




Letter

Ground-state inversion: The monopole-force governance in neutron mid-shell region

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ABSTRACT

This study analyzes the evolution of single-proton states within the odd-*A* isotopes of ^{117–133}Sb. We identified the monopole force interaction between the $\pi g_{7/2}$ and $\nu h_{11/2}$ orbitals as the key driver of the ground-state inversion observed in ¹²¹Sb. Moreover, we provide an additional asset for this monopole force by investigating the negative-parity states in ¹³²Sb and demonstrating its manifestation through a comparative assessment of $\pi g-\nu h$ multiplets. Our results provide conclusive evidence, for the first time, explaining the ground-state inversion in ¹²¹Sb through observations in ¹³²Sb. The increasing occupation and configurations of the orbit $\pi d_{5/2}$ revealed that the single-proton property of state $5/2^+$ remained robust even in the neutron mid-shell region.

Over a century ago, the Rutherford scattering experiment provided useful insights into the atomic structure by revealing that atoms contain a tiny core that primarily accounts for their mass (Rutherford, 1911) [1]. In nuclear structures, the magic numbers of protons and neutrons (Z and $N = 2, 8, 20, 28, 50, 82$, or $N = 126$) are analogous to noble gases in atomic physics. These magic numbers signify the effects of the filled major shells, and the shell model explains the existence of shell closures at these occupation numbers [2]. In the shell model theory, protons (neutrons) move freely in orbits with discrete quantum numbers, as independent particles subject to a mean field caused by all other nucleons. If a nucleon exists outside the double-shell closure, different single-particle states can be formed. Investigating these single-particle states is essential for improving nuclear models and predicting the properties of unmeasured nuclei.

In recent years, nuclei near the doubly magic nucleus ¹³²Sn have been widely investigated, both experimentally and theoretically [3–7]. The properties of neutron-rich nuclei near the $N = 82$ shell are important for understanding the formation of the second peak $A \approx 130$ in the *r*-process [8,9]. Approximately half of the heavy elements (heavier than iron) are synthesized during this process [10,11]. The shell model

has been successfully applied to neutron-rich nuclei near the doubly magic ¹³²Sn [12–16]. However, large-scale shell model calculations are unsuitable for nuclei far away from the shell closure because of the tremendous dimensions caused by more valence nucleons and the sharp increase in errors. Thus, the present Hamiltonian must be improved through the monopole force and evolution of single-particle states outside the shell closure [17,18]. The effects of tensor forces have been discussed for monopole-driven shell evolution [19–24]. The observed ground-state inversions from ¹³⁰In (¹²⁹Cd) to ¹²⁸In (¹²⁷Cd) can be explained by the monopole force between the neutron orbits $h_{11/2}$ and $d_{3/2}$ in the hole nucleus region of ¹³²Sn [25]. The ground-state inversion in ¹²⁹Cd₈₁ was predicted to be driven by an attractive monopole force [26] and subsequently verified in Ref. [27].

In this study, we investigated the proton single-particle evolution in odd-*A* Sb isotopes to extend the interaction model to include the nuclei region near the middle of the shell closure. With one proton above the $Z = 50$ shell, the Sb isotopes allow the study of single-proton evolution in the proton model space between $Z = 50$ and 82. We used a model space with a proton (neutron) shell $Z(N) = 50 \sim 82$. Two additional neutron orbits above the $N = 82$ shell, ($1f_{7/2}$ and $2p_{3/2}$), were

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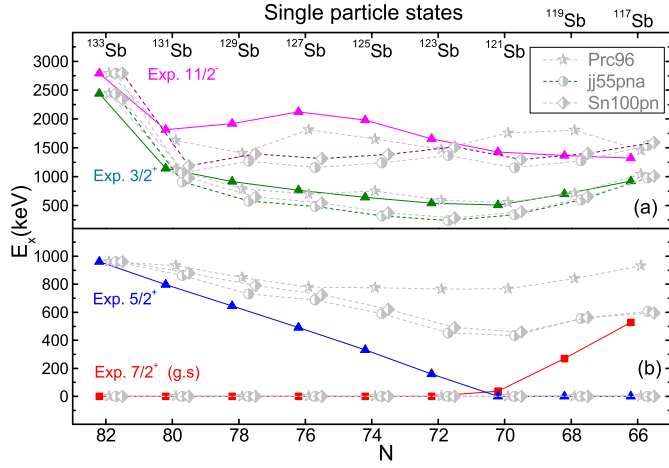


Fig. 1. Single particle states in Sb isotopes. The shell model calculations are from Ref. [31,28] while the corresponding data are from Ref. [32].

included to allow neutron cross-shell excitation. The single-particle energies and employed two-body force strengths employed were consistent with those used in our previous study [28]. The shell-model code NUSHELLX@MSU was used for the calculations [29].

The single-particle energy state in an orbit j is obtained from the kinetic energy and closed-shell effects on this orbit. The single-particle energy changes when the nucleons occupy another orbit, j' . The monopole component of an interaction is [30]:

$$V_{jj'}^T = \frac{\sum_J (2J+1) \langle jj' | V | jj' \rangle_{JT}}{\sum_J (2J+1)}. \quad (1)$$

Here, $\langle jj' | V | jj' \rangle_{JT}$ is a matrix element of the coupled state with angular momentum J and isospin T . If one proton is in the orbit j and some neutrons occupy orbit $j' (j \neq j')$, the energy shift of a single-particle state is given by

$$\Delta \varepsilon_p(j) = 1/2 \{ V_{jj'}^{T=0} + V_{jj'}^{T=1} \} n_n(j'). \quad (2)$$

Here, $n_n(j')$ denotes the number of neutrons in orbit j' . The monopole effects of other orbits should be considered when they are filled with neutrons. The single-particle energy with monopole effects is called the effective single-particle energy and depends on its configuration. The monopole effects were subsequently studied by employing monopole correction (Mc) terms:

$$Mc = k_{mc}(j, j') \sum_{JM} A_{JM}^\dagger(j, j') A_{JM}(j, j'). \quad (3)$$

Here, A_{JM}^\dagger and A_{JM} are the pair operators, while k_{mc} is the monopole force strength. The force strength k_{mc} with a positive (negative) value indicates repulsive (attractive) monopole effects when nucleons are considered to occupy the particle states. If protons (or neutrons) are considered as the hole states, the force direction should be changed to the opposite direction.

As shown in Fig. 1, the low-lying states are experimentally available for Sb isotopes. The shell model calculations were based on three different interactions, namely, jj55pna, Sn100pn [31] and Prc96 [28], which share a common model space defined by proton (and neutron) shells spanning $Z = [50, 82]$ ($N = [50, 82]$). Owing to computational constraints, we restricted the lower neutron orbits, specifically $\nu g_{7/2}$ and $\nu d_{5/2}$. The results of these calculations were consistent with the experimental data for nuclei close to the double-magic nucleus, ^{132}Sn . The theoretical difference increased when the number of neutrons decreased from $N = 82$ as the number of neutron holes increased. For instance, we considered the $5/2^+$ state of ^{129}Sb , which had an energy of 847 keV in our calculations, compared with its experimental value

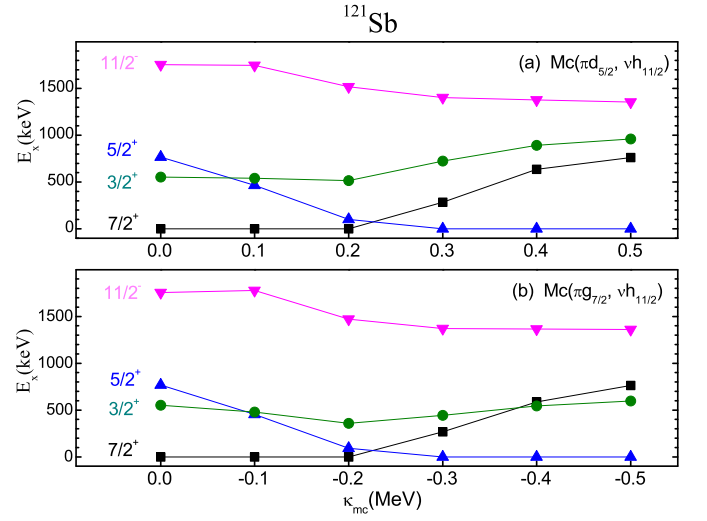


Fig. 2. Single particle states in ^{121}Sb with monopole corrections.

of 645 keV. Although a 202 keV difference is acceptable for the shell model theory, the ground state inversion observed in ^{121}Sb (Fig. 1) remains beyond the capabilities of the current shell model framework. To date, computational resources limit large-scale shell model calculations for describing nuclei far from the shell closure. This necessitates innovative techniques and improvements using our modeling techniques to thoroughly understand and predict the behavior of exotic nuclei.

To address the behavior of the nuclei situated in the middle of the major shell, $N = [50, 82]$, we determined the implicit monopole effects within the experimental data. In the context of this investigation, the monopole force is important for improving the configurations of the $\pi d_{5/2}$ orbit, eventually leading to the ground-state inversion observed in ^{121}Sb . We primarily focused on the monopole interaction between the proton orbit $\pi g_{7/2}$ (or $\pi d_{5/2}$) and the neutron orbit $\nu h_{11/2}$. As illustrated in Fig. 2(a), the monopole terms denoted as $\text{Mc}(\pi d_{5/2}, \nu h_{11/2})$ effectively reproduce the ground-state inversion when the force strength parameter κ_{mc} exceeds 0.2 MeV. A positive value of the force strength indicates repulsive monopole effects when the nucleons are considered particle states. In the present interaction, protons and neutrons were particle and hole states, respectively, and the monopole force changed direction. Thus, a positive value of κ_{mc} represented attractive monopole effects in the particle-hole nuclei region ($Z \geq 50, N \leq 82$).

In Fig. 2(b), another monopole term is denoted as $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$. This can also reverse the ground state of ^{121}Sb when $\kappa_{mc} < -0.2$ MeV. Here, the negative value denotes a repulsive monopole force between the proton $\pi d_{5/2}$ and neutron $\nu h_{11/2}$ orbits. It is noteworthy that two distinct monopole effects can account for the ground-state inversion observed in ^{121}Sb . The first originates from the attractive monopole effect between $\pi d_{5/2}$ and $\nu h_{11/2}$, while the second originates from the repulsive monopole effect between $\pi g_{7/2}$ and $\nu h_{11/2}$. For the odd-A Sb isotopes, two monopole shifts were incorporated: $\text{Mc}(\pi g_{7/2}, \nu h_{11/2}) = -0.23$ MeV and $\text{Mc}(\pi d_{5/2}, \nu h_{11/2}) = 0.24$ MeV. As shown in Fig. 3, the decreasing trend in the energies of the $5/2^+$ states is effectively reproduced across the Sb isotopes from ^{133}Sb to ^{123}Sb by either $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ or $\text{Mc}(\pi d_{5/2}, \nu h_{11/2})$. The most significant discrepancy occurred in the $5/2^+$ state of ^{125}Sb , where the experimental and calculated values were 332 keV and 265 keV, respectively, with $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ set to -0.23 MeV. Similarly, the inversion of the $7/2^+$ state was also observed for neutron numbers below $N = 70$. In the case of ^{121}Sb , the ground state $7/2^+$ of the Sb isotopes transformed into the first excited state at a low energy of 17 keV when $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ was set to -0.23 MeV. The corresponding experimental value for this state ($7/2^+$) was 37 keV. These results emphasize the significant effect

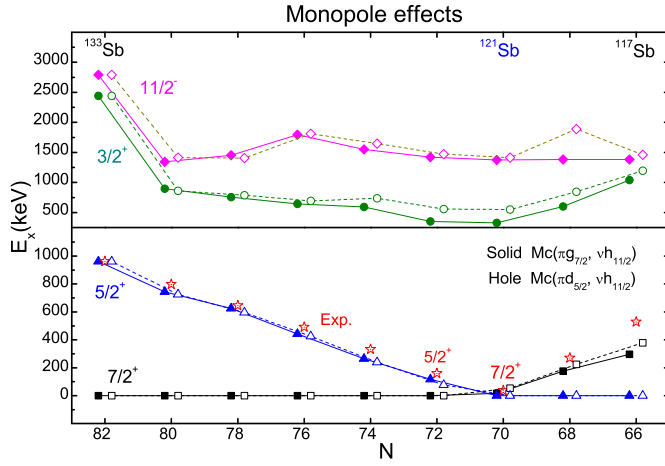


Fig. 3. Single particle states in Sb isotopes with monopole corrections.

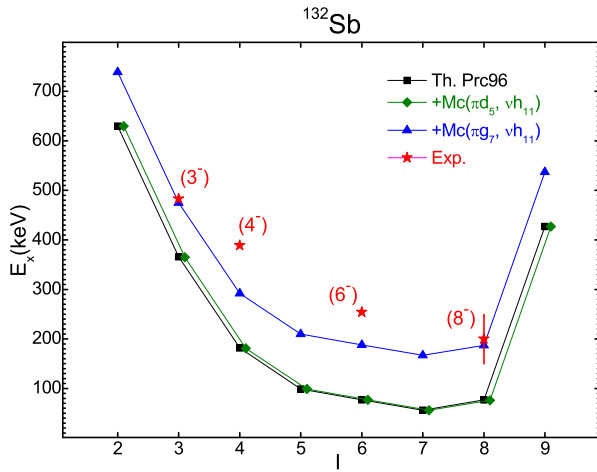


Fig. 4. Level spectra of ^{132}Sb with monopole corrections. The corresponding data are from Ref. [32].

of monopole interactions on the nuclear structure and energy levels of the Sb isotopes.

To determine whether the repulsive or attractive monopole force reverses the single-particle state $5/2^+$, we investigated the data for the odd-odd nucleus ^{132}Sb and confirmed that the monopole force was mainly repulsive. For the tensor interaction of the $\pi + \rho$ meson exchanges, the monopole term between $\pi d_{5/2}$ and $\nu h_{11/2}$ has the opposite sign to the one between $\pi g_{7/2}$ and $\nu h_{11/2}$. Due to the node existence in the $d_{5/2}$ orbital, the magnitude of $\text{Mc}(\pi d_{5/2}, \nu h_{11/2})$ is smaller than $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ by about 5 times in $A \approx 120$ [5]. So, the monopole effect of $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ is dominant over the other one. The odd-odd nuclear ^{132}Sb is suitable to determine their quantitative parameter κ_{mc} . This further highlights the key role of the monopole force in shaping nuclear structure and energy levels.

Fig. 4 shows the calculations using the Prc96 interaction, indicating minimal differences when compared to the experimental values. For example, the 3^- level revealed an energy of 366 keV, whereas the experimental value for the 3^- state was 483 keV. Similarly, the 8^- level was calculated to be 77 keV, with the experimental data denoting a range of 150 ~ 250 keV. The magnitudes of these differences were within the range of approximately 100 keV, which is considered acceptable for shell model calculations. Therefore, they have received little attention in our previous study [21]. Remarkably, the discrepancies in the level spectra of ^{132}Sb were effectively resolved when evaluating the presence of the repulsive monopole term $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ at -0.23 MeV. The quantitative parameter of $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ is confirmed by the data

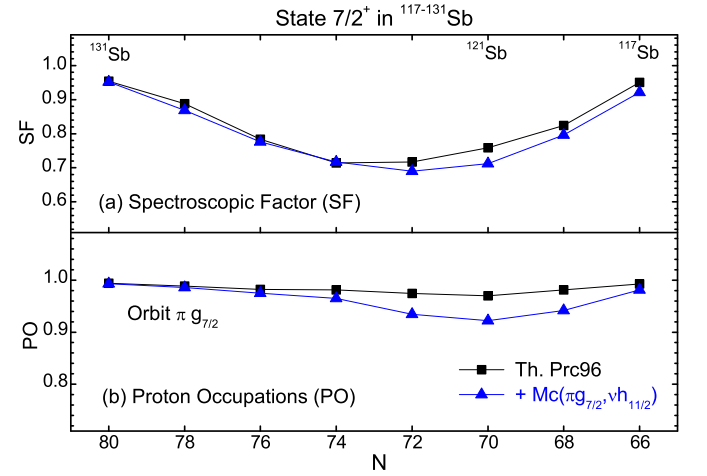


Fig. 5. SFs and POs in the $7/2^+$ state of $^{117-131}\text{Sb}$.

in ^{132}Sb . Meanwhile, the attractive monopole term $\text{Mc}(\pi d_{5/2}, \nu h_{11/2})$ had no noticeable effect on these energy levels (its effects will turn up on the second states of 3^- to 8^-). Consequently, the monopole term $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ appears to be a conclusive and well-supported factor influencing the energy states of (3^-) , (4^-) , (6^-) , and (8^-) in ^{132}Sb .

Interestingly, the repulsive monopole force that reverses the ground state of ^{121}Sb has already been implied in the level spectra of ^{132}Sb . Considering the nuclei region $N < 82$ as the occupation of the hole states (h_n), the hole number increased from $h_n = 0$ ($N = 82$) to $h_n = 16$ ($N = 66$). As shown in Fig. 3(b), the energy difference between levels $7/2^+$ and $5/2^+$ decreases when the number of neutron holes increases. The repulsive monopole force $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ shifted the level $7/2^+$ upwards as the neutron holes increased. This repulsive monopole force is a tensor force between protons and neutrons that quenches the shell gap $Z = 58$. Considering the nuclei region $N < 82$ as being occupied by the particle states, the proton orbit $g_{7/2}$ was shifted down by the attractive tensor force when more neutrons occupied the orbit $h_{11/2}$ (the neutron-particle number increased). The (j_-) and (j'_+) orbits attracted each other. The orbits $g_{7/2}$ and $h_{11/2}$ were from $j = l - 1/2$ ($l = 4$) (j_-) and $j' = 5 + 1/2$ ($l = 5$) (j'_+), respectively. This attractive tensor force continued to shift the proton orbit $g_{7/2}$ downwards as the neutron number increased to the closed shell $N = 82$. As shown in Fig. 3(b), the distance between levels $7/2^+$ and $5/2^+$ increases when the proton orbit $g_{7/2}$ drops owing to an attractive tensor force from the orbit $\nu h_{11/2}$ after more neutrons occupy it.

The monopole term $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ was further studied by investigating the spectroscopic factors (SFs) and proton occupations (POs) in the states $7/2^+$ and $5/2^+$ of $^{117-133}\text{Sb}$ with $\text{Mc}(\pi g_{7/2}, \nu h_{11/2}) = -0.23$ MeV. The SFs of the final state $7/2^+$ (or $5/2^+$) were theoretically derived from the transfer reaction $^A\text{Sn} + p \rightarrow ^{A+1}\text{Sb}$, and the initial state was ground state 0^+ of ^ASn . As shown in Fig. 5, the SFs of the state $7/2^+$ decrease until $N = 72$, and increase when the neutron number $N \leq 70$. The monopole effects of $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ slightly reduce the SFs after the neutron number $N \leq 72$. For example, the SF value of 0.759 decreases to 0.712 in state $7/2^+$ of ^{121}Sb . Considering $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$, the POs in orbit $\pi g_{7/2}$ decrease slightly in $^{123,121,119}\text{Sb}$. The PO 0.97 decreases to 0.92 in ^{121}Sb . This monopole term slightly affected the configuration of state $7/2^+$ in $^{117-133}\text{Sb}$. For example, the state $7/2^+$ in ^{123}Sb had mixed configurations of 36% (26%) $\pi g_{7/2} \nu h_{11/2}^{-8} d_{3/2}^{-2}$, 17% (21%) $\pi g_{7/2} \nu h_{11/2}^{-6} d_{3/2}^{-4}$, and 19% (23%) $\pi g_{7/2} \nu h_{11/2}^{-6} d_{3/2}^{-2} s_{1/2}^{-2}$. The state $7/2^+$ in ^{121}Sb had mixed configurations of 34% (34%) $\pi g_{7/2} \nu h_{11/2}^{-8} d_{3/2}^{-4}$, 11% (7%) $\pi g_{7/2} \nu h_{11/2}^{-10} d_{3/2}^{-2}$, and 31% (30%) $\pi g_{7/2} \nu h_{11/2}^{-8} d_{3/2}^{-2} s_{1/2}^{-2}$. Here, the numbers in brackets include the monopole term of $\text{Mc}(\pi g_{7/2}, \nu h_{11/2}) = -0.23$ MeV.

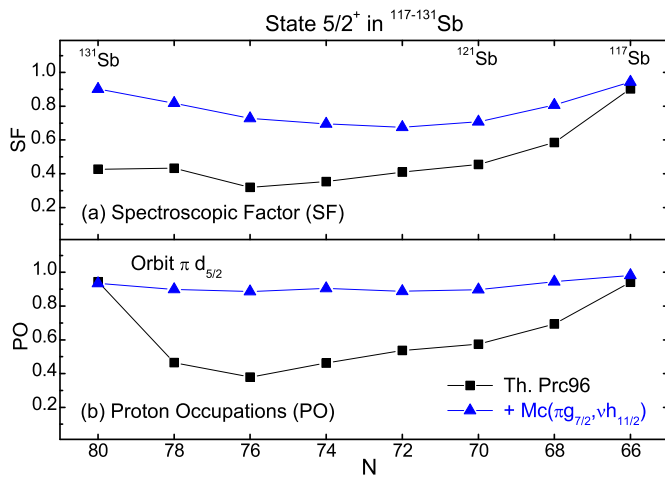


Fig. 6. SFs and POs in the $5/2^+$ state of $^{117-131}\text{Sb}$.

For the states $5/2^+$ from ^{131}Sb to ^{117}Sb , the monopole term $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$ drastically affected both the SF and POs in orbit $\pi d_{5/2}$. In the absence of $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$, the value of SF was approximately 0.40 in $^{131,129,127,125,123}\text{Sb}$, and 0.46 in ^{121}Sb . After including the monopole correction, $\text{Mc}(\pi g_{7/2}, \nu h_{11/2}) = -0.23$ MeV, the SF values drastically increased to 0.90, 0.82, and 0.73 in ^{131}Sb , ^{129}Sb , and ^{127}Sb , respectively. In ^{121}Sb , the SF increased to 0.71 in the state $5/2^+$. The PO of $^{117-131}\text{Sb}$ in orbit $\pi d_{5/2}$ was approximately 1.0 when the above monopole correction was included (Fig. 6). Surprisingly, the single-particle properties remained very good even in the middle of the shell closure after considering $\text{Mc}(\pi g_{7/2}, \nu h_{11/2})$. The monopole term significantly influenced the configuration of state $5/2^+$ in $^{117-131}\text{Sb}$. The state $5/2^+$ of ^{131}Sb had 63% $\pi g_{7/2} \nu h_{11/2}^{-2}$, which increased to 79% upon including $\text{Mc}(\pi g_{7/2}, \nu h_{11/2}) = -0.23$ MeV. In ^{121}Sb , the 20% of $\pi d_{5/2} \nu h_{11/2}^{-8} d_{3/2}^{-4}$ increased to 31%, while the 19% of $\pi d_{5/2} \nu h_{11/2}^{-8} d_{3/2}^{-2} s_{1/2}^{-2}$ increased to 30%. Meanwhile, there were two configurations coupled by orbit $\pi g_{7/2}$ that decreased drastically to approximately 1%, from 10% of $\pi g_{7/2} \nu h_{11/2}^{-8} d_{3/2}^{-4}$ and 11% of $\pi g_{7/2} \nu h_{11/2}^{-8} d_{3/2}^{-2} s_{1/2}^{-2}$. The repulsive monopole force between $\pi g_{7/2}$ and $\nu h_{11/2}$ lowered the configuration with the orbit $\pi g_{7/2}$, whereas the percentage with orbit $\pi d_{5/2}$ increased. These changes in the configurations of the state $5/2^+$ are the structural reasons for the ground-state inversion in ^{121}Sb .

In summary, the evolution of single-particle states in odd-even Sb isotopes was investigated for ^{131}Sb to ^{117}Sb . The shrinking tendency of the single-particle state $5/2^+$ was sufficiently reproduced by the monopole force between $\pi g_{7/2}$ and $\nu h_{11/2}$. The monopole effect provided conclusive evidence for the (3^-) , (4^-) , (6^-) , and (8^-) states of ^{132}Sb . For the first time, the ground-state inversion of ^{121}Sb was explained as an increasing configurative component coupled with the orbit $\pi d_{5/2}$. It shrunk the proton single-particle state $5/2^+$ when $N < 82$ and reversed the excited level $5/2^+$ to the ground state at $N = 70$. During this process, the monopole force governed and significantly increased the SF of the single-particle state $5/2^+$ as well as the PO in orbit $\pi d_{5/2}$. The increasing occupation and configurations of the orbit $\pi d_{5/2}$ revealed that the single-proton property of the state $5/2^+$ remained robust, even far from $N = 82$ shell closure. This study also revealed that the monopole force can sufficiently extend the present Hamiltonian to include the mid-shell region.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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