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Electroweak effects in the extraction of the CKM angle γ from $B \to D\pi$ decays

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ABSTRACT

The angle γ of the standard CKM unitarity triangle can be determined from tree-level B-meson decays essentially without hadronic uncertainties. We calculate the second-order electroweak corrections for the $B \to D\pi$ modes and show that their impact on the determination of γ could be enhanced by an accidental cancellation of poorly known hadronic matrix elements. However, we do not expect the resulting shift in γ to exceed $|\delta\gamma^{D\pi}/\gamma| \lesssim \mathcal{O}(10^{-4})$.

1. Introduction

The Cabibbo–Kobayashi–Maskawa (CKM) angle $\gamma \equiv \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ can be extracted from $B \to DK$ and $B \to D\pi$ decays that receive contributions only from tree operators [1]. The absence of penguin contributions and the fact that all relevant hadronic matrix elements can be obtained from data makes this determination theoretically extremely clean, thus providing a standard candle for the search for physics beyond the standard model (SM).

The sensitivity to γ arises from the interference of $b \to c\bar{u}q$ and $b \to u\bar{c}q$ decay amplitudes (see Fig. 1), which have a relative weak phase γ. Here, q denotes either a strange or a down quark. The quark-level transitions with $q = s$ mediate the $B^- \to D^0K^-$ and $B^- \to \bar{D}^0K^-$ decays, whereas the transitions with $q = d$ induce the $B^- \to D^0\pi^-$ and $B^- \to \bar{D}^0\pi^-$ decays. In both cases the D^0 and $\bar{D}^0$ mesons can decay into a common final state f, leading to the interference of the two decay channels. Several variants of this method have been formulated, distinguished by the final state f [2–7]. Alternatively, one can also use decays of neutral B^0 or B_s^0 mesons [8,9], multibody B decays [10–13], and D^* or D^{**} decays [14,15] (see also the reviews in [16]).

Whereas in most analyses γ has been extracted only from the $B \to DK$ modes, the LHCb Collaboration recently included also the $B \to D\pi$ modes in their full combination [17,18]. The sensitivity to γ of these modes is smaller than that of the $B \to DK$ modes, due to a smaller interference term; this effect is, however, partially compensated by the larger $B \to D\pi$ branching ratio.

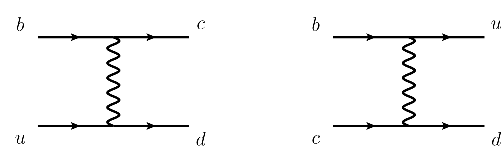

Fig. 1. Tree contributions (with single W exchange) that mediate $b \to c\bar{u}d$ (left) and $b \to u\bar{c}d$ (right) quark-level processes, which lead to $B^- \to D^0\pi^-$ and $B^- \to \bar{D}^0\pi^-$ decays, respectively.

The extraction of γ from tree-level decays suffers from various uncertainties. Some of them can be reduced once more statistics becomes available, for instance, those related to a Dalitz-plot analysis [6,19–21]. Other sources of reducible uncertainties are D–$\bar{D}$ mixing and, for final states with a K_S, also K–$\bar{K}$ mixing. Both of these effects can be taken into account by measuring the mixing parameters and appropriately modifying the expressions for the decay amplitudes [22–25]. In a similar manner, the effects of nonzero $\Delta\Gamma_s$ can be included into the γ extraction from untagged $B_s \to D\phi$ decays [26]. It is also possible to allow for CP violation in the D-meson decays [17,27–30]. The effects of CP violation in kaon mixing have recently been discussed in [31]. Finally, the impact on γ of new-physics contributions to tree-level Wilson coefficients has been estimated in [32].

As shown in [33], the first irreducible theory error on the determination of γ arises from higher-order electroweak corrections. It has been calculated for the $B \to DK$ modes, resulting in an upper bound on the shift in γ of $\delta\gamma^{DK}/\gamma \lesssim \mathcal{O}(10^{-7})$ [33]. The shift due to electroweak corrections for the extraction of γ from the $B \to D\pi$ modes has not yet been computed; we close the gap in this Letter.

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The main difference between the $B \to D\pi$ and the $B \to DK$ modes lies in their CKM structure. Consequently, as we will see later, the effect of the electroweak corrections for the $B \to D\pi$ modes could potentially be much larger than for the $B \to DK$ modes, due to an approximate cancellation of hadronic matrix elements. However, we do not expect the final shift in γ to exceed $|\delta\gamma^{D\pi}/\gamma| \lesssim \mathcal{O}(10^{-4})$ without some accidental fine tuning – well below the precision of any current or future measurement.

This Letter is organized as follows. In Section 2 we calculate the electroweak corrections to the relevant Wilson coefficients and estimate the resulting shift in γ in Section 3. We conclude in Section 4.

2. Calculation of the electroweak corrections

We calculate the shift in γ due to electroweak corrections in close analogy to the procedure in Ref. [33]. The sensitivity of the $B \to D\pi$ modes to γ enters through the amplitude ratio

$$r_B^{D\pi} e^{i(\delta_B^{D\pi} - \gamma)} \equiv \frac{A(B^- \to \bar{D}^0\pi^-)}{A(B^- \to D^0\pi^-)}, \tag{1}$$

where $r_B^{D\pi} \in [0.001, 0.040]$ at 95% CL [18] reflects the CKM and color suppression of the amplitude $A(B^- \to \bar{D}^0\pi^-)$ relative to the amplitude $A(B^- \to D^0\pi^-)$ (there is currently no constraint on the strong phase $\delta_B^{D\pi}$ at 95% CL). Note that the corresponding ratio r_B^{DK} for the $B \to DK$ modes is known more precisely, $r_B^{DK} \in [0.0732, 0.1085]$ at 95% CL [18]. Naive scaling by CKM factors leads to the generic expectation $r_B^{D\pi} \approx 5 \times 10^{-3}$.

The equality (1) is valid only at leading order in the weak interactions, $\mathcal{O}(G_F)$, where both the $b \to c\bar{u}d$ and $b \to u\bar{c}d$ transitions are mediated by a tree-level W exchange.[1] At a scale of order m_b the two transitions are then described by the leading nonleptonic weak effective Hamiltonians [34]

$$\mathcal{H}_{\bar{c}u}^{(0)} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[C_1(\mu) Q_1^{\bar{c}u} + C_2(\mu) Q_2^{\bar{c}u} \right], \tag{2}$$

$$\mathcal{H}_{\bar{u}c}^{(0)} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* \left[C_1(\mu) Q_1^{\bar{u}c} + C_2(\mu) Q_2^{\bar{u}c} \right] \tag{3}$$

which involve the usual four-fermion operators defined by

$$Q_1^{\bar{c}u} = (\bar{c}b)_{V-A} (\bar{d}u)_{V-A}, \qquad Q_2^{\bar{c}u} = (\bar{d}b)_{V-A} (\bar{c}u)_{V-A}, \tag{4}$$

$$Q_1^{\bar{u}c} = (\bar{u}b)_{V-A} (\bar{d}c)_{V-A}, \qquad Q_2^{\bar{u}c} = (\bar{d}b)_{V-A} (\bar{u}c)_{V-A}. \tag{5}$$

Here, $(\bar{q}q')_{V-A}$ denotes the left-handed structure $\bar{q}\gamma^\mu(1-\gamma_5)q'$, for quark fields q, q'. The Wilson coefficients, evaluated at a scale of the order of the b-quark mass $\mu \sim m_b$, are given by $C_1(m_b) = 1.10$ and $C_2(m_b) = -0.24$ at leading-log order, for $m_b(m_b) = 4.163\,\text{GeV}$ [35] and the strong coupling constant $\alpha_s(M_Z) = 0.1184$ [36]. The decay amplitudes in Eq. (1) are then given, at leading order in the electroweak interactions, by

$$A(B^- \to \bar{D}^0\pi^-) = \langle \bar{D}^0\pi^- | \mathcal{H}_{\bar{u}c}^{(0)} | B^- \rangle,$$
$$A(B^- \to D^0\pi^-) = \langle D^0\pi^- | \mathcal{H}_{\bar{c}u}^{(0)} | B^- \rangle. \tag{6}$$

[1] In Ref. [31] it has been pointed out that the weak phase entering the $B \to DK$ modes differs from γ by subleading corrections of order $\lambda^4 \approx 2.6 \times 10^{-3}$, where $\lambda \equiv |V_{us}|$ is the Wolfenstein parameter (note that in [31] λ erroneously appears raised to the power of 5). A similar observation applies for the $B \to D\pi$ modes. The relation of the phase of $r_B^{D\pi}$ to γ involves the ratio $V_{cd}^2/V_{ud}^2 = \lambda^2[1 - \lambda^4 A^2(1 - 2(\rho + i\eta)) + \mathcal{O}(\lambda^6)]$. This introduces another small $\mathcal{O}(\lambda^4)$ uncertainty into the extraction of γ which can, in principle, be removed by measuring the phase of V_{cd}/V_{ud} independently.

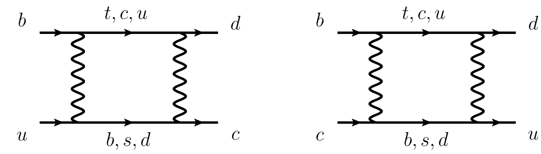

Fig. 2. The electroweak corrections to the $b \to c\bar{u}d$ and $b \to u\bar{c}d$ processes at order $\mathcal{O}(G_F^2)$. Curly lines represent W bosons and the corresponding pseudo-Goldstone bosons.

Electroweak corrections, of the order of $\mathcal{O}(G_F^2)$, to the amplitudes will induce a shift $\delta\gamma^{D\pi}$ in the extracted value of γ if the $\mathcal{O}(G_F)$ and $\mathcal{O}(G_F^2)$ contributions differ in their weak phase. As argued in [33], the only second-order weak corrections to (1) and (6) that need to be considered are those arising from W box diagrams that have a different CKM structure than the corresponding tree amplitude, see Fig. 2. (Diagrams with photon or Z-boson exchange do not lead to a different CKM structure, whereas W vertex corrections can be absorbed into a universal renormalization of the CKM matrix elements.)

For instance, the CKM structures of the $b \to u\bar{c}d$ transition (left diagrams in Figs. 1 and 2) are given by $V_{ub}V_{cd}^*$ for the tree-level diagram and $(V_{tb}V_{td}^*)(V_{ub}V_{cb}^*)$ for the box diagram. They differ in their weak phases and thus lead to a shift in the extracted value of γ.

The $b \to c\bar{u}d$ transition receives a similar correction (right diagrams in Figs. 1 and 2), with CKM structures $V_{cb}V_{ud}^*$ at tree level and $(V_{tb}V_{td}^*)(V_{cb}V_{ub}^*)$ for the box diagram. The effects of these diagrams are CKM suppressed with respect to the previous contribution by two orders of magnitude and can be safely neglected.

To a very good approximation, the only effect of the box diagrams is thus a correction to the Wilson coefficients in the effective Hamiltonian (3). Keeping only the local parts of the box diagrams we can write

$$\mathcal{H}_{\bar{u}c}^{(1)} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* \Big[\big(C_1(\mu) + \Delta C_1(\mu)\big) Q_1^{\bar{u}c} + \big(C_2(\mu) + \Delta C_2(\mu)\big) Q_2^{\bar{u}c} \Big]. \tag{7}$$

The Wilson coefficients $C_{1,2}(\mu)$ are the same as in Eqs. (2) and (3), while $\Delta C_{1,2}(\mu)$ are corrections of $\mathcal{O}(G_F)$ relative to the tree-level diagrams. They depend on the CKM elements and carry a weak phase different from that of $C_{1,2}(\mu)$ (which are real in our convention). While the precise absolute values of the Wilson coefficients are irrelevant for the experimental analysis, where all branching fractions and amplitude ratios are fitted from data, a contribution with a relative weak phase will induce a shift in the extracted value of γ.

To get a first estimate of the size of the effect we will perform a matching calculation from the SM directly onto the weak effective Hamiltonian where the W boson, the top quark, and the bottom quark have been integrated out simultaneously. To this end, we evaluate the box diagrams in Fig. 2 at $\mu \sim M_W$, treating the top and bottom quarks as massive and all remaining quarks as massless, and setting all external momenta to zero. Because of the Glashow–Iliopoulos–Maiani mechanism acting on both the internal up-quark and down-quark lines the result is proportional to $x_t y_b$, where $x_t \equiv m_t^2/M_W^2$, $y_b \equiv m_b^2/M_W^2$, and we find for the shift ΔC_2 of the Wilson coefficient C_2 in Eq. (7)

$$\Delta C_2 = -\sqrt{2} G_F \frac{M_W^2}{4\pi^2} \frac{V_{tb}V_{td}^*V_{cb}^*}{V_{cd}^*} \hat{C}(x_t, y_b)$$
$$= -\sqrt{2} G_F \frac{M_W^2}{4\pi^2} \left| \frac{V_{tb}V_{td}V_{cb}}{V_{cd}} \right| e^{i\beta} \hat{C}(x_t, y_b), \tag{8}$$

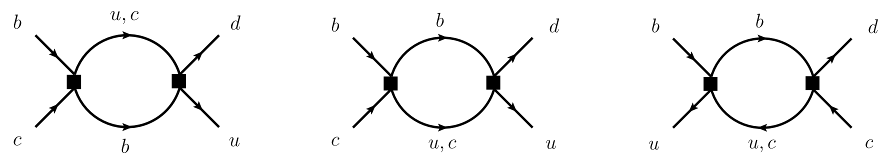

Fig. 3. The double insertion $T\{Q_1, Q_1\}$ (left) and $T\{Q_2, Q_2\}$ (middle and right), contributing to the mixing into $\tilde{Q}_2$.

with the CKM angle $\beta \equiv \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$ and the loop function

$$\hat{C}(x_t, y_b) = \frac{x_t\, y_b}{8}\left[\frac{9}{(x_t-1)(y_b-1)} + \left(\frac{(x_t-4)^2}{(x_t-1)^2(x_t-y_b)}\log x_t + (x_t \leftrightarrow y_b)\right)\right]. \tag{9}$$

The result of our calculation agrees with the corresponding loop function extracted from [37]. In this first estimate, the shift of the Wilson coefficient C_1 is zero. Using the input from [36] we find

$$\Delta C_2 = -(1.18 \pm 0.11)\cdot 10^{-7} \times e^{i\beta}\,, \tag{10}$$

where the shown error is dominated by the uncertainty on the CKM elements V_{tb}, V_{cb}, and V_{td}.

The loop function $\hat{C}(x_t, y_b)$ is dominated by the term proportional to $\log y_b$:

$$\hat{C}(x_t, y_b) = -2y_b \log y_b + \mathcal{O}(y_b)\,, \tag{11}$$

where the subleading terms amount to a 10% correction. In order to capture also the leading QCD corrections, we now refine our analysis and perform a resummation of the terms proportional to $\log y_b$ to all orders in the strong coupling constant. To achieve this, we first match the SM to the effective theory below the scale $\mu_W = \mathcal{O}(M_W)$, where the top quark and the heavy gauge bosons are integrated out, but the bottom quark is still a dynamical degree of freedom. In fact, the matching correction at μ_W vanishes to leading order. However, the renormalization-group (RG) running will generate this term at the bottom-quark scale $\mu_b = \mathcal{O}(m_b)$ via bilocal insertions of the effective Hamiltonian

$$\begin{aligned}\mathcal{H}_{\text{eff}}^{f=5} = {}& \frac{G_F}{\sqrt{2}} \sum_{\substack{u_{1,2}=u,c\\ d_{1,2}=s,d,b}} V_{u_1d_2}V_{u_2d_1}^* \sum_{i,j=1}^{2} C_i(\mu)Z_{ij}Q_j^{(u_1d_2;d_1u_2)}\\ & - 2G_F^2 V_{ub}V_{cd}^* \cdot \left|\frac{V_{tb}V_{td}V_{cb}}{V_{cd}}\right|\\ & \times e^{i\beta}\left[\sum_{i,j,k=1}^{2} C_iC_j\hat{Z}_{ij,k}\tilde{Q}_k + \sum_{l,k=1}^{2}\tilde{C}_l\tilde{Z}_{lk}\tilde{Q}_k\right].\end{aligned} \tag{12}$$

Here, Z and $\hat{Z}$ are the renormalization constants for the local and bilocal insertions, respectively. The first line in Eq. (12) contains the four-quark operators obtained by integrating out the W and Z bosons. We denote them by

$$\begin{aligned}Q_1^{(u_1d_2;d_1u_2)} &= (\bar{u}_1d_2)_{V-A}(\bar{d}_1u_2)_{V-A}\,,\\ Q_2^{(u_1u_2;d_1d_2)} &= (\bar{u}_1u_2)_{V-A}(\bar{d}_1d_2)_{V-A}\,.\end{aligned} \tag{13}$$

The second line in Eq. (12) contains the operators

$$\begin{aligned}\tilde{Q}_1 &= \frac{m_b^2}{\mu^{2\epsilon}g_s^2}(\bar{u}b)_{V-A}(\bar{d}c)_{V-A}\,,\\ \tilde{Q}_2 &= \frac{m_b^2}{\mu^{2\epsilon}g_s^2}(\bar{d}b)_{V-A}(\bar{u}c)_{V-A}\,.\end{aligned} \tag{14}$$

They arise as counterterms to the bilocal insertions and are thus formally of dimension eight; this is made explicit by the m_b^2 prefactor. These operators have the same four-quark structure as the leading-power operators $Q_{1,2}$. We neglect the six-quark operators which arise from integrating out the W boson and the top quark, as they are suppressed by an additional factor of $1/M_W^2$.

To arrive at the CKM structure of the second line in Eq. (12) we note first that the two diagrams in Fig. 3 (right) have exactly the same phase as the corresponding tree-level diagram, so we can drop them. For the remaining diagrams we use the unitarity relation $V_{ub}V_{ud}^* + V_{cb}V_{cd}^* = -V_{tb}V_{td}^*$, combining pairs of diagrams with internal up and charm quarks as shown in Figs. 3 and 4, and then factor out the tree-level coefficient $V_{ub}V_{cd}^*$.

The relevant diagrams in Figs. 3 and 4 yield the following mixing (we use $\hat{\gamma}_{i,j;k} = 2\hat{Z}_{i,j;k}$ and expand $\hat{\gamma}_{i,j;k} = \frac{\alpha_s}{4\pi}\hat{\gamma}_{i,j;k}^{(0)} + \ldots$, where i, j denote the $Q_{1,2}$ insertions, and k is the label of the $\tilde{Q}_k$ operators):

$$\hat{\gamma}_{1,1;2}^{(0)} = \hat{\gamma}_{2,2;2}^{(0)} = \hat{\gamma}_{1,2;1}^{(0)} = \hat{\gamma}_{2,1;1}^{(0)} = -8\,, \tag{15}$$

with all the remaining entries either vanishing or not contributing. The value of the Wilson coefficients $\tilde{C}_k$ at the scale m_b can now be calculated in complete analogy to the procedure in Ref. [33], where we refer the interested reader for details. Our final result, using $m_b(m_b) = 4.163$ GeV [35], $\alpha_s(M_Z) = 0.1184$ [36], and employing "RunDec" [38] for the numerical running of the strong coupling constant, is

$$\big(\tilde{C}_1(m_b), \tilde{C}_2(m_b)\big) = (0.03, 0.31)\,. \tag{16}$$

Finally, at the bottom-quark scale we need to match the matrix elements of the two Hamiltonians (12) and (3). This will yield the leading y_b behavior with resummed logarithms. We write the matrix elements as

$$\begin{aligned}&\sum_k \Delta C_k(\mu_b)\langle Q_k\rangle(\mu_b)\\ &\quad = -2\sqrt{2}G_F\left|\frac{V_{tb}V_{td}V_{cb}}{V_{cd}}\right| e^{i\beta}\sum_{i=1,2}\tilde{C}_i(\mu_b)\langle\tilde{Q}_i\rangle(\mu_b)\,,\end{aligned} \tag{17}$$

where we expand $\Delta C_k = \frac{4\pi}{\alpha_s}\Delta C_k^{(0)} + \ldots$ in such a way that the artificially inserted factor of $1/g_s^2$ in the definition of $\tilde{Q}_k$ (14) is canceled. In Eq. (17) we have dropped the double insertions $\langle Q_iQ_j\rangle$ as they enter at higher order in α_s and need not be calculated in our approximation. Therefore, we effectively obtain the matching condition for the Wilson coefficients of the local operators (7) in the form

$$\Delta C_k^{(0)}(\mu_b) = -2m_b^2\frac{\sqrt{2}G_F}{16\pi^2}\left|\frac{V_{tb}V_{td}V_{cb}}{V_{cd}}\right| e^{i\beta}\tilde{C}_k^{(0)}(\mu_b)\,. \tag{18}$$

Numerically, we find

$$\begin{aligned}\Delta C_1 &= -(1.14 \pm 0.10)\cdot 10^{-8} \times e^{i\beta}\,,\\ \Delta C_2 &= -(1.09 \pm 0.09)\cdot 10^{-7} \times e^{i\beta}\,;\end{aligned} \tag{19}$$

the quoted errors reflect the uncertainty in the electroweak input parameters. This should be compared to the unresummed

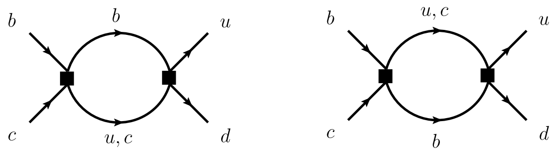

Fig. 4. The double insertions $T\{Q_1, Q_2\}$ contributing to the mixing into the operator $\tilde{Q}_1$.

result (10): we see that, indeed, the RG running has induced a nonzero correction to the Wilson coefficient C_2 in (7). Moreover, also C_1 gets a small correction, in contrast to the unresummed result. As a check of our calculation we expand the solution of the RG equations about $\mu = M_W$ and recover exactly the logarithm in Eq. (11),

$$\Delta C_1 = 0\,, \quad \Delta C_2 \propto -\sqrt{2} G_F \frac{M_W^2}{4\pi^2}(-2y_b \log y_b)\,, \tag{20}$$

where we dropped the CKM factors.

3. The induced shift in γ

The imaginary part of the shift in the Wilson coefficients calculated in the previous two sections induces a shift in γ via a modification of the ratio $r_B^{D\pi}$, Eq. (1):

$$\begin{aligned} & r_B^{D\pi} e^{i(\delta_B^{D\pi}-\gamma)} \\ & \to r_B^{D\pi} e^{i(\delta_B^{D\pi}-\gamma)}\left(1 + \frac{\Delta C_1}{C_1 + C_2 r_{A'}} + \frac{\Delta C_2}{C_1/r_{A'} + C_2}\right), \end{aligned} \tag{21}$$

where we expanded in the small corrections ΔC_1, ΔC_2 to linear order. The resulting shift in the extracted value of γ is

$$\delta\gamma^{D\pi} = -\frac{\mathrm{Im}(\Delta C_1)}{C_1 + C_2 r_{A'}} - \frac{\mathrm{Im}(\Delta C_2)}{C_1/r_{A'} + C_2}\,. \tag{22}$$

To estimate its size we need to evaluate the amplitude ratio $r_{A'}$, defined as

$$r_{A'} \equiv \frac{\langle \pi^- \bar{D}^0 | Q_2^{\bar{u}c} | B^- \rangle}{\langle \pi^- \bar{D}^0 | Q_1^{\bar{u}c} | B^- \rangle}\,. \tag{23}$$

The amplitudes contain the $\bar{D}^0$ meson in the final state; this is directly related to the fact that the electroweak corrections affect only the numerator of the ratio (1). By contrast, in the case of $B \to DK$ only the denominator of the corresponding amplitude ratio r_B^{DK} is modified (the reason being the different CKM structure of the $B \to DK$ modes).

Keeping in mind that the D meson is much heavier than the pion we see that both numerator and denominator in $r_{A'}$ are suppressed by powers of $\Lambda_{\mathrm{QCD}}/m_b$ [39]. Using color counting and neglecting annihilation topologies yields $r_{A'} \sim N_c = 3$ as a naive estimate, with large uncertainties. A crude numerical estimate treating both final-state particles as light [40] and using an asymmetric D-meson wave function [39] suggests that the annihilation contribution is indeed negligible and that $r_{A'} \approx 1$.

Interestingly, for a value of $r_{A'} \approx 4.6$ the two terms in the denominators in Eq. (22) cancel each other, so that the electroweak correction to the ratio $r_B^{D\pi}$ could, in principle, become arbitrarily large. The reason, of course, is that this cancellation would imply the vanishing of $r_B^{D\pi}$. Ignoring differences in the matrix elements related to the replacement of pions by kaons, this would also imply the vanishing of the ratio r_B^{DK}, in contradiction to the measured value (cf. the discussion below Eq. (1)). A complete cancellation can thus be safely excluded, although a more quantitative statement is difficult to obtain. To be conservative we will take $r_{A'} = 4.5$ for our estimate of $\delta\gamma^{D\pi}$. Using $\sin 2\beta = 0.682$ [36] we then obtain

$$\begin{aligned} \delta\gamma^{D\pi} &\simeq 9.7 \cdot 10^{-6}\,(\text{unresummed})\,, \\ \delta\gamma^{D\pi} &\simeq 9.2 \cdot 10^{-6}\,(\text{resummed})\,. \end{aligned} \tag{24}$$

Large uncertainties are associated with these numbers due to the poorly known value of $r_{A'}$ and missing nonlocal contributions, but it seems very unlikely that the shift in γ exceeds $\left|\delta\gamma^{D\pi}/\gamma\right| \lesssim 10^{-4}$. Note that for values of $r_{A'} \lesssim 3$ the shift $\left|\delta\gamma^{D\pi}/\gamma\right|$ drops below 10^{-6}. On the other hand, considerable fine tuning would be required for an almost complete cancellation of the denominators in Eq. (22). For instance, to find $|\delta\gamma^{D\pi}/\gamma|$ larger than 10^{-3} would require a tuning of $r_{A'}$ of the order of 10^{-4}.

4. Summary and conclusion

The determination of the CKM phase γ from tree-level decays is theoretically exceptionally clean, as all necessary branching fractions and amplitude ratios can be obtained from experimental data. In the SM, the only shift in γ is induced by electroweak corrections to the effective Hamiltonian that carry a weak phase relative to the leading contributions. In this Letter we have estimated the shift for the extraction of γ from the $B \to D\pi$ decay modes. We calculated the electroweak corrections in two ways, first integrating out the bottom quark together with the top quark and the W boson, then also summing leading QCD logarithms of m_b/M_W in a two-step matching procedure.

Interestingly, the different CKM structure compared to the $B \to DK$ modes could lead to a moderately large shift in γ via an approximate cancellation of hadronic matrix elements. Whereas these matrix elements are hard to estimate, we find that without large accidental fine tuning the expected shift in γ is very unlikely to exceed

$$\left|\delta\gamma^{D\pi}/\gamma\right| \lesssim 10^{-4}\,. \tag{25}$$

A better estimate of the hadronic matrix elements seems worthwile and could reduce this uncertainty.

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