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More on fake GUT

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Abstract: It is remarkable that the matter fields in the Standard Model (SM) are apparently unified into the SU(5) representations. A straightforward explanation of this fact is to embed all the SM gauge groups into a simple group containing SU(5), i.e., the grand unified theory (GUT). Recently, however, a new framework "fake GUT" has been proposed. In this new framework, the apparent matter unification can be explained by a chiral gauge group G, $G \supset \mathrm{SU}(5)$. We emphasize that the SM matter fields are not necessarily embedded into the chiral representations to explain the apparent unification. In this paper, we discuss details of concrete realizations of the fake GUT model. We first study the model based on $\mathrm{SU}(5) \times \mathrm{U}(2)_H$, where $\mathrm{SU}(3)_c$ in the SM is from SU(5) while $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are from the diagonal subgroups of $\mathrm{SU}(5) \times \mathrm{U}(2)_H$. We also extend this model to the one based on a semi-simple group, $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$, so that $\mathrm{U}(2)_H$ is embedded in $\mathrm{SU}(3)_H$. We also show that this framework predicts rather different decay patterns of the proton, compared to the conventional GUT.

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Contents

1 Introduction

In the Standard Model (SM), the matter fields are apparently unified into the SU(5) representations. In general, chiral fermions consistent with SM gauge symmetry do not necessarily satisfy this property [1–3]. Therefore, the apparent SM matter unification into the SU(5) multiplets is quite remarkable. In fact, the matter unification has been propelling the study of the grand unified theory (GUT) for a long time [4–6] (see ref. [7] for reviews). In the conventional GUT, the SM matter fields are unified into common representations of a simple group containing SU(5). As a result of the matter unification, the GUT predicts the proton decay, which are extensively searched for in a variety of experiments (see e.g. refs. [8–11]).

Recently, we have proposed a new framework "fake GUT" [12]. This framework can explain the apparent unification of the SM matter fields into the SU(5) multiplets in a different way than conventional GUT models. In the fake GUT, the SM matter fields are not necessarily embedded into common SU(5) multiplets at the high energy. Although the quarks and leptons can have different origins, they form complete SU(5) multiplets at the low energy as if they originate from the same multiplets. In the fake GUT, the prediction of

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the proton decay can be significantly different from that in conventional GUT models. As an extreme example, the quarks and leptons may reside in completely different multiplets. In such a case, the proton decay does not occur.

In ref. [12], a fake GUT model based on a non-simple $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ group has been sketched. In this model, the leptons mainly reside in the part of the vector-like fermions of $\mathrm{U}(2)_H$, while the quarks reside in the $\overline{\mathbf{5}} \oplus \mathbf{10}$ representations. After the spontaneous symmetry breaking of $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ down to the SM gauge group, only the leptons and the quarks remain massless which apparently form $\overline{\mathbf{5}} \oplus \mathbf{10}$ representations. Although the "GUT scale" is predicted to be much lower than the conventional GUT scale, the proton decay rate is suppressed as the quarks and the leptons are not in the same multiplets. In this paper, we study details of the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model.

We also extend the non-simple $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model to that based on a semi-simple group $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$. With this extension, we can successfully explain the charge quantization and avoid the Landau pole problem of $\mathrm{U}(1)_H$ gauge interaction. We also discuss the symmetry which determines the fraction of the leptons originating from $\overline{\mathbf{5}} \oplus \mathbf{10}$, which in turn controls the proton decay rate.

The organization of this paper is as follows. In section 2, we first review the idea of the fake GUT. In section 3, we discuss details of the non-simple $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ group. In section 4, we extend the model to that based on a semi-simple $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ model. The final section is devoted to our conclusions.

2 Fake GUT

In this section, we review the idea of the fake GUT. Let us consider a high energy theory which satisfies the following conditions;

1. The gauge group is $G = \mathrm{SU}(5) \times H$, which is spontaneously broken down to the SM gauge group $G_{\mathrm{SM}} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ at the fake GUT scale.

2. Three copies of the chiral fermions in the $\overline{\mathbf{5}} \oplus \mathbf{10}$ representations of $\mathrm{SU}(5)$ which are neutral under H.

3. Additional fermions which consist of the vector-like representations of G.

4. All the Cartan subgroups of $\mathrm{SU}(5)$ remain unbroken and all of them take part in G_{SM}.

4′. Some of $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ may be diagonal subgroups of $\mathrm{SU}(5) \times H$.

We call the theory which satisfies conditions 1 to 4 the "fake GUT." In this work, we also assume the additional condition 4′, which realizes more viable models in view of the proton decay and the coupling unification. As proven in ref. [12], the fake GUT model guarantees that the low energy fermions completely match with the $\overline{\mathbf{5}} \oplus \mathbf{10}$ representations, while the quarks and the leptons do not necessarily originate from $\overline{\mathbf{5}} \oplus \mathbf{10}$ multiplets (see the appendix A). When some of quarks/leptons originate from fields other than $\overline{\mathbf{5}} \oplus \mathbf{10}$, some

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of the $\overline{\mathbf{5}} \oplus \mathbf{10}$ fermions become massive whose mass partners are parts of the vector-like fermions of G after G is spontaneously broken down to G_{SM}. In this way, only the SM fermions remain massless at the GUT scale, which form the $\overline{\mathbf{5}}\oplus\mathbf{10}$ representations regardless of how they are embedded in the fake GUT representations at the high energy [12]. This feature strongly owes to the nature of the chiral fermion and does not apply to the bosonic fields. In the case of supersymmetry (SUSY), this argument applies to the chiral superfields.

Let us emphasize that the apparent matter unification in the fake GUT is guaranteed by the matching of the characters of the chiral representations and does not depend on the details of the models. This argument is more versatile than the 't Hooft anomaly (i.e. the anomaly of the global symmetry) matching conditions to restrict the fermions in the low energy theory [13], since the latter relies on the global symmetries of the model.

We may consider the fake GUT with a larger group G_{UV} than $\mathrm{SU}(5) \times H$, as long as G_{UV} is broken to $\mathrm{SU}(5) \times H$ satisfying the above conditions. For instance, $\mathrm{SO}(10) \times H \to \mathrm{SU}(5) \times H$ with the $\mathbf{16}$ representations satisfies the above conditions and works as a fake GUT model.

Another interesting feature of the fake GUT model is that it does not necessarily require the SM gauge coupling unification for a non-trivial H when it satisfies the condition $4'$. Thus, although the coupling unification fails in the non-supersymmetric SM, the fake GUT framework works with a proper choice of H.

There are many possible choices of H. For example, $H = 1$, $\mathrm{U}(1)_H$, $\mathrm{SU}(N)_H \cdots$, are possible candidates. It is also possible to consider $H = \mathrm{U}(2)_H$, $\mathrm{U}(3)_H$, $\mathrm{SU}(3)_H \times \mathrm{SU}(2)_H \times \mathrm{U}(1)_H$, where $\mathrm{U}(2)_H \supset \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, $\mathrm{U}(3)_H \supset \mathrm{SU}(3)_c \times \mathrm{U}(1)_Y$ etc. In fact, many extensions of the GUT models fit into the fake GUT framework. For instance, massive fermion extensions play important role in the Yukawa coupling unification [14, 15], (see also refs. [16–18] for recent applications), and in suppressing lepton/baryon number violation in GUT [19–21].

Not all possibilities are, however, phenomenologically viable due to the constraints from the gauge coupling matching conditions and the proton lifetime. As we will see later, we find that the minimal viable choice is $H = \mathrm{U}(2)_H$, if the running of the gauge coupling constants is the same as the SM below the fake GUT scale. In this case, the smaller choices such as $H = 1$, $H = \mathrm{U}(1)_H$ or $H = \mathrm{SU}(2)_H$ are not phenomenologically compatible. In the next section, we discuss a $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model. We also extend the model so that $\mathrm{U}(2)_H$ is embedded into $\mathrm{SU}(3)_H$ in section 4.

3 $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model

In this section, we discuss a model with $G = \mathrm{SU}(5) \times \mathrm{U}(2)_H$, where $\mathrm{SU}(5) \supset \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and $\mathrm{U}(2)_H \supset \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Below the fake GUT scale, $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ appear as the diagonal subgroups of $\mathrm{SU}(5) \times \mathrm{U}(2)_H$, while $\mathrm{SU}(3)_c$ appears solely from $\mathrm{SU}(5)$. As we emphasized above, this choice is the minimal gauge group for the fake GUT model which is phenomenologically compatible. We summarize the matter contents of this model in table 1.

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	$(\mathrm{SU}(5), \mathrm{SU}(2)_H)_{\mathrm{U}(1)_H}$	$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$
fermions		
$\overline{\mathbf{5}}$	$(\overline{\mathbf{5}}, \mathbf{1})_0$	$(\overline{\mathbf{3}}, \mathbf{1})_{+1/3} \oplus (\mathbf{1}, \mathbf{2})_{-1/2}$
$\mathbf{10}$	$(\mathbf{10}, \mathbf{1})_0$	$(\mathbf{3}, \mathbf{2})_{+1/6} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-2/3} \oplus (\mathbf{1}, \mathbf{1})_{+1}$
L_H	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{2})_{-1/2}$
$\overline{L}_H$	$(\mathbf{1}, \mathbf{2})_{+1/2}$	$(\mathbf{1}, \mathbf{2})_{+1/2}$
E_H	$(\mathbf{1}, \mathbf{1})_{-1}$	$(\mathbf{1}, \mathbf{1})_{-1}$
$\overline{E}_H$	$(\mathbf{1}, \mathbf{1})_{+1}$	$(\mathbf{1}, \mathbf{1})_{+1}$
scalars		
ϕ_2	$(\mathbf{5}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{3})_0 \oplus (\mathbf{1}, \mathbf{1})_0 \oplus [(\mathbf{3}, \mathbf{2})_{-5/6} \oplus (\mathbf{1}, \mathbf{3})_0 \oplus (\mathbf{1}, \mathbf{1})_0]$
H_5	$(\mathbf{5}, \mathbf{1})_0$	$(\mathbf{3}, \mathbf{1})_{-1/3} \oplus (\mathbf{1}, \mathbf{2})_{+1/2}$
H_2	$(\mathbf{1}, \mathbf{2})_{+1/2}$	$(\mathbf{1}, \mathbf{2})_{+1/2}$
vectors		
V_5	$(\mathbf{24}, \mathbf{1})_0$	$(\mathbf{8}, \mathbf{1})_0 \oplus (\mathbf{3}, \mathbf{2})_{-5/6} \oplus (\mathbf{1}, \mathbf{3})_0 \oplus (\mathbf{1}, \mathbf{1})_0$
V_{2H}	$(\mathbf{1}, \mathbf{3})_0$	$(\mathbf{1}, \mathbf{3})_0$
V_{1H}	$(\mathbf{1}, \mathbf{1})_0$	$(\mathbf{1}, \mathbf{1})_0$

Table 1. The content of the fermions, the scalar fields and the gauge bosons in the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model is shown in the group representation, $(\mathrm{SU}(5), \mathrm{SU}(2)_H)_{\mathrm{U}(1)_H}$ and $(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$. Each fermion has three generations. The $\mathrm{U}(1)_Y$ neutral scalar fields and gauge bosons are given by the real scalar fields and real gauge bosons. The scalar fields enclosed in square brackets are the Goldstone modes associated with $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ breaking to G_{SM}, which are eaten by the gauge bosons.

Note that the idea of the fake GUT is completely different from that of the SUSY GUT model based on the product group [22], although they are both based on the product gauge group. In the product group SUSY GUT model, the product group is used to solve the so-called the doublet-triplet mass splitting problem, while the effective gauge coupling unification is taken seriously and the matter multiplets are assumed as the GUT multiplets.

3.1 Origin of SM fermions

We introduce the three generations of the chiral multiplets $\overline{\mathbf{5}} \oplus \mathbf{10}$ of SU(5). The SM right-handed down quarks $\overline{d}_R$ sector fully come from the $\overline{\mathbf{5}}$, and the right-handed up quarks $\overline{u}_R$ and the left-handed $\mathrm{SU}(2)_L$ doublet quarks q_L from $\mathbf{10}$.[1] In addition, we introduce three pairs of vector-like multiplets charged under $H = \mathrm{U}(2)_H$,

$$(L_H : (\mathbf{1}, \mathbf{2})_{-1/2}, \qquad \overline{L}_H : (\mathbf{1}, \mathbf{2})_{+1/2}) \times 3, \tag{3.1}$$

$$(E_H : (\mathbf{1}, \mathbf{1})_{-1}, \qquad \overline{E}_H : (\mathbf{1}, \mathbf{1})_{+1}) \times 3. \tag{3.2}$$

Here, the group representations are denoted by $(\mathrm{SU}(5), \mathrm{SU}(2)_H)_{\mathrm{U}(1)_H}$.

[1]In this paper, we use Weyl fermion notation thoroughly.

vectors	$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$	mass
X_μ	$(\mathbf{3},\mathbf{2})_{-5/6}$	$g_5 v_2/\sqrt{2}$
$\Omega_{3\mu}$	$(\mathbf{1},\mathbf{3})_0$	$\sqrt{g_5^2+g_{2H}^2}v_2$
$\Omega_{1\mu}$	$(\mathbf{1},\mathbf{1})_0$	$\sqrt{3g_5^2/5+g_{1H}^2}v_2$

Table 2. The mass spectrum of the gauge bosons. X_μ, $\Omega_{3\mu}$ and $\Omega_{1\mu}$ are the SU(5) gauge fields, $\mathrm{SU}(2)_H$ gauge fields, and $\mathrm{U}(1)_H$ gauge fields respectively.

In this model, spontaneous breaking of $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ into G_{SM} is achieved by a vacuum expectation value (VEV) of a complex scalar field ϕ_2, which is a bi-fundamental representation, $(\mathbf{5},\mathbf{2})_{-1/2}$. Explicitly, the VEV of ϕ_2 (see ref. [23])

$$\langle\phi_2\rangle = \begin{pmatrix} 0 & 0 & 0 & v_2 & 0 \\ 0 & 0 & 0 & 0 & v_2 \end{pmatrix}, \tag{3.3}$$

breaks $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ into G_{SM}. Here, $v_2 > 0$ is a constant with mass dimension and much larger than the electroweak scale. In this case, $\mathrm{SU}(3)_c$ appears as an unbroken subgroup of SU(5), while $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ appear as diagonal subgroups of SU(5) and $\mathrm{U}(2)_H$. After the symmetry breaking, the gauge bosons corresponding to the broken generators obtain masses of $\mathcal{O}(v_2)$. The gauge charges and the masses of them are given in table 2.

Once $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ is broken, the massless fermions coincide to the SM chiral fermions, while the other fermions become heavy. To see this point explicitly, let us consider the following interactions between the fermions and ϕ_2,

$$\mathcal{L} = m_{L,ij}\overline{L}_{Hi}L_{Hj} + \lambda_{L,ij}\overline{L}_{Hi}\phi_2\overline{\mathbf{5}}_j + m_{E,ij}E_{Hi}\overline{E}_{Hj} + \frac{\lambda_{E,ij}}{\Lambda_{\mathrm{cut}}}E_{Hi}\phi_2^\dagger\phi_2^\dagger\mathbf{10}_j + h.c. \tag{3.4}$$

Here, $\lambda_{L,E}$ are coupling constants, and Λ_{cut} a cutoff scale larger than the fake GUT scale. The summation of the flavor indices $i, j = 1, 2, 3$ is understood. The above interactions are the most general forms of fermion bilinears up to mass dimension five. The higher dimensional operator can be generated by integrating over the massive fermions in $(\mathbf{5},\mathbf{2})_{-1/2}$ representation coupling to ϕ_2, where Λ_{cut} is given by the mass of the massive fermions. Or, we may also consider a complex scalar ϕ_2' in $(\overline{\mathbf{10}},\mathbf{1})_1$ which has a Yukawa coupling $E_H\phi_2'\mathbf{10}$ and a trilinear coupling $\phi_2\phi_2'\phi_2$ with a dimensionful coupling, $a_{22'2}$. With this coupling, the VEV of ϕ_2' is aligned to $\langle\phi_2^\dagger\phi_2^\dagger\rangle$ as long as the mass of ϕ_2', $m_{\phi_2'}^2$, is larger than v_2. In this case, Λ_{cut} is given by $m_{\phi_2'}^2/a_{22'2}$.

After the fake GUT symmetry breaking, the above interactions lead to the mass terms in the leptonic sector. The quarks are, on the other hand, fully contained in $\overline{\mathbf{5}}\oplus\mathbf{10}$, and hence, remain massless. The mass terms of the leptonic sector are given by,

$$\mathcal{L}_{\mathrm{mass}} = \overline{L}_{Hi}\,\mathcal{M}_{L,ij}\begin{pmatrix}\overline{\mathbf{5}}_L \\ L_H\end{pmatrix}_j + E_{Hi}\,\mathcal{M}_{E,ij}\begin{pmatrix}\mathbf{10}_{\overline{E}} \\ \overline{E}_H\end{pmatrix}_j + h.c., \tag{3.5}$$

where the mass matrices are given written as

$$\mathcal{M}_{L,ij} = \begin{pmatrix}\lambda_{L,ij}v_2 & m_{L,ij}\end{pmatrix}, \quad \mathcal{M}_{E,ij} = \begin{pmatrix}\dfrac{\lambda_{E,ij}v_2^2}{\Lambda_{\mathrm{cut}}} & m_{E,ij}\end{pmatrix}. \tag{3.6}$$

Here, $\overline{\mathbf{5}}_L$ and $\mathbf{10}_{\overline{E}}$ denote the components of the $G_{\rm SM}$ gauge charges corresponding to those of the doublet and the singlet leptons. Since the mass matrices are given by 3×6 matrices, we find that three leptons remain massless due to the rank conditions. As a result, the three generations of the Weyl fermions of the SM can be realized, although the leptons do not fully belong to $\overline{\mathbf{5}}\oplus\mathbf{10}$.

To see the origin of the massless leptons, let us omit the flavor mixing and focus on a single generation. In this case, the mass eigenstates are given by

$$\begin{pmatrix} L_M \\ \ell_L \end{pmatrix} = \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L \end{pmatrix} \begin{pmatrix} \overline{\mathbf{5}}_L \\ L_H \end{pmatrix}, \tag{3.7}$$

$$\begin{pmatrix} \overline{E}_M \\ \bar{e}_R \end{pmatrix} = \begin{pmatrix} \cos\theta_E & \sin\theta_E \\ -\sin\theta_E & \cos\theta_E \end{pmatrix} \begin{pmatrix} \mathbf{10}_{\overline{E}} \\ \overline{E}_H \end{pmatrix}, \tag{3.8}$$

where ℓ_L and $\bar{e}_R$ are the doublet and the singlet massless eigenstates, respectively. The mixing angles, $\theta_{L,E}$, are

$$\tan\theta_L = \frac{m_L}{\lambda_L v_2}, \tag{3.9}$$

$$\tan\theta_E = \frac{m_E\,\Lambda_{\rm cut}}{\lambda_E v_2^2}, \tag{3.10}$$

where we have taken the parameters real positive. The masses of the heavy leptons are given by,

$$M_L = \sqrt{\lambda_L^2 v_2^2 + m_L^2}\,, \quad M_E = \sqrt{\lambda_E^2 v_2^4/\Lambda_{\rm cut}^2 + m_E^2}\,. \tag{3.11}$$

As an extreme example, the SM leptons are fully contained in L_H and $\overline{E}_H$ for $m_L = m_E = 0$, while the heavy leptons remain massive. In this case, although the quarks and the leptons in the SM originate from completely separate multiplets in the fake GUT, the quarks and the leptons apparently form $\overline{\mathbf{5}}\oplus\mathbf{10}$ multiplets at the low energy. Note that the limit $m_L = m_E = 0$ enhances a global symmetry under which L_H and $\overline{E}_H$ are charged (see section 3.4).

3.2 Origin of SM Higgs and Yukawa interactions

In the previous subsection, we discussed the origin of the SM Weyl fermions. Here, we discuss the origin of the SM Higgs boson and the Yukawa interactions. As the quarks and the leptons can originate from the separate multiplets, the SM Yukawa interactions consist of various contributions. Here, we consider a case that only one SM Higgs doublet remains in the low energy.

Concretely, we introduce H_5 and H_2 scalar fields, which are of $(\mathbf{5},\mathbf{1})_0$ and $(\mathbf{1},\mathbf{2})_{1/2}$ representations. These scalar fields contain the SM Higgs scalars as well as the colored Higgs,

$$H_2 = h_2^{\rm SM}\,, \quad H_5 = \begin{pmatrix} h_5^{\rm color} \\ h_5^{\rm SM} \end{pmatrix}. \tag{3.12}$$

The SM Higgs components in H_5 and H_2 mix with each other, via an interaction,

$$\mathcal{L}_{52\mathrm{mix}} = \mu_{\mathrm{mix}} H_2 \phi_2 H_5^* + h.c., \tag{3.13}$$

once ϕ_2 develops the VEV. As a result, the effective mass terms of H_5 and H_2 are given by,

$$\mathcal{L} = -m_5'^2 |h_5^{\mathrm{color}}|^2 - m_5^2 |h_5^{\mathrm{SM}}|^2 - m_2^2 |h_2^{\mathrm{SM}}|^2 + (\mu_{\mathrm{mix}} v_2 h_2^{\mathrm{SM}} h_5^{\mathrm{SM}*} + h.c.)\,. \tag{3.14}$$

Accordingly, the SM Higgs, h^{SM}, is given by a linear combination of h_2^{SM} and h_5^{SM},

$$h^{\mathrm{SM}} = \cos\theta_h\, h_2^{\mathrm{SM}} - \sin\theta_h\, h_5^{\mathrm{SM}}\,. \tag{3.15}$$

Here, m_5^2, $m_5'^2$, and m_2^2 are orders of $\mathcal{O}(v_2^2)$ and include the contribution from the VEV of ϕ_2. We also assume μ_{mix} is of $\mathcal{O}(v_2)$. To achieve the mass of h^{SM} in $\mathcal{O}(100)\,\mathrm{GeV}$, we require severe fine-tuning as in the case of the conventional GUT.

By using H_5 and H_2, the origins of the SM Yukawa interactions are given by,

$$\mathcal{L}_{YQ} = -(y_5)_{ij}\, \overline{\mathbf{5}}_i\, \mathbf{10}_j\, H_5^* - (y_{10})_{ij}\, \mathbf{10}_i\, \mathbf{10}_j\, H_5 + h.c.\,, \tag{3.16}$$

$$\mathcal{L}_{YL} = -(y_{LE})_{ij}\, L_{Hi}\, \overline{E}_{Hj}\, H_2^* + h.c.\,, \tag{3.17}$$

where i and j run the number of generations. The SM Yukawa couplings are obtained by substituting $h_5^{\mathrm{SM}} \to -\sin\theta_h h^{\mathrm{SM}}$, $h_2^{\mathrm{SM}} \to \cos\theta_h h^{\mathrm{SM}}$ after diagonalizing the mass matrices in eq. (3.6).

As we will see in section 3.5, the lepton components in $\overline{\mathbf{5}} \oplus \mathbf{10}$ should be highly suppressed to evade the constrains from the proton decay, i.e., $\theta_{L,E} \ll 1$. In this case, the SM Yukawa couplings are approximately given by,

$$\begin{aligned}
(y_u^{\mathrm{SM}})_{ij} &= -\sin\theta_h (y_{10})_{ij}\,,\\
(y_d^{\mathrm{SM}})_{ij} &= -\sin\theta_h (y_5)_{ij}\,,\\
(y_e^{\mathrm{SM}})_{ij} &= \cos\theta_h (y_{LE})_{ij} + \mathcal{O}(\theta_L\theta_E)\sin\theta_h (y_5)_{ij}\,.
\end{aligned} \tag{3.18}$$

3.3 Gauge coupling constants

In the present setup, $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetries are the diagonal subgroups of $\mathrm{SU}(5)\times\mathrm{U}(2)_H$, and $\mathrm{SU}(3)_c$ gauge symmetry is the remaining unbroken subgroup of $\mathrm{SU}(5)$. The tree-level matching conditions of the gauge coupling constants of the SM and the $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ model at the fake GUT scale are given by [24],

$$\begin{aligned}
\alpha_1^{-1}(M_X) &= \alpha_5^{-1}(M_X) + \frac{3}{5}\alpha_{1H}^{-1}(M_X),\\
\alpha_2^{-1}(M_X) &= \alpha_5^{-1}(M_X) + \alpha_{2H}^{-1}(M_X),\\
\alpha_3^{-1}(M_X) &= \alpha_5^{-1}(M_X),
\end{aligned} \tag{3.19}$$

where $\alpha_i = g_i^2/(4\pi)$, and g_i's are the gauge coupling constants. Here, M_X is the mass of X_μ in table 2, and we assume that all the supermassive particles have the masses of the same order of the magnitude. The fine structure constants α_5, α_{2H} and α_{1H} are those of

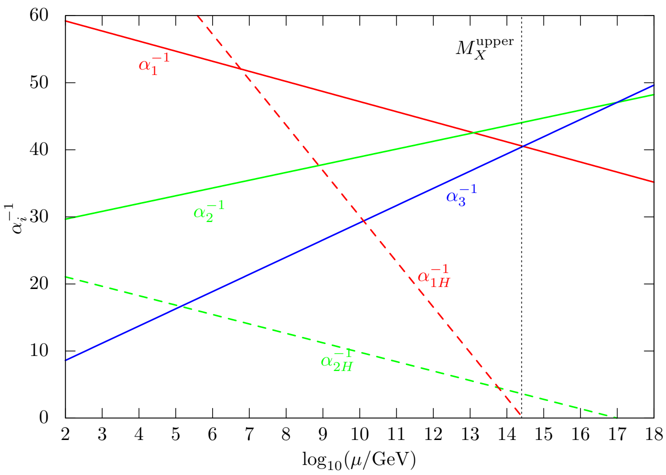

Figure 1. The running of the SM gauge couplings (solid lines). The red and green dashed lines denote the matched values of α_{2H} and α_{1H} for a given fake GUT scale (see eqs. (3.19)). The fake GUT scale is required to be below $M_X^{\rm upper} \simeq 10^{14.4}$ GeV.

	$\bar{\mathbf{5}}$	$\mathbf{10}$	L_H	$\bar{L}_H$	E_H	$\bar{E}_H$	ϕ_2	H_5	H_2	m_L	m_E
$\mathrm{U}(1)_5$	-3	1	-3	3	-1	1	0	-2	-2	0	0
$\mathrm{U}(1)_{LH}$	0	0	1	0	0	-1	0	0	0	-1	1

Table 3. The global $\mathrm{U}(1)_5$ and $\mathrm{U}(1)_{LH}$ charges of fermions and scalars are shown. Here, $\mathrm{U}(1)_{LH}$ charges of $m_{L,E}$ in eq. (3.4) are charges of spurions.

SU(5) and $\mathrm{SU}(2)_H \times \mathrm{U}(1)_H$, respectively.[2] Thus, the gauge couplings in the SM do not unify at the fake GUT scale.

As an interesting feature of $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model, there is an upper-limit on the fake GUT scale. From the matching conditions in eq. (3.19), the fake GUT scale is consistent only for $\alpha_1^{-1} > \alpha_3^{-1}$ and $\alpha_2^{-1} > \alpha_3^{-1}$, since otherwise either α_{1H} or α_{2H} is negative. From figure 1, we find the fake GUT scale is lower than $M_X^{\rm upper} \simeq 10^{14.4}$ GeV at which $\alpha_1^{-1} = \alpha_3^{-1}$. On the other hand, there is no lower limit on the fake GUT scale, M_X, from the coupling matching condition. The phenomenological lower limits on M_X are of $\mathcal{O}(10^{4-5})$ GeV from collider experiments and precision measurements.

Here, let us comment on the coupling matching condition for other choice of the gauge group H. For the model with $\mathrm{SU}(5) \times \mathrm{U}(3)_H$ where $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Y \subset \mathrm{U}(3)_H$, for example, the matching condition of the gauge couplings similar to eq. (3.19), requires that $\alpha_{1,3}^{-1} > \alpha_2^{-1}$. This is not possible unless $G_{\rm SM}$ charged particles other than the SM fields appear below the fake GUT scale (see figure 1). In this way, the matching condition and the gauge coupling running of the SM constraints the choice of H.

3.4 Global lepton and baryon symmetries

The present model possesses the global $\mathrm{U}(1)_5$ symmetry (fiveness) as in the conventional GUT model. The charge assignment of the fiveness enlarged to the $\mathrm{U}(2)_H$ sector is given

[2] Here, α_1 satisfies $\alpha_1 = 5/3\,\alpha_Y$, where α_Y is the fine structure constant of $\mathrm{U}(1)_Y$.

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in table 3. Note that the interaction terms in eqs. (3.4), (3.13), (3.16), and (3.17) allow two independent U(1) symmetries one of which is gauged as $\mathrm{U}(1)_H$ symmetry. Thus, the uniqueness of charge assignment of $\mathrm{U}(1)_5$ is up to the $\mathrm{U}(1)_H$ charge. We fix the $\mathrm{U}(1)_5$ charge by taking the $\mathrm{U}(1)_5$ charge of ϕ_2 vanishing.

After the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ breaking, $\mathrm{U}(1)_5$ results in the low energy $\mathrm{U}(1)_{B-L}$ symmetry, with the charge

$$Q_{B-L} = \frac{1}{5}(Q_5 + 4Q_Y)\,, \tag{3.20}$$

where Q_5 and Q_Y are the charges of $\mathrm{U}(1)_5$ and $\mathrm{U}(1)_Y$. The $B-L$ applies to the fermion fields in the same way of the SM.

As mentioned earlier, an additional global symmetry is enhanced in the limit of $m_{L,E} \to 0$, which we call $\mathrm{U}(1)_{LH}$ symmetry. The charge assignment is given in table 3. The $\mathrm{U}(1)_{LH}$ symmetry also remains in the low energy which gives an additional lepton symmetry, $\mathrm{U}(1)_L$. This lepton symmetry is, however, not exact since it is broken by the gauge/gravitational anomalies. This additional symmetry can be identified with the lepton symmetry of the SM.

Up to the gauge/gravitational anomalies the $\mathrm{U}(1)_{B-L}$ and $\mathrm{U}(1)_L$ symmetries are conserved separately in the limit of $m_{L,E} \to 0$, which may be rearranged as $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_L$. As far as these symmetries are respected, no visible low energy baryon/lepton violating processes are expected. Note that the effects of the breaking of $\mathrm{U}(1)_{B-L}$ and $\mathrm{U}(1)_L$ by the gauge and the gravitational anomalies are highly suppressed for the nucleus decays [25].

In the conventional SU(5) GUT both the baryon symmetry $\mathrm{U}(1)_B$ and the lepton symmetry $\mathrm{U}(1)_L$ are embedded in the GUT fiveness. Thus, at the low energy, only the linear combination, $\mathrm{U}(1)_{B-L}$, is conserved. As a result, the conventional GUT predicts the proton decay which violates the $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_L$ symmetries while $\mathrm{U}(1)_{B-L}$ is conserved. The proton lifetime is dominantly determined by the GUT gauge boson mass scale.

In the present fake GUT model, the proton decay is forbidden when the $\mathrm{U}(1)_{LH}$ symmetry emerges, i.e. $m_{L,E} = 0$. In fact, this symmetry is crucial for the successful $\mathrm{U}(2)_H$ fake GUT model as the X gauge boson mass scale $M_X < 10^{14.4}$ GeV, which would lead to too rapid proton decay without this symmetry.

It is, however, highly non-trivial whether we can impose such global symmetries on the model. In general, it is argued that all global symmetries are broken by quantum gravity effects (see e.g. [26–32]). Moreover, the gauge/gravitational anomalies already break the $\mathrm{U}(1)_{B-L}$ and $\mathrm{U}(1)_L$ symmetries explicitly in this model. In addition, the origins of the neutrino masses and the baryon asymmetry in the Universe may also indicate the breaking of those symmetries. Therefore, we expect that small violation of $\mathrm{U}(1)_{B-L}$ and $\mathrm{U}(1)_L$ exist which generate tiny $m_{L,E}$. The breaking of the $\mathrm{U}(1)_{B-L}$ and $\mathrm{U}(1)_L$ symmetries depend on the further high energy physics. In the following analysis, we simply regard $m_{L,E}$ (or equivalently $\theta_{L,E}$ in eqs. (3.9) and (3.10)) as the parameter of the explicit symmetry breaking.[3]

Let us also comment on the origin of the neutrino mass. The observations of the neutrino oscillations show that the neutrinos have tiny masses. In the present model, there are various ways to realize the active neutrino masses. First, we may consider the active

[3]We discuss one example of the origin of the $\mathrm{U}(1)_{LH}$ symmetry and its breaking based on the $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ model in the appendix C.

neutrinos as the Dirac neutrinos, where the right-handed neutrinos couple to the leptons and Higgs in the $\mathrm{U}(2)_H$ sector. In this case, there is no lepton symmetry violation, and hence, the proton decay is also suppressed for $m_{L,E} \to 0$ as explained above.

Another possibility is to couple the right-handed neutrinos to the SU(5) sector. In this case, although the active neutrino masses are the Dirac type, we require the lepton symmetry breaking, that is, the tiny mixing between the lepton components of $\overline{\mathbf{5}}$ and L_H. It is an interesting feature of this model, that both the neutrino masses and the proton decay rate are generated by the effects of the lepton symmetry breaking, i.e., m_L.

More attractive possibility is to consider the Majorana neutrino masses, which break $\mathrm{U}(1)_{LH}$ symmetry down to $\mathbb{Z}_2$ subgroup. When $\mathbb{Z}_2$ subgroup of $\mathrm{U}(1)_{LH}$ is conserved separately from $\mathrm{U}(1)_{B-L}$, the proton is stable as long as Z_2 is unbroken. For instance, it is possible to consider the seesaw mechanism [33–38] in the $\mathrm{U}(2)_H$ sector where the model possesses the Z_2 symmetry while $m_{L,E}$ are suppressed.

3.5 Nucleon decay

The nucleons can decay through the heavy SU(5) gauge bosons X exchange. For simplicity, we assume $\theta_{E,L}$ in eq. (3.7) and (3.8) do not depend on the generations. In this case, the proton lifetime of the $p \to \pi^0 + e^+$ mode is

$$\tau(p \to \pi^0 + e^+) \simeq \frac{5 \times 10^{26}\,\mathrm{yrs}}{\sin^2\theta_E + 0.2\sin^2\theta_L}\left(\frac{M_X/g_5}{10^{14}\,\mathrm{GeV}}\right)^4 . \tag{3.21}$$

In order to estimate the proton lifetime, we adopt the method described in ref. [39], using the hadron matrix elements of refs. [40, 41]. By comparing to the current limit, $\tau(p \to \pi^0 + e^+) > 2.4 \times 10^{34}\,\mathrm{yrs}$ [8], this model seemingly predicts too short proton lifetime as in the case of the conventional non-supersymmetric SU(5) GUT model. However, the proton decay width can be suppressed by the mixing angles $\theta_{E,L}$ in the fake GUT. In the above example, small mixings, $\sin\theta_{E,L} \lesssim 10^{-4}$, are consistent with the current limit on the proton lifetime. This suppression means that the SM quarks and leptons mostly have different origins. Such small mixing angles correspond to $m_{E,L} \ll v_2$, where the global symmetry is enhanced, as discussed in the previous subsection.

The proton lifetime in eq. (3.21) shows that we need small mixings of both the $\overline{\mathbf{5}}_L$ and $\mathbf{10}_{\overline{E}}$ with the SM leptons to evade the constraints. Thus, for the choices such as $H = 1$, $\mathrm{U}(1)_H$, $\mathrm{SU}(2)_H$, for example, the proton lifetime is predicted to be too short unless we introduce a large number of SU(5) charged fermions [20]. This is the reason why $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ is the minimal choice which is phenomenologically viable. Note that we estimate the above proton lifetime without considering the effects of the flavor mixings for simplicity. In general, however, the lepton mass matrices in eq. (3.6) are flavor dependent. Hence, an SU(5) gauge interaction eigenstate does not coincide with one generation of the SM fermions but consists of the admixture of the multiple SM generations. Therefore, the predictions of the nucleon decay rates and the branching fractions in the fake GUT are different from those in the conventional GUT. For example, the decay rate of the $p \to \pi^0 + \mu^+$ mode can be larger than that of the $p \to \pi^0 + e^+$ mode. This is a contrary to the conventional GUT, where the nucleon decay modes which include different generations

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are suppressed by the Cabibbo-Kobayashi-Maskawa (CKM) mixing angle. In this way, a variety of the nucleon decay modes will provide striking signatures of the fake GUT.

To see the effects of the flavor dependence of the mass matrices in eq. (3.6), let us consider the following example of the fermion mass terms at the fake GUT scale.

$$\mathcal{M}_L = M_0 \begin{pmatrix} 1 & 0 & 0 & \delta_{L,11} & \delta_{L,12} & \delta_{L,13} \\ 0 & 1 & 0 & \delta_{L,21} & \delta_{L,22} & \delta_{L,23} \\ 0 & 0 & 1 & \delta_{L,31} & \delta_{L,32} & \delta_{L,33} \end{pmatrix} , \quad \mathcal{M}_E = M_0 \begin{pmatrix} 1 & 0 & 0 & \delta_{E,11} & \delta_{E,12} & \delta_{E,13} \\ 0 & 1 & 0 & \delta_{E,21} & \delta_{E,22} & \delta_{E,23} \\ 0 & 0 & 1 & \delta_{E,31} & \delta_{E,32} & \delta_{E,33} \end{pmatrix} . \tag{3.22}$$

Here, we introduce small parameters $\delta_{L,E,ij}$ with $|\delta_{L,E,ij}| \ll 1$, which represents explicit breaking of the global symmetry. We take flavor indices of $\overline{\mathbf{5}}_i$ and $\mathbf{10}_i$ to match the generations of the quarks, and those of L_{Hi} and $\overline{E}_{Hi}$ approximately corresponds to the generations of the leptons (see eq. (3.18)). In our analysis, we adopt a flavor basis that the up-type Yukawa matrix is diagonal, while the down-type Yukawa matrix has CKM-related off diagonal elements.

In figure 2, we show the proton lifetime for each mode. In each figure, we switch on the mixing parameters in eq. (3.22) as shown in the caption. The predictions are proportional to $\delta_{L,E}^{-2}$ and M_X^4. Black lines denote the Super-Kamiokande constraints at 90% C.L. [8, 42–46] and yellow ones represent the future prospects of Hyper-Kamiokande [9]. The figures show the model predicts various leading proton decay modes depending on the mixing parameters. For the case of the figure 2 (b), for example, $p \to \pi^0 + \mu^+$ mode is the leading decay mode, which is not expected in the conventional GUT model with minimal flavor violation.[4]

Finally, let us comment on the proton decay through the colored Higgs exchanges. In the conventional GUT model, these contributions is always subdominant compared with those of the X boson exchanges due to the Yukawa suppression [47],

$$\tau(p \to K^+ + \nu)|_{\text{colored Higgs exchange}} \sim 10^{45}\,\text{yrs} \times \theta_{L,E}^{-2} \left(\frac{M_{H_c}}{10^{14}\,\text{GeV}}\right)^4 \sin^4\theta_h \,. \tag{3.23}$$

Here, we multiply the factor $\sin^4\theta_h$ which stems from the relative enhancement of $y_{5,10}$, compared with the SM Yukawa couplings given in eq. (3.18). To reproduce the masses of the SM fermions, we find that, the largest components of $y_{10,5,LE}$ are

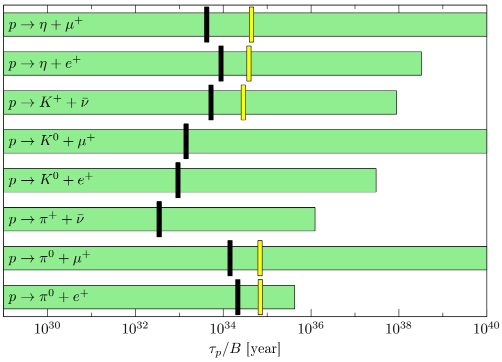
$$y_{10} \sim y_t/\sin\theta_h \,, \quad y_5 \sim y_b/\sin\theta_h \,, \quad y_{LE} \sim y_\tau/\cos\theta_h \,. \tag{3.24}$$

Thus, by requiring $|y_{10,5,LE}| \lesssim 1$,

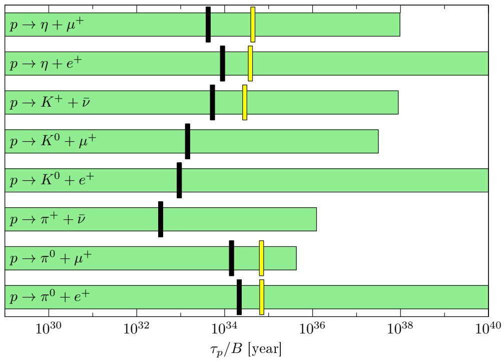
$$\sin\theta_h \sim y_t/y_{10} \gtrsim 0.5 \,, \quad \cos\theta_h \sim y_\tau/y_{LE} \gtrsim 10^{-2} \,. \tag{3.25}$$

Therefore, there is no large enhancement from $\sin^4\theta_h$ and the proton decay rate from the colored Higgs exchange is negligible.

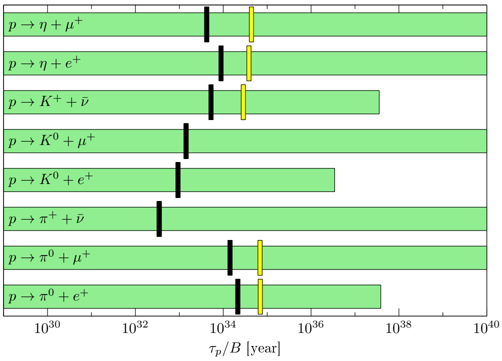
[4]The GUT models with non-minimal flavor violation can also lead to the branching ratio patterns differing from the GUT models with minimal flavor violation (see e.g. ref. [39]).



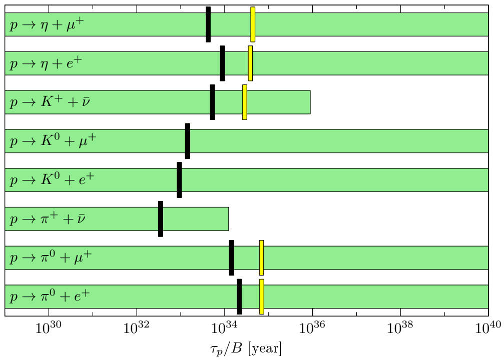
(a) $\delta_{L,11} = \delta_{E,11} = 10^{-4}$.



(b) $\delta_{L,12} = \delta_{E,12} = 10^{-4}$.



(c) $\delta_{L,21} = \delta_{E,21} = 10^{-4}$.



(d) $\delta_{L,13} = \delta_{E,13} = 10^{-3}$.

Figure 2. The proton lifetime of each decay mode. Here, we switch on the mixing parameters in eq. (3.22) as indicated by the subcaptions. We take $M_X = 10^{14}$ GeV.

4 SU(5) × SU(3)$_H$ model as UV completion of SU(5) × U(2)$_H$ model

In this section, we consider $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ model where $\mathrm{U}(2)_H$ is embedded into $\mathrm{SU}(3)_H$. Note that $\mathrm{SU}(3)_H$ does not include $\mathrm{SU}(3)_c$ of the SM. As we will construct, $\mathrm{SU}(3)_H$ is spontaneously broken down to $\mathrm{U}(2)_H$ by the VEV of the adjoint scalar of $\mathrm{SU}(3)_H$, A. The remaining $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ is subsequently broken down to G_{SM} by the VEV of the scalar field in the bi-fundamental representation of $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$, ϕ_3. That is,

$$\mathrm{SU}(5) \times \mathrm{SU}(3)_H \xrightarrow{\langle A \rangle} \mathrm{SU}(5) \times \mathrm{U}(2)_H \xrightarrow{\langle \phi_3 \rangle} G_{\mathrm{SM}}. \tag{4.1}$$

In the previous section, we showed that $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model is phenomenologically viable. However, since H includes U(1) gauge symmetry, the model cannot explain the charge quantization unlike the conventional GUT. The U(1) gauge symmetry also exhibits the Landau pole problem. These drawbacks can be solved by the extension to $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$. We summarize the matter contents in table 4.

4.1 Spontaneous symmetry breaking of SU(5) × SU(3)$_H$

In this model, we introduce a real scalar field, A, in $(\mathbf{1}, \mathbf{8})$ representation, and a complex scalar field, ϕ_3, in $(\mathbf{5}, \mathbf{3})$ representation of $(\mathrm{SU}(5), \mathrm{SU}(3)_H)$, respectively. The scalar

	$(\mathrm{SU}(5), \mathrm{SU}(3)_H)$	$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$
fermions		
$\overline{\mathbf{5}}$	$(\overline{\mathbf{5}}, \mathbf{1})$	$(\overline{\mathbf{3}}, \mathbf{1})_{+1/3} \oplus (\mathbf{1}, \mathbf{2})_{-1/2}$
$\mathbf{10}$	$(\mathbf{10}, \mathbf{1})$	$(\mathbf{3}, \mathbf{2})_{+1/6} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-2/3} \oplus (\mathbf{1}, \mathbf{1})_{+1}$
L_T	$(\mathbf{1}, \mathbf{3})$	$(\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{1}, \mathbf{1})_{+1}$
$\overline{L}_T$	$(\mathbf{1}, \overline{\mathbf{3}})$	$(\mathbf{1}, \mathbf{2})_{+1/2} \oplus (\mathbf{1}, \mathbf{1})_{-1}$
scalars		
A	$(\mathbf{1}, \mathbf{8})$	$(\mathbf{1}, \mathbf{3})^R_0 \oplus (\mathbf{1}, \mathbf{2})_{+3/2} \oplus (\mathbf{1}, \mathbf{1})^R_0$
ϕ_3	$(\mathbf{5}, \mathbf{3})$	$(\mathbf{3}, \mathbf{2})_{-5/6} \oplus (\mathbf{3}, \mathbf{1})_{+2/3} \oplus (\mathbf{1}, \mathbf{2})_{+3/2} \oplus (\mathbf{1}, \mathbf{3})_0 \oplus (\mathbf{1}, \mathbf{1})_0$
H_5	$(\mathbf{5}, \mathbf{1})$	$(\mathbf{3}, \mathbf{1})_{-1/3} \oplus (\mathbf{1}, \mathbf{2})_{+1/2}$
H_3	$(\mathbf{1}, \mathbf{3})$	$(\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{1}, \mathbf{1})_{+1}$
vectors		
V_5	$(\mathbf{24}, \mathbf{1})$	$(\mathbf{8}, \mathbf{1})^R_0 \oplus (\mathbf{3}, \mathbf{2})_{-5/6} \oplus (\mathbf{1}, \mathbf{3})^R_0 \oplus (\mathbf{1}, \mathbf{1})^R_0$
V_{3H}	$(\mathbf{1}, \mathbf{8})$	$(\mathbf{1}, \mathbf{3})^R_0 \oplus (\mathbf{1}, \mathbf{2})_{+3/2} \oplus (\mathbf{1}, \mathbf{1})^R_0$

Table 4. The content of fermions, scalar fields and gauge bosons in the $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ model is shown in the group representation, $(\mathrm{SU}(5), \mathrm{SU}(3)_H)$ and $(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$. Each fermion has three generations. The superscript R denotes the real field, while the other fields are complex fields. Some components of the fake GUT Higgs scalars ϕ_3 and A become the longitudinal modes of the fake GUT gauge boson when $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ is broken to G_{SM} (see text).

potential is

$$\begin{aligned} V(A,\, \phi_3) = &-\mu_A^2 \mathrm{Tr}\left[A^2\right] + \lambda_{1A}\left(\mathrm{Tr}\left[A^2\right]\right)^2 + \lambda_{2A}\mathrm{Tr}\left[A^4\right] + \mu_{3A}\mathrm{Tr}\left[A^3\right] \\ &- \mu_\phi^2 \mathrm{Tr}\left[\phi_3^\dagger \phi_3\right] + \lambda_{1\phi}\left(\mathrm{Tr}\left[\phi_3^\dagger\phi_3\right]\right)^2 + \lambda_{2\phi}\mathrm{Tr}\left[\phi_3^\dagger\phi_3\phi_3^\dagger\phi_3\right] \\ &+ \lambda_{1\phi A}\mathrm{Tr}\left[\phi_3^\dagger\phi_3\right]\mathrm{Tr}\left[A^2\right] + \lambda_{2\phi A}\mathrm{Tr}\left[\phi_3^\dagger A^2 \phi_3\right] - \mu_{3\phi A}\mathrm{Tr}\left[\phi_3^\dagger A \phi_3\right]. \end{aligned} \tag{4.2}$$

Here, μ's are parameters with mass dimension one, and λ's are dimensionless parameters. We take all the parameters are positive for simplicity. The vacuum in $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ model in eq. (3.3) is reproduced when A and ϕ_3 take the VEVs in the following form;

$$\langle A \rangle = \begin{pmatrix} v_A & 0 & 0 \\ 0 & v_A & 0 \\ 0 & 0 & -2v_A \end{pmatrix}, \quad \langle \phi_3 \rangle = \begin{pmatrix} 0 & 0 & 0 & v_3 & 0 \\ 0 & 0 & 0 & 0 & v_3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.3}$$

To achieve this form, $v_A \neq 0$ is crucial, since otherwise we find that either only a single row of $\langle\phi_3\rangle$ or all the three rows of $\langle\phi_3\rangle$ are non-vanishing.[5] Once $v_A \neq 0$, the last two terms in eq. (4.2) can make the mass squared of the third row of ϕ_3 positive while those

[5]The symmetry is broken as $\mathrm{SU}(5)\times\mathrm{SU}(3)_H \to \mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ for the former case, and $\mathrm{SU}(5)\times\mathrm{SU}(3)_H \to \mathrm{SU}(3)\times\mathrm{SU}(2)$ for the latter case.

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vectors	$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$	mass
X_μ	$(\mathbf{3},\mathbf{2})_{-5/6}$	$g_5 v_3/\sqrt{2}$
$\Omega_{2\mu}$	$(\mathbf{1},\mathbf{2})_{+3/2}$	$3g_{3H}\sqrt{v_A^2+v_3^2/18}$
$\Omega_{3\mu}$	$(\mathbf{1},\mathbf{3})_0$	$\sqrt{g_5^2+g_{2H}^2}v_3$
$\Omega_{1\mu}$	$(\mathbf{1},\mathbf{1})_0$	$\sqrt{3g_5^2/5+g_{1H}^2}v_3$

Table 5. The mass spectrum of the gauge bosons. The X_μ is the SU(5) gauge fields. $\Omega_{2\mu}$, $\Omega_{3\mu}$ and $\Omega_{1\mu}$ are the $\mathrm{SU}(3)_H$ gauge fields.

of the first and the second rows are kept negative. For $v_A \gtrsim v_3$, we may regard that the symmetry breaking takes place in the two steps as follows;

$$\mathrm{SU}(5)\times\mathrm{SU}(3)_H \xrightarrow{\langle A\rangle} \mathrm{SU}(5)\times\mathrm{U}(2)_H \xrightarrow{\langle\phi_3\rangle} G_{\mathrm{SM}}, \tag{4.4}$$

where the second step corresponds to the symmetry breaking in the $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ model. Note that the model does not require the hierarchy between v_A and v_3, and it is possible to consider situation where $\mathrm{SU}(5)\times\mathrm{SU}(3)_H$ breaks down directly to G_{SM} for $v_A < v_3$.

In table 5, we show the gauge boson masses. The X boson absorbs $(\mathbf{3},\mathbf{2})_{-5/6}$ scalar in table 4 as a longitudinal mode. The Ω_2 boson absorbs a linear combination of two $(\mathbf{1},\mathbf{2})_{+3/2}$ scalar fields. The Ω_3 (Ω_1) absorb a linear combination of $(\mathbf{1},\mathbf{3})_0$ ($(\mathbf{1},\mathbf{1})_0$) in the bi-fundamental fake GUT Higgs ϕ_3. Accordingly, the physical fake GUT Higgs bosons appearing from ϕ_3 and A consist of a $(\mathbf{1},\mathbf{2})_{+3/2}$ scalar field, two $(\mathbf{1},\mathbf{3})_0^R$ scalars, two $(\mathbf{1},\mathbf{1})_0^R$ scalars, and a $(\mathbf{3},\mathbf{1})_{+2/3}$ scalar.

In the previous $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ model, the matching conditions between the SM and $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ gauge couplings are given in eq. (3.19). For $v_A \gtrsim v_3$ the matching conditions of the gauge coupling constants at the symmetry breaking in the second step in eq. (4.4) are also given by eq. (3.19) with M_X replaced by $M_X = g_5 v_3/\sqrt{2}$.

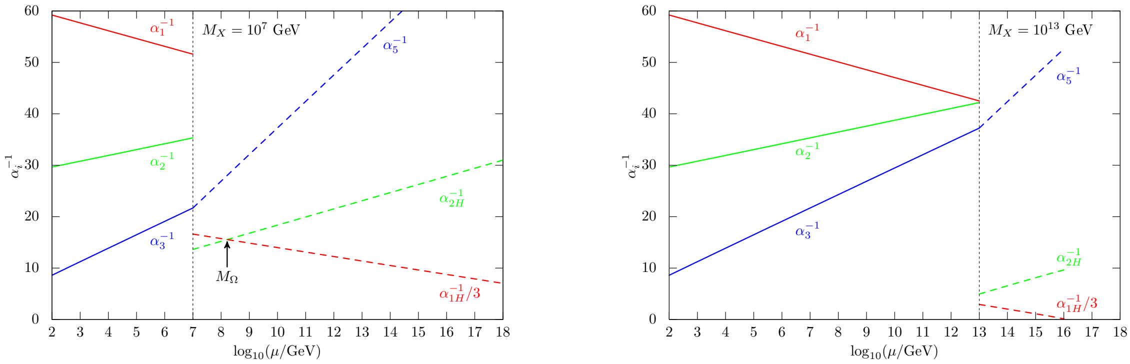
The matching conditions between $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ and $\mathrm{SU}(5)\times\mathrm{SU}(3)_H$ models are, on the other hand, given by

$$\begin{aligned}\frac{1}{3}\alpha_{1H}^{-1}(M_\Omega) &= \alpha_{3H}^{-1}(M_\Omega),\\ \alpha_{2H}^{-1}(M_\Omega) &= \alpha_{3H}^{-1}(M_\Omega)\,.\end{aligned} \tag{4.5}$$

Here, $M_\Omega = 3g_{3H}\sqrt{v_A^2+v_3^2/18}$ is the energy scale at which these two models are matched. The factor 1/3 in eq. (4.5) appears since the $\mathrm{U}(1)_H$ charge is embedded in the fundamental representation of $\mathrm{SU}(3)_H$ with the normalization, $\mathrm{tr}[t^a t^b] = \delta^{ab}/2$, with t^a being the generator of $\mathrm{SU}(3)_H$.[6]

For a given M_X, the gauge couplings α_{1H} and α_{2H} are determined by eq. (3.19). Above M_X, α_5, α_{1H}, and α_{2H} evolves following the renormalization group (RG) equations in the $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ model for $v_A \gtrsim v_3$. The RG equations of the present model are given in the

[6]The generator of $\mathrm{U}(1)_H$ embedded in $\mathrm{SU}(3)_H$ corresponds to $-t_8$ with t_8 being the eighth Gell-Mann matrix divided by 2.



(a) $M_X = 10^7$ GeV.

(b) $M_X = 10^{13}$ GeV.

Figure 3. The left (right) figure shows the running of gauge couplings when $M_X = 10^7$ GeV ($M_X = 10^{13}$ GeV). The red, green, and blue solid lines denote the runnings of α_1^{-1}, α_2^{-1}, and α_3^{-1}, respectively. The red, green, and blue dashed lines are, respectively, the runnings of $\alpha_{1H}^{-1}/3$, α_{2H}^{-1}, and α_5^{-1}. Here we assume that only the particles contained in the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model contribute to the running of gauge couplings. The figure shows that the matching between $\mathrm{U}(2)_H$ and $\mathrm{SU}(3)_H$ is possible for the left panel while it is not possible in the right panel.

appendix B. We show examples of the RG running in figure 3. By using these running gauge couplings, we can find the matching scale $M_\Omega^{\mathrm{match}}$ which satisfies the conditions in eq. (4.5).

For $v_A \lesssim v_3$, all the gauge boson masses are dominated by v_3 contribution, and hence, $M_X \simeq M_\Omega$. Accordingly, the matching conditions in eqs. (3.19) and (4.5) are combined where the matching conditions are given at the same scale, $M_X \simeq M_\Omega$.

Since $M_\Omega \gtrsim M_X$ in the present model, we find that there is an upper limit on M_X which can be seen from figure 4;

$$M_X \leq 4 \times 10^{10}\,\mathrm{GeV}\,. \tag{4.6}$$

Due to this severer constraints on M_X than that in $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model, $M_X \lesssim 10^{14}$ GeV, (see figure 1), this model requires smaller mixing angles of the lepton components of $\overline{\mathbf{5}} \oplus \mathbf{10}$ to evade the constraints from the proton lifetime;

$$\theta_{L,E} \lesssim 10^{-12} \left(\frac{M_X/g_5}{10^{10}\,\mathrm{GeV}}\right)^2 , \tag{4.7}$$

(see eq. (3.21)). To achieve the highly suppressed lepton mixing angle, we need a high quality lepton symmetry. In the appendix C, we give an example of a model in which such a high quality lepton symmetry originates from a discrete gauge symmetry.

4.2 SM fermions, SM Higgs and Yukawa interactions

In the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model, the vector-like fermions are charged under $H = \mathrm{U}(2)_H$ are $(L_H, \overline{L}_H)$ and $(E_H, \overline{E}_H)$. In the $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ model, they are straightforwardly embedded into the $\mathrm{SU}(3)_H$ fundamental representations,

$$(\, L_T = (L_H\, \overline{E}_H) : (\mathbf{1}, \mathbf{3}),\;\; \overline{L}_T = (\overline{L}_H\, E_H) : (\mathbf{1}, \overline{\mathbf{3}})\,) \quad \times 3. \tag{4.8}$$

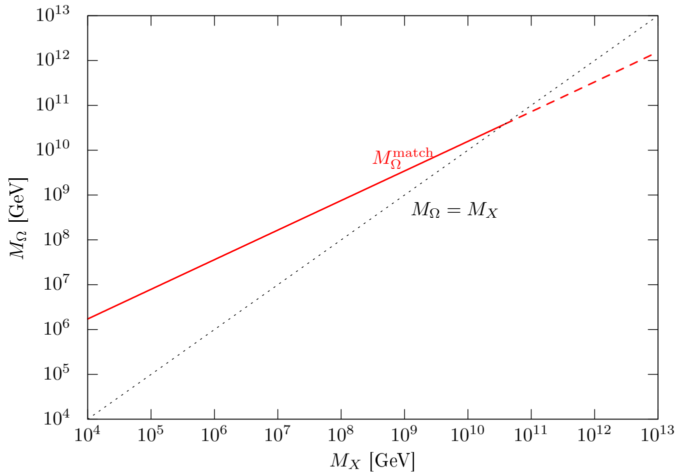


Figure 4. The matching scale M_Ω^{match} as a function of M_X (red solid line).

As in the case of $\text{SU}(5) \times \text{U}(2)_H$ model, the leptonic components, $\overline{\mathbf{5}}_L$ and $\mathbf{10}_E$ become the mass partners of L_T and $\overline{L}_T$ through the coupling to ϕ_3,

$$\mathcal{L} = m_T \overline{L}_T L_T + \lambda_5 \overline{L}_T \phi_3 \overline{\mathbf{5}} + \frac{\lambda_{10}}{\Lambda_{\text{cut}}} \overline{L}_T \phi_3^\dagger \phi_3^\dagger \mathbf{10} + \lambda_{TAT} \overline{L}_T A L_T + h.c. \tag{4.9}$$

Here, m_T is a mass parameter of the vector-like fermions and λ's are dimensionless couplings. The flavor indices are omitted for simplicity. The $\text{SU}(3)_H$ indices of $\overline{L}_T \phi_3^\dagger \phi_3^\dagger$ are contracted by the totally anti-symmetric invariant tensor of $\text{SU}(3)_H$, while its SU(5) indices are contracted with those of $\mathbf{10}$ which is the anti-symmetric representation. The cutoff scale Λ_{cut} is a scale larger than the fake GUT scale v_3 and v_A corresponding to some heavier particles. An example of such particles is Dirac fermion with $(\mathbf{5}, \overline{\mathbf{3}})$. We may also consider a complex scalar ϕ_3' in $(\overline{\mathbf{10}}, \mathbf{3})$ which has a Yukawa coupling $\overline{L}_T \phi_3' \mathbf{10}$ and a trilinear coupling $\phi_3 \phi_3' \phi_3$ (see the appendix C for a concrete model.).

As in the previous section, the SM quarks originate from $\overline{\mathbf{5}} \oplus \mathbf{10}$. The lepton components in $\overline{\mathbf{5}}$ and $\mathbf{10}$ and L_T's obtain 3×6 mass matrices from the interactions in eq. (4.9), and the three leptons remain massless due to the rank conditions. Since the $\overline{\mathbf{5}} \oplus \mathbf{10}$ contributions to the SM leptons are highly suppressed (see eq. (4.7)), we require $m_T \ll v_{3,A}$ and $\lambda_{TAT} \ll 1$. As in the case of the $\text{SU}(5) \times \text{U}(2)_H$ model, the global $\text{U}(1)_{LT}$ symmetry is enhanced in addition to the global $\text{U}(1)_5$ symmetry in the limit of vanishing m_T and λ_{TAT}. In the appendix C, we discuss a model where the lepton symmetry originates from a discrete gauge symmetry.

Next, we discuss the origin of the SM Higgs and Yukawa interactions in the $\text{SU}(5) \times \text{SU}(3)_H$ model. As in the $\text{SU}(5) \times \text{U}(2)_H$ model, the SM Yukawa interactions come from various contributions, and we consider a case that only one $\text{SU}(2)_L$ Higgs doublet remains in the low energy. We introduce H_5 and H_3 scalar fields, which are $(\mathbf{5}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{3})$ representations. The H_3 and H_5 are decomposed as,

$$H_3 = \begin{pmatrix} h_3^{\text{SM}\dagger} \\ h_3^{\text{singlet}} \end{pmatrix}, \quad H_5 = \begin{pmatrix} h_5^{\text{color}} \\ h_5^{\text{SM}} \end{pmatrix}, \tag{4.10}$$

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where a linear combination of $h_3^{\rm SM}$ and $h_5^{\rm SM}$ becomes the SM Higgs doublet. The mixing term between $h_3^{\rm SM}$ and $h_5^{\rm SM}$ comes from

$$\mathcal{L}_{53\rm mix} = \mu_{\phi 53}\, H_3^\dagger \phi_3 H_5^\dagger + h.c., \tag{4.11}$$

where $\mu_{\phi 53}$ is the mass parameter of the order of the fake GUT scale. The SM Higgs doublet in the electroweak scale requires fine-tuning. The singlet and the colored Higgs as well as a heavier combination of $h_3^{\rm SM}$ and $h_5^{\rm SM}$ obtain masses of the order of the fake GUT scale.

The Yukawa interactions of the quarks are given by those of SU(5) multiplets with H_5 as in eq. (3.16). Since the SM leptons dominantly come from L_T's, we expect that the SM lepton Yukawa couplings are dominated by the couplings to H_3,

$$\mathcal{L}_{YL} = -(y_{LT})_{ij}\epsilon_{\alpha\beta\gamma} H_3^\alpha L_{Ti}^\beta L_{Tj}^\gamma + h.c., \tag{4.12}$$

which is the naive extension of eq. (3.17). Here, the subscript α is the index of the $\mathrm{SU}(3)_H$ and i and j run the number of generations. Since $\epsilon_{\alpha\beta\gamma}$ is totally antisymmetric, we find $(y_{LT})_{ij} = -(y_{LT})_{ji}$. The 3-by-3 antisymmetric Yukawa coupling results in the massless electron and the μ and τ lepton in the same mass. Thus, the Yukawa coupling given by eq. (4.12) does not reproduce the lepton masses in the SM.

As in eq. (3.18), the lepton Yukawa couplings receive the contribution from y_5, which are suppressed by the lepton mixing angles $\theta_L \times \theta_E$. The lepton mixing angles are, however, required to be highly suppressed, $\theta_{L,E} \lesssim 10^{-12}$, to evade the constraints from the proton lifetime (see eq. (4.7)). Thus, those contributions are too small to reproduce the lepton masses in the SM, and hence, we need other origins of the lepton Yukawa interactions.

As an example to generate appropriate lepton Yukawa coupling, we consider the following higher dimensional operator,

$$\mathcal{L}_{YL} = -\frac{(Y_{LT})_{ij}}{\Lambda_Y} L_{Ti}^\alpha A_\alpha^\beta L_{Tj}^\gamma H_3^\delta \epsilon_{\beta\gamma\delta} + h.c., \tag{4.13}$$

where Λ_Y is a cutoff scale. The scale Λ_Y can be given by the mass of heavy particles such as Dirac fermions in $(\mathbf{1},\mathbf{3})$ representation (see appendix C). Once A obtains the VEV, this operator generates the lepton Yukawa couplings. Unlike the Yukawa couplings in eq. (4.12), the 3-by-3 coefficient matrix is not anti-symmetric, and hence, this operator can reproduce the SM lepton spectrum, when Λ_Y is smaller than $\mathcal{O}(10^2) \times v_A$.

5 Conclusions and discussions

The fake GUT is a framework which has been proposed to explain the perfect fit of the SM matter fields into the SU(5) multiplets [12]. Unlike the conventional GUT [4], the quarks and leptons are not necessarily embedded in common GUT multiplets but embedded in different multiplets although they appear to form complete GUT multiplets at the low energy. In this paper, we studied details of the model based on $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ gauge symmetry and its extension with $\mathrm{SU}(5)\times\mathrm{SU}(3)_H$ gauge symmetry. We discussed the nature of the fake GUT symmetry breaking. We also studied the origins of the SM quarks/leptons, the Higgs fields, and the SM Yukawa interactions.

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In the present models, the global lepton and baryon symmetries play a crucial role to avoid too rapid nucleon decays. However, these global symmetries are less likely exact ones, due to theoretical and phenomenological reasons. With the violation of the global symmetries, the nucleons are no longer stable. The decay rates and decay modes strongly depend on its size and flavor structure. Observations of multiple nucleon decay modes are the smoking guns of the present models.

We extended the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model to the $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ model to explain the $\mathrm{U}(1)_H$ charge quantization and avoid Landau pole problem. As for $\mathrm{SU}(3)_H$ model, we discussed the scalar potential with which the $\mathrm{SU}(2)_L$ appears in the diagonal subgroup of $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$. As a result, we find that the model with the scalar fields in the bi-fundamental representation of $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ and in the adjoint representation of $\mathrm{SU}(3)_H$ provides the successful symmetry breaking. We also studied the upper limit on the fake GUT scale, $M_X \lesssim 10^{10}\,\mathrm{GeV}$. Due to the low fake GUT scale, the model requires rather strict lepton symmetry to suppress the proton decay.

Let us briefly discuss alternative extension of $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model, instead of $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$. For example, one may consider models based on the gauge groups $\mathrm{SU}(6) \times \mathrm{SU}(2)_H \to \mathrm{SU}(5) \times \mathrm{U}(2)_H$ or $\mathrm{SU}(7) \to \mathrm{SU}(5) \times \mathrm{U}(2)_H$. Those groups have chiral representations $\overline{\mathbf{6}} \oplus \overline{\mathbf{6}} \oplus \mathbf{15}$ for $\mathrm{SU}(6)$ and $\overline{\mathbf{7}} \oplus \overline{\mathbf{7}} \oplus \overline{\mathbf{7}} \oplus \mathbf{21}$ for $\mathrm{SU}(7)$, respectively. However, the resultant $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ models do not satisfy the conditions of the fake GUT model, and hence, they do not provide viable UV completions of the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model.

Finally let us comment on the topological objects generated in the early Universe in these models. Unlike the conventional GUT based on simple groups, the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model has no dangerous topological objects. Therefore high-scale inflation and high reheating temperature are allowed in the model. In the case of $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$, on the other hand, it is possible to generate monopoles if the fake GUT breaking takes place after the inflation. In order to avoid this monopole problem, the reheating temperature should be much less than the fake GUT scale. Detailed constraints and the possible detection of the monopole will be explored elsewhere. Besides, let us also comment on the monopole catalysis of proton decay. In the conventional GUT model, the monopoles induce the baryon-number non-conserving reactions [48, 49]. In the fake GUT model, on the other hand, baryon-number violating processes, e.g. proton decay, are also controlled by the lepton symmetry unlike the conventional GUT. This may provide different features of the monopole in the fake GUT model compared to that in the conventional GUT.

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A Characters of SM and SU(5) groups

In order to see that the SM fermions are apparently unified into the SU(5) multiplets, it is convenient to use the character of the gauge groups G defined by the trace of a representation matrix, $R(g)$ $(g \in G)$

$$\chi_R(g) := \mathrm{tr}[R(g)] = R(g)_{ii} \,. \tag{A.1}$$

In this paper, we use the left-handed Weyl fermions, and the representation matrices are defined for those left-handed Weyl fermions. In this definition, the chiral nature of the representations of the fermions is encoded in the following quantity,

$$\Delta\chi_R(g) := \chi_R(g) - \chi_{R^\dagger}(g) \,, \tag{A.2}$$

to which only the chiral fermions give non-zero contributions [50].

Now, let us consider $\Delta\chi_R$ in the SM model. Since the SM model fermions are the chiral fermions, all of them contributes to $\Delta\chi_R$ which amounts to

$$A_{\mathrm{SM}}(g_{\mathrm{SM}}) := n_{\mathrm{gen}} \times \sum_{i=L,\bar{d},Q,\bar{u},\bar{e}} \Delta\chi_i(g_{\mathrm{SM}}) \,, \tag{A.3}$$

where $n_{\mathrm{gen}} = 3$ is the number of the fermion generations. As a surprising feature of the SM fermions, $A_{\mathrm{SM}}(g_{\mathrm{SM}})$ coincides with the $\Delta\chi_R$ of the $\overline{\mathbf{5}} \oplus \mathbf{10}$ representation of SU(5), that is

$$A_{\mathrm{SM}}(g_{\mathrm{SM}}) = n_{\mathrm{gen}} \times [\Delta\chi_{\overline{\mathbf{5}}}(g_{\mathrm{SM}}) + \Delta\chi_{\mathbf{10}}(g_{\mathrm{SM}})] \,, \tag{A.4}$$

where g_{SM} is the SU(5) elements restricted to the SM gauge group. By remembering the orthogonality and the completeness of the characters, eq. (A.4) means that the SM fermions can be exactly embedded into the three copies of $\overline{\mathbf{5}} \oplus \mathbf{10}$. This amazing feature cannot be explained within the SM, because, in general, chiral fermions consistent with SM gauge symmetry do not necessarily satisfy this property.

The fake GUT conditions in section 2 guarantee the relation in eq. (A.4) automatically. Let us emphasize again that the quarks and leptons are not required to reside in the same SU(5) multiplets in the fake GUT model unlike the conventional GUT model. Once eq. (A.4) is guaranteed by the fake GUT model, it uniquely determines the gauge charges of the low energy fermions under G_{SM} due to the orthogonality and the completeness of the characters. In this way, the fake GUT model explains why the SM fermions form the apparently complete SU(5) multiplets.

In the following, we list the characters of the SM and SU(5). Here, we parameterize the Cartan subgroups of G_{SM} with four parameters, $\theta_{\mathrm{U}(1)}$, $\theta_{\mathrm{SU}(2)}$, $\theta_{\mathrm{SU}(3),3}$, and $\theta_{\mathrm{SU}(3),8}$. In this case, the character of the SM leptons and quarks are computed as follows:

$$\begin{aligned}\chi_L(g_{\mathrm{SM}}) &= \mathrm{Tr}\left[\exp\left(i\theta_{\mathrm{SU}(2)}\frac{\sigma^3}{2}\right)\right] \times \mathrm{Tr}\left[\exp\left(i\theta_{\mathrm{U}(1)}\left(-\frac{1}{2}\right)\right)\right] \\ &= \left[\exp\left(\frac{i\theta_{\mathrm{SU}(2)}}{2}\right) + \exp\left(-\frac{i\theta_{\mathrm{SU}(2)}}{2}\right)\right] \times \exp\left(-\frac{i\theta_{\mathrm{U}(1)}}{2}\right),\end{aligned} \tag{A.5}$$

$$\begin{aligned}\chi_{\overline{d}}(g_{\rm SM}) &= {\rm Tr}\left[\exp\left(-i\theta_{{\rm SU}(3),3}\frac{\lambda^3}{2}-i\theta_{{\rm SU}(3),8}\frac{\lambda^8}{2}\right)\right]\times{\rm Tr}\left[\exp\left(i\theta_{{\rm U}(1)}\frac{1}{3}\right)\right]\\ &= \left[\exp\left(\frac{i\theta_{{\rm SU}(3),8}}{\sqrt{3}}\right)+\exp\left(\frac{i\theta_{{\rm SU}(3),3}}{2}-\frac{\sqrt{3}i\theta_{{\rm SU}(3),8}}{6}\right)+\exp\left(-\frac{i\theta_{{\rm SU}(3),3}}{2}-\frac{\sqrt{3}i\theta_{{\rm SU}(3),8}}{6}\right)\right]\\ &\quad\times\exp\left(\frac{i\theta_{{\rm U}(1)}}{3}\right), \end{aligned}\tag{A.6}$$

$$\begin{aligned}\chi_{Q}(g_{\rm SM}) &= {\rm Tr}\left[\exp\left(i\theta_{{\rm SU}(3),3}\frac{\lambda^3}{2}+i\theta_{{\rm SU}(3),8}\frac{\lambda^8}{2}\right)\right]\times{\rm Tr}\left[\exp\left(i\theta_{{\rm SU}(2)}\frac{\sigma^3}{2}\right)\right]\times{\rm Tr}\left[\exp\left(i\theta_{{\rm U}(1)}\frac{1}{6}\right)\right]\\ &= \left[\exp\left(\frac{-i\theta_{{\rm SU}(3),8}}{\sqrt{3}}\right)+\exp\left(\frac{-i\theta_{{\rm SU}(3),3}}{2}+\frac{\sqrt{3}i\theta_{{\rm SU}(3),8}}{6}\right)+\exp\left(\frac{i\theta_{{\rm SU}(3),3}}{2}+\frac{\sqrt{3}i\theta_{{\rm SU}(3),8}}{6}\right)\right]\\ &\quad\times\left[\exp\left(\frac{i\theta_{{\rm SU}(2)}}{2}\right)+\exp\left(-\frac{i\theta_{{\rm SU}(2)}}{2}\right)\right]\times\exp\left(\frac{i\theta_{{\rm U}(1)}}{6}\right), \end{aligned}\tag{A.7}$$

$$\begin{aligned}\chi_{\overline{u}}(g_{\rm SM}) &= {\rm Tr}\left[\exp\left(-i\theta_{{\rm SU}(3),3}\frac{\lambda^3}{2}-i\theta_{{\rm SU}(3),8}\frac{\lambda^8}{2}\right)\right]\times{\rm Tr}\left[\exp\left(i\theta_{{\rm U}(1)}\left(-\frac{2}{3}\right)\right)\right]\\ &= \left[\exp\left(\frac{i\theta_{{\rm SU}(3),8}}{\sqrt{3}}\right)+\exp\left(\frac{i\theta_{{\rm SU}(3),3}}{2}-\frac{\sqrt{3}i\theta_{{\rm SU}(3),8}}{6}\right)+\exp\left(-\frac{i\theta_{{\rm SU}(3),3}}{2}-\frac{\sqrt{3}i\theta_{{\rm SU}(3),8}}{6}\right)\right]\\ &\quad\times\exp\left(-\frac{2i\theta_{{\rm U}(1)}}{3}\right), \end{aligned}\tag{A.8}$$

$$\chi_{\overline{e}}(g_{\rm SM}) = {\rm Tr}\left[\exp\left(i\theta_{{\rm U}(1)}(1)\right)\right] = \exp\left(i\theta_{{\rm U}(1)}\right). \tag{A.9}$$

Next, let us compute the characters of the $\overline{\mathbf{5}}$ and $\mathbf{10}$. Here, we parameterize the Cartan subgroups of SU(5) with four parameters, θ_3, θ_8, θ_{23}, θ_{24}. At this time, the diagonal generators of $\overline{\mathbf{5}}$ and $\mathbf{10}$ representations correponding each parameters are the following forms:

$$T^3_{\overline{\mathbf{5}}} = -\frac{1}{2}\,{\rm diag}\left(1,-1,0,0,0\right), \tag{A.10}$$

$$T^8_{\overline{\mathbf{5}}} = -\frac{1}{2\sqrt{3}}\,{\rm diag}\left(1,1,-2,0,0\right), \tag{A.11}$$

$$T^{23}_{\overline{\mathbf{5}}} = -\frac{1}{2}\,{\rm diag}\left(0,0,0,1,-1\right), \tag{A.12}$$

$$T^{24}_{\overline{\mathbf{5}}} = -\frac{1}{2\sqrt{15}}\,{\rm diag}\left(-2,-2,-2,3,3\right), \tag{A.13}$$

$$T^3_{\mathbf{10}} = \frac{1}{2}\,{\rm diag}\left(1,-1,0,1,-1,0,-1,1,0,0\right), \tag{A.14}$$

$$T^8_{\mathbf{10}} = \frac{1}{2\sqrt{3}}\,{\rm diag}\left(1,1,-2,1,1,-2,-1,-1,2,0\right), \tag{A.15}$$

$$T^{23}_{\mathbf{10}} = \frac{1}{2}\,{\rm diag}\left(1,1,1,-1,-1,-1,0,0,0,0\right), \tag{A.16}$$

$$T^{24}_{\mathbf{10}} = \frac{1}{2\sqrt{15}}\,{\rm diag}\left(1,1,1,1,1,1,-4,-4,-4,6\right). \tag{A.17}$$

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In this case, $\chi_{\overline{\mathbf{5}}}$ and $\chi_{\mathbf{10}}$ are

$$\begin{aligned}
\chi_{\overline{\mathbf{5}}}(g_{\mathrm{SM}}) &= \mathrm{Tr}\left[\exp\left(i\theta_3 T^3_{\overline{\mathbf{5}}} + i\theta_8 T^8_{\overline{\mathbf{5}}} + i\theta_{23} T^{23}_{\overline{\mathbf{5}}} + i\theta_{24}\sqrt{\frac{5}{3}}T^{24}_{\overline{\mathbf{5}}}\right)\right] \\
&= \left[\exp\left(\frac{i\theta_{23}}{2}\right) + \exp\left(-\frac{i\theta_{23}}{2}\right)\right] \times \exp\left(-\frac{i\theta_{24}}{2}\right) \\
&\quad + \left[\exp\left(\frac{i\theta_8}{\sqrt{3}}\right) + \exp\left(\frac{i\theta_3}{2} - \frac{\sqrt{3}i\theta_8}{6}\right) + \exp\left(-\frac{i\theta_3}{2} - \frac{\sqrt{3}i\theta_8}{6}\right)\right] \\
&\quad \times \exp\left(\frac{i\theta_{24}}{3}\right), \qquad \text{(A.18)} \\
\chi_{\mathbf{10}}(g_{\mathrm{SM}}) &= \mathrm{Tr}\left[\exp\left(i\theta_3 T^3_{\mathbf{10}} + i\theta_8 T^8_{\mathbf{10}} + i\theta_{23} T^{23}_{\mathbf{10}} + i\theta_{24}\sqrt{\frac{5}{3}}T^{24}_{\mathbf{10}}\right)\right] \\
&= \left[\exp\left(\frac{-i\theta_8}{\sqrt{3}}\right) + \exp\left(\frac{-i\theta_3}{2} + \frac{\sqrt{3}i\theta_8}{6}\right) + \exp\left(\frac{i\theta_3}{2} + \frac{\sqrt{3}i\theta_8}{6}\right)\right] \\
&\quad \times \left[\exp\left(\frac{i\theta_{23}}{2}\right) + \exp\left(-\frac{i\theta_{23}}{2}\right)\right] \times \exp\left(\frac{i\theta_{24}}{6}\right) \\
&\quad + \left[\exp\left(\frac{i\theta_8}{\sqrt{3}}\right) + \exp\left(\frac{i\theta_3}{2} - \frac{\sqrt{3}i\theta_8}{6}\right) + \exp\left(-\frac{i\theta_3}{2} - \frac{\sqrt{3}i\theta_8}{6}\right)\right] \\
&\quad \times \exp\left(-\frac{2i\theta_{24}}{3}\right) \\
&\quad + \exp\left(i\theta_{24}\right). \qquad \text{(A.19)}
\end{aligned}$$

If we identify θ_3, θ_8, θ_{23} and θ_{24} with $\theta_{\mathrm{SU}(3),3}$, $\theta_{\mathrm{SU}(3),8}$, $\theta_{\mathrm{SU}(2)}$ and $\theta_{\mathrm{U}(1)}$ respectively, we can find that $\chi_{\overline{\mathbf{5}}}(g_{\mathrm{SM}})$ and $\chi_{\mathbf{10}}(g_{\mathrm{SM}})$ correspond to $\chi_L(g_{\mathrm{SM}}) + \chi_{\overline{d}}(g_{\mathrm{SM}})$ and $\chi_Q(g_{\mathrm{SM}}) + \chi_{\overline{u}}(g_{\mathrm{SM}}) + \chi_{\overline{e}}(g_{\mathrm{SM}})$ respectively.

B RG equations

We show the RG equations of the gauge couplings of the $\mathrm{SU}(5) \times \mathrm{U}(2)_H$ model and $\mathrm{SU}(5) \times \mathrm{SU}(3)_H$ model up to two-loop level. We used the program PyR@TE 3 [51] for the calculation. Here, we neglect the Yukawa and Higgs couplings.

$\mathbf{SU(5) \times U(2)}_{\boldsymbol{H}}$ model. The matter fields are given in table 1.

$$(4\pi)^2 \frac{dg_5}{d\log\mu} = -\frac{83}{6}g_5^3 + \frac{1}{(4\pi)^2}\left(-\frac{373}{3}g_5^5 + 3g_5^3 g_{2H}^2 + g_5^3 g_{1H}^2\right), \qquad \text{(B.1)}$$

$$(4\pi)^2 \frac{dg_{2H}}{d\log\mu} = -\frac{13}{3}g_{2H}^3 + \frac{1}{(4\pi)^2}\left(24g_5^2 g_{2H}^3 - \frac{47}{6}g_{2H}^5 + \frac{9}{2}g_{2H}^3 g_{1H}^2\right), \qquad \text{(B.2)}$$

$$(4\pi)^2 \frac{dg_{1H}}{d\log\mu} = 7g_{1H}^3 + \frac{1}{(4\pi)^2}\left(24g_5^2 + 24g_5^2 g_{1H}^3 + \frac{27}{2}g_{2H}^2 g_{1H}^3 + \frac{33}{2}g_{1H}^5\right). \qquad \text{(B.3)}$$

$\mathbf{SU(5) \times SU(3)}_{\boldsymbol{H}}$ model. The matter fields are given in table 4.

$$(4\pi)^2 \frac{dg_5}{d\log\mu} = -\frac{41}{3}g_5^3 + \frac{1}{(4\pi)^2}\left(-\frac{1768}{15}g_5^5 + 8g_{3H}^2 g_5^3\right), \qquad \text{(B.4)}$$

$$(4\pi)^2 \frac{dg_{3H}}{d\log\mu} = -\frac{15}{2}g_{3H}^3 + \frac{1}{(4\pi)^2}\left(24g_{3H}^3 g_5^2 - 21g_{3H}^5\right). \qquad \text{(B.5)}$$

C On global lepton and baryon symmetry

As discussed in subsection 3.4, the global lepton symmetry plays a crucial role to make the $\mathrm{SU}(5)\times\mathrm{U}(2)_H$ model phenomenologically viable. The global lepton symmetry is more important for $\mathrm{SU}(5)\times\mathrm{SU}(3)_H$ model due to the smaller X gauge boson mass. In this appendix, we give an example of the model in which the high quality lepton symmetry originates from a discrete gauge symmetry in $\mathrm{SU}(5)\times\mathrm{SU}(3)_H$ model.

Before discussing the lepton symmetry, we first provide a concrete model to generate the higher dimensional operators used in eqs. (4.9) and (4.13). As for the mixing term in eq. (4.9), we consider a complex scalar ϕ_3' in $(\overline{\mathbf{10}},\mathbf{3})$ representation which has a Yukawa coupling $\overline{L}_T\phi_3'\mathbf{10}$ and a trilinear coupling $\phi_3\phi_3'\phi_3$. When the mass of ϕ_3' is larger than the fake GUT scale while the coefficient of the trilinear coupling is of order of the fake GUT scale, the VEV of ϕ_3' is aligned to the $\phi_3'\sim\phi_3^\dagger\phi_3^\dagger$. In this way, the higher dimensional lepton mixing mass is achieved by

$$\mathcal{L}=\lambda_{10,ij}\overline{L}_T\phi_3'\mathbf{10}_j\to\mathcal{L}_{\text{eff}}\sim\frac{\mu_{\phi\phi'\phi}}{M_{\phi'}^2}\lambda_{10,ij}\overline{L}_T\phi_3^\dagger\phi_3^\dagger\mathbf{10}_j\,. \tag{C.1}$$

Here, $M_{\phi'}$ and $\mu_{\phi\phi'\phi}$ are the mass and the coefficient of the trilinear coupling of ϕ', respectively.

To generate the origin of the Yukawa interactions in eq. (4.13), we introduce other Dirac fermions, $(H_F,\overline{H}_F)$ with the Yukawa interactions,

$$\mathcal{L}_{YL}=-M_{H\overline{H}}H_F\overline{H}_F+a_iL_{Ti}A\overline{H}_F+b_jH_FL_{Tj}H_3+h.c. \tag{C.2}$$

We again assume that $(H_F,\overline{H}_F)$ is heavier than the fake GUT scale. By integrating out H_F, $\overline{H}_F$, we obtain,

$$\mathcal{L}_{\text{eff}}=\frac{a_ib_j}{M_{H\overline{H}}}L_{Ti}^\alpha A_\alpha^\beta L_{Tj}^\gamma H_3^\delta\epsilon_{\beta\gamma\delta}+h.c. \tag{C.3}$$

Since we have the other Yukawa coupling in eq. (4.12), one pair of $(H_F,\overline{H}_F)$ can reproduce the masses of the SM leptons with appropriate choice of a_i, b_i and $(Y_{LT})_{ij}$.

In section 3.4 we assumed a global lepton symmetry to suppress the proton decay rate. In the $\mathrm{SU}(5)\times\mathrm{SU}(3)_H$ model, the lepton symmetry is required to suppress m_T and λ_T. In the present model, however, the lepton Yukawa interactions in eq. (4.12) and (4.13) include L_T^2, and hence, the lepton symmetry can not be the continuous U(1) symmetry. Instead, we consider a discrete $\mathbb{Z}_{2n}$ ($n\in\mathbb{N}$) symmetry, under which L_T's have charge n. The discrete $\mathbb{Z}_{2n}$ symmetry forbids only the unwanted m_T and λ_T terms when $(H_F,\overline{H}_F)$ also have $\mathbb{Z}_{2n}$ charge n while other fields are neutral under $\mathbb{Z}_{2n}$ (see table 6).

As mentioned earlier, it is argued that all global symmetries are broken by quantum gravity effects (see e.g., refs. [26–32]). Thus, we seek a possibility to realize $\mathbb{Z}_{2n}$ symmetry. The anomaly coefficients of the gauged discrete symmetry come only from $L_T(n)$ since $(H_F,\overline{H}_F)$ contributions trivially cancels;

$$\mathbb{Z}_{2n}\times[\mathrm{SU}(3)_H]^2=n\times3\equiv n\ (\mathrm{mod}\,2n)\,, \tag{C.4}$$

$$\mathbb{Z}_{2n}\times[\text{gravity}]^2=n\times3\times3\equiv0\ (\mathrm{mod}\,n)\,, \tag{C.5}$$

$$\mathbb{Z}_{2n}^3=n^3\times3\times3\equiv0\ (\mathrm{mod}\ n^3\ \text{or}\ 2n)\,, \tag{C.6}$$

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which should be vanishing for the $\mathbb{Z}_{2n}$ symmetry to be a gauge symmetry [52–55]. The third condition in eq. C.6 can be always satisfied by choosing the normalization of n appropriately [56]. Thus, it does not lead to useful constraints on the particle contents. Here, the first, second and third quantities in the each multiplication represent $\mathbb{Z}_{2n}$ charge, the number of particles with same charges and degrees of freedom of $\mathrm{SU}(3)_H$. Thus, to realize the gauged $\mathbb{Z}_{2n}$ symmetry, we need additional $\mathrm{SU}(3)_H$ charged fermions which are chiral under $\mathbb{Z}_{2n}$.

To cancel the anomaly, $\mathbb{Z}_{2n} \times [\mathrm{SU}(3)_H]^2$, we introduce a pair of $(\Theta, \overline{\Theta})$ which are (anti)-fundamental representation of $\mathrm{SU}(3)_H$ and have $\mathbb{Z}_{2n}$ charges $(n,0)$. In this case, all the anomaly coefficients vanish, and $\mathbb{Z}_{2n}$ can be the gauge symmetry. However, $\mathbb{Z}_{2n}$ must be broken to give a mass to the pair, $(\Theta, \overline{\Theta})$, by the VEV of a complex scalar ϕ_Z with the $\mathbb{Z}_{2n}$ charge n. Such a VEV of ϕ_Z also generates m_T, which results in the large lepton mixing angles.

To avoid this problem, we assume that the additional chiral fermions have smaller $\mathbb{Z}_{2n}$ charges. Concretely, we consider $n = 4$, and hence, $\mathbb{Z}_8$ symmetry, with the $\mathbb{Z}_8$ charges of $(\Theta, \overline{\Theta})$ being $(2,0)$ (see table 6). In this case, the anomaly coefficients of $\mathbb{Z}_8 \times [\mathrm{SU}(3)_H]^2$ and $\mathbb{Z}_8 \times [\text{gravity}]^2$ require two pairs of $(\Theta, \overline{\Theta})$. In this way, we achieve the anomaly-free $\mathbb{Z}_8$ symmetry.

The masses of the pairs of $(\Theta, \overline{\Theta})$ are given by the VEV of the complex field ϕ_Z with a $\mathbb{Z}_8$ charge 6, $v_Z = \langle \phi_Z \rangle$. The mass of a pair $(H_F, \overline{H}_F)$ is not forbidden by the $\mathbb{Z}_8$ symmetry.[7] The lepton symmetry breaking parameters m_T and λ_{TAT} are, on the other hand, suppressed by the cutoff scale which is now taken to be the Planck scale, M_{Pl},

$$m_T \sim \frac{v_Z^2}{M_{\mathrm{Pl}}}\,, \quad \lambda_{TAT} v_A \sim \frac{v_Z^2}{M_{\mathrm{Pl}}^2} v_A\,. \tag{C.7}$$

To achieve those mass parameters smaller than $10^{-4}\,\mathrm{GeV}$, we find that the breaking scale of $\mathbb{Z}_8$ is limited from above,

$$v_Z < \mathcal{O}\left(10^7\right)\mathrm{GeV}\,. \tag{C.8}$$

In summary, the extra particles $(\Theta, \overline{\Theta})$ have masses of $v_Z \lesssim 10^7\,\mathrm{GeV}$, while the lepton symmetry breaking mass parameter m_T is highly suppressed.

When a discrete symmetry is broken spontaneously, the domain walls are formed. To avoid the cosmological domain wall problem, we need to assume that at least the reheating temperature after inflation should be lower than the $\mathbb{Z}_8$ breaking scale, v_Z. Combined with the upper limit on v_Z in eq. (C.8), the upper limit on the reheating temperature is given by $T_{\mathrm{rh}} \lesssim \mathcal{O}(10^7)\,\mathrm{GeV}$.

The running of the gauge coupling of the $\mathrm{SU}(3)_H$ is modified because of the addition of multiple Dirac fermions and scalar particles in this example model. However, the asymptotic freedom $\mathrm{SU}(3)_H$ gauge interaction is still preserved in the present setup.

[7] The fields L_T and H_F have the same gauge charges, and hence, they mix with each other. Such mixing does not alter our conclusions.

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	SU(5)	$SU(3)_H$	$\mathbb{Z}_8$
$L_{T1,2,3}$	$\mathbf{1}$	$\mathbf{3}$	4
$\overline{L}_{T1,2,3}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	0
H_F	$\mathbf{1}$	$\mathbf{3}$	4
$\overline{H}_F$	$\mathbf{1}$	$\overline{\mathbf{3}}$	4
$\Theta_{1,2}$	$\mathbf{1}$	$\mathbf{3}$	2
$\overline{\Theta}_{1,2}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	0
ϕ_Z	$\mathbf{1}$	$\mathbf{1}$	6

Table 6. Field contents of fermions and a scalar with the $\mathbb{Z}_{2n}$ charge and pairing fermions for $n = 4$. SM lepton doublets are in L_T's. The $\mathbb{Z}_8$ symmetry is free from anomaly including $\mathbb{Z}_8^3$. In practice, this $\mathbb{Z}_8$ symmetry is equivalent to $\mathbb{Z}_4$ symmetry.

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