing from that in light sector. We also employed a slowly-running strong coupling $\alpha_s(R)$ to reflect its dependence upon the hadron sizes proportional to the average distance between two

TABLE XII. Computed mass (in GeV) and other properties of doubly heavy tetraquarks $nn\bar{c}\bar{c}$, $nn\bar{b}\bar{b}$, and $nn\bar{c}\bar{b}$. Magnetic moments and charge radii are organized in the order of $I_3 = 1, 0, -1$ for $I = 1$. The bag radius R_0 is in GeV^{-1}. The mass and magnetic moment of T_{cc}^+ are predicted to be 3.925 GeV and $0.88\mu_N$ which is comparable to $0.66\mu_N$ in Ref. [49].

State	J^P	Eigenvector	R_0	M_{bag}	μ_{bag}	r_E (fm)
$(nn\bar{c}\bar{c})^{I=1}$	0^+	(0.40, 0.92)	5.40	4.342	⋯	0.56, 0.54, 0.94
		(−0.91, 0.41)	5.04	4.032	⋯	0.52, 0.50, 0.88
	1^+	1.00	5.22	4.117	1.36, −0.03, −1.43	0.54, 0.52, 0.91
	2^+	1.00	5.32	4.179	2.80, −0.05, −2.90	0.55, 0.53, 0.93
$(nn\bar{c}\bar{c})^{I=0}$	1^+	(0.97, 0.25)	5.15	3.925	−0.88	0.51
		(−0.24, 0.97)	5.30	4.205	0.83	0.53
$(nn\bar{b}\bar{b})^{I=1}$	0^+	(0.17, 0.99)	4.90	11.092	⋯	0.92, 0.59, 0.38
		(−0.98, 0.18)	4.77	10.834	⋯	0.90, 0.58, 0.37
	1^+	1.00	4.83	10.854	1.81, 0.52, −0.78	0.91, 0.58, 0.38
	2^+	1.00	4.88	10.878	3.65, 1.04, −1.57	0.92, 0.59, 0.38
$(nn\bar{b}\bar{b})^{I=0}$	1^+	(1.00, 0.08)	4.76	10.654	0.18	0.57
		(−0.08, 1.00)	4.83	10.982	0.86	0.58
$(nn\bar{c}\bar{b})^{I=1}$	0^+	(0.30, 0.95)	5.16	7.714	⋯	0.78, 0.25, 0.70
		(−0.95, 0.31)	4.91	7.438	⋯	0.74, 0.23, 0.67
	1^+	(0.65, 0.76, 0.09)	5.04	7.509	2.33, 0.21, −1.90	0.76, 0.24, 0.68
		(0.13, −0.22, 0.97)	5.10	7.699	−0.17, −0.32, −0.47	0.77, 0.24, 0.69
		(−0.75, 0.61, 0.24)	4.96	7.465	2.57, 0.82, −0.93	0.75, 0.24, 0.67
	2^+	1.00	5.12	7.531	3.24, 0.50, −2.24	0.78, 0.25, 0.69
$(nn\bar{c}\bar{b})^{I=0}$	0^+	(0.93, 0.37)	4.96	7.502	⋯	0.24
		(−0.38, 0.93)	4.84	7.260	⋯	0.23
	1^+	(0.91, −0.36, 0.21)	4.93	7.518	0.55	0.23
		(−0.39, −0.92, 0.10)	5.07	7.605	0.50	0.24
		(−0.16, 0.18, 0.97)	4.93	7.288	−0.33	0.23
	2^+	1.00	5.14	7.483	0.50	0.25

TABLE XIII. Computed mass (in GeV), magnetic moments and charge radii of doubly heavy tetraquarks $ss\bar{c}\bar{c}$, $ss\bar{b}\bar{b}$ and $ss\bar{c}\bar{b}$. The bag radius R_0 is in GeV^{-1}.

State	J^P	Eigenvector	R_0	M_{bag}	μ_{bag}	r_E (fm)
$ss\bar{c}\bar{c}$	0^+	(0.49, 0.87)	5.48	4.521	···	0.92
		(−0.87, 0.50)	5.13	4.300	···	0.87
	1^+	1.00	5.30	4.382	−1.22	0.89
	2^+	1.00	5.39	4.433	−2.46	0.91
$ss\bar{b}\bar{b}$	0^+	(0.24, 0.97)	5.01	11.232	···	0.32
		(−0.97, 0.25)	4.88	11.078	···	0.31
	1^+	1.00	4.94	11.099	−0.60	0.31
	2^+	1.00	4.98	11.119	−1.20	0.32
$ss\bar{c}\bar{b}$	0^+	(0.40, 0.92)	5.26	7.875	···	0.67
		(−0.91, 0.40)	5.01	7.693	···	0.64
	1^+	(0.70, 0.71, 0.11)	4.98	7.757	−1.54	0.64
		(0.17, −0.32, 0.93)	5.06	7.858	−0.48	0.64
		(−0.69, 0.63, 0.35)	4.88	7.716	−0.66	0.62
	2^+	1.00	5.20	7.779	−1.84	0.66

TABLE XIV. Computed mass (in GeV) and other properties of doubly heavy tetraquarks $ns\bar{c}\bar{c}$, $ns\bar{b}\bar{b}$. Magnetic moments and charge radii are organized in the order of $I_3 = \frac{1}{2}, -\frac{1}{2}$ for $I = \frac{1}{2}$. The bag radius R_0 is in GeV^{-1}.

State	J^P	Eigenvector	R_0	M_{bag}	μ_{bag}	r_E (fm)
$ns\bar{c}\bar{c}$	0^+	(0.44, 0.90)	5.44	4.429	···	0.51, 0.93
		(−0.89, 0.45)	5.09	4.165	···	0.48, 0.88
	1^+	(0.99, −0.07, 0.09)	5.16	4.247	0.32, −1.33	0.49, 0.89
		(−0.11, −0.28, 0.95)	5.23	4.314	0.86, −1.58	0.49, 0.90
		(0.04, 0.96, 0.29)	5.04	4.091	−0.95, −1.03	0.48, 0.87
	2^+	1.00	5.36	4.305	0.19, −2.68	0.50, 0.92
$ns\bar{b}\bar{b}$	0^+	(0.20, 0.98)	4.96	11.160	-	0.62, 0.35
		(−0.98, 0.20)	4.83	10.955	-	0.60, 0.34
	1^+	(1.00, −0.01, 0.03)	4.79	10.974	0.65, −0.68	0.60, 0.34
		(−0.03, −0.09, 1.00)	4.77	11.068	1.02, −1.50	0.60, 0.34
		(0.01, 1.00, 0.10)	4.66	10.811	0.14, 0.16	0.58, 0.33
	2^+	1.00	4.93	10.997	1.25, −1.39	0.62, 0.35

interacted quarks (or antiquark) in a hadron. The strong coupling $\alpha_s(R)$ runs from 0.4 to 0.6 as the bag radius R varies between 3–6 GeV^{-1}.

We remark that the MIT bag model can reproduce the measured masses of heavy hadrons within the accuracy of 40 MeV–50 MeV, from which we proceed to predict the masses and other properties of the tetraquarks with one and two open heavy quarks. For the DH tetraquarks, we reduce the error limit to about 40 MeV and exclude $X_0(2900)$ to be an isosinglet tetraquark of $nn\bar{s}\,\bar{c}$ due to the mismatch with the measured data as high as 70 MeV.

Owing to the uncertainty of model computations, we are not able to discuss the near-threshold effect. The mismatch of our predictions with the measured data may come from the limitations of the bag model in this work: (1) the bag may deform into an elliptic shape in the case of the DH hadrons, and (2) the constant approximation of the short-range binding energy may not be sufficient as the latter may depend upon hadron's size R implicitly, for instance, in the form of a Coulomb-like $\sim 1/R$, and needs to be determined variationally. These effects go beyond the scope of this work and await the further exploration in the future.

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APPENDIX A: COLOR WAVE FUNCTIONS

For meson $q_1\bar{q}_2$ (denoted by M), baryon $(q_1q_2)q_3$ (denoted by B), and tetraquark systems $q_1q_2\bar{q}_3\bar{q}_4$ (denoted by T), the full color wave functions, which respect $SU(3)_c$ symmetry, can be written as

TABLE XV. Comparison of calculated mass (in GeV) among different calculations for double heavy tetraquarks. The masses before and after the slash stand for that of the color states split chromomagnetically. Refs. [7,50] employ the CMI model.

State	J	This work	Scheme 1[7]	Scheme 2[7]	[50]	[51]
$(nn\bar{c}\bar{c})^{I=1}$	0	4.032/4.342	4.078/4.356	3.850/4.128	4.195/4.414	4.056
	1	4.117	4.201	3.973	4.268	4.079
	2	4.179	4.271	4.044	4.318	4.118
$(nn\bar{c}\bar{c})^{I=0}$	1	3.925/4.205	4.007/4.204	3.779/3.977	4.041/4.313	3.935
$(nn\bar{b}\bar{b})^{I=1}$	0	10.834/11.092	10.841/10.937	10.637/10.734	10.765/11.019	10.648
	1	10.854	10.875	10.671	10.779	10.657
	2	10.878	10.897	10.694	10.799	10.673
$(nn\bar{b}\bar{b})^{I=0}$	1	10.654/10.982	10.686/10.821	10.483/10.617	10.550/10.951	10.502
$(nn\bar{c}\bar{b})^{I=1}$	0	7.438/7.714	7.457/7.643	7.241/7.428	7.519/7.740	7.383
	1	7.465/7.509	7.473/7.548	7.258/7.332	7.537/7.561	7.396/7.403
		7.699	7.609	7.393	7.729	
	2	7.531	7.582	7.367	7.586	7.422
$(nn\bar{c}\bar{b})^{I=0}$	0	7.260/7.502	7.256/7.429	7.041/7.213	7.297/7.580	7.239
	1	7.288/7.518	7.321/7.431	7.106/7.215	7.325/7.607	7.246
		7.605	7.516	7.301	7.666	
	2	7.483	7.530	7.315	7.697	-
$ns\bar{c}\bar{c}$	0	4.165/4.429	4.236/4.514	3.933/4.210	4.323/4.512	4.221
	1	4.091/4.247	4.225/4.363	3.921/4.060	4.232/4.394	4.143/4.239
		4.314	4.400	4.096	4.427	
	2	4.305	4.434	4.131	4.440	4.271
$ns\bar{b}\bar{b}$	0	10.955/11.160	10.999/11.095	10.707/10.804	10.883/11.098	10.802
	1	10.811/10.974	10.911/11.010	10.619/10.718	10.734/10.897	10.706/10.809
		11.068	11.037	10.745	11.046	
	2	10.997	11.060	10.769	10.915	10.823
$ss\bar{c}\bar{c}$	0	4.300/4.521	4.395/4.672	4.016/4.293	4.417/4.587	4.359
	1	4.382	4.526	4.146	4.493	4.375
	2	4.433	4.597	4.218	4.536	4.402
$ss\bar{b}\bar{b}$	0	11.078/11.232	11.157/11.254	10.777/10.875	10.972/11.155	10.932
	1	11.099	11.199	10.820	10.986	10.939
	2	11.119	11.224	10.844	11.004	10.950
$ss\bar{c}\bar{b}$	0	7.693/7.875	7.774/7.960	7.394/7.581	7.735/7.894	7.673
	1	7.716/7.757	7.793/7.872	7.414/7.493	7.752/7.775	7.683/7.684
		7.858	7.924	7.545	7.881	
	2	7.779	7.908	7.529	7.798	7.701

$$\phi^M = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b}), \tag{A1}$$

$$\phi^B = \frac{1}{\sqrt{6}}(gbr - bgr + brg - rbg + rgb - grb),$$

$$\phi_1^T = \frac{1}{\sqrt{6}}(rr\bar{r}\,\bar{r} + gg\bar{g}\,\bar{g} + bb\bar{b}\,\bar{b}) + \frac{1}{2\sqrt{6}}(rb\bar{b}\,\bar{r} + br\bar{b}\,\bar{r} + gr\bar{g}\,\bar{r} + rg\bar{g}\,\bar{r} + gb\bar{b}\,\bar{g} + bg\bar{b}\,\bar{g} + gr\bar{r}\,\bar{g} + rg\bar{r}\,\bar{g} + gb\bar{g}\,\bar{b} + bg\bar{g}\,\bar{b} + rb\bar{r}\,\bar{b} + br\bar{r}\,\bar{b}), \tag{A2}$$

$$\phi_2^T = \frac{1}{2\sqrt{3}}(rb\bar{b}\,\bar{r} - br\bar{b}\,\bar{r} - gr\bar{g}\,\bar{r} + rg\bar{g}\,\bar{r} + gb\bar{b}\,\bar{g} - bg\bar{b}\,\bar{g} + gr\bar{r}\,\bar{g} - rg\bar{r}\,\bar{g} - gb\bar{g}\,\bar{b} + bg\bar{g}\,\bar{b} - rb\bar{r}\,\bar{b} + br\bar{r}\,\bar{b}), \tag{A3}$$

respectively. Here, the wave function ϕ_1^T in Eq. (16) corresponds to the configuration $6_c \otimes \bar{6}_c$ while ϕ_2^T there corresponds to $3_c \otimes \bar{3}_c$.

Using the color wave functions above and Eq. (19), one can compute the matrices of color factors. The results can be given explicitly by

$$\langle \boldsymbol{\lambda_1} \cdot \boldsymbol{\lambda_2} \rangle = -\frac{16}{3}, \tag{A4}$$

for meson with the wave function (ϕ^M) and

$$\langle \boldsymbol{\lambda_1} \cdot \boldsymbol{\lambda_2} \rangle = \langle \boldsymbol{\lambda_1} \cdot \boldsymbol{\lambda_3} \rangle = \langle \boldsymbol{\lambda_2} \cdot \boldsymbol{\lambda_3} \rangle = -\frac{8}{3}, \tag{A5}$$

for baryon with (ϕ^B). For tetraquarks with the two-component wave functions (ϕ_1^T, ϕ_2^T), the matrices of color factors are

$$\langle \boldsymbol{\lambda_1}\cdot\boldsymbol{\lambda_2}\rangle = \langle \boldsymbol{\lambda_3}\cdot\boldsymbol{\lambda_4}\rangle = \begin{bmatrix} \frac{4}{3} & 0 \\ 0 & -\frac{8}{3} \end{bmatrix},$$
$$\langle \boldsymbol{\lambda_1}\cdot\boldsymbol{\lambda_3}\rangle = \langle \boldsymbol{\lambda_2}\cdot\boldsymbol{\lambda_4}\rangle = \begin{bmatrix} -\frac{10}{3} & 2\sqrt{2} \\ 2\sqrt{2} & -\frac{4}{3} \end{bmatrix},$$
$$\langle \boldsymbol{\lambda_1}\cdot\boldsymbol{\lambda_4}\rangle = \langle \boldsymbol{\lambda_2}\cdot\boldsymbol{\lambda_3}\rangle = \begin{bmatrix} -\frac{10}{3} & -2\sqrt{2} \\ -2\sqrt{2} & -\frac{4}{3} \end{bmatrix}, \quad \text{(A6)}$$

all of which are 2×2 matrices in the space of the two-component wavefunction (ϕ_1^T, ϕ_2^T).

APPENDIX B: SPIN WAVE FUNCTIONS

For meson $q_1\bar{q}_2$ (denoted by M), baryon $(q_1q_2)q_3$ (denoted by B), and tetraquark systems $q_1q_2\bar{q}_3\bar{q}_4$ (denoted by T), one can write the spin wave functions for them, with the help of the Clebsch-Gordan coefficients. The results are

$$\chi_1^M = \uparrow\uparrow, \qquad \chi_2^M = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow), \quad \text{(B1)}$$

for the mesons, and

$$\chi_1^B = \uparrow\uparrow\uparrow,$$
$$\chi_2^B = \sqrt{\frac{2}{3}}\uparrow\uparrow\downarrow - \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow),$$
$$\chi_3^B = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \text{(B2)}$$

for the baryons. For the tetraquark there are six spin wave functions,

$$\chi_1^T = \uparrow\uparrow\uparrow\uparrow,$$
$$\chi_2^T = \frac{1}{2}(\uparrow\uparrow\uparrow\downarrow + \uparrow\uparrow\downarrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow),$$
$$\chi_3^T = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow\downarrow + \downarrow\downarrow\uparrow\uparrow), -\frac{1}{2\sqrt{3}}(\uparrow\downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\uparrow\downarrow + \downarrow\uparrow\downarrow\uparrow)$$
$$\chi_4^T = \frac{1}{\sqrt{2}}(\uparrow\uparrow\uparrow\downarrow - \uparrow\uparrow\downarrow\uparrow),$$
$$\chi_5^T = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow),$$
$$\chi_6^T = \frac{1}{2}(\uparrow\downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow\uparrow - \downarrow\uparrow\uparrow\downarrow + \downarrow\uparrow\downarrow\uparrow), \quad \text{(B3)}$$

which correspond to the states (12), (14) and (17), respectively.

Given the spin wave functions above, one can also compute the matrices of spin factors with the help of Eq. (20). There is one spin matrix

$$\langle \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}, \quad \text{(B4)}$$

for meson in (χ_1^M, χ_2^M) space, and three spin matrices

$$\langle \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}\rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \text{(B5)}$$

$$\langle \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_3}\rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -\sqrt{3} \\ 0 & -\sqrt{3} & 0 \end{bmatrix}, \quad \text{(B6)}$$

$$\langle \boldsymbol{\sigma_2}\cdot\boldsymbol{\sigma_3}\rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & \sqrt{3} \\ 0 & \sqrt{3} & 0 \end{bmatrix}, \quad \text{(B7)}$$

for a baryon in $(\chi_1^B, \chi_2^B, \chi_3^B)$ space. In the case of a tetraquark, there are six spin matrices,

$$\langle \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 \end{bmatrix}, \quad \text{(B8)}$$

$$\langle \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_3}\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & \sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & -2 & 0 & 0 & -\sqrt{3} \\ 0 & \sqrt{2} & 0 & 0 & 1 & 0 \\ 0 & -\sqrt{2} & 0 & 1 & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 0 & 0 & 0 \end{bmatrix}, \tag{B9}$$

$$\langle \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_4}\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & -2 & 0 & 0 & \sqrt{3} \\ 0 & -\sqrt{2} & 0 & 0 & -1 & 0 \\ 0 & -\sqrt{2} & 0 & -1 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 & 0 \end{bmatrix}, \tag{B10}$$

$$\langle \boldsymbol{\sigma_2}\cdot\boldsymbol{\sigma_3}\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & \sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & -2 & 0 & 0 & \sqrt{3} \\ 0 & \sqrt{2} & 0 & 0 & -1 & 0 \\ 0 & \sqrt{2} & 0 & -1 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 & 0 \end{bmatrix}, \tag{B11}$$

$$\langle \boldsymbol{\sigma_2}\cdot\boldsymbol{\sigma_4}\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & -2 & 0 & 0 & -\sqrt{3} \\ 0 & -\sqrt{2} & 0 & 0 & 1 & 0 \\ 0 & \sqrt{2} & 0 & 1 & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 0 & 0 & 0 \end{bmatrix}, \tag{B12}$$

$$\langle \boldsymbol{\sigma_3}\cdot\boldsymbol{\sigma_4}\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 \end{bmatrix}, \tag{B13}$$

in the subspace of $(\chi_1^T, \chi_2^T, \chi_3^T, \chi_4^T, \chi_5^T, \chi_6^T)$.

One can then use these factors of color and spin space to find the matrix representation of the CMI in Eq. (4) for a given hadronic state. The color and spin factors are the diagonal elements of the matrices in Eqs. (A4)–(A6) and (B4)–(B13), respectively. The off-diagonal elements of these matrices lead to chromomagnetic mixing of these basis functions given in Appendix A and at the beginning of this section.

APPENDIX C: CHROMOMAGNETIC MIXING SYSTEMS

Based on Appendixes A and B, one can use Eq. (4) to calculate the matrices of the CMI in the hadronic basis of the wave functions involved in this work. We list some of them whose nondiagonal elements are nonvanishing. For instance, for the $(\phi^B\chi_2^B, \phi^B\chi_3^B)$ mixed state of baryons, the CMI matrix is

$$\begin{pmatrix} \frac{8}{3}C_{12} - \frac{16}{3}C_{13} - \frac{16}{3}C_{23} & -\frac{8\sqrt{3}}{3}C_{13} + \frac{8\sqrt{3}}{3}C_{23} \\ -\frac{8\sqrt{3}}{3}C_{13} + \frac{8\sqrt{3}}{3}C_{23} & -8C_{12} \end{pmatrix}, \tag{C1}$$

and for the $(\phi_2^T\chi_3^T, \phi_1^T\chi_6^T)$ state of tetraquarks, it is

$$\begin{pmatrix} \frac{8}{3}(\alpha-\beta) & 2\sqrt{6}\beta \\ 2\sqrt{6}\beta & 4\alpha \end{pmatrix}. \tag{C2}$$

For other cases of tetraquarks, the CMI matrices can be obtained similarly. They are

$$\begin{pmatrix} -\frac{8}{3}\theta & -2\sqrt{2}\beta \\ -2\sqrt{2}\beta & -\frac{4}{3}\eta \end{pmatrix}, \tag{C3}$$

for the tetraquark wave function $(\phi_2^T\chi_5^T, \phi_1^T\chi_4^T)$, and

$$\begin{pmatrix} -\frac{4}{3}(\alpha+5\beta) & 2\sqrt{6}\beta \\ 2\sqrt{6}\beta & -8\alpha \end{pmatrix}, \tag{C4}$$

for the tetraquark wave function $(\phi_1^T\chi_3^T, \phi_2^T\chi_6^T)$. In the case of a three-dimensional subspace, one can find the CMI matrix to be

$$\begin{pmatrix} \frac{4}{3}(2\alpha-\beta) & \frac{4\sqrt{2}}{3}\delta & 4\delta \\ \frac{4\sqrt{2}}{3}\delta & \frac{8}{3}\eta & -2\sqrt{2}\beta \\ 4\delta & -2\sqrt{2}\beta & \frac{4}{3}\beta \end{pmatrix}, \tag{C5}$$

for the mixed state of $(\phi_2^T\chi_2^T, \phi_2^T\chi_4^T, \phi_1^T\chi_5^T)$ of tetraquarks, and

$$\begin{pmatrix} \frac{4}{3}(2\alpha-\beta) & -\frac{4\sqrt{2}}{3}\gamma & -4\gamma \\ -\frac{4\sqrt{2}}{3}\gamma & -\frac{8}{3}\theta & -2\sqrt{2}\beta \\ -4\gamma & -2\sqrt{2}\beta & -\frac{4}{3}\eta \end{pmatrix}, \tag{C6}$$

for the tetraquark state of $(\phi_2^T\chi_2^T, \phi_2^T\chi_5^T, \phi_1^T\chi_4^T)$, in addition to

$$\begin{pmatrix} -\frac{2}{3}(2\alpha+5\beta) & \frac{10\sqrt{2}}{3}\delta & 4\delta \\ \frac{10\sqrt{2}}{3}\delta & -\frac{4}{3}\eta & -2\sqrt{2}\beta \\ 4\delta & -2\sqrt{2}\beta & -\frac{8}{3}\theta \end{pmatrix}, \tag{C7}$$

for the tetraquark state of $(\phi_1^T\chi_2^T, \phi_1^T\chi_4^T, \phi_2^T\chi_5^T)$. In these matrices, one used $\alpha = C_{12} + C_{34}$, $\beta = C_{13} + C_{14} + C_{23} + C_{24}$, $\gamma = C_{13} + C_{14} - C_{23} - C_{24}$, $\delta = C_{13} - C_{14} + C_{23} - C_{24}$, $\eta = C_{12} - 3C_{34}$, and $\theta = 3C_{12} - C_{34}$, from Ref. [7].

In the following we list the expressions for overall binding energy of the hadrons involved in this work. They are

$$B_{12} + B_{13} + B_{23}, \tag{C8}$$

for the baryons described by ϕ^B. For the tetraquarks, the overall binding energy is

$$-\frac{1}{2}B_{12} + \frac{5}{4}B_{13} + \frac{5}{4}B_{14} + \frac{5}{4}B_{23} + \frac{5}{4}B_{24} - \frac{1}{2}B_{34}, \tag{C9}$$

for the configuration ϕ_1^T, and

$$B_{12} + \frac{1}{2}B_{13} + \frac{1}{2}B_{14} + \frac{1}{2}B_{23} + \frac{1}{2}B_{24} + B_{34}, \tag{C10}$$

for the configuration ϕ_2^T, respectively. Here, the notation B_{ij} ($i, j = 1, 2, 3, 4$, corresponding to b, c, s, and $n = u, d$) stands for the binding energies in Eq. (32), and is assumed to vanish if $i, j = nn, sn$, or ss, in which case there is no short-distance binding in hadrons [14].

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