Letter

Generalizing eleven-dimensional supergravity

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(Received 7 March 2022; accepted 10 April 2022; published 27 April 2022)

We develop a procedure to reproduce the ten-dimensional generalized supergravity equations from T-duality covariant equations that facilitates generalization to U-duality covariant formulations of eleven-dimensional supergravity. The latter leads to a modification of the eleven-dimensional supergravity equations with terms that contain a rank-2 tensor field, which is the eleven-dimensional analog of the nonunimodularity Killing vector I^m in ten dimensions.

DOI: 10.1103/PhysRevD.105.L081904

I. INTRODUCTION

An old and important problem in string/M theory is description of consistent backgrounds for the fundamental string/membrane dynamics. A partial answer is given by the supergravity equations in 10 and 11 spacetime dimensions, which ensure kappa symmetry of the Green-Schwarz (GS) superstring and of the membrane, respectively. At the quantum level, cancellation of the superstring Weyl anomaly is achieved by the supergravity as well. An intriguing observation made recently is that κ symmetry of the GS superstring still holds if equations of motion for background fields are generalized to include a Killing vector I^m [1,2]. This set of equations is usually referred to as generalized supergravity and has the following form:

$$
\begin{aligned}
R_{mn} - \frac{1}{4} H_{mpq} H_n{}^{pq} + 2\nabla_{(m} Z_{n)} &= T_{mn}, \\
-\frac{1}{2}\nabla^k H_{kmn} + Z^k H_{kmn} + 2\nabla_{[m} I_{n]} &= K_{mn}, \\
R - \frac{1}{2}|H_3|^2 + 4(\nabla^m Z_m - I^m I_m - Z^m Z_m) &= 0, \\
d * F_p - H_3 \wedge *F_{p+2} - \iota_I B_2 \wedge *F_p - \iota_I * F_{p-2} &= 0, \qquad (1)
\end{aligned}
$$

where the expressions on the rhs read

$$
\begin{aligned}
T_{mn} &= \frac{1}{4} e^{2\Phi} \sum_p \left[\frac{1}{p!} F_m{}^{k_1\ldots k_p} F_{nk_1\ldots k_p} - \frac{1}{2} g_{mn} |F_{p+1}|^2\right], \\
K_{mn} &= \frac{1}{4} e^{2\Phi} \sum_p \frac{1}{p!} F_{k_1\ldots k_p} F_{mn}{}^{k_1\ldots k_p}. \qquad (2)
\end{aligned}
$$

Here $|\omega_p|^2 = \frac{1}{p!}\omega_{i_1\ldots i_p}\omega^{i_1\ldots i_p}$ for a p-form ω_p, $Z_m = \partial_m \Phi + I^n B_{nm}$, and $I = I^m \partial_m$ is a Killing vector field that is also a symmetry of all fields in the theory, including the dilaton Φ. Setting I^m to zero one gets the usual equations of $d = 10$ supergravity.

Although some subtleties arise at the quantum level related to the issue of locality of the generalized Fradkin-Tseytlin term [3,4], the classical κ symmetry ensures one-loop scale invariance (UV finiteness) for a superstring propagating in generalized supergravity backgrounds [1,2]. While they are lacking full conformal symmetry, their relation to consistent string theory backgrounds is known. Solutions to the generalized equations (1) can be obtained

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from the standard supergravity solutions by T-duality transformations in the direction of an isometry that is broken by a linear dependence of the dilaton [5,6]. More generally, a sequence of (possibly non-Abelian) T dualities or a Yang-Baxter (YB) deformation parametrized by a bivector $\beta^{mn} = \frac{1}{2} r^{\alpha\beta} k_\alpha{}^m k_\beta{}^n$ is required. Here $k_\alpha{}^m$ are the Killing vectors of the *initial* background and $r^{\alpha\beta} = r^{[\alpha\beta]}$ is a constant matrix (greek indices label the isometry algebra elements, while small latin indices refer to the spacetime coordinates). For the deformed background to be a solution of (1) the r matrix must satisfy the classical YB equation [7–10]

$$r^{\alpha[\gamma} r^{\delta|\beta|} f_{\alpha\beta}{}^{\epsilon]} = 0, \tag{3}$$

where $f_{\alpha\beta}{}^\gamma$ are the structure constants of the isometry algebra $[k_\alpha, k_\beta] = f_{\alpha\beta}{}^\gamma k_\gamma$. The vector I^m is then simply

$$I^m = r^{\alpha\beta} f_{\alpha\beta}{}^\gamma k_\gamma{}^m, \tag{4}$$

and thus generalized supergravity solutions correspond to the nonunimodular YB deformations. The integrability preservation property of the YB deformation [11] extends to generalized supergravity solutions, the renowned example being the Arutyunov-Borsato-Frolov background [12,13].

Translating the above narrative into the 11-dimensional language encounters certain difficulties. In contrast to superstring, there are no known integrability properties and no conformal versus scale invariance in the case of the supermembrane. Classical κ symmetry of the membrane action leads to the torsion constraints that imply the standard 11D supergravity equations for the background fields [14]. Both for the superstring and the supermembrane the κ symmetry implies that the dimension $\frac{1}{2}$ torsion component is expressed in terms of a spinor superfield χ_α. While in 10D, in addition to κ symmetry, one may or may not require this to be a spinor derivative of the dilaton, $\chi_\alpha = \nabla_\alpha \Phi$, leading to the difference between the usual and generalized supergravities [2], there is no such freedom in 11D as there is no dilaton. Instead, the scalar superfield Φ in this case can be gauged away by a super-Weyl transformation [15].

In this Letter, we propose a possible way of circumventing these obstacles using the methods of exceptional field theories (EFTs, see [16,17] for review). We show that the EFTs, yielding U-duality covariant formulations of 11-dimensional supergravity, provide a highly efficient tool for constructing a generalization of 11D supergravity equations, such that broken symmetry is the global GL(11) symmetry. We develop an algorithm that yields a set of generalized equations for the fields of 11D supergravity, which include an additional tensor J^{mn}, whose properties are very similar to those of the I^m above (4). Specifically, the 11D generalization of YB deformation, now parametrized by a trivector $\Omega^{mnk} = \frac{1}{3!} \rho^{\alpha\beta\gamma} k_\alpha{}^m k_\beta{}^n k_\gamma{}^k$ with some constant $\rho^{\alpha\beta\gamma} = \rho^{[\alpha\beta\gamma]}$, is required to satisfy an analog of the unimodularity constraint in order to generate solutions of the standard supergravity equations [18]. This has the form

$$J^{mn} = \rho^{\alpha\beta\gamma} f_{\beta\gamma}{}^\delta k_\alpha{}^m k_\delta{}^n = 0. \tag{5}$$

Together with the so-called generalized YB equation [19]

$$6\rho^{\alpha\beta[\gamma} \rho^{\delta\epsilon|\zeta|} f_{\alpha\zeta}{}^{\eta]} + \rho^{[\gamma\delta\epsilon} \rho^{\eta]\alpha\zeta} f_{\alpha\zeta}{}^\beta = 0, \tag{6}$$

the above condition is sufficient for the generalized fluxes of EFT to be invariant under the generalized YB deformation [20]. Given that the supergravity equations of motion can be written in terms of generalized fluxes and their derivatives, this proves that such trivector deformations always produce solutions of the usual 11D supergravity. Relaxing the unimodularity condition (5) one obtains a transformation of generalized fluxes parametrized by J^{mn} and the corresponding generalized supergravity field equations, which are *by construction* satisfied by the deformed backgrounds.

After briefly describing our procedure, we will present the resulting generalized supergravity equations together with consistency conditions on the tensor J^{mn}. We will consider examples of nontrivial deformed backgrounds that solve these equations, but which are not solutions of the ordinary 11-dimensional supergravity. Details will be presented in an upcoming paper [22].

It is important to mention that at this stage the exact role played by these equations in the fundamental membrane dynamics is not clear. Our result certainly has two necessary features for *the* theory of generalized 11D supergravity: (i) It reproduces the equations of 10D generalized supergravity (1) upon dimensional reduction, and (ii) the tensor J^{mn} appears in the field equations, carrying additional information about the background field configuration. However, until there are some further checks that show that these are also sufficient, it may be appropriate to think of what we have as *a* trivector deformation of the supergravity equations.

II. GENERALIZED SUPERGRAVITY FROM DOUBLE FIELD THEORY

Generalized supergravity can be obtained from the modified double field theory (DFT) construction of [23]. Let us shortly review this construction in a form more suitable for generalization to 11 dimensions. DFT is a T-duality covariant representation of supergravity that by definition requires an extended space parametrized by the doubled set of $d + d$ coordinates $\mathbb{X}^M = (x^m, \tilde{x}_m)$. In what follows, we always assume that the fields do not depend on the dual coordinates $\tilde{x}_m$, thus the section constraint $\eta^{MN} \partial_M \otimes \partial_N = 0$ is satisfied. The extended space indices are raised and lowered using the invariant tensor η_{MN}.

The bosonic sector of the theory is encoded in the generalized metric $\mathcal{H}_{MN} \in O(d,d)/O(d) \times O(d)$, and the invariant dilaton $d = \Phi - \frac{1}{4}\log\det g_{mn}$. Local symmetries of the theory are the generalized diffeomorphisms, which act as

$$\mathbb{L}_\Lambda V^M = \Lambda^N \partial_N V^M - V^N \partial_N \Lambda^M + \eta^{MN}\eta_{KL}\partial_N \Lambda^K V^L, \qquad (7)$$

on a generalized vector V^M of weight zero. The dilaton d has weight $1/2$. These include standard diffeomorphisms, Kalb-Ramond gauge transformations, and T-duality transformations. For our purposes it is necessary to formulate the theory in terms of the generalized fluxes, or the anholonomicity coefficients $\mathcal{F}_{AB}{}^C$,

$$[E_A, E_B]^M = \mathcal{F}_{AB}{}^C E_C{}^M, \qquad \mathbb{L}_{E_A} d = \frac{1}{2}\mathcal{F}_A, \qquad (8)$$

where $E_A{}^M$ is the inverse of the generalized vielbein defined as usual as $\mathcal{H}_{MN} = E_M{}^A E_N{}^B \mathcal{H}_{AB}$ for a flat $O(d,d)$ metric $\mathcal{H}_{AB}$. The fluxes satisfy generalized Bianchi identities

$$\begin{aligned} 0 &= \partial_{[A}\mathcal{F}_{BCD]} - \frac{3}{4}\mathcal{F}_{[AB}{}^E \mathcal{F}_{CD]E}, \\ 0 &= 2\partial_{[A}\mathcal{F}_{B]} + \partial^C \mathcal{F}_{CAB} - \mathcal{F}^C \mathcal{F}_{CAB}, \\ 0 &= \partial^A \mathcal{F}_A - \frac{1}{2}\mathcal{F}^A \mathcal{F}_A + \frac{1}{12}\mathcal{F}^{ABC}\mathcal{F}_{ABC}, \end{aligned} \qquad (9)$$

which may be understood as conditions of covariance under local transformations.

The generalized supergravity equations can be obtained by looking into the transformation of generalized fluxes under a bivector YB deformation, which is an $O(d,d)$ rotation. Given that the r matrix defining the bivector β^{mn} satisfies the classical YB equation (3), the transformation of the fluxes reads

$$\mathcal{F}'_{ABC} = \mathcal{F}_{ABC}, \qquad \mathcal{F}'_A = \mathcal{F}_A + X_A, \qquad (10)$$

where $X_A = E_A{}^M X_M = E'_A{}^M X_M$ with $X_M = (0, I^m)$. In order to be able to interpret the deformed fluxes $\mathcal{F}'_{ABC}$ and $\mathcal{F}'_A$ as generalized fluxes of a new vielbein, we must ensure that the Bianchi identities (9) hold both for the initial and the deformed fluxes. This results in the condition $\mathbb{L}_X E'_A{}^M = 0$; i.e., it is enough to take $I^m = r^{\alpha\beta} f_{\alpha\beta}{}^\gamma k^m_\gamma$ to be a Killing vector of the *deformed* background. It is important to mention that, generally speaking, Bianchi identities produce both linear and quadratic constraints for X^M. The former gives the condition above, while the latter is $X^M X_M = 0$ and is satisfied by the choice $X_M = (0, I^m)$. As will be seen later, such generalized Killing vector property of X^M is a feature of bivector deformations only and does not hold for the 11-dimensional case.

To obtain the field equations for the NS-NS sector of generalized supergravity, one starts with DFT equations of motion in the flux formulation [24] and according to (10) substitute $\mathcal{F}_A = \mathcal{F}'_A - X_A$. The result will be a set of equations for the new vielbein $E'_M{}^A$, which are satisfied by construction given that $E_M{}^A$ satisfies the undeformed DFT equations. An explicit check shows that these are precisely the equations of generalized supergravity (1).

The suggested approach is very close to the idea of the deformed generalized Lie derivative considered in [25,26], where the deformation is proportional to the Romans mass m_R of the resulting massive type IIA theory. The difference is that we deform not the Lie derivative but the fluxes, which results in conditions on the deformation rather than differential conditions on the fields. As discussed in [25], the nonderivative terms in the Lie derivative that determine the mass deformation can also be obtained by a Scherk-Schwarz type ansatz. Likewise, generalized supergravity equations in ten dimensions can be derived by imposing such an ansatz on certain fields, either within EFT [27] or DFT [28]. Hence, it is natural to expect the deformation induced from J^{mn} to be derivable via this mechanism. The relationship between these approaches will be investigated further in [22].

III. ELEVEN DIMENSIONS

The DFT procedure described above can also be carried over to the case of trivector deformations, which is relevant to 11D backgrounds. Consider the SL(5) EFT, which is a U-duality covariant formulation of supergravity in terms of the fields of the maximal $D = 7$ supergravity [29]. Its bosonic sector

$$\{g_{\mu\nu}, A_\mu{}^{[MN]}, B_{\mu\nu\,M}, m_{MN}\}, \qquad \mu,\nu = 0,\dots,6, \quad M,N = 1,\dots,5, \qquad (11)$$

contains the so-called external metric $g_{\mu\nu}$, generalized metric $m_{MN} \in \mathrm{SL}(5)/\mathrm{SO}(5)$, and 1- and 2-form fields $A_\mu{}^{MN}$, $B_{\mu\nu M}$ transforming in the $\mathbf{10}$ and $\bar{\mathbf{5}}$ of SL(5). All fields depend on $7 + 10$ coordinates $\{x^\mu, X^{[MN]}\}$ and are subject to the section constraint $\partial_{[MN} \otimes \partial_{KL]} = 0$. The theory is defined by the Lagrangian [30,31] covariant with respect to to diffeomorphisms in the external space, gauge transformations, and generalized diffeomorphisms defined as

$$\begin{aligned}\mathbb{L}_\Lambda V^M = {} & \frac{1}{2}\Lambda^{KL}\partial_{KL}V^M - V^L \partial_{LK}\Lambda^{MK} + \frac{1}{4}V^M \partial_{KL}\Lambda^{KL} \\ & + \lambda_V \partial_{KL}\Lambda^{KL} V^M, \end{aligned} \qquad (12)$$

when acting on a generalized vector V^M of weight λ_V. To keep the setup as simple as possible, we consider the following truncation of the theory [32,33]:

$$g_{\mu\nu}(x,X) = e^{-2\phi}h^{\frac{1}{5}}\bar{g}_{\mu\nu}(x), \qquad A_\mu{}^{MN} = 0,$$
$$m_{MN} = e^{-\phi}h^{\frac{1}{5}}M_{MN}, \qquad B_{\mu\nu\,M} = 0. \tag{13}$$

Here $h = \det\|h_{mn}\|$ denotes the determinant of the 4×4 block of the full 11-dimensional metric and the fields ϕ, h, M_{MN} are restricted to depend only on the extended coordinates $X^{[MN]}$. The 7×7 block $\bar{g}_{\mu\nu}$ of the full metric has only dependence on external coordinates.

This allows to one reformulate the theory in terms of only the metric $M_{MN} \in \mathrm{SL}(5)/\mathrm{SO}(5)\times\mathbb{R}^+$ or, equivalently, in terms of generalized fluxes $\mathcal{F}_{AB,C}{}^D$ defined by $\mathbb{L}_{E_{AB}}E^M{}_C = \mathcal{F}_{AB,C}{}^D E^M{}_D$. The fluxes satisfy Bianchi identities

$$\begin{aligned}&\frac{3}{2}\partial_{[AB}\mathcal{F}_{|DF|C]}{}^E - \frac{1}{2}\partial_{DF}\mathcal{F}_{ABC}{}^E + \partial_{CG}\mathcal{F}_{DF[A}{}^G\delta_{B]}{}^E\\ &-\frac{1}{4}\delta_C{}^E\partial_{BG}\mathcal{F}_{DFA}{}^G + \frac{1}{4}\delta_C{}^E\partial_{AG}\mathcal{F}_{DFB}{}^G\\ &-\mathcal{F}_{BGC}{}^E\mathcal{F}_{DFA}{}^G + \mathcal{F}_{AGC}{}^E\mathcal{F}_{DFB}{}^G\\ &+\mathcal{F}_{ABG}{}^E\mathcal{F}_{DFC}{}^G - \mathcal{F}_{ABC}{}^G\mathcal{F}_{DFG}{}^E = 0,\end{aligned} \tag{14}$$

where we define $\partial_{AB} = E_{AB}{}^{MN}\partial_{MN}$ for clarity of notations.

Generalized Yang-Baxter deformation parametrized by a trivector $\Omega^{mnk} = \frac{1}{3!}\rho^{\alpha\beta\gamma}k_\alpha{}^m k_\beta{}^n k_\gamma{}^k$ is an SL(5) transformation [33]

$$O_M{}^N = \begin{bmatrix}\delta_m{}^n & 0\\ \frac{1}{3!}\epsilon_{mpqr}\Omega^{pqr} & 1\end{bmatrix}, \tag{15}$$

after which the flux components transform as

$$\begin{aligned}\delta_\rho\mathcal{F}_{ABC}{}^D = {}& E^m{}_C E^n{}_A E^k{}_B E_l{}^D J^{lp}\epsilon_{kmnp}\\ &+\delta^{(2)}_\rho\mathcal{F}_{ABC}{}^D.\end{aligned} \tag{16}$$

Here we introduce the nonunimodularity parameter

$$J^{mn} = \rho^{\alpha\beta\gamma}f_{\beta\gamma}{}^\delta k_\alpha{}^m k_\delta{}^n = S^{mn} + I^{mn} \tag{17}$$

that encodes terms in the transformation linear in $\rho^{\alpha\beta\gamma}$. S^{mn} and I^{mn} are the symmetric and antisymmetric parts of J^{mn}, respectively. The terms $\delta^{(2)}_\rho\mathcal{F}_{ABC}{}^D$ are quadratic in $\rho^{\alpha\beta\gamma}$ and proportional to the generalized YB equation (6) that is equivalent to vanishing of the R flux defined as $R^{p,mnkl} = 4\Omega^{pq[m}\nabla_q\Omega^{nkl]}$ [19]. Algebraically, this is the condition for exceptional Drinfeld algebra to preserve its structure under a trivector deformation [34]. Interestingly, it can be shown that setting $I^{mn} = 0$ is sufficient for the generalized YB equation to be satisfied [19]. This is purely a matter of the low internal space dimension $d = 4$ and the same is true for bivector YB deformations in $d = 3$. Instead, we assume that the weaker quadratic condition is satisfied while $J^{mn} \neq 0$, which allows one to arrive at a $d = 11$ version of the generalized supergravity equations (1).

To do this, consider the generalized fluxes $\mathcal{F}'_{ABC}{}^D = \mathcal{F}_{ABC}{}^D + X_{ABC}{}^D$, shifted precisely so as they transform under a nonunimodular generalized YB deformation, i.e.,

$$X_{mnk}{}^l = \epsilon_{mnkp}J^{lp}. \tag{18}$$

One observes that $X_{MNK}{}^L$ cannot be interpreted as a generalized Killing vector like in the ten-dimensional case. Moreover, the nonunimodularity parameter J^{mn} is essentially a $d = 4$ tensor, hence, it breaks global GL(11) to $\mathrm{GL}(7)\times\mathrm{GL}(4)$.

We now proceed with construction of the deformed theory. Following the analogy with the ten-dimensional case, we consider quadratic and linear conditions following from the Bianchi identities (14) on $\mathcal{F}'_{ABC}{}^D$ separately. For the former we have

$$L_{e_a}J^{kl} + J^{nl}\partial_n\phi e_a{}^k = 0, \qquad J^{mn}\partial_n\phi = 0,$$
$$\nabla_m(e^{-\phi}I^{mn}) = 0, \qquad J^{m[n}J^{kl]} = 0, \tag{19}$$

which, in particular, implies the constraint $I^{[mn}I^{kl]} = 0$, which is like a section condition. This gives a hint for the possibility to interpret the antisymmetric part I^{mn} as a dual derivative of some field. These equations are straightforward generalization of the definition for I^m (4) of the 10D case. The first equation above can be rewritten as $\nabla_{(m}J_{nk)} = 0$, which implies that the symmetric part $S^{mn} = J^{(mn)}$ is a Killing tensor of the deformed background.

Additionally, we have conditions that are linear in the gauge field $V^m = \frac{1}{3!}\varepsilon^{mnkl}C_{nkl}$,

$$\begin{aligned}\nabla_{[m}Z_{n]} - \frac{1}{3}J^{kl}F_{mnkl} &= 0,\\ \nabla_k(e^{-\phi}J^{k[l}V^{p]}) &= 0,\\ \nabla_k(J^{(pl)}V^k) - \nabla_k(V^{(p}J^{l)k}) &= 0,\end{aligned} \tag{20}$$

where

$$Z_m = \partial_m\phi - \frac{2}{3}\varepsilon_{mnkl}I^{nk}V^l, \tag{21}$$

following a straightforward generalization of the ten-dimensional case. We also note that the first condition can be nicely rewritten as

$$L_V I^{mn} + 2I^{p[m}V^{n]}\partial_p\phi = 0, \tag{22}$$

which together with the first line of (19) can be understood as action of I^{mn} on the background via the Schouten-Nijenhuis bracket.

To obtain equations of 11-dimensional supergravity generalized by adding a nonvanishing J^{mn}, one shifts fluxes in equations of motion for SL(5) exceptional field theory by $X_{MNK}{}^L$ that gives

$$
\begin{aligned}
0=&\mathcal{R}_{mn}[h_{(4)}]-7\tilde{\nabla}_{(m}Z_{n)}-\frac{1}{3}h_{mn}(\nabla V)\\
&+8(1+V^2)(S_{mn}J^k{}_k-2J^k{}_{(m}J_{n)k})\\
&+4V_mV_n(J^{kl}J_{kl}-2J^{kl}J_{lk})\\
&+4V_kV_l(4J_{(m}{}^kJ_{n)}{}^l-J^k{}_{(m}J^l{}_{n)}-2J^{kl}S_{mn})\\
&+8V_kV_{(m}(2J^l{}_{n)}J^k{}_l-2S_{n)}{}^kJ^l{}_l+J^{kl}J_{n)l}),\\
0=&\frac{1}{7}e^{2\phi}\mathcal{R}[\bar{g}_{(7)}]+\frac{1}{6}(\nabla V)^2+\tilde{\nabla}^mZ_m-6Z_mZ^m\\
&-2J^{mn}J_{mn}+\frac{4}{3}J_{mn}J^{nm},\\
0=&\left(\frac{1}{6}\tilde{\nabla}^m-Z^m\right)F_{mnkl}+J^{pm}(2C_{m[nk}J_{l]p}-J_{p[n}C_{kl]m}),
\end{aligned}\tag{23}
$$

where $F_{mnkl}=4\partial_{[m}C_{nkl]}$ and $\tilde{\nabla}_m=\nabla_m-\partial_m\phi$. When $J^{mn}=0$, these equations reproduce the truncated version of 11D supergravity equations given in [33].

Terms in the equations above that are quadratic in J^{mn} are the consequence of the fact that an analog of $X_m=I_m+Z_m$ of the ten-dimensional case cannot be defined here since J^{mn} has two indices. Another reason for these to be expected is that, after the dimensional reduction, various powers of e^{ϕ} appear both in the Einstein and Maxwell equations. It is easy to see that dimensional reduction of the generalized equations (23) reproduce the known result (1). Indeed, suppose one of the Killing vectors $k_\alpha{}^m$ commutes with the remaining set, forming a separate U(1) isometry to be identified with the M-theory circle. Then keeping nonzero only $I^{\bar{m}}=J^{4\bar{m}}\neq 0$, we are left with $X_{4\bar{m}\,\bar{n}}{}^4\neq 0$, which can contribute only to $\mathcal{F}_A$ simply by the index count. Finally, since the SL(5) theory after the reduction reproduces the O(3,3) DFT [35], one just repeats the construction in the beginning of the letter. More details on the reduction will be given in an upcoming paper [22].

IV. EXAMPLES

The constraints (20) on J^{mn} may look too restrictive, however, the theory is not void and contains nontrivial solutions. As an illustration let us consider two examples of nonunimodular generalized Yang-Baxter deformations of the $AdS_4\times\mathbb{S}^7$ solution. These are solutions to the equations (23) by construction and the fields satisfy all the conditions above. We will use the Killing vectors of the AdS_4 space of radius R, which include three momentum generators P_a, dilatation D, angular momenta M_{ab}, and special conformal transformations K_a, where $a,b=0,1,2$.

Our first example is the trivector deformation obtained in [33] for which the three-vector is

$$
\Omega=\frac{2}{R^3}D\wedge(\rho_a\epsilon^{abc}P_b\wedge P_c).\tag{24}
$$

The deformation is nonunimodular with $J^{ab}=-\frac{4}{3R^3}\epsilon^{abc}\rho_c$, and the generalized YB equation (6) is satisfied for arbitrary values of ρ_a. In general, the above does not solve equations of motion of the ordinary 11-dimensional supergravity, except for the special case $\rho^2=\rho_a\rho_b\eta^{ab}=0$ [33]. This simply means that terms with and without J^{mn} vanish separately, providing a trivial solution to generalized equations in the sense of [36].

Our next example is a solution of the generalized theory only and corresponds to the deformation cubic in x^a,

$$
\begin{aligned}
\Omega&=\frac{4}{R^3}\rho_a\epsilon^{abc}D\wedge M_{bd}\wedge M_c{}^d\\
&=\frac{4\rho_ax^a}{R^3}\left(x^bx_b\partial_0\wedge\partial_1\wedge\partial_2-\frac{z}{2}x_b\epsilon^{bcd}\partial_c\wedge\partial_d\wedge\partial_z\right).
\end{aligned}\tag{25}
$$

The generalized YB equation (6) constrains the ρ matrix as $\rho^2=\rho_a\rho_b\eta^{ab}=0$. The background is then given by

$$
\begin{aligned}
ds^2&=\frac{R^2}{4z^2}K^{-\frac{2}{3}}\left\{dx_adx^a+\frac{1}{z^2}\rho_ax^ax^bdx_bdz\right.\\
&\quad\left.+\left(1-\frac{x_ax^a\rho_bx^b}{z^3}\right)dz^2\right\}+R^2K^{\frac{1}{3}}d\Omega^2_{(7)},\\
F_{012z}&=-\frac{3}{8}\frac{R^3}{z^4}K^{-2}\left(1+\frac{1}{12}\frac{x_ax^a\rho_b\rho_cx^bx^c}{z^4}\right),\\
J^{ma}&=\frac{32}{R^3}\rho_b\epsilon^{abc}x_cx^m,\\
K&=1+\frac{x_ax^a}{z^3}\rho_bx^b\left(1-\frac{\rho_cx^c}{4z}\right),
\end{aligned}\tag{26}
$$

where $m=0,1,2,z$ and $a,\ b=0,\ 1,\ 2$.

V. CONCLUSIONS

We develop an effective method that allows one to deform the equations of 11-dimensional supergravity by including an additional tensor J^{mn}, which is related to the nonunimodularity of trivector deformed 11-dimensional backgrounds and is a generalization of the Killing vector I^m (4) of the ten-dimensional generalized supergravity. J^{mn} (17) has similar properties to I^m, in particular, its symmetric part must be a Killing tensor. Using this method, we find a set of modified field equations of 11-dimensional supergravity (23), which is an important step in connecting 10-dimensional generalized supergravity [1,2] to 11 dimensions. We also give two solutions that are obtained by the trivector generalized YB deformations. By construction, they are always solutions to the proposed equations (23).

Our algorithm is equally applicable to 10- and 11-dimensional supergravities, reproducing the known generalized supergravity equations (1) for the former. In 10D such possibility is allowed by the κ-symmetry constraints of the GS superstring and is associated with the breaking of conformal symmetry down to scale symmetry. The common lore is that a similar procedure cannot be done for the supermembrane due to the lack of conformal symmetry. However, our newly introduced tensor J^{mn} breaks the global GL(11) diffeomorphism symmetry to $\mathrm{GL}(7) \times \mathrm{GL}(4)$, allowing one to avoid this obstruction. Hence, one expects that the fundamental supermembrane is κ symmetric on solutions of (23) at the cost of such global symmetry breaking. This is reserved for future work.

ACKNOWLEDGMENTS

This work has been supported by Russian Science Foundation Grant No. RSCF-20-72-10144. We would like to thank Gleb Arutyunov and David Berman for useful comments. N. S. D. and E. T. M. are grateful to Albert-Einstein-Institute, Potsdam, where this work was initiated. We thank Istanbul Center for Mathematical Sciences, Istanbul for hospitality during the final phase of this work.

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