



# Structure and production mechanism of the enigmatic $X(3872)$ in high-energy hadronic reactions

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**Abstract** We calculate the total cross section and transverse momentum distributions for the production of the enigmatic  $\chi_{c1}(3872)$  (or  $X(3872)$ ) assuming different scenarios:  $c\bar{c}$  state and  $D^{0*}\bar{D}^0 + D^0\bar{D}^{0*}$  molecule. The derivative of the  $c\bar{c}$  wave function needed in the first scenario is taken from a potential model calculations. Compared to earlier calculations of molecular state we include not only single parton scattering (SPS) but also double parton scattering (DPS) contributions. The latter one seems to give smaller contribution than the SPS one. The upper limit for the DPS production of  $\chi_{c1}(3872)$  is much below the CMS data. We compare results of our calculations with existing experimental data of CMS, ATLAS and LHCb collaborations. Reasonable cross sections can be obtained in either  $c\bar{c}$  or molecular  $D\bar{D}^*$  scenarios for  $X(3872)$ , provided one takes into account both directly produced  $D^0, \bar{D}^0$ , as well as  $D^0, \bar{D}^0$  from the decay of  $D^*$ . However arguments related to the lifetime of  $D^*$  suggest that the latter component is not active.

## 1 Introduction

The  $X(3872)$  state was discovered already some time ago by the Belle collaboration [1]. Since then its existence has been confirmed in several other processes and numerous theoretical studies have been performed, see for example the review articles [2–4]. There is at present agreement that the  $X(3872)$  has the axial vector quantum numbers  $J^{PC} = 1^{++}$ , and accordingly the state is named as  $\chi_{c1}(3872)$  [5].

The internal structure of  $X(3872)$  stays rather enigmatic. While its quantum numbers are not exotic – it could indeed be a quarkonium  $c\bar{c}$  state, e.g. a radial excitation of the  $\chi_{c1}$ ,

there are strong arguments for a non- $c\bar{c}$  scenario, manifested e.g. by the violation of isospin in its decays [5].

More importantly, the mass of  $X$  is very close to the  $D\bar{D}^*$  threshold. It is therefore rather popular to consider  $X(3872)$  as very weakly bound state of the  $D\bar{D}^*$  system – a hadronic molecule, see the review [6]. A tetraquark scenario was considered in [7]. The  $c\bar{c}$  quarkonium scenario, where  $X(3872)$  is the  $\chi_{c1}(2P)$  state has been advocated in [8]. Other approaches treat the  $X(3872)$  as a  $c\bar{c}$  bound state in the meson-meson continuum taking into account coupling of  $c\bar{c}$  and meson-meson channels [9–11]. The possible mixture of quarkonium and molecule/virtual state is considered in [12] and found to be consistent with current data.

Recently, the transverse momentum distributions of  $X(3872)$  were measured at the LHC [13–15] by the CMS, ATLAS and LHCb collaborations.

There is a debate in the literature [16–19], whether the rather large production rate at large  $p_T$  allows one to exclude the molecular scenario. With few exceptions [20–22] the discussion in the literature is limited to estimates of orders of magnitude. An estimate of the total cross section in a tetraquark scenario has been given in Ref. [23].

In this paper we shall consider two scenarios of prompt  $X(3872)$  production, which are not mutually exclusive and in fact both can contribute depending on the structure of  $X(3872)$ . Both scenarios have in common that the production is initiated by the production of a  $c\bar{c}$  pair in a hard process.

In the first scenario we shall consider that the  $X(3872)$  is a pure  $c\bar{c}$  state, the first radial excitation of  $\chi_{c1}$ . The corresponding wave function and its derivative were calculated in potential models e.g. in [24].

In the second scenario, where the  $X(3872)$  is treated as a weakly bound  $s$ -wave state in the  $D\bar{D}^* + D^*\bar{D}$ -system, we exploit the connection between the low-energy scattering amplitude in the continuum and at the bound state pole below threshold, well known from effective range theory. In this

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work we follow [17] and give an estimate of a  $p_T$ -dependent upper bound for  $X(3872)$  production in the molecule scenario.

As far as the hard production mechanism is concerned, we employ the  $k_T$ -factorization framework [25–27]. For the quarkonium scenario, the dominant mechanism of  $C = +1$  is probably color singlet two virtual gluon fusion. The production of  $\chi_{c0}, \chi_{c1}, \chi_{c2}$  production was considered e.g. in [28–30]. Recently we have shown that the transverse momentum distribution of  $\eta_c$  measured by the LHCb [31] can be nicely described as  $g^*g^* \rightarrow \eta_c$  fusion within  $k_T$ -factorization approach [32].

For the molecular scenario, in addition to the single-parton scattering (SPS) mechanism of fusion of two off-shell gluons  $g^*g^* \rightarrow c\bar{c}$ , we will also consider production through the double-parton scattering (DPS) mode.

## 2 Formalism

In Fig.1 we show two generic Feynman diagrams for  $X(3872)$  quarkonium production in proton–proton collision via gluon–gluon fusion: for the quarkonium (left) and molecule (right). These diagrams illustrate the situation adequate for the  $k_T$ -factorization calculations used in the present paper.

The inclusive cross section for  $X(3872)$ -production via the  $2 \rightarrow 1$  gluon–gluon fusion mode is obtained from

$$\begin{aligned} \frac{d\sigma}{dyd^2\vec{p}_T} &= \int \frac{d^2\vec{q}_{T1}}{\pi\vec{q}_{T1}^2} \mathcal{F}(x_1, \vec{q}_{T1}^2, \mu_F^2) \\ &\times \int \frac{d^2\vec{q}_{T2}}{\pi\vec{q}_{T2}^2} \mathcal{F}(x_2, \vec{q}_{T2}^2, \mu_F^2) \delta^{(2)}(\vec{q}_{T1} + \vec{q}_{T2} - \vec{p}_T) \\ &\times \frac{\pi}{(x_1x_2s)^2} |\mathcal{M}_{g^*g^* \rightarrow X(3872)}|^2. \end{aligned} \tag{2.1}$$

Here the matrix element squared for the fusion of two off-shell gluons into the  $^3P_1$  color singlet  $c\bar{c}$  charmonium is obtained from (see e.g. [28,33] for a derivation):

$$\begin{aligned} \mathcal{M}_{g^*g^* \rightarrow X(3872)} &= \frac{q_{1T}^\mu q_{2T}^\nu}{|\vec{q}_{T1}||\vec{q}_{T2}|} \mathcal{M}_{\mu\nu} \\ &= \frac{x_1x_2s}{|\vec{q}_{T1}||\vec{q}_{T2}|} n_+^\mu n_-^\nu \mathcal{M}_{\mu\nu}, \end{aligned} \tag{2.2}$$

and reads explicitly:

$$\begin{aligned} &|n_+^\mu n_-^\nu \mathcal{M}_{\mu\nu}|^2 \\ &= (4\pi\alpha_S)^2 \frac{4|R'(0)|^2}{\pi M_X^3} \frac{\vec{q}_{T1}^2 \vec{q}_{T2}^2}{(M_X^2 + \vec{q}_{T1}^2 + \vec{q}_{T2}^2)^4} \end{aligned}$$

$$\begin{aligned} &\times \left( (\vec{q}_{T1}^2 + \vec{q}_{T2}^2)^2 \sin^2 \phi \right. \\ &\left. + M_X^2 (\vec{q}_{T1}^2 + \vec{q}_{T2}^2 - 2|\vec{q}_{T1}||\vec{q}_{T2}|\cos \phi) \right), \end{aligned} \tag{2.3}$$

where  $\phi$  is the azimuthal angle between  $\vec{q}_{T1}, \vec{q}_{T2}$ . The momentum fractions of gluons are fixed as  $x_{1,2} = m_T \exp(\pm y)/\sqrt{s}$ , where  $m_T^2 = \vec{p}_T^2 + M_X^2$ . The derivative of the radial quarkonium wave function at the origin is taken for the first radial  $p$ -wave excitation from Ref. [24],  $|R'(0)|^2 = 0.1767 \text{ GeV}^5$ .

The unintegrated gluon parton distribution functions (gluon uPDFs) are normalized such, that the collinear glue is obtained from

$$xg(x, \mu_F^2) = \int^{\mu_F^2} \frac{d^2\vec{k}_T}{\pi\vec{k}_T^2} \mathcal{F}(x, \vec{k}_T^2, \mu_F^2). \tag{2.4}$$

The hard scale is taken to be always  $\mu_F = m_T$ , the transverse mass of the  $X(3872)$ . In order to estimate the production cross section for the molecule we also start from a hard production of a  $c\bar{c}$ -pair, which we then hadronize into a  $D\bar{D}^* + h.c$  system using a prescription given below.

The parton-level differential cross section for the  $c\bar{c}$  production, formally at leading-order, reads:

$$\begin{aligned} \frac{d\sigma(pp \rightarrow Q\bar{Q}X)}{dy_1dy_2d^2\vec{p}_{T1}d^2\vec{p}_{T2}} &= \int \frac{d^2\vec{k}_{T1}}{\pi\vec{k}_{T1}^2} \mathcal{F}(x_1, \vec{k}_{T1}^2, \mu_F^2) \\ &\times \int \frac{d^2\vec{k}_{T2}}{\pi\vec{k}_{T2}^2} \mathcal{F}(x_2, \vec{k}_{T2}^2, \mu_F^2) \\ &\times \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{p}_{T1} - \vec{p}_{T2}) \\ &\times \frac{1}{16\pi^2(x_1x_2s)^2} |\mathcal{M}_{g^*g^* \rightarrow c\bar{c}}^{\text{off-shell}}|^2. \end{aligned} \tag{2.5}$$

where  $\mathcal{M}_{g^*g^* \rightarrow Q\bar{Q}}^{\text{off-shell}}$  is the off-shell matrix element for the hard subprocess [25], we use its implementation from [34].

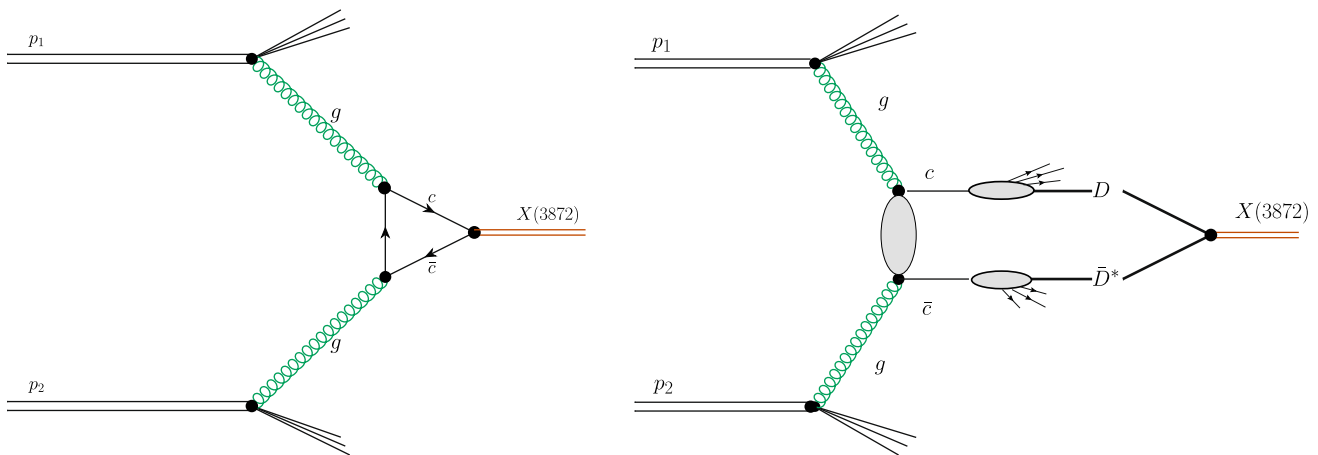
Here, one keeps exact kinematics from the very beginning and additional hard dynamics coming from transverse momenta of incident partons. Explicit treatment of the transverse momenta makes the approach very efficient in studies of correlation observables. The two-dimensional Dirac delta function assures momentum conservation. The gluon uPDFs must be evaluated at longitudinal momentum fractions:

$$x_1 = \frac{m_{T1}}{\sqrt{s}} \exp(+y_1) + \frac{m_{T2}}{\sqrt{s}} \exp(+y_2), \tag{2.6}$$

$$x_2 = \frac{m_{T1}}{\sqrt{s}} \exp(-y_1) + \frac{m_{T2}}{\sqrt{s}} \exp(-y_2), \tag{2.7}$$

where  $m_{Ti} = \sqrt{p_{Ti}^2 + m_c^2}$  is the quark/antiquark transverse mass.

In the present analysis we employ the heavy  $c$ -quark approximation and assume that three-momenta in the  $pp$ -



**Fig. 1** Generic diagrams for the inclusive process of  $X(3872)$  production in proton–proton scattering via two gluons fusion

cm frame are equal:

$$\vec{p}_D = \vec{p}_c. \tag{2.8}$$

This approximation could be relaxed in future.

We now should take into account the fragmentation into  $D, D^*$ -mesons. The fragmentation fractions fulfill the sum rule:

$$\sum_i f(c \rightarrow H_i) = 1. \tag{2.9}$$

In this formula  $H_i$  are the final (after strong decays) hadrons. Therefore the spin-1  $D^*$  mesons should not be included here as it would lead to double counting. The final charmed particles are only those which have only weak decays:  $D^+, D^0, D_s^+, \Lambda_c$ , etc.

The  $D^0$  (or  $\bar{D}^0$ ) are produced directly or come from the decays of spin-1 mesons (see e.g. [5]):

$$\text{Br}(D^{*0} \rightarrow D^0) = 1, \quad \text{Br}(D^{*+} \rightarrow D^0) = 0.68. \tag{2.10}$$

In Ref. [35] the total fragmentation probability for  $c \rightarrow D^0$  is extracted. Different values are given in the literature:

$$f(c \rightarrow D^0) = 0.54 - 0.63. \tag{2.11}$$

The total uncertainties is however less than  $\sim 10\%$ .

The total probability can be decomposed as the sum:

$$f(c \rightarrow D^0) = f(c \rightarrow D^0)|_{\text{direct}} + f(c \rightarrow D^0)|_{\text{feeddown}}. \tag{2.12}$$

The direct component can be approximated as:

$$f(c \rightarrow D^0)|_{\text{direct}} \approx f(c \rightarrow D^\pm)|_{\text{direct}}, \tag{2.13}$$

assuming isospin symmetry.

Let us calculate therefore the feeddown probability:

$$\begin{aligned} f(c \rightarrow D^0)|_{\text{feeddown}} &= f(c \rightarrow D^{*0})\text{Br}(D^{*0} \rightarrow D^0) \\ &\quad + f(c \rightarrow D^{*+})\text{Br}(D^{*+} \rightarrow D^0), \\ f(c \rightarrow D^+)|_{\text{feeddown}} &= f(c \rightarrow D^{*+})\text{Br}(D^{*+} \rightarrow D^+). \end{aligned} \tag{2.14}$$

Then the direct contributions can be calculated from

$$f(c \rightarrow D^0)|_{\text{direct}} = f(c \rightarrow D^0) - f(c \rightarrow D^0)|_{\text{feeddown}}, \tag{2.15}$$

$$f(c \rightarrow D^+)|_{\text{direct}} = f(c \rightarrow D^+) - f(c \rightarrow D^+)|_{\text{feeddown}}. \tag{2.16}$$

For definiteness, in our numerical calculations we will use

$$f(c \rightarrow D^0) = f(\bar{c} \rightarrow \bar{D}^0) = 0.547, \tag{2.17}$$

$$f(c \rightarrow D^+) = f(\bar{c} \rightarrow D^-) = 0.227, \tag{2.18}$$

$$f(c \rightarrow D^{*0}) = f(\bar{c} \rightarrow \bar{D}^{*0}) = 0.237, \tag{2.19}$$

$$f(c \rightarrow D^{*+}) = f(\bar{c} \rightarrow \bar{D}^{*-}) = 0.237. \tag{2.20}$$

These numbers are from a fit to the fragmentation ratios found in Table 5 of Ref. [35], and we have assumed that  $f(c \rightarrow D^{*0}) = f(c \rightarrow D^{*+})$ . These fractions are close to the ones found in Table 1 of Ref. [36] where they have overlap.

We then obtain an isospin symmetric result:

$$f(c \rightarrow D^0)|_{\text{direct}} = 0.15, \quad f(c \rightarrow D^+)|_{\text{direct}} = 0.15. \tag{2.21}$$

We summarize that the direct contribution is much smaller than the total one including feeddown. One may debate which of the two should be used, given the fact that in the feeddown contribution the  $D^0$  will be accompanied by a decay pion close by in phase space. In the following we shall show results obtained from the total fragmentation fraction for

$D^0$ 's as well as using only the direct production probabilities. Arguably, the true result should be in between the two predictions.

The cross section for  $c\bar{c}$  production are then multiplied by

$$\frac{1}{2}[f(c \rightarrow D^0)f(\bar{c} \rightarrow \bar{D}^{*0}) + f(c \rightarrow D^{*0})f(\bar{c} \rightarrow \bar{D}^0)] = \begin{cases} 0.036 \text{ direct} \\ 0.13 \text{ including feeddown.} \end{cases} \quad (2.22)$$

Furthermore, when comparing to experimental data we need to multiply by the relevant branching fraction i.e. by  $\text{BR}(X \rightarrow J/\psi\pi^+\pi^-)$  for the case of CMS and LHCb, and by  $\text{BR}(X \rightarrow J/\psi\pi^+\pi^-)\text{BR}(J/\psi \rightarrow \mu^+\mu^-)$  for the ATLAS data. The branching fractions are taken from [5]. The factor  $\frac{1}{2}$  is related to the factor  $\frac{1}{\sqrt{2}}$  in the definition of the molecular wave function,

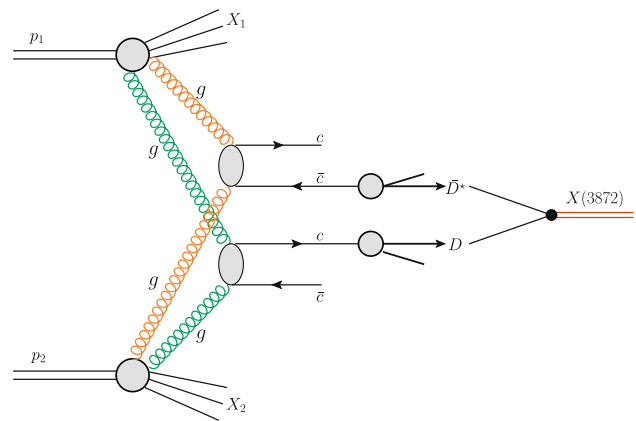
$$|\Psi_{mol}\rangle = \frac{1}{\sqrt{2}} (|D^0\bar{D}^{*0}\rangle + |D^{*0}\bar{D}^0\rangle). \quad (2.23)$$

According to our knowledge, both directly produced  $D^0$  mesons as well as  $D^0$  mesons coming from the decay of  $D^*$  mesons are included in the calculation of formation of the  $X(3872)$  molecule in the literature, see e.g. [16,22]. There is a question whether the  $D^0$  mesons coming from decays of  $D^*$  mesons should be included in the formation of the  $D^0 - \bar{D}^{*0}$  molecules. The lifetime of  $D^*$  mesons is rather short and cannot be directly measured. However, in the literature [37] a calculation of the decay width for  $D^{0*} \rightarrow D^0 + \pi^0$  is available. The authors find  $\Gamma(D^{0*} \rightarrow D^0\pi^0) = 0.05 \text{ MeV}$ . The branching fraction for this channel is about 60 % [5]. Therefore one can estimate that the total decay width of  $D^{0*}$  is of the order of 0.1 MeV. Then the lifetime of the vector  $D$  mesons is about  $10^{-20} \text{ s}$ , which corresponds to a decay length of  $D^{0*} c\tau \sim 2000 \text{ fm}$ , long enough for the  $D^{*0}$  to form the molecule. However, it is not clear whether  $D^0$ s from the decay of the vector mesons may participate in forming the molecule. It seems a sizeable part of the  $D^0$  mesons from the decay cannot contribute to the  $D^0 - \bar{D}^0$  molecule production. However, exact estimation of the probability goes beyond the scope of the present paper. Therefore in the following we shall present both limits:

1. including all  $D^0$ s (as done previously in the literature),
2. including only directly produced  $D^0$ s.

In our calculation, we control the dependence on the relative momentum of quark and antiquark in the rest frame of the pair:

$$k_{rel} = \frac{1}{2}\sqrt{M_{c\bar{c}}^2 - 4m_c^2}, \quad (2.24)$$



**Fig. 2** A generic diagram for the inclusive process of  $X(3872)$  production in proton-proton scattering via the double parton scattering mode

where  $M_{c\bar{c}}$  is invariant mass of the  $c\bar{c}$  system and  $m_c$  is the quark mass. In order to obtain an upper bound for the molecule production cross section, we should integrate the  $D\bar{D}^*$  cross section over the relative momentum  $k_{rel}^{D\bar{D}^*}$  up to a cutoff  $k_{max}^{D\bar{D}^*}$  [17]. We will instead impose a cutoff  $k_{max}$  on the relative momentum  $k_{rel}$ . Within our kinematics the latter will be similar to  $k_{rel}^{D\bar{D}^*}$ , but somewhat larger. In reality, for larger  $\vec{p}_{TX} \approx \vec{p}_{T1} + \vec{p}_{T2}$ , we have  $k_{DD} < k_{rel}$ . We therefore estimate, that  $k_{max} = 0.2 \text{ GeV}$  corresponds roughly to  $k_{max}^{DD} \approx 0.13 \text{ GeV}$ . A better approximation would be to add simultaneous  $c \rightarrow D, \bar{c} \rightarrow \bar{D}$  fragmentation to our Monte Carlo code, which however means at least two more integrations. We do not consider here any model of the  $D\bar{D}^*$  wave function.

What is the appropriate choice for  $k_{max}^{D\bar{D}^*}$  was a matter of discussion in the literature. In Ref. [16] it was suggested, that  $k_{max}^{D\bar{D}^*}$  should be of the order of the binding momentum  $k_X = \sqrt{2\mu\varepsilon_X}$ , where  $\mu$  is the reduced mass of the  $D\bar{D}^*$ -system, and  $\varepsilon_X$  is the binding energy of  $X(3872)$ . This would lead to a very small value for  $k_{max}^{D\bar{D}^*}$ , similar to  $k_{max}^{D\bar{D}^*} = 35 \text{ MeV}$  used in [16]. However problems with this estimate have been pointed out in [17–19]. As has been argued in these works, the integral should be extended rather to a scale  $k_{max}^{D\bar{D}^*} \sim m_\pi$ . In our choice of  $k_{max}$ , we follow this latter prescription.

In the following for illustration we shall therefore assume  $k_{max} = 0.2 \text{ GeV}$ . The calculation for the SPS molecular scenario is done using the VEGAS algorithm for Monte-Carlo integration [38].

We also include double parton scattering contributions (see Fig. 2).

The corresponding cross section is calculated in the so-called factorized ansatz as:

$$\Delta\sigma = \frac{1}{2\sigma_{\text{eff}}} \int \frac{d\sigma_{c\bar{c}}}{dy_1 d^2\vec{p}_{T1}} \times \frac{d\sigma_{c\bar{c}}}{dy_2 d^2\vec{p}_{T2}} dy_1 d^2\vec{p}_{T1} dy_2 d^2\vec{p}_{T2} \Big|_{k_{rel} < k_{max}}. \quad (2.25)$$

Above the differential distributions of the first and second parton scattering  $\frac{d\sigma}{dy_i d^2 p_{T_i}}$  are calculated in the  $k_T$ -factorization approach as explained above. In the following we take  $\sigma_{\text{eff}} = 15 \text{ mb}$  as in [39]. The differential distributions (in  $p_T$  of the  $X(3872)$  or  $y_{\text{diff}} = y_1 - y_2$ , etc.) are obtained by binning in the appropriate variable. We include all possible fusion combinations leading to  $X(3872)$ :

$$c_1 \rightarrow D^0, \bar{c}_2 \rightarrow \bar{D}^{*0}, \tag{2.26}$$

$$c_1 \rightarrow D^{*0}, \bar{c}_2 \rightarrow \bar{D}^0, \tag{2.27}$$

$$\bar{c}_1 \rightarrow \bar{D}^0, c_2 \rightarrow D^{*0}, \tag{2.28}$$

$$\bar{c}_1 \rightarrow \bar{D}^{*0}, c_2 \rightarrow D^0. \tag{2.29}$$

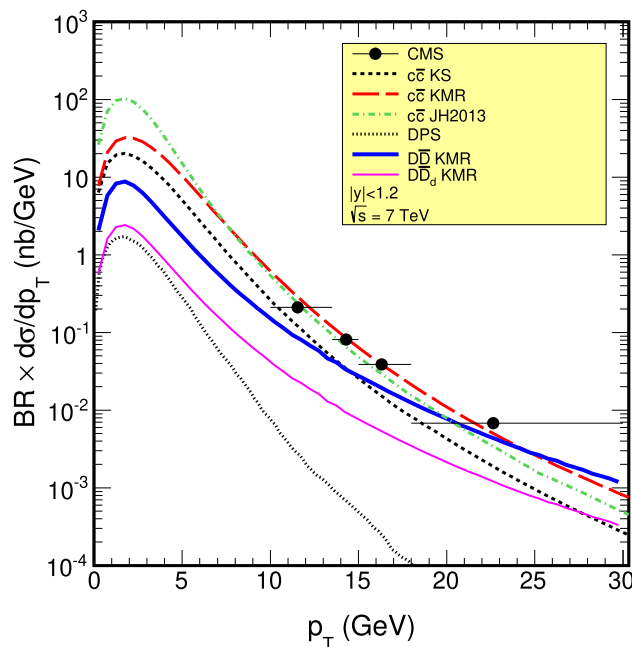
This leads to the multiplication factor two times bigger than for the SPS contribution (see Eq. (2.22)).

### 3 Results

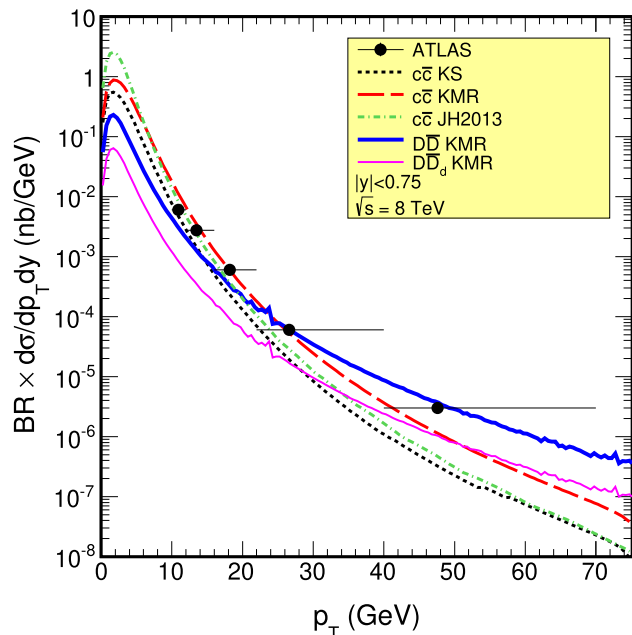
In this section we shall show our results for recent CMS [13] ATLAS [14] and LHCb [15] data. The CMS data is for  $\sqrt{s} = 7 \text{ TeV}$  and  $-1.2 < y_X < 1.2$ , the ATLAS data for  $\sqrt{s} = 8 \text{ TeV}$ ,  $-0.75 < y_X < 0.75$  and the LHCb data for  $\sqrt{s} = 13 \text{ TeV}$ ,  $2 < y_X < 4.5$ . In all cases the  $X(3872)$  was measured in the  $J/\psi \pi^+ \pi^-$  channel. We have used a number of different unintegrated gluon distributions, firstly a distribution obtained from the solution of a BFKL equation with kinematic constraints by Kutak and Stařto denoted KS [40], secondly a gluon uPDF obtained from a modified Kimber–Martin–Ryskin (KMR) procedure [41–43] based on Durham group collinear PDFs [44]. Finally we also employ a gluon uPDF obtained by Hautmann and Jung [45] from a description of precise HERA data on deep inelastic structure function by a solution of the CCFM evolution equations.

#### 3.1 $c\bar{c}$ state

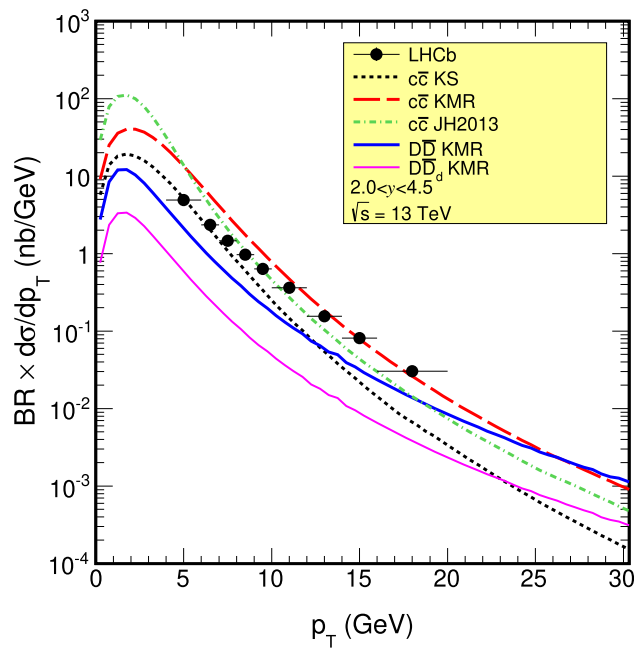
Here we show our results for  $\chi_{c,1}(3872)$  production treated as a pure  $c\bar{c}$  state. In Figs. 3, 4 and 5 we show the transverse momentum distribution of the  $X(3872)$  together with the data from the CMS, ATLAS and LHCb experimental data. A surprisingly good description is obtained with different gluon uPDFs specified in the figure legend without any free parameters. It is worth to mention approximately good slope of the  $p_{TX}$  distributions which is due to effective inclusion of higher-order corrections. The corresponding result within the collinear leading-order approximation would be equal to zero! A slightly different slope is obtained for the molecular scenario (solid line) discussed in detail somewhat below.



**Fig. 3** Transverse momentum distribution of  $X(3872)$  for the CMS experiment. Shown are results for 3 different gluon uPDFs. Here  $\text{BR} = 0.038$ . The upper limit for the SPS molecular scenario is shown as the thick solid line. The thin solid line shows the molecular scenario neglecting the feeddown component of  $D^0, \bar{D}^0$ . We also show corresponding distribution for the DPS mechanism (dotted line)



**Fig. 4** Transverse momentum distribution of  $X(3872)$  for the ATLAS experiment. Shown are results for 3 different gluon uPDFs. Here  $\text{BR} = 0.038 \cdot 0.0596$ . The upper limit for the SPS molecular scenario is shown as the thick solid line. The thin solid line shows the molecular scenario neglecting the feeddown component of  $D^0, \bar{D}^0$



**Fig. 5** Transverse momentum distribution of  $X(3872)$  for the LHCb experiment. Shown are results for 3 different gluon uPDFs. Here  $BR = 0.038$ . The upper limit for the SPS molecular scenario is shown as the thick solid line. The thin solid line shows the molecular scenario neglecting the feeddown component of  $D^0, \bar{D}^0$

### 3.2 Molecular picture

Here we show our predictions for  $\chi_{c1}(3872)$  production treated as the  $\bar{D}^{*0}D^0 + h.c.$  molecule. Then the  $\chi_{c1}(3872)$  can be produced if  $k_{rel}$  is small, and an estimate (or upper bound) for its production cross section can be obtained from the continuum cross section at small  $k_{rel}$ . In Fig. 6 we show the distribution in this variable for different windows of  $|\vec{p}_{T,c\bar{c}}|$  (0,5 GeV), (5,10 GeV), (10,15) GeV, (15,20) GeV, (20,25) GeV, (25,30) GeV.

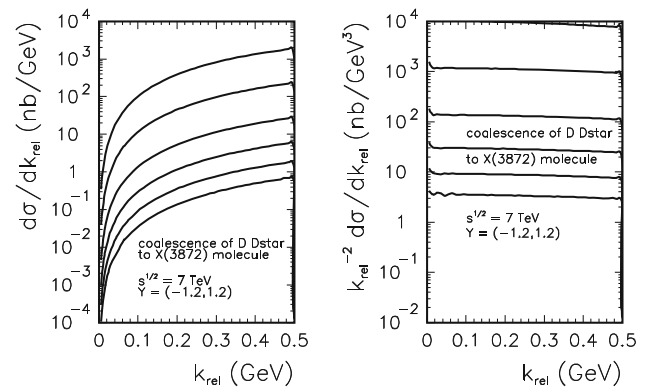
Similar shapes in  $k_{rel}$  are obtained for the different windows of  $p_{T,c\bar{c}}$ .

To visualize this better we show in the right panel of Fig. 6  $k_{rel}^{-2}d\sigma/dk_{rel}$ . As expected, the so-obtained distributions closely follow phase-space, and are almost flat in a broad range of  $k_{rel}$ . Therefore, the cross section has essentially the phase-space behaviour

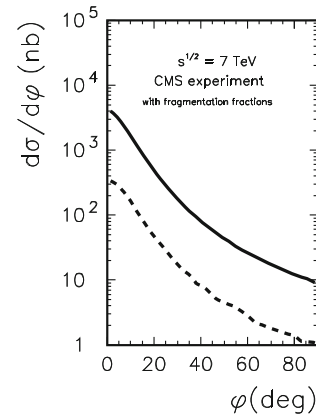
$$d\sigma \propto k_{rel}^2 dk_{rel}, \tag{3.1}$$

which implies the strong, cubic dependence  $\propto k_{max}^3$  on the upper limit  $k_{max}$  of the  $k_{rel}$ -integration.

The calculation in the whole phase space  $p_{T1} \in (0, 20)$  GeV and  $p_{T2} \in (0, 20)$  GeV leads to fluctuations at large  $p_{T,c\bar{c}} > 20$  GeV. This can be understood as due to steep dependence of the cross section on  $p_{T1}, p_{T2}, y_1, y_2, \phi$ . Only a narrow range in the  $p_{T1} \otimes p_{T2}$  space with  $p_{T1} \approx p_{T2}$  fulfills the condition  $k_{rel} < k_{max} = 0.2$  GeV.



**Fig. 6** Distribution in  $k_{rel}$  for different windows of  $p_{T,c\bar{c}} = p_{T,X}$  (left panel) for the CMS kinematics as specified in the main text. In the right panel we show the cross sections divided by  $k_{rel}^2$ . In these calculations the KMR UGDF with the MMHT NLO collinear gluon distribution was used. Branching fractions are included here



**Fig. 7** Azimuthal correlations between  $c\bar{c}$  that fulfill the condition  $k_{rel} < 0.2$  GeV. Here the CMS cuts were imposed. We show contribution of SPS (solid line) and DPS (dashed line)

We remind the reader that this value of  $k_{max}$  is imposed on the  $c\bar{c}$  final state, and would correspond to a smaller  $k_{max}^{DD^*} \sim 0.14$  GeV for the  $DD^*$  mesons. Due to the behaviour shown in Eq. (3.1), the cross section for  $k_{max} = 0.14$  GeV or  $k_{max} = 0.1$  GeV imposed on the  $c\bar{c}$  state would go down by a factor three or eight, respectively.

The distribution in relative azimuthal angle between  $c\bar{c}$  in the  $pp$ -frame with the cut  $k_{rel} < 0.2$  GeV is shown in Fig. 7. This is rather a steep distribution around  $\phi = 0^\circ$ . This is not a typical region of the phase space. Recall, that in the leading-order collinear approach  $\phi = 180^\circ$ . It is obvious that the region of  $\phi \approx 0$  cannot be obtained easily in the collinear approach. As discussed in [34] the  $k_T$ -factorization approach gives a relatively good description of  $D^0\bar{D}^0$  correlations in this region of the phase space.

Having understood the kinematics of the  $X(3872)$  production we can improve the description of transverse momentum distribution of  $X(3872)$ . The rather strong correlation

$p_{T1} \approx p_{T2}$  allows to perform VEGAS calculation simultaneously limiting to  $p_{T1}, p_{T2} > p_{Tmin}$ . We have also verified that

$$p_{T1} + p_{T2} \approx p_{Tc\bar{c}} = |\vec{p}_{T1} + \vec{p}_{T2}|. \quad (3.2)$$

Therefore imposing lower limits on  $p_{T1} > p_{Tmin}$  and  $p_{T2} > p_{Tmin}$  means also lower limit on  $p_{Tc\bar{c}} > 2p_{Tmin}$ . The solid line in Fig. 3 shows result of such a calculation. The fluctuations are gone. Combining the two calculations at say  $p_{Tc\bar{c}} = 15 \text{ GeV}$  gives a smooth result everywhere.

The condition on relative momentum of  $D$  and  $D^*$  mesons selects mesons flying almost parallel to each other. In such a system, in somewhat naive calculation (non-interacting mesons), the probability that together with a  $D^0$  there exists a  $D^{*0}$  that has not decayed yet to produce a  $X(3872)$  at a time  $t$  can be estimated as  $P = (1 - \exp(-t/\tau)) \exp(-t/\tau) < 0.25$ .

This suggests that in reality one should rather include only directly produced  $D^0$  (or  $\bar{D}^0$ ). This strongly reduces the cross section and causes that the purely molecular scenario is disfavoured – the corresponding  $d\sigma/dp_T$  is below the experimental data. Finally we conclude that the mechanism of reaction prefers  $X(3872)$  to be produced via its  $c\bar{c}$  component.

## 4 Conclusions

We have performed the calculation of  $X(3872)$  production at the LHC energies. We have performed two independent calculations: one within nonrelativistic QCD approach assuming pure  $c\bar{c}$  state and second assuming a coalescence of  $D$  and  $\bar{D}^*$  or  $\bar{D}$  and  $D^*$ , consistent with molecular state assumption. The first calculation requires usage of derivative of the  $c\bar{c}$  wave function. In the present analysis we have used the wave function obtained in [24]. The resulting cross section was calculated within the  $k_T$ -factorization approach with a few unintegrated gluon distributions. In the second approach first the hard production of a  $c\bar{c}$  pair is calculated. Next a simple hadronization is performed giving a correlation distribution of  $D$  and  $D^*$  mesons. Imposing limitations (upper limit) on relative momenta of  $D$  and  $\bar{D}^*$  we get a  $p_T$ -dependent upper limit of the cross section for  $D-\bar{D}^*$  or  $\bar{D}-D^*$  fusion (coalescence). We compare, for the first time, to all available experimental data on the  $p_T$ -dependent cross section.

In the case of the molecular scenario we have presented results including both direct fragmentation of  $D^0$  (or  $\bar{D}^0$ ) and feeddown contributions as well as results with direct contribution alone. In the latter case the molecular scenario would be rather in disagreement with the CMS, ATLAS and LHCb data.

In the molecular scenario there is an important issue whether one should take all  $D^0$  (or  $\bar{D}^0$ ) or only the directly produced ones  $D^0$  (or  $\bar{D}^0$ ) into account. In the literature calculations based on Monte Carlo generators include both feeddown and direct components. However, these standard evaluations do not take into account time scales of the processes involved. The  $D^0$  from the decay is produced after some time, which is the lifetime of vector  $D^*$  mesons. The associated vector  $\bar{D}^{*0}$  (or  $D^{*0}$ ) mesons must, however, survive during this time in order to form the molecule. On average these two facts do not happen in coincidence. At short times  $D^0$  (or  $\bar{D}^0$ ) is not produced, at large times  $\bar{D}^{*0}$  (or  $D^{*0}$ ) are not existing. Only at intermediate times the two facts could happen in coincidence. As discussed in our paper such cases (coincidences) are strongly reduced compared to the standard calculations.

A better study would require a detailed treatment of fragmentation and decays of spin-one  $D^*$  mesons which however goes beyond the scope of the present analysis.

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**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors' comment: Theoretical curves can be obtained from the authors upon request.]

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