

Unbinned angular analysis of $B \rightarrow D^* \ell \nu_\ell$ and the right-handed current

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In this article, we perform a sensitivity study of an unbinned angular analysis of the $B \rightarrow D^* \ell \nu_\ell$ decay, including the contributions from the right-handed current. We show that the angular observable can constrain very strongly the right-handed current without the intervention of the yet unsolved V_{cb} puzzle.

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I. INTRODUCTION

The $B \rightarrow D^* \ell \nu_\ell$ ($\ell = e$ or μ) has been receiving a great deal of attention in recent years. Two main reasons are the so-called V_{cb} puzzle and the $R(D^{(*)})$ anomalies. The former is the problem that there is a tension between the values of $|V_{cb}|$ determined by using the experimental measurements of the inclusive and the exclusive $b \rightarrow c \ell \nu_\ell$ decays. The $|V_{cb}|$ determination from the exclusive $B \rightarrow D^{(*)} \ell \nu_\ell$ decay depends on the $B \rightarrow D^{(*)}$ form factors. Simultaneous measurements [1–3] of the $|V_{cb}|$ and the form factors using the angular distribution [4–9] of the $B \rightarrow D^* \ell \nu_\ell$ have been attempted, which were followed by intensive theoretical interpretations [10–23]. The most exciting progress we foresee in this study is that new average of lattice QCD results will become possible [24]. The latter $R(D^{(*)})$ anomalies are the discrepancies between the Standard Model (SM) predictions and the experimental results on the ratios of $\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)$ and $\mathcal{B}(B \rightarrow D^{(*)} \ell \nu_\ell)$. Various investigations assuming that this is an appearance of the physics beyond the SM are ongoing [25–35]. In order to confirm that these phenomena are indeed the new physics discoveries, detailed studies need to be carried out both theoretically and experimentally.

In this article, motivated by these phenomenological problems, we examine the usefulness of *the unbinned*

angular distribution measurements to scrutinize the $B \rightarrow D^* \ell \nu_\ell$ decay. The existing experimental analysis mentioned above utilized four one dimensional binned distributions: they are the projections of one of the three angles (θ_l , θ_V , χ) and one momentum (w). On the other hand, once a larger amount of data becomes available at the Belle II experiment [36], the unbinned analysis with simultaneous fit of three angular distributions will become possible. The method is similar to the one which was applied for the $B \rightarrow K^* \mu^+ \mu^-$ decay where another anomaly is found [37]. We expect that the angular distribution would be most useful to distinguish the new physics contributions which carry opposite chirality to the SM, i.e., the right-handed vector contribution [38–40], which can be induced in some NP scenarios, e.g., from the $W_L - W_R$ mixing in the left-right symmetric model [41]. Furthermore, in [42,43], it is pointed out that the right-handed vector current contribution is lepton flavor universal at tree level in the context of the linear electroweak symmetry breaking. Thus, the NP effects in $B \rightarrow D^* \ell \nu_\ell$ may have a strong implication for $B \rightarrow D^* \tau \nu_\tau$ and $B_c \rightarrow \tau \nu_\tau$ processes.

In this article, we will investigate the impact of the unbinned angular distribution measurements to the investigation of new physics solely from the right-handed vector current involving a light charged lepton. We will utilize the pseudodata generated using hadronic form factors from the Belle analysis [1] for the light modes and discuss the role of the lattice QCD results which will become available soon.

II. THE UNBINNED ANGULAR ANALYSIS

The weak Hamiltonian for $B \rightarrow D^* \ell \nu_\ell$ decay including the left-handed (SM) operator and the right-handed operator (assuming no right-handed neutrino) is

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$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} [C_{V_L} \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R}] + \text{H.c.}, \quad (1)$$

where G_F and V_{cb} are respectively the Fermi Constant and the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, and C_{V_L} and C_{V_R} are the Wilson coefficients of the left-handed and the right-handed vector operators with \mathcal{O}_{V_L} and \mathcal{O}_{V_R} defined as

$$\mathcal{O}_{V_L} = (\bar{c}_L \gamma^\mu b_L) (\bar{\ell}_L \gamma_\mu \nu_L), \quad \mathcal{O}_{V_R} = (\bar{c}_R \gamma^\mu b_R) (\bar{\ell}_L \gamma_\mu \nu_L). \quad (2)$$

In the SM, we have $C_{V_L} = 1$ and $C_{V_R} = 0$, while in some NP scenarios such as the left-right symmetric model [41], C_{V_R} can be nonzero.

Having the above weak Hamiltonian, let us then build our probability density function (PDF) in terms of 11 independent angular observables J_i , defined as functions of Wilson coefficients and helicity amplitudes in Eqs. (A13) and (A14) in Appendix. First, we integrate out all the angles to obtain the normalization [44]:

$$\begin{aligned} \frac{d\Gamma}{dw} &= \frac{6m_B m_{D^*}^2}{8(4\pi)^4} G_F^2 \eta_{\text{EW}}^2 |V_{cb}|^2 \times \mathcal{B}(D^* \rightarrow D\pi) \\ &\times \frac{8\pi}{9} \{6J'_{1s} + 3J'_{1c} - 2J'_{2s} - J'_{2c}\}, \end{aligned} \quad (3)$$

where $J'_i \equiv J_i \sqrt{w^2 - 1} (1 - 2wr + r^2)$. In the following, to take into account the w dependence, we separate w in 10 bins and prepare the PDF for each bin. We express the decay rate for each bin as

$$\begin{aligned} \langle \Gamma \rangle_{w\text{-bin}} &= \frac{6m_B m_{D^*}^2}{8(4\pi)^4} G_F^2 \eta_{\text{EW}}^2 |V_{cb}|^2 \times \mathcal{B}(D^* \rightarrow D\pi) \\ &\times \frac{8\pi}{9} \{6\langle J'_{1s} \rangle_{w\text{-bin}} + 3\langle J'_{1c} \rangle_{w\text{-bin}} - 2\langle J'_{2s} \rangle_{w\text{-bin}} \\ &- \langle J'_{2c} \rangle_{w\text{-bin}}\}. \end{aligned} \quad (4)$$

Hereafter, the index w -bin is implicit. Now, the PDF is written by new *normalized* angular coefficients g_i as:

$$\begin{aligned} \hat{f}_{\langle \vec{g} \rangle}(\cos \theta_V, \cos \theta_\ell, \chi) &= \frac{9}{8\pi} \left\{ \frac{1}{6} (1 - 3\langle g_{1c} \rangle + 2\langle g_{2s} \rangle + \langle g_{2c} \rangle) \sin^2 \theta_V + \langle g_{1c} \rangle \cos^2 \theta_V + (\langle g_{2s} \rangle \sin^2 \theta_V + \langle g_{2c} \rangle \cos^2 \theta_V) \cos 2\theta_\ell \right. \\ &+ \langle g_3 \rangle \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi + \langle g_4 \rangle \sin 2\theta_V \sin 2\theta_\ell \cos \chi + \langle g_5 \rangle \sin 2\theta_V \sin \theta_\ell \cos \chi \\ &+ (\langle g_{6s} \rangle \sin^2 \theta_V + \langle g_{6c} \rangle \cos^2 \theta_V) \cos \theta_\ell + \langle g_7 \rangle \sin 2\theta_V \sin \theta_\ell \sin \chi + \langle g_8 \rangle \sin 2\theta_V \sin 2\theta_\ell \sin \chi \\ &\left. + \langle g_9 \rangle \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \right\}, \end{aligned} \quad (5)$$

where

$$\langle g_i \rangle \equiv \frac{\langle J'_i \rangle}{6\langle J'_{1s} \rangle + 3\langle J'_{1c} \rangle - 2\langle J'_{2s} \rangle - \langle J'_{2c} \rangle}. \quad (6)$$

Notice that $\langle g_{6s} \rangle$ is equivalent to the forward-backward asymmetry (FBA) up to a constant:

$$\begin{aligned} \langle \mathcal{A}_{\theta_\ell} \rangle &\equiv \frac{\int_0^1 \frac{d\Gamma}{d\cos \theta_\ell} d\cos \theta_\ell - \int_{-1}^0 \frac{d\Gamma}{d\cos \theta_\ell} d\cos \theta_\ell}{\int_0^1 \frac{d\Gamma}{d\cos \theta_\ell} d\cos \theta_\ell + \int_{-1}^0 \frac{d\Gamma}{d\cos \theta_\ell} d\cos \theta_\ell} \\ &= 3\langle g_{6s} \rangle. \end{aligned} \quad (7)$$

Now having the PDF, the experimental determination of the $\langle g_i \rangle$ can be pursued by the maximum likelihood method:

$$\mathcal{L}(\langle \vec{g} \rangle) = \sum_{i=1}^N \ln \hat{f}_{\langle \vec{g} \rangle}(e_i), \quad (8)$$

where e_i indicates the experimental events and N is the number of events.

The error matrix for $\langle g_i \rangle$ can be obtained via the covariance matrix, V_{ij} , which is the 11×11 matrix for each w -bin (so we need 10 of these matrices if we have 10 bins). In this work,

based on the truth values of $\langle g_i \rangle$ obtained using the measured form factors in [1], we use the toy Monte-Carlo method to generate the covariance matrices.

Using the pseudodata of $\langle g_i \rangle$, the Wilson coefficient C_{V_R} and the parameters in hadronic form factors can be fitted. Following the Belle analysis [1], we use two sets of parametrization for $B \rightarrow D^*$ form factors, i.e., the CLN parametrization [45] based on heavy quark expansion (HQE) and the BGL parametrization [46] based on analyticity, despite the fact that there is updated HQE parametrization [18,47] which is more flexible by including higher-order terms in $1/m_{b,c}$ expansion and z expansion. The theoretical parameters \vec{v} , which is $\vec{v} = (h_{A_1}(1), \rho_{D^*}^2, R_1(1), R_2(1), V_{cb})$ for the CLN parametrization and $\vec{v} = (a_{0,1,\dots}^g, a_{0,1,\dots}^f, a_{1,2,\dots}^f, V_{cb})$ for the BGL parametrization [46] are fitted by minimizing the following χ^2 :

$$\chi^2(\vec{v}) = \chi_{\text{angle}}^2(\vec{v}) + \chi_{w\text{-bin}}^2(\vec{v}) + \chi_{\text{lattice}}^2(\vec{v}), \quad (9)$$

where $\chi_{\text{angle}}^2(\vec{v})$ takes into account the angular distribution and $\chi_{w\text{-bin}}^2(\vec{v})$ does the w dependence. The χ_{lattice}^2 is the constraint from the lattice QCD computation, which we explain more in detail below.

The first term can be given as

$$\chi_{\text{angle}}^2(\vec{v}) = \sum_{w\text{-bin}=1}^{10} \left[\sum_{ij} N_{\text{event}} \hat{V}_{ij}^{-1} (\langle g_i \rangle^{\text{exp}} - \langle g_i^{\text{th}}(\vec{v}) \rangle) (\langle g_j \rangle^{\text{exp}} - \langle g_j^{\text{th}}(\vec{v}) \rangle) \right]_{w\text{-bin}}, \quad (10)$$

where \hat{V} and $\langle g_i \rangle^{\text{exp}}$ are the covariance matrix and the w -bin integrated g_i functions. The second term is given as

$$\chi_{w\text{-bin}}^2(\vec{v}) = \sum_{w\text{-bin}=1}^{10} \frac{([N]_{w\text{-bin}} - \alpha \langle \Gamma \rangle_{w\text{-bin}})^2}{[N]_{w\text{-bin}}}, \quad (11)$$

where $\langle \Gamma \rangle_{w\text{-bin}}$ is in Eq. (4). In reality, there might be an experimental correlation between different w -bin, which must be taken into account.

The factor α is a constant, which relates the number of events and the decay rate:

$$\alpha \equiv \frac{4N_{B\bar{B}}}{1 + f_{+0}} \tau_{B^0} \times \epsilon \mathcal{B}(D^0 \rightarrow K^- \pi^+), \quad (12)$$

where $N_{B\bar{B}} = (772 \pm 11) \times 10^6$ is the number of $B\bar{B}$ pairs produced from $\Upsilon(4S)$, which corresponds to 711 fb^{-1} of data at Belle [48], f_{+0} is the constant defined as $f_{+0} = \frac{\Gamma(\Upsilon(4S) \rightarrow B^+ B^-)}{\Gamma(\Upsilon(4S) \rightarrow B^0 \bar{B}^0)}$, τ_{B^0} is the lifetime of B^0 , and ϵ is the experimental efficiency (the values are from PDG [49]).

In the following sections, we will investigate the sensitivity of the unbinned angular analysis proposed above to the right-handed current, i.e., C_{V_R} . We use the best-fit values of the CLN and BGL parameters in the Belle '18 paper [1], in order to generate the *pseudo*-experimental data. The total number of events is also adjusted to $\sim 95 \text{ k}$ as in [1], corresponding to roughly the universal efficiency of $\epsilon \sim 4.8 \times 10^{-2}$. Thus, the α parameter is computed as $6.616(6.613) \times 10^{18}$ for CLN(BGL) parametrization.

For illustration, the pseudodata with CLN parameters leads to the number of events for w -bin as

$$N_{\text{event}} = (5306, 8934, 10525, 11241, 11392, 11132, \\ \times 10555, 9726, 8693, 7497). \quad (13)$$

Similarly, using the pseudodata with BGL parameters we find

$$N_{\text{event}} = (5239, 8868, 10500, 11264, 11455, 11217, \\ \times 10638, 9776, 8676, 7368). \quad (14)$$

As an example, we show the $\langle g_i \rangle$ distribution with CLN pseudodata in Fig. 1.

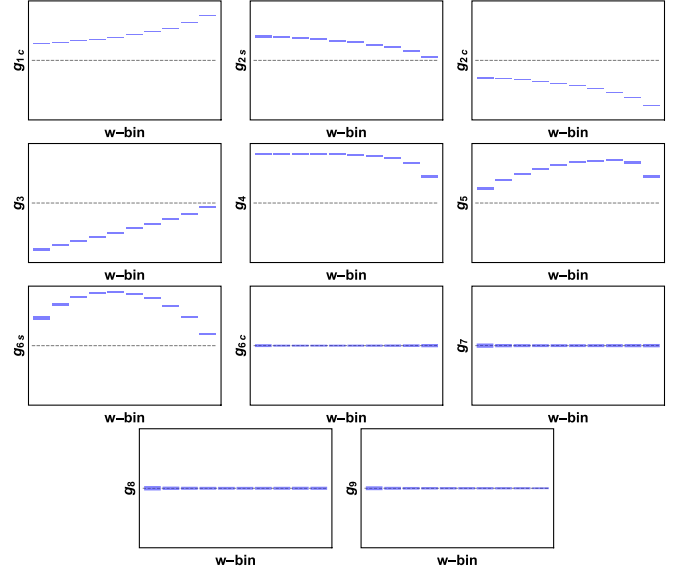


FIG. 1. Distribution of $\langle g_i \rangle$ in ten w -bins.

III. SENSITIVITY TO THE C_{V_R}

In this section, we will show that using the two sets of pseudodata discussed in the previous section, how precisely the C_{V_R} parameter can be determined assuming the right-handed vector current is the only source of new physics contribution (i.e., $C_{V_L} = 1$ and C_{V_R} nonzero). But first, for a sanity check, we study the SM case and compare to the result in [1]. Let us start with the CLN pseudodata. We use the lattice input $h_{A_1}(1) = 0.906 \pm 0.013$ as done in [1] by including the following χ_{lattice}^2 term

$$\chi_{\text{lattice}}^2(v) = \left(\frac{v^{\text{lattice}} - v}{\sigma_v^{\text{lattice}}} \right)^2, \quad (15)$$

with $v = h_{A_1}(1)$. Then, we find

$$\vec{v} = (h_{A_1}(1), \rho_{D^*}^2, R_1(1), R_2(1), V_{cb}) \\ = (0.906, 1.106, 1.229, 0.852, 0.0387), \\ \sigma_{\vec{v}} = (0.013, 0.019, 0.011, 0.011, 0.0006), \quad (16)$$

using $\alpha = 6.616 \times 10^{18}$ and $\eta_{\text{EW}} = 1.006$.

The fitted values coincide well with our input from [1] while we cannot directly compare the errors with [1] as the experimental efficiency is not correctly taken into account here. Nevertheless, our error is $\sim 50\%$ smaller and a partial reason might be the unbinned analysis we have applied here. Next, we use the BGL data. The lattice data for $h_{A_1}(1)$ is again used to constrain the BGL parameter a_0^f via a relation:

$$h_{A_1}(1) = \frac{1}{2m_B \sqrt{r} P_f(0) \phi_f(0)} a_0^f, \quad (17)$$

which leads to $a_0^f = 0.0132 \pm 0.0002$. The fit result yields

$$\begin{aligned} \vec{v} &= (a_0^f, a_1^f, a_1^{\mathcal{F}1}, a_2^{\mathcal{F}1}, a_0^g, V_{cb}) \\ &= (0.0132, 0.0169, 0.0070, \\ &\quad -0.0853, 0.0242, 0.0384), \end{aligned} \quad (18)$$

$$\sigma_{\vec{v}} = (0.0002, 0.0028, 0.0011, 0.0199, 0.0004, 0.0006), \quad (19)$$

using $\alpha = 6.613 \times 10^{18}$ and $\eta_{\text{EW}} = 1.006$. Thus, the similar conclusion applies for the BGL case as well.

Now, let us move to our main topic, the sensitivity study of the right-handed current, C_{V_R} . By simply adding $C_{V_R} \neq 0$, we immediately encounter two fundamental problems: (i) the fit does not converge as C_{V_R} is not independent of the vector form factor, which means without knowing the SM value of the vector form factor, we cannot determine C_{V_R} , (ii) C_{V_R} and V_{cb} are also depending as the changes in both parameters directly impact on the branching ratio of $B \rightarrow D^* \ell \nu_\ell$. Thus, to obtain the allowed range of C_{V_R} , we need a precise information of the SM value of V_{cb} . However, as it is manifested in the V_{cb} puzzle, there is a controversy in the experimental determination of V_{cb} from the semileptonic $b \rightarrow c \ell \nu_\ell$ transitions.

Fortunately, the first problem will be soon resolved as the lattice QCD result on the vector form factor will be available [24]. It is important to emphasize that the right-handed current contribution cannot be determined from experimental data without this lattice QCD result. For the second problem, we may also use the SM value of V_{cb} obtained indirectly from the unitarity relation with other measurements. However, ignoring the V_{cb} determination from the semileptonic decays enlarges the error on V_{cb} and the correlation between V_{cb} and C_{V_R} is so strong that the obtained fit result becomes unstable, especially when we have many numbers of parameters to fit. On the other hand, we found that we can circumvent this problem entirely and determine C_{V_R} at a high precision when we use only the angular distribution, i.e., $\chi_{\text{angle}}^2(\vec{v})$, in which an overall factor such as V_{cb} is canceled out in the normalized $\langle g_i \rangle$ functions. That is, we ignore the $\chi_{w\text{-bin}}^2(\vec{v})$ term, which is useful solely to determine the overall factor and the w dependence of the form factors. Thus, in the following studies, we use these strategies: (a) we assume that the vector form factor is known at a certain precision (we use the expected lattice QCD precision, 4% in $R_1(1)$ and 7% in $h_V(1)$ [50]), (b) we use only the first and the third terms of Eq. (9) and evaluate the compatibility with V_{cb} after the fit.

We first consider a scenario where C_{V_R} is real, i.e., $C_{V_R} = \text{Re}(C_{V_R})$. We start with the CLN pseudodata. The central value of the lattice input for the vector form factor, which is represented by the $R_1(1)$ in the CLN parametrization, is chosen to be our input, with 4% error as

mentioned above, and thus, $R_1(1) = 1.229 \pm 0.049$. Our fit result yields

$$\vec{v} = (\rho_{D^*}^2, R_1(1), R_2(1), C_{V_R}) = (1.106, 1.229, 0.852, 0), \quad (20)$$

$$\sigma_{\vec{v}} = (3.177, 0.049, 0.018, 0.021),$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & -0.016 & -0.763 & 0.095 \\ -0.016 & 1. & 0.006 & -0.973 \\ -0.763 & 0.006 & 1. & -0.117 \\ 0.095 & -0.973 & -0.117 & 1. \end{pmatrix}. \quad (21)$$

The most important finding here is that the C_{V_R} can be determined at a 2.1% precision. Note that the central values here are simply due to our input, where the SM is assumed, and we will know the true C_{V_R} value only if an experimental data analysis is performed considering the right-handed contribution and including the lattice QCD results on the vector form factor. In this fit, the obtained uncertainties in the form factor parameters, especially for $\rho_{D^*}^2$ and $R_2(1)$, are very large. However, the small correlation between those parameters and C_{V_R} implies that ignorance of these parameters has little impact on the determination of the C_{V_R} . In Fig. 2 left, we show a contour plot on the $R_1(1) - C_{V_R}$ plane. We repeat that the centre of this plot is at SM due to its initial assumption. Only when the lattice QCD value of $R_1(1)$ is obtained, we will be able to tell whether C_{V_R} is deviated from the SM value, $C_{V_R} = 0$, or not. This plot shows that one day if the lattice QCD result on $R_1(1)$ becomes available and it turns out to be different from the experimental fitted value (assuming SM), nonzero C_{V_R} can be hinted. We found that our result does not change significantly even if we have lattice input for $\rho_{D^*}^2$, $R_2(1)$, except that the errors on these parameters become smaller.

Next we study the BGL pseudodata. We again fix the vector form factor at our input value. In the BGL parametrization, the vector form factor is related to the a_0^g parameter

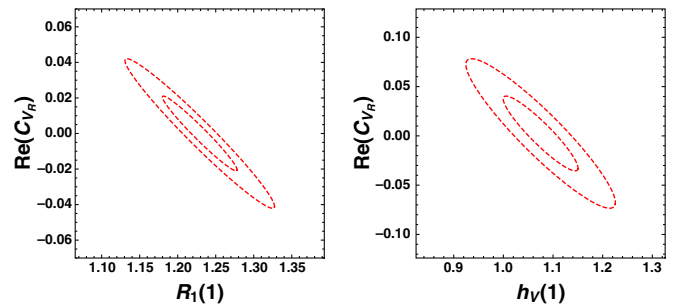


FIG. 2. $R_1(1) - C_{V_R}$ and $h_V(1) - C_{V_R}$ contours.

$$h_V(1) = \frac{m_B \sqrt{r}}{P_g(0) \phi_g(0)} a_0^g. \quad (22)$$

By adding the error of 7% (larger than the ratio $R_1(1)$ which can be determined at a higher precision by lattice QCD), we use a constraint of the a_0^g parameter, $a_0^g = 0.0241 \pm 0.0017$. We also note that the BGL fit

requires $h_{A_1}(1)$ lattice QCD input as well. The fit result yields

$$\begin{aligned} \vec{v} &= (a_0^f, a_1^f, a_1^{\mathcal{F}^1}, a_2^{\mathcal{F}^1}, a_0^g, C_{V_R}) \\ &= (0.0132, 0.0169, 0.0070, \\ &\quad -0.0852, 0.0241, 0.0024), \end{aligned} \quad (23)$$

$$\begin{aligned} \sigma_{\vec{v}} &= (0.0002, 0.0109, 0.0026, 0.0352, 0.0017, 0.0379), \\ \rho_{\vec{v}} &= \begin{pmatrix} 1. & 0.022 & 0.039 & -0.035 & 0.000 & 0.189 \\ 0.022 & 1. & 0.860 & -0.351 & 0.000 & 0.316 \\ 0.039 & 0.860 & 1. & -0.762 & 0.000 & 0.283 \\ -0.035 & -0.351 & -0.762 & 1. & 0.000 & -0.119 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1. & -0.923 \\ 0.189 & 0.316 & 0.283 & -0.119 & -0.923 & 1. \end{pmatrix}. \end{aligned} \quad (24)$$

We find again, despite the fact that the hadronic parameters except for a_0^f and a_0^g have large uncertainties, the right-handed contribution, C_{V_R} , is constrained at a high precision, $\sim 4\%$ level. In Fig. 2 right, we show a contour plot on the $h_V(1) - C_{V_R}$ plane, which again suggests the importance of the lattice calculation of the vector form factor in the determination of C_{V_R} .

Now, let us discuss the impact of the right-handed contributions to the V_{cb} puzzle. As mentioned earlier, the V_{cb} determination is in a contradictory situation. It is determined by the inclusive method at a $\sim 2\%$ precision and by the exclusive method at a $\sim 1\%$ precision, while their central values are deviated by $\sim 7\%$. The most interesting question is whether the right-handed contribution fill this gap. Reference [39] pointed out that to match the exclusive $B \rightarrow D^* \ell \nu_\ell$ to the inclusive one requires $C_{V_R} \simeq -5\%$ while the exclusive $B \rightarrow D \ell \nu_\ell$ requires $C_{V_R} \simeq +5\%$. On the other hand, if we consider only these two exclusive processes, they allow C_{V_R} to be $\sim 5\%$. Thus, [39] concluded that it is difficult to explain the V_{cb} puzzle by the right-handed contributions. However, if some problem is found in one of these three measurements, or the lattice QCD input utilized to obtain V_{cb} , the situation could be reversed. Our proposed method using only the angular distribution allows us to pin down the C_{V_R} parameter at a few % level without intervention of the controversial V_{cb} determinations, including its sign as shown in Fig. 2. Thus, this method will provide an important step toward revealing the nature of the right-handed contribution.

Now, let us investigate the role of the w -dependent FBA. This observable is particularly interesting to measure: it requires only one angle measurement and many experimental errors can cancel out. As mentioned earlier, FBA is

proportional to $\langle g_{6s} \rangle$ and for curiosity, we investigate what constraint on C_{V_R} we would obtain from this single angular observable. The fit method is the same as before: we use the vector form factor as input. The result for the CLN case yields:

$$\begin{aligned} \vec{v} &= (\rho_{D^*}^2, R_1(1), R_2(1), C_{V_R}) \\ &= (1.106, 1.229, 0.852, 0.000), \end{aligned} \quad (25)$$

$$\sigma_{\vec{v}} = (2.200, 0.049, 0.031, 0.022), \quad (26)$$

$$\rho_{\vec{v}} = \begin{pmatrix} 1. & 0.008 & -0.873 & 0.262 \\ 0.008 & 1. & -0.040 & -0.931 \\ -0.873 & -0.040 & 1. & -0.296 \\ 0.262 & -0.931 & -0.296 & 1. \end{pmatrix}. \quad (27)$$

It is quite intriguing that the C_{V_R} can be determined at a 2.2% precision, which is almost as good as the case where we use the full angular coefficients. For the BGL parametrization, we find the situation is similar: FBA alone can constrain C_{V_R} at a precision of $\sim 4\%$. Since this measurement can be made as a very simple extension of the work, e.g., in [1], we highly suggest it be done in the near future.

Finally, we discuss the models in which the right-handed interaction contain the CP violating phase, i.e., C_{V_R} is a complex number. The imaginary part of the C_{V_R} can be determined thanks to the angular observables $\langle g_8 \rangle$ and $\langle g_9 \rangle$, which are the triple product observables that can detect the CP violation without having a source of a strong phase. These observables were not included in the previous analysis, e.g., [1] as they are always zero in the SM. They are important observables determining the CP

violation and they should be introduced in the future study. In our fit, we find that $\text{Im}(C_{V_R})$ can be determined at a 0.7% precision, both for CLN and BGL.

IV. CONCLUSIONS

In this article, motivated by the various interesting problems observed recently in the semileptonic $b \rightarrow c \ell \nu$ transitions, we investigated an application of the unbinned angular analysis of the $B \rightarrow D^* \ell \nu_\ell$ and its impact on pinning down the new physics signals. We proposed the detailed processes to apply the unbinned angular analysis to search for the right-handed contributions, in the future experimental analysis. We introduced the 11 angular coefficients and their distributions in w -bin, i.e., the $\langle g_i \rangle$ functions. The $\langle g_i \rangle$ functions are normalized functions, in which the overall factor in the decay rate expression, including V_{cb} , is canceled. Therefore, we can determine the right-handed vector contributions by circumventing the problem of the controversial values of V_{cb} .

We use pseudodata computed using the theoretical parameters, from both CLN and BGL parametrizations, obtained by the fit of the Belle data [1] assuming the SM, with 95 k events and performed a sensitivity study of the parameters which represent the right-handed contribution, C_{V_R} , assuming it is the only/dominant source of new physics. The very important finding of this study is that the new physics parameter C_{V_R} and the SM parameter coming from the vector form factor cannot be separately measured. Fortunately, the latter can be obtained by the lattice QCD computation and the result is expected very soon. Its central value is not known yet while the precision that the current lattice QCD computation can achieve is known to be 4% for $R_1(1)$ and 7% for $h_V(1)$. Using these values, we found that the real part of the C_{V_R} can be determined at a precision of 2–4%. Furthermore, the imaginary part of C_{V_R} can also be determined once we include the two CP violating angular coefficients, which are neglected in the previous experimental analysis. We found the imaginary part of C_{V_R} can be determined at a $\sim 1\%$ precision.

An additional result is obtained from a sensitivity study of the well-known FBA observable to the real part of the C_{V_R} parameter. The FBA turned out to be proportional to the angular coefficient $\langle g_{6s} \rangle$. We performed the same fit as above but with this single angular observable. In the CLN (BGL) parametrization, we found that the FBA alone can determine the real part of the C_{V_R} at a $\sim 2(4)\%$ precision, which is almost equally good as the full angular coefficient fit. Therefore future measurements of FBA will be particularly useful for constraining C_{V_R} .

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APPENDIX: THEORETICAL FRAMEWORK

We define the helicity amplitudes of left and right handed currents as follows

$$\begin{aligned} H_\lambda^{\lambda_{D^*}}(q^2) &= \bar{\epsilon}_\mu^*(\lambda) \langle D^*(\lambda_{D^*}) | \bar{c} \gamma^\mu (1 - \gamma_5) b | \bar{B} \rangle, \\ \hat{H}_\lambda^{\lambda_{D^*}}(q^2) &= \bar{\epsilon}_\mu^*(\lambda) \langle D^*(\lambda_{D^*}) | \bar{c} \gamma^\mu (1 + \gamma_5) b | \bar{B} \rangle, \end{aligned} \quad (\text{A1})$$

where $\bar{\epsilon}_\mu(\lambda)$ is the polarization vector of the virtual W boson.

The hadronic matrix elements describing the $\bar{B} \rightarrow D^*$ decay can be parametrized in terms of four Lorentz invariant transition form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, and $A_2(q^2)$ [51]:

$$\begin{aligned} &\langle D^*(p_{D^*}, \epsilon) | \bar{c} \gamma_\mu (1 - \gamma_5) b | \bar{B}(p_B) \rangle \\ &= \frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_{D^*}^\alpha p_B^\beta - 2m_{D^*} A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q_\mu \\ &\quad - (m_B + m_{D^*}) A_1(q^2) \left(\epsilon_\mu^* - \frac{\epsilon^* \cdot q}{q^2} q_\mu \right) \\ &\quad + A_2(q^2) \frac{\epsilon^* \cdot q}{m_B + m_{D^*}} \left[(p_B + p_{D^*})_\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q_\mu \right], \end{aligned} \quad (\text{A2})$$

where we use $\epsilon_{0123} = 1$.

The nonzero helicity amplitudes $H_0 \equiv H_0^0$, $H_\pm \equiv H_\pm^\pm$, $\hat{H}_0 \equiv \hat{H}_0^0$ and $\hat{H}_\pm \equiv \hat{H}_\pm^\pm$ of left-handed and right-handed currents satisfy the following relations using form factors in Eq. (A2):

$$\begin{aligned} H_\pm(q^2) &= -\hat{H}_\mp(q^2) \\ &= (m_B + m_{D^*}) A_1(q^2) \mp \frac{2m_B |\mathbf{p}_{D^*}|}{m_B + m_{D^*}} V(q^2), \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} H_0(q^2) &= -\hat{H}_0(q^2) \\ &= \frac{m_B + m_{D^*}}{2m_{D^*} \sqrt{q^2}} \left[(m_B^2 - m_{D^*}^2 - q^2) A_1(q^2) \right. \\ &\quad \left. - \frac{4m_B^2 |\mathbf{p}_{D^*}|^2}{(m_B + m_{D^*})^2} A_2(q^2) \right], \end{aligned} \quad (\text{A4})$$

where $|\mathbf{p}_{D^*}| = \frac{\sqrt{[(m_B - m_{D^*})^2 - q^2][(m_B + m_{D^*})^2 - q^2]}}{2m_B}$. In the following, we use w variable instead of q^2 , with $w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$

such that $|\mathbf{p}_{D^*}| = m_{D^*} \sqrt{w^2 - 1}$ and $w = 1$ corresponds to the zero-recoil momentum.

In the CLN parametrization, the helicity amplitudes are written as

$$H_\pm(w) = m_B \sqrt{r} (w+1) h_{A_1}(w) \left[1 \mp \sqrt{\frac{w-1}{w+1}} R_1(w) \right], \quad (\text{A5})$$

$$H_0(w) = m_B^2 \sqrt{r} (w+1) \frac{1-r}{\sqrt{q^2}} h_{A_1}(w) \times \left[1 + \frac{w-1}{1-r} (1 - R_2(w)) \right], \quad (\text{A6})$$

where we have $r = m_{D^*}/m_B$ and

$$\begin{aligned} h_{A_1}(w) &= h_{A_1}(1) (1 - 8\rho_{D^*}^2 z + (53\rho_{D^*}^2 - 15)z^2 \\ &\quad - (231\rho_{D^*}^2 - 91)z^3), \\ R_1(w) &= R_1(1) - 0.12(w-1) + 0.05(w-1)^2, \\ R_2(w) &= R_2(1) + 0.11(w-1) - 0.06(w-1)^2, \end{aligned} \quad (\text{A7})$$

where $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$.

In the BGL parametrization, the helicity amplitudes are written as [14]

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D^*(\rightarrow D\pi)\ell^-\bar{\nu}_\ell)}{dw d\cos\theta_V d\cos\theta_\ell d\chi} &= \frac{6m_B m_{D^*}^2}{8(4\pi)^4} \sqrt{w^2 - 1} (1 - 2wr + r^2) G_F^2 |V_{cb}|^2 \mathcal{B}(D^* \rightarrow D\pi) \\ &\times \{ J_{1s} \sin^2\theta_V + J_{1c} \cos^2\theta_V + (J_{2s} \sin^2\theta_V + J_{2c} \cos^2\theta_V) \cos 2\theta_\ell + J_3 \sin^2\theta_V \sin^2\theta_\ell \cos 2\chi \\ &+ J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi + (J_{6s} \sin^2\theta_V + J_{6c} \cos^2\theta_V) \cos \theta_\ell \\ &+ J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi + J_9 \sin^2\theta_V \sin^2\theta_\ell \sin 2\chi \}, \end{aligned} \quad (\text{A13})$$

where in the massless limit of leptons the J_i ($i = 1 \sim 9$) can be written by the helicity amplitudes and the Wilson coefficients of left- and right-handed currents as

$$\begin{aligned} J_{1s} &= \frac{3}{2} (H_+^2 + H_-^2) (|C_{V_L}|^2 + |C_{V_R}|^2) - 6H_+ H_- \text{Re}[C_{V_L} C_{V_R}^*], \\ J_{1c} &= 2H_0^2 (|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L} C_{V_R}^*]), \\ J_{2s} &= \frac{1}{2} (H_+^2 + H_-^2) (|C_{V_L}|^2 + |C_{V_R}|^2) - 2H_+ H_- \text{Re}[C_{V_L} C_{V_R}^*], \\ J_{2c} &= -2H_0^2 (|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L} C_{V_R}^*]), \\ J_3 &= -2H_+ H_- (|C_{V_L}|^2 + |C_{V_R}|^2) + 2(H_+^2 + H_-^2) \text{Re}[C_{V_L} C_{V_R}^*], \\ J_4 &= (H_+ H_0 + H_- H_0) (|C_{V_L}|^2 + |C_{V_R}|^2 - 2\text{Re}[C_{V_L} C_{V_R}^*]), \\ J_5 &= -2(H_+ H_0 - H_- H_0) (|C_{V_L}|^2 - |C_{V_R}|^2), \\ J_{6s} &= -2(H_+^2 - H_-^2) (|C_{V_L}|^2 - |C_{V_R}|^2), \\ J_{6c} &= 0, \\ J_7 &= 0, \\ J_8 &= 2(H_+ H_0 - H_- H_0) \text{Im}[C_{V_L} C_{V_R}^*], \\ J_9 &= -2(H_+^2 - H_-^2) \text{Im}[C_{V_L} C_{V_R}^*]. \end{aligned} \quad (\text{A14})$$

$$H_\pm(w) = f(w) \mp m_B |\mathbf{p}_{D^*}| g(w), \quad (\text{A8})$$

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{q^2}}, \quad (\text{A9})$$

where the form factors are the expansion in terms of the z variable

$$\begin{aligned} g(z) &= \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^N a_n^g z^n, \quad f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^N a_n^f z^n, \\ \mathcal{F}_1(z) &= \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^N a_n^{\mathcal{F}_1} z^n. \end{aligned} \quad (\text{A10})$$

The full expressions of P_{g,f,\mathcal{F}_1} and ϕ_{g,f,\mathcal{F}_1} can be found in [1]. The f and \mathcal{F}_1 are not completely independent and we have

$$\mathcal{F}_1(0) = (m_B - m_{D^*}) f(0), \quad (\text{A11})$$

which leads to a relation of their leading order coefficients

$$a_0^{\mathcal{F}_1} = (m_B - m_{D^*}) \frac{\phi_{\mathcal{F}_1}(0)}{\phi_f(0)} a_0^f. \quad (\text{A12})$$

We write contributions from the left-handed current, the right-handed current, and the interference terms in terms of J_i parameters:

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