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Holographic conductivity for logarithmic charged dilaton-Lifshitz solutions

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ABSTRACT

We disclose the effects of the logarithmic nonlinear electrodynamics on the holographic conductivity of Lifshitz dilaton black holes/branes. We analyze thermodynamics of these solutions as a necessary requirement for applying gauge/gravity duality, by calculating conserved and thermodynamic quantities such as the temperature, entropy, electric potential and mass of the black holes/branes. We calculate the holographic conductivity for a $(2+1)$-dimensional brane boundary and study its behavior in terms of the frequency per temperature. Interestingly enough, we find out that, in contrast to the Lifshitz–Maxwell-dilaton black branes which have conductivity for all z, here in the presence of nonlinear gauge field, the holographic conductivity does exist provided $z \leq 3$ and vanishes for $z > 3$. It is shown that independent of the nonlinear parameter β, the real part of the conductivity is the same for a specific value of frequency per temperature in both AdS and Lifshitz cases. Besides, the behavior of real part of conductivity for large frequencies has a positive slope with respect to large frequencies for a system with Lifshitz symmetry whereas it tends to a constant for a system with AdS symmetry. This behavior may be interpreted as existence of an additional charge carrier rather than the AdS case, and is due to the presence of the scalar dilaton field in model. Similar behavior for optical conductivity of single-layer graphene induced by mild oxygen plasma exposure has been reported.

1. Introduction

The idea of correspondence between gravity in an anti-de Sitter (AdS) spaces and a conformal field theory (CFT) living on its boundary (AdS/CFT) [1–3] has been successful in many theories like superconductors, quark-gluon plasma and entanglement entropy. In recent years, the extension of AdS/CFT correspondence to other gauge field theories and various spacetimes (gravity theories) have got a lot of enthusiasm and usually is called gauge/gravity duality in the literatures. It has been well established that the gauge/gravity duality provides powerful tools for exploring dynamics of strongly coupled field theories and physics of our real Universe. Recently, an interesting application of gauge/gravity duality in condensed matter physics was suggested by Hartnoll, et al. [4,5] who demonstrated that some properties of strongly coupled superconductors have dual gravitational descriptions. Such strongly coupled superconducting phases of the boundary field theory are termed *holographic superconductors* in the literatures.

On the other hand, the dynamics of many condensed matter systems near the critical point can be described by a relativistic CFT or a more subtle scaling theory respecting the Lifshitz symmetry [6]

$$t \to \lambda^z t, \quad \vec{\mathbf{x}} \to \lambda \vec{\mathbf{x}}. \tag{1}$$

The spacetime which supports the above symmetry on its r-infinity boundary is known as Lifshitz spacetime and has the line element [6]

$$ds^2 = -\frac{r^{2z}}{l^{2z}}dt^2 + \frac{l^2 dr^2}{r^2} + r^2 d\vec{\mathbf{x}}^2, \tag{2}$$

where z is dynamical critical exponent. Black hole spacetime with asymptotic Lifshitz symmetry has been widely investigated in the literature. For example, thermodynamics of asymptotic Lifshitz black solutions in the presence of massive gauge fields have been studied in [7]. The generalization to include the higher curvature corrections terms to Einstein gravity and thermodynamics of

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asymptotically Lifshitz black hole spacetimes was explored in [8]. The studies were also extended to dilaton gravity. In this regards, thermal behavior of uncharged [9] and linearly charged [10] Lifshitz black holes in the context of dilaton gravity has been explored. When the gauge field is in the form of power-law Maxwell field, a new class of analytic topological Lifshitz black holes with constant curvature horizon in four and higher dimensional spacetime were constructed in [11]. A class of black brane solutions of an effective supergravity action in the presence of a massless gauge field, which contains Gauss–Bonnet term as well as a dilaton field, with Lifshitz asymptotic have been investigated in [12].

It is also interesting to study other physical properties of systems with Lifshitz symmetry such as conductivity by applying the gauge/gravity duality [13–17]. The holographic conductivity of an Abelian Higgs model in a gravity background which is dual to a strongly coupled system at a Lifshitz-fixed point, was explored in [14]. Other studies on the holographic superconductors with asymptotic Lifshitz symmetry were carried out in [18–21]. The behavior of holographic conductivity for linearly charged Lifshitz black branes has been studied in [22,23] for $1 \leq z \leq 2$. It is also of great importance to investigate the effects of nonlinear electrodynamics on the holographic conductivity. In [17], the holographic conductivity for 4-dimensional Lifshitz black branes in the presence of nonlinear exponential electrodynamics [24] has been explored. The pioneering study on the nonlinear electrodynamics was done by Born and Infeld (BI) in 1934 [25] who considered a Lagrangian of the form [25]

$$L_{\rm BI} = 4\beta^2 \left(1 - \sqrt{1 + \frac{F^2}{2\beta^2}}\right), \tag{3}$$

where β is called the nonlinear parameter with dimension of mass, $F = F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$ in which A_ν is the gauge potential. It has been shown that BI nonlinear electrodynamics is capable to remove the divergency of the electric field of a point-like charged particle at its location as well as the divergency of its self-energy. In addition to BI Lagrangian, other BI-like nonlinear electrodynamics in the context of gravitational field have been introduced. Among them, the so called logarithmic nonlinear, which introduced by Soleng [26], have got a lot of attention, in recent years. The Lagrangian density of the logarithmic gauge field is given by [26]

$$L(F) = -8\beta^2 \ln\left(1 + \frac{F}{8\beta^2}\right). \tag{4}$$

In the framework of dilaton gravity, thermal stability and thermodynamic geometry of a class of black hole spacetimes in the presence of logarithmic nonlinear electrodynamics have been explored in four [27] and higher dimensional spacetime [28]. Also, a class of spinning magnetic dilaton string solutions which produces a longitudinal nonlinear electromagnetic field, in the presence of logarithmic nonlinear source, were explored in [29]. These solutions have no curvature singularity and no horizon, but have a conic geometry [29]. In this paper, we would like to consider a class of asymptotically Lifshitz black hole/brane solutions of Einstein-dilaton gravity in the presence of logarithmic nonlinear electrodynamics and study the thermodynamics of them as a necessary requirement for a system on which we intend to apply gauge/gravity duality. We shall also calculate the holographic conductivity of linearly and nonlinearly charged 4-dimensional black brane solutions for all values of z and disclose the effects of nonlinear gauge field on the conductivity.

This paper is structured as follows. In the next section, we introduce the action and construct a new class of asymptotic Lifshitz black hole/brane solutions of Einstein-dilaton gravity in the presence of logarithmic nonlinear electrodynamics. In section 3, we study thermodynamics of the nonlinear Lifshitz black hole/brane solutions and calculate conserved and thermodynamics quantities. We also verify the validity of the first law of thermodynamics on the horizon. In section 4, we study the holographic conductivity of two-dimensional systems for both linear Maxwell and logarithmic nonlinear electrodynamics. We also plot the behavior of real and imaginary parts of holographic conductivity for asymptotic AdS and Lifshitz solutions. We finish our paper with concluding remarks in the last section.

2. Action and Lifshitz solutions

One of the properties of dilaton field is that it couples with gauge fields. In the presence of dilaton field Φ, the Lagrangian of the logarithmic electrodynamics get modified as well. In this case, the Lagrangian density of the logarithmic gauge field coupled to the dilaton field in $(n+1)$-dimensions can be written as [28]

$$L(F,\Phi) = -8\beta^2 e^{4\lambda\Phi/(n-1)} \ln\left(1 + \frac{e^{-8\lambda\Phi/(n-1)}F}{8\beta^2}\right), \tag{5}$$

where β is the nonlinear parameter and λ is a constant. The large β limit of $L(F,\Phi)$ reproduces the linear Maxwell electrodynamics coupled to the dilaton field [30,31]

$$L(F,\Phi) = -e^{-4\lambda\Phi/(n-1)}F + \frac{e^{-12\lambda\Phi/(n-1)}F^2}{16\beta^2} + O\left(\frac{1}{\beta^4}\right). \tag{6}$$

In this paper, we look for asymptotic Lifshitz topological black hole solutions. Thus we assume the line elements of the metric is [10, 32]

$$ds^2 = -\frac{r^{2z}}{l^{2z}} f(r)dt^2 + \frac{l^2}{r^2}\frac{dr^2}{f(r)} + r^2 d\Sigma_k^{(n-1)}, \tag{7}$$

where z is dynamical critical exponent and $k = 0, \pm 1$ determines the sign of constant curvature $(n-1)(n-2)k$ of $(n-1)$-dimensional hypersurface with the line element $d\Sigma_k^{(n-1)}$ and volume ω_{n-1}. In order to respect the Lifshitz symmetry, we require $f(r) \to 1$ as $r \to \infty$. We desire to consider the string-generated Einstein-dilaton model [33] with two Maxwell and one logarithmic gauge fields. The Lagrangian density of this theory in Einstein frame is

$$\mathcal{L} = \frac{1}{16\pi}\Bigg(\mathcal{R} - \frac{4}{n-1}(\nabla\Phi)^2 - 2\Lambda + L(F,\Phi) - \sum_{i=1}^{2} e^{-4/(n-1)\lambda_i\Phi} H_i\Bigg), \tag{8}$$

where $\mathcal{R}$ is Ricci scalar and Λ and λ_i's are some constants. $H_i = (H_i)_{\mu\nu}(H_i)^{\mu\nu}$ in which $(H_i)_{\mu\nu} = \partial_{[\mu}(B_i)_{\nu]}$ where $(B_i)_\nu$ are the gauge potentials. Varying the action $S = \int d^{n+1}x\sqrt{-g}\mathcal{L}$ with respect to metric $g_{\mu\nu}$, dilaton field Φ and gauge potentials A_μ and $(B_i)_\mu$'s, one can derive the corresponding equations of motion as

$$\mathcal{R}_{\mu\nu} = \frac{g_{\mu\nu}}{n-1}\left[2\Lambda + 2L_F F - L(F,\Phi) - \sum_{i=1}^{2} H_i e^{-4\lambda_i\Phi/(n-1)}\right] + \frac{4}{n-1}\partial_\mu\Phi\partial_\nu\Phi - 2L_F F_{\mu\lambda}F_\nu{}^\lambda + 2\sum_{i=1}^{2} e^{-4\lambda_i\Phi/(n-1)}(H_i)_{\mu\lambda}(H_i)_\nu{}^\lambda, \tag{9}$$

$$\nabla^2\Phi+\frac{n-1}{8}L_\Phi+\sum_{i=1}^{2}\frac{\lambda_i}{2}e^{-4\lambda_i\Phi/(n-1)}H_i=0, \tag{10}$$

$$\nabla_\mu\left(L_F F^{\mu\nu}\right)=0, \tag{11}$$

$$\nabla_\mu\left(e^{-4\lambda_i\Phi/(n-1)}\left(H_i\right)^{\mu\nu}\right)=0, \tag{12}$$

where $L_F=\partial L/\partial F$ and $L_\Phi=\partial L/\partial\Phi$. Lifshitz black hole solutions of Einstein-dilaton gravity with Maxwell [10], power Maxwell [11] and exponential nonlinear [17] electrodynamics have been explored. Clearly, in the limiting case where $\beta\to\infty$, one may expect that our solutions reduce to linear Maxwell case constructed in [10] and in [11] when the power of electrodynamics Lagrangian is equal to 1. It is notable to mention that our definitions are so that in the linear limit, our solutions directly reproduce the results of [11] for $p=1$. First of all, we solve differential equations (11) and (12) by using the metric (7). We find

$$F_{rt}=\frac{2qe^{4\lambda\Phi/(n-1)}}{(\Upsilon+1)r^{n-z}}, \tag{13}$$

$$(H_i)_{rt}=\frac{q_i}{r^{z-n}}e^{4\lambda_i\Phi/(n-1)}, \tag{14}$$

where

$$\Upsilon\equiv\sqrt{1+\frac{q^2l^{2z-2}}{\beta^2r^{2n-2}}} \tag{15}$$

and q and q_i's are some integration constants. As we will see, q is related to the electric charge of the black hole. Expanding (13) for $\beta\to\infty$, yields

$$F_{rt}=\frac{qe^{4\lambda\Phi/(n-1)}}{r^{n-z}}-\frac{q^3l^{2z-2}e^{4\lambda\Phi/(n-1)}}{4r^{3n-z-2}\beta^2}+O\left(\frac{1}{\beta^4}\right). \tag{16}$$

The first term in Eq. (16) is the linear Maxwell one presented in [11]. The second term is the leading order nonlinear correction term to the Maxwell field. Substituting solutions (13) and (14) into the field equations (9) and (10), one arrives at four differential equations

$$\frac{(n-1)\left(r^nf\right)'}{2l^2r^{n-1}}+\frac{2r^2f\Phi'^2}{(n-1)l^2}+\Lambda-\frac{(n-1)(n-2)k}{2r^2}+\sum_{i=1}^{2}\frac{q_i^2e^{4\lambda_i\Phi/(n-1)}}{l^{2(1-z)}r^{2(n-1)}}+\Xi=0, \tag{17}$$

$$\frac{(n-1)\left(r^{n+2(z-1)}f\right)'}{2l^2r^{n+2z-3}}-\frac{2r^2f\Phi'^2}{(n-1)l^2}+\Lambda-\frac{(n-1)(n-2)k}{2r^2}+\sum_{i=1}^{2}\frac{q_i^2e^{4\lambda_i\Phi/(n-1)}}{l^{2(1-z)}r^{2(n-1)}}+\Xi=0, \tag{18}$$

$$\frac{r^2f''}{2l^2}+\frac{(2n+3z-3)rf'}{2l^2}+\frac{2r^2f\Phi'^2}{(n-1)l^2}+\frac{(2z^2+2(n-2)z+(n-1)(n-2))f}{2l^2}+\Lambda-\frac{(n-3)(n-2)k}{2r^2}-\sum_{i=1}^{2}\frac{q_i^2e^{4\lambda_i\Phi/(n-1)}}{l^{2(1-z)}r^{2(n-1)}}-4\beta^2e^{4\lambda\Phi/(n-1)}\ln\left(\frac{\Upsilon+1}{2}\right)=0, \tag{19}$$

$$\frac{\left(r^{n+z}f\Phi'\right)'}{l^2r^{n+z-2}}-\sum_{i=1}^{2}\frac{q_i^2\lambda_ie^{4\lambda_i\Phi/(n-1)}}{l^{2(1-z)}r^{2(n-1)}}-\lambda\Xi=0, \tag{20}$$

where $\Phi=\Phi(r)$ and

$$\Xi\equiv4\beta^2e^{4\lambda\Phi/(n-1)}\left[\Upsilon-1-\ln(\frac{\Upsilon+1}{2})\right].$$

Combining (17) and (18), we find

$$4r^2\Phi'^2=(n-1)^2(z-1), \tag{21}$$

which has the solution

$$\Phi(r)=\ln\left(\frac{r}{b}\right)^{\xi},\quad \xi=\frac{(n-1)\sqrt{z-1}}{2}, \tag{22}$$

where b is an integration constant with dimension of length. Solution (22) implies $z\geq1$. With $\Phi(r)$ at hand, we can obtain the function $f(r)$ form Eqs. (17)–(20) as

$$f(r)=-\frac{2l^2\Lambda}{(n+z-1)(n+z-2)}-\frac{m}{r^{n+z-1}}+\frac{(n-2)^2kl^2}{(n+z-3)^2r^2}+\frac{8\beta^2l^2b^{2z-2}}{(n-1)(n-z+1)r^{2z-2}}\times\left\{\frac{2n-z}{n-z+1}+\ln\left(\frac{1+\Upsilon}{2}\right)-\frac{(n-1)}{(n-z+1)}\mathbf{F}\left(\frac{1}{2},X,X+1,1-\Upsilon^2\right)-\mathbf{F}\left(-\frac{1}{2},X,X+1,1-\Upsilon^2\right)\right\}, \tag{23}$$

where

$$X=\frac{z-n-1}{2n-2},$$

$\mathbf{F}$ is the hypergeometric function and m is a constant related to the mass of the black hole. Using the hypergeometric identities [34]

$$(y-w-1)\mathbf{F}(w,x,y,s)+w\mathbf{F}(w+1,x,y,s)-(y-1)\mathbf{F}(w,x,y-1,s)=0, \tag{24}$$

and

$$\mathbf{F}(w,x,x,s)=(1-s)^{-w}, \tag{25}$$

one can rewrite $f(r)$ in a more simple form

$$f(r)=-\frac{2l^2\Lambda}{(n+z-1)(n+z-2)}-\frac{m}{r^{n+z-1}}+\frac{(n-2)^2kl^2}{(n+z-3)^2r^2}+\frac{8\beta^2l^2b^{2z-2}}{(n-1)(n-z+1)r^{2z-2}}\times\left\{\frac{2n-z}{n-z+1}+\ln\left(\frac{1+\Upsilon}{2}\right)+\frac{(n-z+1)\Upsilon}{z-2}-\frac{(n-1)^2}{(z-2)(n-z+1)}\times\mathbf{F}\left(\frac{1}{2},\frac{z-n-1}{2n-2},\frac{n+z-3}{2n-2},1-\Upsilon^2\right)\right\}. \tag{26}$$

Let us note that although at the first glance relation (26) seems divergent in $z=2$, $f(r)$ is not really diverging at this point as one can see from (23). The above solutions will fully satisfy the system of Eqs. (17)–(20) provided,

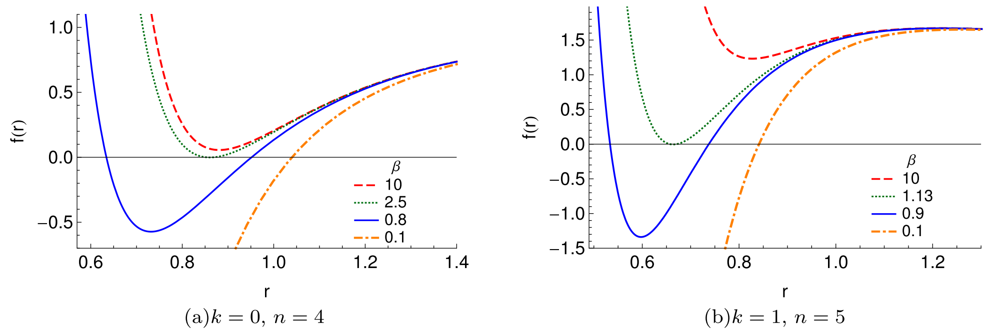


Fig. 1. The behavior of $f(r)$ versus r for $l = 1.5$, $b = 0.4$, $q = 1.4$, $z = 1.5$ and $m = 1.5$.

$$\lambda = -\sqrt{z-1}, \quad \lambda_1 = \frac{n-1}{\sqrt{z-1}}, \quad \lambda_2 = \frac{n-2}{\sqrt{z-1}},$$
$$q_1^2 = -\frac{\Lambda(z-1)b^{2(n-1)}}{(z+n-2)l^{2(z-1)}},$$
$$q_2^2 = \frac{k(n-1)(n-2)(z-1)b^{2(n-2)}}{2(z+n-3)l^{2(z-1)}}. \tag{27}$$

Note that q is hidden in $\Upsilon = \sqrt{1+q^2 l^{2z-2}/(\beta^2 r^{2n-2})}$ in (26). The reality of q_2 requires that $k \neq -1$ except for $z = 1$. Thus, hereafter, we consider the black branes ($k = 0$) and black holes ($k = 1$) in the general cases with $z \neq 1$. Also, reality of q_1 implies $\Lambda < 0$. As we mentioned above the asymptotic Lifshitz behavior implies that $f(r) \to 1$ as $r \to \infty$. However, from Eq. (26) we have

$$\lim_{r\to\infty} f(r) = -\frac{2l^2\Lambda}{(n+z-1)(n+z-2)}.$$

Therefore, in order to have appropriate asymptotic behavior for $f(r)$ we fix Λ as

$$\Lambda = -\frac{(n+z-1)(n+z-2)}{2l^2}, \tag{28}$$

which is negative ($\Lambda < 0$), as the reality of q_1 implies. Hence, the final form of $f(r)$ is

$$f(r) = 1 - \frac{m}{r^{n+z-1}} + \frac{(n-2)^2 k l^2}{(n+z-3)^2 r^2} + \frac{8\beta^2 l^2 b^{2z-2}}{(n-1)(n-z+1)r^{2z-2}} \times \left\{ \frac{2n-z}{n-z+1} + \frac{(n-z+1)\Upsilon}{z-2} + \ln\left(\frac{1+\Upsilon}{2}\right) - \frac{(n-1)^2}{(z-2)(n-z+1)} \times \mathbf{F}\left(\frac{1}{2}, \frac{z-n-1}{2n-2}, \frac{n+z-3}{2n-2}, 1-\Upsilon^2\right)\right\}. \tag{29}$$

The behavior of $f(r)$ for large β, may be written

$$f(r) = 1 - \frac{m}{r^{n+z-1}} + \frac{(n-2)^2 k l^2}{(n+z-3)^2 r^2} + \frac{2q^2 b^{2z-2} l^{2z}}{(n-1)(n+z-3)\, r^{2n+2z-4}} - \frac{q^4 b^{2z-2} l^{4z-2}}{4(n-1)(3n+z-5)\,\beta^2 r^{4n+2z-6}} + O\left(\frac{1}{\beta^4}\right). \tag{30}$$

When $\beta \to \infty$, this solution recovers the Lifshitz black holes in Einstein–Maxwell-dilaton gravity [10,11], as expected. Fig. 1 shows the behavior of $f(r)$ for different values of β correspond to Lifshitz black branes ($k = 0$) and black holes ($k = 1$). This figure exhibits that it is possible to have black solutions with one or two horizons.

We can calculate the Hawking temperature of outermost horizon r_+ as

$$T = \frac{r_+^{z+1} f'(r_+)}{4\pi l^{z+1}} = \frac{(n+z-1)r_+^z}{4\pi l^{z+1}} + \frac{(n-2)^2 k r_+^{z-2}}{4\pi(n+z-3)l^{z-1}} + \frac{2\beta^2 l^{1-z} b^{2z-2}}{\pi(n-1)r_+^{z-2}}\left[1 - \Upsilon_+ + \ln\left(\frac{\Upsilon_+ + 1}{2}\right)\right], \tag{31}$$

where $\Upsilon_+ = \Upsilon(r_+)$. In the next section we shall study thermodynamics of Lifshitz black branes/holes we obtained in this section.

3. Thermodynamics OF Lifshitz solutions

In this section, we want to study thermodynamics of Lifshitz black holes/branes. The temperature of our solutions on the horizon was calculated in previous section. In order to find the entropy of these Lifshitz solutions we can use the so-called area law which states that the entropy of the black hole is quarter of the event horizon area [35]. The entropy of almost all kinds of black holes in Einstein gravity including dilaton ones is computed by using this near universal law [36]. Hence, the entropy of the obtained Lifshitz solutions per unit volume ω_{n-1} can be calculated as

$$S = \frac{r_+^{n-1}}{4}. \tag{32}$$

Now, we turn to calculation of electric charge of Lifshitz black holes. We use the nonlinear logarithmic electrodynamics. The well-known Gauss law for this nonlinear electrodynamics can be given by

$$Q = \frac{1}{4\pi}\int r^{n-1} L_F F_{\mu\nu} n^\mu u^\nu d\Sigma, \tag{33}$$

where u^ν and u^μ are the unite timelike and spacelike normals to a sphere of radius r given as

$$n^\mu = \frac{1}{\sqrt{-g_{tt}}}dt = \frac{l^z}{r^z\sqrt{f(r)}}dt,$$
$$u^\nu = \frac{1}{\sqrt{g_{rr}}}dr = \frac{r\sqrt{f(r)}}{l}dr. \tag{34}$$

Therefore, the electric charge per unit volume ω_{n-1} is obtained as

$$Q = \frac{q l^{z-1}}{4\pi}. \tag{35}$$

Another conserved quantity of our solutions is mass. We can obtain this conserved quantity by applying the modified subtraction method of Brown and York [37]. Thus, the mass per unit volume is computed as (see Ref. [11] for more details)

$$M=\frac{(n-1)m}{16\pi l^{z+1}}, \tag{36}$$

where m can be calculated by using this fact that $f(r_+)=0$. Therefore, one obtains

$$m=r_+^{n+z-1}+\frac{(n-2)^2kl^2r_+^{n+z-3}}{(n+z-3)^2}+\frac{8\beta^2l^2b^{2z-2}r_+^{n-z+1}}{(n-1)(n-z+1)}\times\left\{\frac{2n-z}{n-z+1}+\frac{(n-z+1)\Upsilon_+}{z-2}+\ln\left(\frac{1+\Upsilon_+}{2}\right)-\frac{(n-1)^2}{(z-2)(n-z+1)}\times\mathbf{F}\left(\frac{1}{2},\frac{z-n-1}{2n-2},\frac{n+z-3}{2n-2},1-\Upsilon_+^2\right)\right\}. \tag{37}$$

Since from one side $m=m(r_+,q)$ (note that q is hidden in Υ_+) and from another side r_+ and q are related to entropy and charge through (32) and (35), respectively, one can re-express m and consequently M in terms of extensive quantities S and Q. The desired Smarr formula $M(S,Q)$ is therefore

$$M(S,Q)=\frac{(n-1)(4S)^{(n+z-1)/(n-1)}}{16\pi l^{z+1}}+\frac{(n-1)(n-2)^2k(4S)^{(n+z-3)/(n-1)}}{16\pi l^{z-1}(n+z-3)^2}+\frac{\beta^2b^{2z-2}(4S)^{(n-z+1)/(n-1)}}{2\pi(n-z+1)l^{z-1}}\times\left\{\frac{2n-z}{n-z+1}+\frac{(n-z+1)\Gamma}{z-2}+\ln\left(\frac{1+\Gamma}{2}\right)-\frac{(n-1)^2}{(z-2)(n-z+1)}\times\mathbf{F}\left(\frac{1}{2},\frac{z-n-1}{2n-2},\frac{n+z-3}{2n-2},1-\Gamma^2\right)\right\}, \tag{38}$$

where $\Gamma=\sqrt{1+\pi^2Q^2/(\beta^2S^2)}$. One can expand $M(S,Q)$ for large values of β to arrive at

$$M(S,Q)=\frac{(n-1)(4S)^{(n+z-1)/(n-1)}}{16\pi l^{z+1}}+\frac{(n-1)(n-2)^2k(4S)^{(n+z-3)/(n-1)}}{16\pi(n+z-3)^2l^{z-1}}+\frac{2\pi Q^2b^{2z-2}(4S)^{(3-n-z)/(n-1)}}{(n+z-3)l^{z-1}}-\frac{16\pi^3Q^4b^{2z-2}(4S)^{(5-3n-z)/(n-1)}}{4(3n+z-5)l^{z-1}\beta^2}+O\left(\frac{1}{\beta^4}\right), \tag{39}$$

which is the Smarr-type formula obtained for the Lifshitz black holes of EMd theory in the limit of $\beta\to\infty$ [11]. We also introduce the conjugate intensive quantities corresponding to entropy and charge namely temperature and electric potential as

$$T=\left(\frac{\partial M}{\partial S}\right)_Q \quad \text{and} \quad U=\left(\frac{\partial M}{\partial Q}\right)_S. \tag{40}$$

The electric potential U, measured at infinity with respect to the horizon r_+, is principally defined as

$$U=A_\mu\chi^\mu\big|_{r\to\infty}-A_\mu\chi^\mu\big|_{r=r_+}, \tag{41}$$

where $\chi=\partial_t$ is the null generator of the horizon. In order to find electric potential U, we first have to calculate the gauge potential A_t. The gauge potential A_t corresponding to the electromagnetic field (13) is given by $A_t(r)=\int F_{rt}dr$. It is a matter of calculations to show that

$$A_t=\mu+\frac{2\beta^2b^{2z-2}l^{2-2z}r^{n-z+1}}{(n-z+1)q}(\Upsilon-1)-\frac{2q(n-1)b^{2z-2}}{(1+n-z)(n+z-3)r^{n+z-3}}\times\mathbf{F}\left(\frac{1}{2},\frac{z+n-3}{2n-2},\frac{n+z-5}{n-1},1-\Upsilon^2\right). \tag{42}$$

One can check that A_t reduces to finite value μ at infinity. Requiring the fact that $A_t(r_+)=0$, one gets

$$\mu=-\frac{2\beta^2b^{2z-2}l^{2-2z}r_+^{n-z+1}}{(n-z+1)q}(\Upsilon_+-1)+\frac{2q(n-1)b^{2z-2}}{(1+n-z)(n+z-3)r_+^{n+z-3}}\times\mathbf{F}\left(\frac{1}{2},\frac{z+n-3}{2n-2},\frac{n+z-5}{n-1},1-\Upsilon_+^2\right). \tag{43}$$

Note that μ is commonly referred to as chemical potential of the thermodynamical system lives on boundary. Using Eqs. (41) and (42) the electric potential may be obtained as

$$U=\mu. \tag{44}$$

If we consider S and Q as a complete set of extensive quantities for $M(S,Q)$, it is confirmed numerically that the intensive quantities corresponding to S and Q namely temperature T and electric potential U, coincide with Eqs. (31) and (44), respectively. Thus, the first law of thermodynamics

$$dM=TdS+UdQ, \tag{45}$$

is satisfied for our obtained Lifshitz black branes/holes. In the remaining part of this paper, we turn to study the holographic conductivity of a $(2+1)$-dimensional system lives on the boundary of brane of a four dimensional bulk.

4. Holographic electrical conductivity

In this section, we will focus on studying gauge/gravity duality for Lifshitz black brane solutions. In particular, we obtain the AC conductivity as a function of frequency for a $(2+1)$-dimensional system lives on the boundary of brane. In order to make the effects of nonlinearity on the conductivity more clear, we first review the calculation of this quantity for the linear Maxwell case [22,23]. Then, we turn to the case with nonlinear logarithmic electrodynamics. In what follows, we set $l=b=r_+=1$. We take the planar $(3+1)$-dimensional metric for the bulk as

$$ds^2=-\mathcal{F}(u)u^{-2z}dt^2+\left[\mathcal{F}(u)u^2\right]^{-1}du^2+u^{-2}(dx^2+dy^2), \tag{46}$$

which can be obtained from (7) by defining $u=1/r$. Therefore, the black brane horizon sits at $u=1$ and the three-dimensional system lives at $u=0$ (brane boundary). For linear Maxwell case, $\mathcal{F}(u)$ in (46) is calculated by substituting $u=1/r$, $n=3$ and $k=0$ in (30) and taking the $\beta\to\infty$ limit. Thus, we get

$$\mathcal{F}(u)=1-mu^{z+2}+q^2z^{-1}u^{2z+2}. \tag{47}$$

Now, we perturb the vector potential and the metric by turning on $A_x(u)e^{-i\omega t}$ and $g_{tx}(u)e^{-i\omega t}$ and arrive at two additional equations as

$$A_x'' + \left[\mathcal{F}'\mathcal{F}^{-1} + 3(1-z)u^{-1}\right]A_x' + u^{2z-2}\mathcal{F}^{-2}\left[\omega^2 A_x - qu^z\mathcal{F}\left(2g_{tx} + ug_{tx}'\right)\right] = 0, \tag{48}$$

and

$$2g_{tx} + ug_{tx}' = 4qu^{2-z}A_x, \tag{49}$$

where the prime indicates the derivative with respect to u. Eliminating g_{tx} from Eq. (48) through (49), we can easily obtain a linearized equation for the gauge field A_x

$$A_x'' + \left[\mathcal{F}'\mathcal{F}^{-1} + 3(1-z)u^{-1}\right]A_x' + u^{2z-2}\mathcal{F}^{-2}\left[\omega^2 - 4q^2u^2\mathcal{F}\right]A_x = 0. \tag{50}$$

From gauge/gravity duality, we know that the expectation value of current is given by [38]

$$\langle J_x\rangle = \left.\frac{\partial\mathfrak{L}}{\partial\left(\partial_u\delta A_x\right)}\right|_{u=0}, \tag{51}$$

where $\mathfrak{L} = \sqrt{-g}\mathcal{L}$ in which $\mathcal{L}$ was introduced in (8) and $\delta A_x = A_x e^{-i\omega t}$. Therefore, we can calculate conductivity through Ohm's law as

$$\sigma(\omega) = \frac{\langle J_x\rangle}{E_x} = -\frac{\langle J_x\rangle}{\partial_t\delta A_x} = -\frac{i\langle J_x\rangle}{\omega\delta A_x}. \tag{52}$$

In order to compute conductivity $\sigma(\omega)$, we need to know the asymptotic behavior of the perturbative field A_x governed by (50) near the boundary $u=0$. This reads

$$A_x'' - 3(z-1)u^{-1}A_x' + \omega^2u^{2(z-1)}A_x + \cdots = 0, \tag{53}$$

which has the solutions

$$A_x(u) = \begin{cases} A^0 + \frac{A^0\omega^2}{2z(z-2)}u^{2z} + A^1u^{3z-2} + \cdots & \text{for } z\neq 2, \\ A^0 - \frac{A^0\omega^2}{4}\ln(u)\,u^4 + A^1u^4 + \cdots & \text{for } z = 2, \end{cases} \tag{54}$$

where A^0 and A^1 are two constants. Thus, the conductivity for the linear Maxwell electrodynamics is obtained as [22,23]

$$\sigma = \begin{cases} \frac{(3z-2)A^1}{4\pi i\omega A^0} & \text{for } z\neq 2, \\ \frac{16A^1 - A^0\omega^2}{16\pi i\omega A^0} & \text{for } z = 2. \end{cases} \tag{55}$$

It is remarkable to note that for $z \geq 2$, there is a divergence term in $\mathfrak{L}$ when we use (51) and (52) to calculate conductivity, σ. However, these terms do not effect on the value of conductivity and can be easily eliminated by using holographic re-normalization approach [22,23,39]. In this method, the divergence is canceled by adding appropriate counterterms to the action. In [23], conductivity has been studied for $(3+1)$-dimensional black branes in the presence of linear Maxwell electrodynamics where $1 \leq z \leq 2$. Here, in (55), we generalize those results to $z > 2$.

Now, we turn to calculate the conductivity on the boundary of the Lifshitz black branes when the bulk gauge field is in the form of logarithmic nonlinear electrodynamics. The motivation is to disclose the effects of nonlinearity on the holographic conductivity in comparison with linear Maxwell case. In this case, by transforming $r \to u = 1/r$ in Eq. (29), $\mathcal{F}(u)$ can be rewritten as

$$\mathcal{F}(u) = 1 - mu^{z+2} + \frac{4\beta^2u^{2z-2}}{(4-z)}\left\{\frac{6-z}{4-z} + \frac{(4-z)\Upsilon_u}{z-2} + \ln\left(\frac{1+\Upsilon_u}{2}\right) - \frac{4}{(z-2)(4-z)} \times \mathbf{F}\left(\frac{1}{2}, \frac{z-4}{4}, \frac{z}{4}, 1-\Upsilon_u^2\right)\right\}, \tag{56}$$

where $\Upsilon_u = \sqrt{1+q^2u^4/\beta^2}$. Perturbative equations of motion coming from turning on $A_x(u)e^{-i\omega t}$ and $g_{tx}(u)\,e^{-i\omega t}$ in the bulk for nonlinear electrodynamics are

$$A_x'' + \left[\frac{3(1-z)}{u} + \frac{\mathcal{F}'}{\mathcal{F}} + \frac{4q^2u^3}{q^2u^4 + \beta^2(1+\Upsilon_u)^2}\right]A_x' + \frac{\omega^2u^{2z-2}}{\mathcal{F}^2}A_x = \frac{2qu^{3z-2}}{(1+\Upsilon_u)\mathcal{F}}\left(2g_{tx} + ug_{tx}'\right), \tag{57}$$

and

$$2g_{tx} + ug_{tx}' = 4qu^{2-z}A_x, \tag{58}$$

which give rise to the decoupled equation for the gauge field A_x

$$A_x'' + \left[\frac{3(1-z)}{u} + \frac{\mathcal{F}'}{\mathcal{F}} + \frac{4q^2u^3l^{2z-2}}{q^2u^4 + \beta^2(1+\Upsilon_u)^2}\right]A_x' + \frac{u^{2z-2}}{\mathcal{F}^2}A_x\left[\omega^2 - \frac{8q^2u^2\mathcal{F}}{(1+\Upsilon_u)}\right] = 0. \tag{59}$$

One can check that the general behavior of Eq. (59) near the boundary $u=0$ is

$$A_x'' - 3(z-1)u^{-1}A_x' + \omega^2u^{2(z-1)}A_x + \cdots = 0, \tag{60}$$

which cause the same behavior for the gauge potential near the boundary as (54). Now, we can compute conductivity in the presence of logarithmic electrodynamics. Using Eq. (52), we arrive at

$$\sigma = \begin{cases} \left.\frac{(3z-2)A^1}{4\pi i\omega A^0}\left[1 + \left(\frac{\omega A^0u^{3-z}}{2\beta}\right)^2\right]^{-1}\right|_{u=0} & \text{for } z\neq 2, \\ \left.\frac{16A^1 - A^0\omega^2}{16\pi i\omega A^0}\left[1 + \left(\frac{\omega A^0u}{2\beta}\right)^2\right]^{-1}\right|_{u=0} & \text{for } z = 2. \end{cases} \tag{61}$$

Consequently, the conductivity σ in this case for different ranges of dynamical critical exponent z is

$$\sigma = \begin{cases} \frac{(3z-2)A^1}{4\pi i\omega A^0} & \text{for } z < 3\ (\neq 2), \\ \frac{16A^1 - A^0\omega^2}{16\pi i\omega A^0} & \text{for } z = 2, \\ \frac{(3z-2)A^1}{4\pi i\omega A^0}\left[1 + \left(\frac{\omega A^0}{2\beta}\right)^2\right]^{-1} & \text{for } z = 3, \\ 0 & \text{for } z > 3. \end{cases} \tag{62}$$

It is notable to mention that the same comments as linear Maxwell case given in the end of previous paragraph about the diverging terms are also valid here. The above result indicates that for $z < 3$, the conductivity has the same expression as linear Maxwell field. For $z = 3$ the conductivity get modified due to the nonlinear parameter β and reduces to the Maxwell case as $\beta \to \infty$. However, in contrast to the linear Maxwell field, the conductivity is zero for $z > 3$. The media show the behavior like the latter case ($z > 3$) in which $\sigma = 0$ are known as "lossless" since conductivity represents power loss within a medium. In such a media which is called also as "perfect dielectrics", $J = 0$ regardless of the electric field E. This means that the electric field E cannot move the charge carriers. This result shows that, for $z > 3$ the media in gauge side of gauge/gravity duality represents lossless behavior if the nonlinear electrodynamics is employed.

In order to have further understanding on the behavior of the conductivity, we depict the conductivity in terms of the frequency by solving Eq. (59), numerically. For this purpose, we need initial conditions. Let us look at the solution for the gauge potential A_x

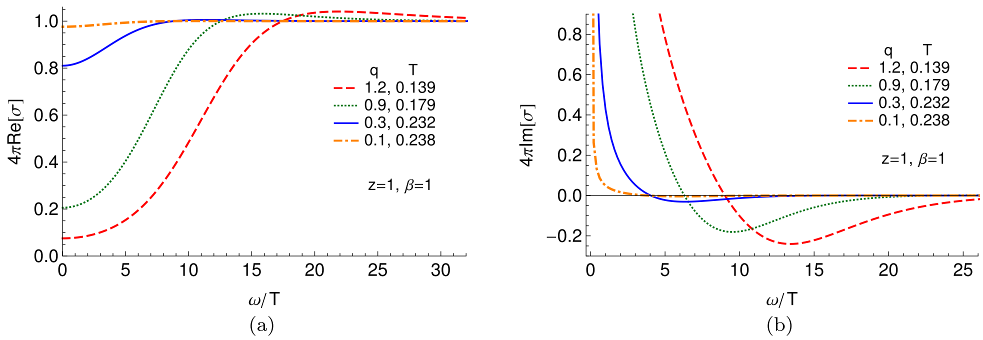


Fig. 2. The behaviors of real and imaginary parts of electrical conductivity σ versus ω/T for $z=1$, $\beta=1$ and different values of q with $l=b=r_+=1$.

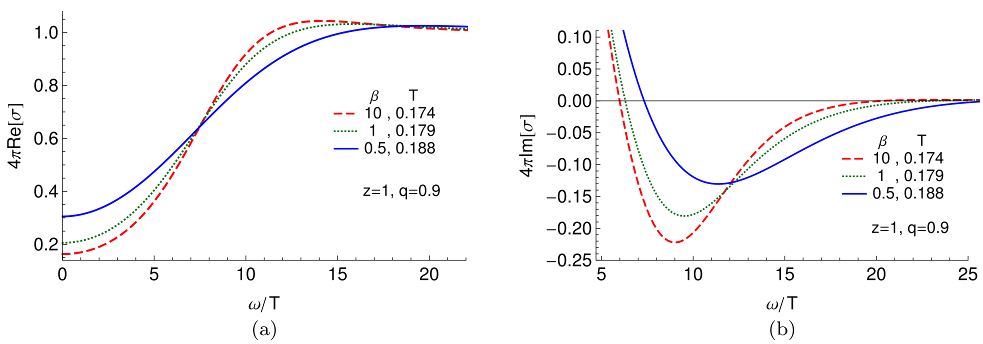


Fig. 3. The behaviors of real and imaginary parts of electrical conductivity σ versus ω/T for $z=1$, $q=0.9$ and different values of β with $l=b=r_+=1$.

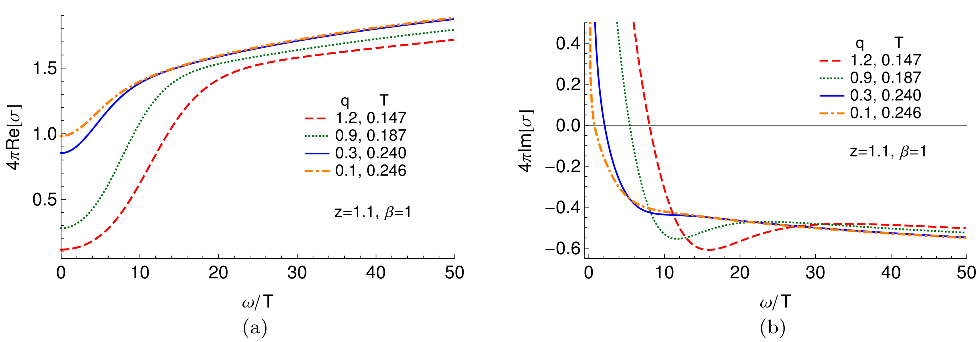


Fig. 4. The behaviors of real and imaginary parts of electrical conductivity σ versus ω/T for $z=1.1$, $\beta=1$ and different values of q with $l=b=r_+=1$.

near the horizon r_+. In order to take into account the causal behavior, we solve Eq. (59) for A_x near horizon by performing ingoing wave boundary condition. Therefore, we receive

$$A_x(u)=\mathcal{F}(u)^{-i4\pi\omega/T}\Psi(u), \tag{63}$$

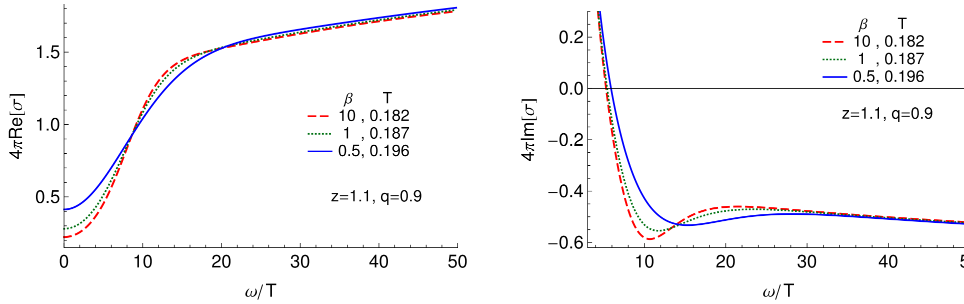
where T is temperature and

$$\Psi(u)=1+a(u-1)+b(u-1)^2+\cdots, \tag{64}$$

where $a, b, \dots$ are some constants to be determined numerically and considered as initial conditions required for solving differential equation (59), numerically. Substituting Eq. (63) into Eq. (59), one can obtain the differential equation for Ψ. Therefore, $a, b, \dots$ can be found by looking for Taylor series expansions of Eq. (59) near the horizon r_+. With these initial conditions, we are able to plot the behavior of the conductivity in terms of frequency.

Figs. 2 and 3 show the behavior of conductivity σ with respect to frequency per temperature, ω/T, for asymptotic AdS case ($z=1$). In Fig. 2, this behavior is depicted for different values of q. From Fig. 2(a) we see that $\sigma_{DC}=\text{Re}[\sigma(0)]$ decreases as q (temperature T) increases (decreases). This figure also shows that the effects of increasing of frequency is more for larger values of q (lower temperatures). Furthermore, inspite of different values of q (T) there is an asymptotic value for Re[σ] in large frequencies. Such behavior for conductivity has been reported in [40] for a graphene system. It is important to note that the behaviors of real and imaginary parts of conductivity are not independent and is related to each other via Kramers–Kronig relations. Fig. 3 illustrates the behavior of conductivity versus frequency for different values of the nonlinear parameter β. This figure again confirms that σ_{DC} decreases with decreasing the temperature. As one can see from Fig. 3(a), there is a specific ω/T (between 5 and 10) that inspite of different values of β, the conductivity is the same for it.

In Figs. 4 and 5, previous cases are illustrated for a system with Schrodinger-like symmetry, namely $z=1.1$. For small frequencies, σ_{DC} has the same behavior as previous case i.e., it increases as temperature does. However, the behavior of conductivity for large frequencies is different. In fact, real part of conductivity has a positive slope with respect to frequency for large ones. This behavior may be interpreted as existence of an additional charge carrier rather than the previous case. This is due to existence of dilaton scalar field in model in comparison with asymptotic AdS case. Similar behavior for optical conductivity of single-layer graphene



(a) (b)

Fig. 5. The behaviors of real and imaginary parts of electrical conductivity σ versus ω/T for $z=1.1$, $q=0.9$ and different values of β with $l=b=r_+=1$.

induced by mild oxygen plasma exposure has been reported in [41]. Fig. 4 shows that we have more near behavior for conductivity for smaller q's (higher temperatures). Finally, from Fig. 5, we see that for a specific value of ω/T (near 10), the conductivity has the same behavior, independent of the nonlinear parameter β. This occurs for isotropic symmetrical systems ($z=1$) too (see Fig. 3(a)).

5. Closing remarks

To summarize, we have constructed a new class of $(n+1)$-dimensional Lifshitz dilaton black holes/branes in the presence of logarithmic nonlinear electrodynamics. The obtained solutions in this paper obey the scaling symmetry $t \to \lambda^z t$ and $x^i \to \lambda x^i$, comes from the generalized gauge/gravity duality, at r-infinity boundary. We found that the horizon of these spacetime can be an $(n-1)$-dimensional hypersurface with positive ($k=1$) or zero ($k=0$), constant curvature. Therefore, our solutions rule out the case with negative curvature ($k=-1$). We studied thermodynamics of Lifshitz dilaton black holes/branes and calculated the temperature, entropy, charge, electric potential and mass of the spacetime. We have also confirmed that these conserved and thermodynamic quantities satisfy the first law of thermodynamics on the horizon. This is a necessary requirement for a system in which one can apply gauge/gravity duality.

Then, we investigated the gauge/gravity duality of Lifshitz black branes by calculating the holographic conductivity as a function of frequency for a $(2+1)$-dimensional system lives on the boundary of a $(3+1)$-dimensional bulk. First, we reviewed the calculations of the conductivity on the boundary of Einstein–Maxwell-dilaton black branes and found that it holds for all values of the dynamical exponent z. In this part, we generalized the study of [23] to $z>2$. Then, we extended our study to the case with logarithmic nonlinear electrodynamics. We found that for $z<3$, the conductivity has the same expression as linear Maxwell field. For $z=3$ the conductivity get modified due to the nonlinear parameter β as given in Eq. (62), and reduces to the Maxwell case as $\beta \to \infty$. However, in contrast to the linear Maxwell case, in the presence of logarithmic nonlinear gauge field the conductivity is zero for $z>3$. Taking suitable initial conditions, we have plotted the behavior of the conductivity in terms of frequency per temperature. We found that for asymptotic AdS case ($z=1$), $\sigma_{DC}=\mathrm{Re}[\sigma(0)]$ decreases as q (temperature T) increases (decreases). Latter behavior holds for asymptotic Lifshitz solutions ($z>1$). However, the behavior of conductivity for large frequencies is different. In fact, real part of conductivity has a positive slope with respect to large frequency for asymptotic Lifshitz solutions whereas it tends to a constant for asymptotic AdS ones. This behavior may be interpreted as existence of an additional charge carrier for systems respecting Lifshitz symmetry rather than the AdS case. This is due to existence of dilaton scalar field in our model, comparing with asymptotic AdS case. Similar behavior for optical conductivity of single-layer graphene induced by mild oxygen plasma exposure has been reported in [41]. Finally, we observed that for a specific value of ω/T, the conductivity has the same value, independent of the nonlinear parameter β, for both asymptotic Lifshitz and asymptotic AdS solutions.

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