

Conditions for suppressing dimension-five proton decay in renormalizable SUSY SO(10) GUT

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ABSTRACT: The SUSY SO(10) GUT is in severe tension with the experimental bounds on proton partial lifetimes because proton decay mediated by colored Higgsinos (dimension-five proton decay) is too rapid. In this paper, we pursue the possibility that a texture of the Yukawa coupling matrices in a renormalizable SUSY SO(10) GUT model suppresses dimension-five proton decay. We focus on a general renormalizable SUSY SO(10) GUT model which contains $\mathbf{10} + \mathbf{126} + \overline{\mathbf{126}} + \mathbf{120}$ representation fields and where the Yukawa coupling matrices of the $\mathbf{16}$ matter fields with the $\mathbf{10}$, $\overline{\mathbf{126}}$, $\mathbf{120}$ fields, Y_{10}, Y_{126}, Y_{120} , provide the quark and lepton Yukawa couplings and Majorana mass of the singlet neutrinos. We find that if components in certain flavor bases, $(Y_{10})_{u_R d_R}, (Y_{126})_{u_R d_R}, (Y_{10})_{u_R s_R}, (Y_{126})_{u_R s_R}, (Y_{10})_{u_L d_L}, (Y_{126})_{u_L d_L}, (Y_{10})_{u_L s_L}, (Y_{126})_{u_L s_L}, (Y_{10})_{u_L u_L}, (Y_{126})_{u_L u_L}$, are all on the order of the up quark Yukawa coupling, dimension-five proton decay can be suppressed while the Yukawa coupling matrices still reproduce the realistic quark and lepton masses and flavor mixings. We numerically obtain specific Yukawa coupling matrices satisfying the above conditions, calculate proton partial lifetimes from them and evaluate how dimension-five proton decay is suppressed when these conditions are met.

KEYWORDS: Grand Unification, Supersymmetry, Theories of Flavour

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1 Introduction

The SO(10) grand unified theory (GUT) [1, 2] is a viable extension of the Standard Model (SM) for its attractive features such as the embedding of the SM gauge groups into an anomaly-free group, the unification of one generation of the matter fields into a **16** representation field, and the automatic realization of the seesaw mechanism [3–6] that naturally explains the tiny neutrino mass. The supersymmetric (SUSY) SO(10) GUT can further alleviate the gauge hierarchy problem, and achieve the gauge coupling unification without intermediate scale. A drawback of the SUSY SO(10) GUT is that proton decay mediated by colored Higgsinos (dimension-five proton decay) [7, 8] is too rapid to be consistent with the current experimental bounds on proton partial lifetimes. In particular, since the unification of the top and bottom quark Yukawa couplings implies $\tan\beta \sim 50$, the contribution of $E^c U^c U^c D^c$ operators [9] to the $p \rightarrow K^+ \bar{\nu}_\tau$ decay is enhanced, and since simultaneous cancellations of $E^c U^c U^c D^c$ and $QQQL$ operators' contributions to $p \rightarrow K^+ \bar{\nu}_\tau$, and $QQQL$

operators' contributions to $p \rightarrow K^+ \bar{\nu}_\mu$ are difficult to realize, the SUSY SO(10) GUT is in severe tension with the experimental bound on the $p \rightarrow K^+ \bar{\nu}$ partial lifetime [10].

However, there is a possibility that a texture of the Yukawa coupling matrices suppresses the troublesome dimension-five proton decay, because the decay amplitudes are proportional to bi-products of Yukawa couplings. In this paper, we pursue the above possibility and find conditions for a texture of the Yukawa coupling matrices suppressing dimension-five proton decay. We further obtain specific Yukawa coupling matrices that satisfy the conditions. We focus on a general renormalizable SO(10) GUT model which contains $\mathbf{10} + \mathbf{126} + \overline{\mathbf{126}} + \mathbf{120}$ representation fields from which the Higgs fields of the minimal SUSY SM (MSSM) originate (a broader class of renormalizable SUSY SO(10) GUT models have been studied in refs. [11]–[37]). In the model, the $\mathbf{16}$ matter fields have Yukawa couplings with the $\mathbf{10}, \overline{\mathbf{126}}, \mathbf{120}$ fields, which provide the quark and lepton Yukawa couplings and Majorana mass of the singlet neutrinos. Note that the Yukawa couplings with the $\mathbf{10}, \overline{\mathbf{126}}, \mathbf{120}$ fields are most general, since $\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120}$ and they are the only allowed renormalizable couplings involving a pair of $\mathbf{16}$ matter fields.

In the main body of the paper, we identify those components of the Yukawa coupling matrices that are involved in dimension-five proton decay and that can be on the order of the up quark Yukawa coupling without contradicting that they give the realistic quark and lepton Yukawa couplings. Here the up quark Yukawa coupling is considered as the smallest scale of the components of the Yukawa coupling matrices because it is a specially small Yukawa coupling in the SUSY SO(10) GUT where $\tan \beta \sim 50$. That the components identified above be on the order of the up quark Yukawa coupling, is the desired conditions for a texture suppressing dimension-five proton decay. Next, we obtain specific Yukawa coupling matrices satisfying these conditions, by fitting the experimental data of quark and lepton masses and flavor mixings with the Yukawa coupling matrices of the $\mathbf{10}, \overline{\mathbf{126}}, \mathbf{120}$ fields under the constraint that the components identified above be on the order of the up quark Yukawa coupling. Then we calculate proton partial lifetimes from these Yukawa coupling matrices, and compare them with those calculated from Yukawa coupling matrices that do not necessarily satisfy the conditions. Thereby we evaluate how dimension-five proton decay is suppressed owing to the conditions.

Previously, suppression of dimension-five proton decay by a texture of the Yukawa coupling matrices in the SUSY SO(10) GUT has been studied in refs. [23, 25, 37]. Those papers deal with the case when the active neutrino mass is dominated by the contribution of the Type-2 seesaw mechanism coming from a tiny VEV of the $SU(2)_L$ -triplet component of the $\overline{\mathbf{126}}$ field. However, the dominance of the Type-2 seesaw contribution is not a general situation, since it requires a fine-tuning of a mass term, coupling constants and VEVs of GUT-breaking fields [37, 38] so as not to spoil the gauge coupling unification. Thus, the present paper considers the case when the active neutrino mass is generated solely by the Type-1 seesaw mechanism, with singlet neutrinos coming from the $\mathbf{16}$ matter fields and their Majorana mass from the GUT-breaking VEV of the $\overline{\mathbf{126}}$ field.

This paper is organized as follows: in section 2, we review the general renormalizable SUSY SO(10) GUT model containing $\mathbf{10} + \mathbf{126} + \overline{\mathbf{126}} + \mathbf{120}$ fields, and write the formulas for partial widths of dimension-five proton decay. In section 3, we derive conditions for a

texture of the Yukawa coupling matrices suppressing dimension-five proton decay. It will turn out that not only the texture of the Yukawa coupling matrices, but also a certain texture of the colored Higgs mass matrix is needed to suppress dimension-five proton decay. The latter texture is studied in section 4. In section 5, we numerically obtain specific Yukawa coupling matrices satisfying the conditions found in section 3. We further calculate proton partial lifetimes from them and evaluate how dimension-five proton decay is suppressed when these conditions are met. Section 6 summarizes the paper.

2 General renormalizable SUSY SO(10) GUT model

2.1 Model description

We consider a SUSY SO(10) GUT model that contains single **10**, single pair of **126** + $\overline{\mathbf{126}}$, single **120** fields, denoted by $H, \Delta + \overline{\Delta}, \Sigma$, respectively. The matter fields of MSSM and a singlet neutrino of each generation are unified into a **16** representation field, denoted by $\mathbf{16}^i$ with i being the flavor index. The Yukawa couplings are given by

$$W = (\tilde{Y}_{10})_{ij} \mathbf{16}^i H \mathbf{16}^j + (\tilde{Y}_{126})_{ij} \mathbf{16}^i \overline{\Delta} \mathbf{16}^j + (\tilde{Y}_{120})_{ij} \mathbf{16}^i \Sigma \mathbf{16}^j, \quad (2.1)$$

where $\tilde{Y}_{10}, \tilde{Y}_{126}, \tilde{Y}_{120}$ are Yukawa coupling matrices in the flavor space, and $\tilde{Y}_{10}, \tilde{Y}_{126}$ are complex symmetric and \tilde{Y}_{120} is complex antisymmetric. The quark and lepton Yukawa couplings are assumed to arise solely from eq. (2.1).

Additionally, we introduce single **210** and single **45** fields, denoted by Φ, A , respectively. The Φ, A develop vacuum expectation values (VEVs) to break SU(5) subgroup of the SO(10) while $\Delta + \overline{\Delta}$ develop VEVs to break U(1) subgroup.

When the SO(10) is broken into the SM gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y$, the $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components of $H, \Delta, \overline{\Delta}, \Sigma, \Phi$ yield the Higgs fields of MSSM. Accordingly, $\tilde{Y}_{10}, \tilde{Y}_{126}, \tilde{Y}_{120}$ give the up-type quark, down-type quark, charged lepton and neutrino Dirac Yukawa coupling matrices, Y_u, Y_d, Y_e, Y_D , as

$$W_{\text{eff}} = (Y_u)_{ij} Q_i H_u U_j^c + (Y_d)_{ij} Q_i H_d D_j^c + (Y_e)_{ij} L_i H_d E_j^c + (Y_D)_{ij} L_i H_u N_j^c, \quad (2.2)$$

where the Yukawa coupling matrices satisfy at a GUT breaking scale μ_{GUT} the following relations:

$$Y_u = Y_{10} + r_2 Y_{126} + r_3 Y_{120}, \quad (2.3)$$

$$Y_d = r_1 (Y_{10} + Y_{126} + Y_{120}), \quad (2.4)$$

$$Y_e = r_1 (Y_{10} - 3Y_{126} + r_e Y_{120}), \quad (2.5)$$

$$Y_D = Y_{10} - 3r_2 Y_{126} + r_D Y_{120}, \quad (2.6)$$

where $Y_{10} \propto \tilde{Y}_{10}, Y_{126} \propto \tilde{Y}_{126}, Y_{120} \propto \tilde{Y}_{120}$, and r_1, r_2, r_3, r_e, r_D are complex numbers determined from the mass matrix of the $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components. Hereafter we perform a phase redefinition of fields to make r_1 real positive.

The GUT-breaking VEV of $\overline{\Delta}$, denoted by \bar{v}_R , provides the singlet neutrinos with Majorana mass as

$$W_{\text{Majorana}} = \frac{1}{2} (M_N)_{ij} N_i^c N_j^c, \quad M_N \propto \tilde{Y}_{126} \bar{v}_R. \quad (2.7)$$

Integrating out N_i^c 's, we get the Weinberg operator

$$W_{\text{eff}} = \frac{1}{2}(C_\nu)_{ij} L_i H_u L_j H_u, \quad (2.8)$$

where the Wilson coefficient C_ν satisfies at the scale of the singular values of M_N ,

$$C_\nu = -Y_D M_N^{-1} Y_D^T. \quad (2.9)$$

Eq. (2.8) gives rise to the Type-1 seesaw contribution to the active neutrino mass. In this paper, we assume that the VEV of the $(\mathbf{1}, \mathbf{3}, 1)$ component of $\bar{\Delta}$ is so small that the Type-2 seesaw contribution is negligible compared to the Type-1 seesaw one.

2.2 Dimension-five proton decay

The $(\mathbf{3}, \mathbf{1}, -\frac{1}{3}) + (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$ components of H , Δ , $\bar{\Delta}$, Σ , Φ , which we call colored Higgs fields and denote by H_C^A, \bar{H}_C^B (A, B are labels), induce dimension-five operators responsible for proton decay. After the GUT breaking, the colored Higgs fields have GUT-scale mass \mathcal{M}_{H_C} and Yukawa couplings with matter fields as

$$W_{\text{coloredHiggs}} = \sum_{A,B} \bar{H}_C^A (\mathcal{M}_{H_C})_{AB} H_C^B + \sum_A \frac{1}{2} (Y_L^A)_{ij} Q_i H_C^A Q_j + (\bar{Y}_L^A)_{ij} Q_i \bar{H}_C^A L_j + (Y_R^A)_{ij} E_i^c H_C^A U_j^c + (\bar{Y}_R^A)_{ij} U_i^c \bar{H}_C^A D_j^c. \quad (2.10)$$

Here $Y_L^A, \bar{Y}_L^A, Y_R^A, \bar{Y}_R^A \propto Y_{10}$ when H_C^A, \bar{H}_C^A are components of H , and they are proportional to Y_{126} when H_C^A, \bar{H}_C^A are components of $\bar{\Delta}$. When H_C^A, \bar{H}_C^A are components of Σ , Yukawa couplings $\bar{Y}_L^A, Y_R^A, \bar{Y}_R^A$ are proportional to Y_{120} while Y_L^A vanishes because Y_L^A must be symmetric. In the other cases, all the Yukawa couplings vanish. By integrating out the colored Higgs fields, we obtain dimension-five operators responsible for proton decay

$$W_5 = -\frac{1}{2} C_{5L}^{ijkl} (Q_k Q_l) (Q_i L_j) - C_{5R}^{ijkl} E_k^c U_l^c U_i^c D_j^c, \quad (2.11)$$

where isospin indices are summed in each bracket in the first term. The Wilson coefficients satisfy at the scale of the singular values of \mathcal{M}_{H_C} ,

$$C_{5L}^{ijkl} = \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_L^A)_{kl} (\bar{Y}_L^B)_{ij} - \frac{1}{2} (Y_L^A)_{li} (\bar{Y}_L^B)_{kj} - \frac{1}{2} (Y_L^A)_{ik} (\bar{Y}_L^B)_{lj} \right\}, \quad (2.12)$$

$$C_{5R}^{ijkl} = \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{kl} (\bar{Y}_R^B)_{ij} - (Y_R^A)_{ki} (\bar{Y}_R^B)_{lj} \right\}. \quad (2.13)$$

We write the partial widths of the $p \rightarrow K^+ \bar{\nu}_\tau$, $p \rightarrow K^+ \bar{\nu}_\mu$, $p \rightarrow K^+ \bar{\nu}_e$, $p \rightarrow K^0 \mu^+$, $p \rightarrow K^0 e^+$ decays induced by dimension-five operators (other decay modes will be commented on in the last paragraph of section 3). The partial widths read, for $\beta = e, \mu$, [39]

$$\Gamma(p \rightarrow K^+ \bar{\nu}_\tau) = \frac{m_N}{64\pi} \left(1 - \frac{m_K^2}{m_N^2} \right)^2 \left| \beta_H(\mu_{\text{had}}) \frac{1}{f_\pi} \left\{ \left(1 + \frac{D}{3} + F \right) C_{LL}^{s\tau ud}(\mu_{\text{had}}) + \frac{2D}{3} C_{LL}^{d\tau us}(\mu_{\text{had}}) \right\} + \alpha_H(\mu_{\text{had}}) \frac{1}{f_\pi} \left\{ \left(1 + \frac{D}{3} + F \right) C_{RL}^{ud\tau s}(\mu_{\text{had}}) + \frac{2D}{3} C_{RL}^{us\tau d}(\mu_{\text{had}}) \right\} \right|^2, \quad (2.14)$$

$$\Gamma(p \rightarrow K^+ \bar{\nu}_\beta) = \frac{m_N}{64\pi} \left(1 - \frac{m_K^2}{m_N^2}\right)^2 \left| \beta_H(\mu_{\text{had}}) \frac{1}{f_\pi} \left\{ \left(1 + \frac{D}{3} + F\right) C_{LL}^{s\beta ud}(\mu_{\text{had}}) + \frac{2D}{3} C_{LL}^{d\beta us}(\mu_{\text{had}}) \right\} \right|^2, \quad (2.15)$$

$$\Gamma(p \rightarrow K^0 \beta^+) = \frac{m_N}{64\pi} \left(1 - \frac{m_K^2}{m_N^2}\right)^2 \left| \beta_H(\mu_{\text{had}}) \frac{1}{f_\pi} (1 - D + F) \bar{C}_{LL}^{u\beta us}(\mu_{\text{had}}) \right|^2, \quad (2.16)$$

where the Wilson coefficients of dimension-six operators $C_{RL}, C_{LL}, \bar{C}_{LL}$ satisfy,¹ for $\alpha = e, \mu, \tau$,

$$C_{RL}^{ud\tau s}(\mu_{\text{had}}) = A_{RL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{\mu_H}{m_{\tilde{t}_R}^2} \frac{1}{16\pi^2} \mathcal{F}'(V_{ts}^{\text{ckm}})^* y_t y_\tau C_{5R}^{ud\tau t} |_{\mu=\mu_{\text{SUSY}}}, \quad (2.17)$$

$$C_{RL}^{us\tau d}(\mu_{\text{had}}) = A_{RL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{\mu_H}{m_{\tilde{t}_R}^2} \frac{1}{16\pi^2} \mathcal{F}'(V_{td}^{\text{ckm}})^* y_t y_\tau C_{5R}^{us\tau t} |_{\mu=\mu_{\text{SUSY}}} \quad (2.18)$$

$$C_{LL}^{s\alpha ud}(\mu_{\text{had}}) = A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \frac{1}{16\pi^2} \mathcal{F} g_2^2 \left(C_{5L}^{s\alpha ud} - C_{5L}^{u\alpha ds} \right) |_{\mu=\mu_{\text{SUSY}}}, \quad (2.19)$$

$$C_{LL}^{d\alpha us}(\mu_{\text{had}}) = A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \frac{1}{16\pi^2} \mathcal{F} g_2^2 \left(C_{5L}^{d\alpha us} - C_{5L}^{u\alpha ds} \right) |_{\mu=\mu_{\text{SUSY}}}, \quad (2.20)$$

$$\bar{C}_{LL}^{u\beta us}(\mu_{\text{had}}) = A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \frac{1}{16\pi^2} \mathcal{F} g_2^2 \left(-C_{5L}^{u\beta us} + C_{5L}^{s\beta uu} \right) |_{\mu=\mu_{\text{SUSY}}}, \quad (2.21)$$

and the Wilson coefficients of dimension-five operators satisfy

$$C_{5R}^{ud\tau t}(\mu_{\text{SUSY}}) = A_R^{\tau t}(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{\tau t} (\bar{Y}_R^B)_{ud} - (Y_R^A)_{\tau u} (\bar{Y}_R^B)_{td} \right\} |_{\mu=\mu_{\text{GUT}}}, \quad (2.22)$$

$$C_{5R}^{us\tau t}(\mu_{\text{SUSY}}) = A_R^{\tau t}(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{\tau t} (\bar{Y}_R^B)_{us} - (Y_R^A)_{\tau u} (\bar{Y}_R^B)_{ts} \right\} |_{\mu=\mu_{\text{GUT}}}, \quad (2.23)$$

$$\begin{aligned} C_{5L}^{s\alpha ud}(\mu_{\text{SUSY}}) - C_{5L}^{u\alpha ds}(\mu_{\text{SUSY}}) \\ = A_L^\alpha(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \frac{3}{2} \left\{ (Y_L^A)_{ud} (\bar{Y}_L^B)_{s\alpha} - (Y_L^A)_{ds} (\bar{Y}_L^B)_{u\alpha} \right\} |_{\mu=\mu_{\text{GUT}}}, \end{aligned} \quad (2.24)$$

$$\begin{aligned} C_{5L}^{d\alpha us}(\mu_{\text{SUSY}}) - C_{5L}^{u\alpha ds}(\mu_{\text{SUSY}}) \\ = A_L^\alpha(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \frac{3}{2} \left\{ (Y_L^A)_{us} (\bar{Y}_L^B)_{d\alpha} - (Y_L^A)_{ds} (\bar{Y}_L^B)_{u\alpha} \right\} |_{\mu=\mu_{\text{GUT}}}, \end{aligned} \quad (2.25)$$

$$\begin{aligned} C_{5L}^{u\beta us}(\mu_{\text{SUSY}}) - C_{5L}^{s\beta uu}(\mu_{\text{SUSY}}) \\ = A_L^\beta(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \frac{3}{2} \left\{ (Y_L^A)_{us} (\bar{Y}_L^B)_{u\beta} - (Y_L^A)_{uu} (\bar{Y}_L^B)_{s\beta} \right\} |_{\mu=\mu_{\text{GUT}}}, \end{aligned} \quad (2.26)$$

where μ_{had} denotes a hadronic scale, μ_{SUSY} a soft SUSY breaking scale and μ_{GUT} a GUT- breaking scale. Here α_H, β_H denote the hadronic matrix elements, D, F are parameters of the baryon chiral Lagrangian, and $C_{LL}, \bar{C}_{LL}, C_{RL}$ are the Wilson coefficients of the effective Lagrangian where the SUSY particles are integrated out, $-\mathcal{L}_6 = C_{LL}^{ijkl}(\psi_{u_L k} \psi_{d_L l})(\psi_{d_L i} \psi_{\nu_L j}) + \bar{C}_{LL}^{ijkl}(\psi_{u_L k} \psi_{d_L l})(\psi_{u_L i} \psi_{e_L j}) + C_{RL}^{ijkl}(\psi_{\nu_L k} \psi_{d_L l})(\psi_{u_R^c i} \psi_{d_R^c j})$ (ψ denote SM Weyl spinors, and spinor indices are summed in each bracket). In eqs. (2.17)–(2.21), y_t, y_τ, g_2 denote the top quark Yukawa, tau lepton Yukawa and weak gauge couplings

¹By writing $C_{5L}^{s\alpha ud}$, we mean that Q_i is in the flavor basis where the down-type quark Yukawa coupling is diagonalized and that the down-type quark component of Q_i is exactly s quark. Likewise, Q_k is in the flavor basis where the up-type quark Yukawa coupling is diagonalized and its up-type component is exactly u quark, and Q_l is in the flavor basis where the down-type quark Yukawa coupling is diagonalized and its down-type quark component is exactly d quark. The same rule applies to other Wilson coefficients.

in MSSM, respectively, and V_{ij}^{ckm} denotes (i, j) -component of CKM matrix. $\mathcal{F}', \mathcal{F}$ are loop functions defined as $\mathcal{F}' = \frac{1}{x-y}(\frac{x}{1-x} \log x - \frac{y}{1-y} \log y)$ and $\mathcal{F} = \frac{1}{z-w}(\frac{z}{1-z} \log z - \frac{w}{1-w} \log w) + \frac{1}{z-1}(\frac{z}{1-z} \log z + 1)$, where $x = |\mu_H|^2/m_{\tilde{t}_R}^2$, $y = m_{\tilde{\tau}_R}^2/m_{\tilde{t}_R}^2$, $z = |M_{\tilde{W}}|^2/m_{\tilde{q}}^2$, $w = m_{\tilde{\ell}_\alpha}^2/m_{\tilde{q}}^2$, and $\mu_H, m_{\tilde{t}_R}, m_{\tilde{\tau}_R}, M_{\tilde{W}}, m_{\tilde{\ell}_\alpha}, m_{\tilde{q}}$ denote the pole masses of Higgsinos, isospin-singlet top squark, isospin-singlet tau slepton, Winos, isospin-doublet slepton of flavor α , and 1st and 2nd generation isospin-doublet squarks, respectively. A_{LL}, A_{RL} account for corrections from renormalization group (RG) evolutions in SM from scale μ_{SUSY} to μ_{had} . Here RG corrections involving SM Yukawa couplings other than the top quark one are neglected and thus A_{LL}, A_{RL} are flavor-universal. $A_R^{\tau t}, A_L^\alpha$ account for corrections from RG evolutions in MSSM from scale μ_{GUT} to μ_{SUSY} .

We rewrite the flavor-dependent part of eqs. (2.22)–(2.26) with the GUT Yukawa coupling matrices Y_{10}, Y_{126}, Y_{120} as ($\alpha = e, \mu, \tau; \beta = e, \mu$)

$$\begin{aligned} & \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{\tau t} (\bar{Y}_R^B)_{ud} - (Y_R^A)_{\tau u} (\bar{Y}_R^B)_{td} \right\} \\ &= \frac{1}{M_{H_C}} [a \{ (Y_{10})_{\tau_R t_R} (Y_{10})_{u_R d_R} - (Y_{10})_{\tau_R u_R} (Y_{10})_{t_R d_R} \} + b \{ (Y_{10})_{\tau_R t_R} (Y_{126})_{u_R d_R} - (Y_{10})_{\tau_R u_R} (Y_{126})_{t_R d_R} \} \\ & \quad + c \{ (Y_{10})_{\tau_R t_R} (Y_{120})_{u_R d_R} - (Y_{10})_{\tau_R u_R} (Y_{120})_{t_R d_R} \} \\ & \quad + d \{ (Y_{126})_{\tau_R t_R} (Y_{10})_{u_R d_R} - (Y_{126})_{\tau_R u_R} (Y_{10})_{t_R d_R} \} + e \{ (Y_{126})_{\tau_R t_R} (Y_{126})_{u_R d_R} - (Y_{126})_{\tau_R u_R} (Y_{126})_{t_R d_R} \} \\ & \quad + f \{ (Y_{126})_{\tau_R t_R} (Y_{120})_{u_R d_R} - (Y_{126})_{\tau_R u_R} (Y_{120})_{t_R d_R} \} \\ & \quad + g \{ (Y_{120})_{\tau_R t_R} (Y_{10})_{u_R d_R} - (Y_{120})_{\tau_R u_R} (Y_{10})_{t_R d_R} \} + h \{ (Y_{120})_{\tau_R t_R} (Y_{126})_{u_R d_R} - (Y_{120})_{\tau_R u_R} (Y_{126})_{t_R d_R} \} \\ & \quad + j \{ (Y_{120})_{\tau_R t_R} (Y_{120})_{u_R d_R} - (Y_{120})_{\tau_R u_R} (Y_{120})_{t_R d_R} \}], \end{aligned} \quad (2.27)$$

$$\sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{\tau t} (\bar{Y}_R^B)_{us} - (Y_R^A)_{\tau u} (\bar{Y}_R^B)_{ts} \right\} = (\text{Above expression with exchange } d_R \leftrightarrow s_R), \quad (2.28)$$

$$\begin{aligned} & \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_L^A)_{ud} (\bar{Y}_L^B)_{s\alpha} - (Y_L^A)_{ds} (\bar{Y}_L^B)_{u\alpha} \right\} \\ &= \frac{1}{M_{H_C}} [a \{ (Y_{10})_{u_L d_L} (Y_{10})_{s_L \alpha_L} - (Y_{10})_{d_L s_L} (Y_{10})_{u_L \alpha_L} \} + b \{ (Y_{10})_{u_L d_L} (Y_{126})_{s_L \alpha_L} - (Y_{10})_{d_L s_L} (Y_{126})_{u_L \alpha_L} \} \\ & \quad + c \{ (Y_{10})_{u_L d_L} (Y_{120})_{s_L \alpha_L} - (Y_{10})_{d_L s_L} (Y_{120})_{u_L \alpha_L} \} \\ & \quad + d \{ (Y_{126})_{u_L d_L} (Y_{10})_{s_L \alpha_L} - (Y_{126})_{d_L s_L} (Y_{10})_{u_L \alpha_L} \} + e \{ (Y_{126})_{u_L d_L} (Y_{126})_{s_L \alpha_L} - (Y_{126})_{d_L s_L} (Y_{126})_{u_L \alpha_L} \} \\ & \quad + f \{ (Y_{126})_{u_L d_L} (Y_{120})_{s_L \alpha_L} - (Y_{126})_{d_L s_L} (Y_{120})_{u_L \alpha_L} \}], \end{aligned} \quad (2.29)$$

$$\sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_L^A)_{us} (\bar{Y}_L^B)_{d\alpha} - (Y_L^A)_{ds} (\bar{Y}_L^B)_{u\alpha} \right\} = (\text{Above expression with exchange } d_L \leftrightarrow s_L), \quad (2.30)$$

$$\begin{aligned} & \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_L^A)_{us} (\bar{Y}_L^B)_{u\beta} - (Y_L^A)_{uu} (\bar{Y}_L^B)_{s\beta} \right\} \\ &= \frac{1}{M_{H_C}} [a \{ (Y_{10})_{u_L s_L} (Y_{10})_{u_L \beta_L} - (Y_{10})_{u_L u_L} (Y_{10})_{s_L \beta_L} \} + b \{ (Y_{10})_{u_L s_L} (Y_{126})_{u_L \beta_L} - (Y_{10})_{u_L u_L} (Y_{126})_{s_L \beta_L} \} \\ & \quad + c \{ (Y_{10})_{u_L s_L} (Y_{120})_{u_L \beta_L} - (Y_{10})_{u_L u_L} (Y_{120})_{s_L \beta_L} \} \\ & \quad + d \{ (Y_{126})_{u_L s_L} (Y_{10})_{u_L \beta_L} - (Y_{126})_{u_L u_L} (Y_{10})_{s_L \beta_L} \} + e \{ (Y_{126})_{u_L s_L} (Y_{126})_{u_L \beta_L} - (Y_{126})_{u_L u_L} (Y_{126})_{s_L \beta_L} \} \\ & \quad + f \{ (Y_{126})_{u_L s_L} (Y_{120})_{u_L \beta_L} - (Y_{126})_{u_L u_L} (Y_{120})_{s_L \beta_L} \}], \end{aligned} \quad (2.31)$$

where M_{H_C} is the scale of the singular values of \mathcal{M}_{H_C} , and $a, b, c, d, e, f, g, h, j$ are $O(1)$ numbers determined from \mathcal{M}_{H_C} as ref. [40]–[45]. Here $(Y_{10})_{\tau_R t_R}$ denotes the component of Y_{10} in the term $(Y_{10})_{ij} \Psi_i H \Psi_j$ that involves the right-handed tau lepton component of Ψ_i and the right-handed top quark component of Ψ_j . Other symbols are defined analogously.

3 Conditions for a texture of the Yukawa coupling matrices suppressing dimension-five proton decay

We identify those components of the Yukawa coupling matrices Y_{10}, Y_{126}, Y_{120} that are involved in dimension-five proton decay, namely, appear in eqs. (2.27)–(2.31), and that can be on the order of the up quark Yukawa coupling y_u . Here the up quark Yukawa coupling is considered as the smallest scale of the components of the Yukawa coupling matrices because it is a specially small Yukawa coupling in the SUSY SO(10) GUT where $\tan \beta \sim 50$. That the components identified above be on the order of the up quark Yukawa coupling, is the desired conditions for a texture suppressing dimension-five proton decay.

- We focus on the first term in each $\{\dots\}$ of eqs. (2.27), (2.28). Components $(Y_{10})_{\tau_R t_R}$ and $(Y_{126})_{\tau_R t_R}$ are almost (3, 3)-components of the symmetric matrices Y_{10}, Y_{126} and hence **always** on the order of the top quark Yukawa coupling y_t . For other components, at most two of $(Y_{10})_{u_R d_R}, (Y_{126})_{u_R d_R}, (Y_{120})_{u_R d_R}$ and at most two of $(Y_{10})_{u_R s_R}, (Y_{126})_{u_R s_R}, (Y_{120})_{u_R s_R}$ can be on the order of the up quark Yukawa coupling y_u . However, all of them cannot be so because of the following equalities that result from eq. (2.4):

$$\begin{aligned} (Y_{10})_{u_R d_R} + (Y_{126})_{u_R d_R} + (Y_{120})_{u_R d_R} &= \frac{1}{r_1} (Y_d)_{u_R d_R} & (3.1) \\ &\simeq \frac{y_t}{y_b} y_d \times (d_L\text{-}u_R \text{ part of the mixing matrix}), \end{aligned}$$

$$\begin{aligned} (Y_{10})_{u_R s_R} + (Y_{126})_{u_R s_R} + (Y_{120})_{u_R s_R} &= \frac{1}{r_1} (Y_d)_{u_R s_R} & (3.2) \\ &\simeq \frac{y_t}{y_b} y_s \times (s_L\text{-}u_R \text{ part of the mixing matrix}), \end{aligned}$$

where r_1 is estimated to be y_b/y_t so that the ratio of the top and bottom quark Yukawa couplings is reproduced. $d_L\text{-}u_R$ part of the mixing matrix is estimated to be about 1 and we get $(Y_{10})_{u_R d_R} + (Y_{126})_{u_R d_R} + (Y_{120})_{u_R d_R} \simeq \frac{y_t}{y_b} y_d$, which is much greater than y_u . Also, $s_L\text{-}u_R$ part of the mixing matrix is estimated to be the Cabibbo angle $\lambda = 0.22$ and hence we get $(Y_{10})_{u_R s_R} + (Y_{126})_{u_R s_R} + (Y_{120})_{u_R s_R} \simeq 0.22 \times \frac{y_t}{y_b} y_s$, which is much greater than y_u . Therefore, to make the entire eqs. (2.27), (2.28) proportional to y_u , we have to tune the colored Higgs mass matrix such that some of $a, b, c, d, e, f, g, h, j$ are much smaller than 1. The most economical choice is to tune the colored Higgs mass matrix to make

$$c = f = 0 \tag{3.3}$$

and at the same time consider the following texture:

$$(Y_{10})_{u_R d_R} = O(y_u), \quad (Y_{126})_{u_R d_R} = O(y_u), \quad (Y_{120})_{u_R d_R} \simeq \frac{y_t}{y_b} y_d, \tag{3.4}$$

$$(Y_{10})_{u_R s_R} = O(y_u), \quad (Y_{126})_{u_R s_R} = O(y_u), \quad (Y_{120})_{u_R s_R} \simeq 0.22 \times \frac{y_t}{y_b} y_s. \tag{3.5}$$

The terms $j(Y_{120})_{\tau_R t_R} (Y_{120})_{u_R d_R}$ and $j(Y_{120})_{\tau_R t_R} (Y_{120})_{u_R s_R}$ appear to be not proportional to y_u . However, since Y_{120} is antisymmetric in the flavor space, $(Y_{120})_{\tau_R t_R}$,

being a nearly diagonal component, is so small that the above terms are suppressed compared to the other remaining terms. Note that $(Y_{120})_{\tau_R t_R}$ is not exactly 0 because the flavor basis of the isospin-singlet charged leptons (e_R, μ_R, τ_R) is not identical with that of the isospin-singlet up-type quarks (u_R, c_R, t_R) . However, these flavor bases are close and thus $(Y_{120})_{\tau_R t_R}$ is suppressed.

- We focus on the second term in each $\{\dots\}$ of eq. (2.27). Each term is estimated to be $\sin^2 \theta_{13}^{\text{ckm}} y_t^2$ (θ_{13}^{ckm} is the (1,3)-mixing angle of CKM matrix), since each term contains two 1st-3rd generation flavor mixings. The value of $\sin^2 \theta_{13}^{\text{ckm}} y_t^2$ is numerically close to $y_u y_t$. Thus, these terms are **always** on the same order as first terms $(Y_{10})_{\tau_R t_R} (Y_{10})_{u_R d_R}$, $(Y_{10})_{\tau_R t_R} (Y_{126})_{u_R s_R}$, $(Y_{126})_{\tau_R t_R} (Y_{10})_{u_R d_R}$, $(Y_{126})_{\tau_R t_R} (Y_{126})_{u_R s_R}$ with the texture of eq. (3.4).

Likewise the second term in each $\{\dots\}$ of eq. (2.28) is estimated to be $\sin \theta_{13}^{\text{ckm}} \sin \theta_{23}^{\text{ckm}} y_t^2$. These terms contribute to the proton decay amplitude **always** by a similar amount to the second terms in $\{\dots\}$'s of eq. (2.27) because they enter the proton decay amplitude in the form $V_{ts}^{\text{ckm}} (Y_A)_{\tau_R u_R} (Y_B)_{t_R d_R}$ and $V_{td}^{\text{ckm}} (Y_A)_{\tau_R u_R} (Y_B)_{t_R s_R}$ and the CKM matrix components satisfy $|V_{ts}^{\text{ckm}}| \sim \sin \theta_{23}^{\text{ckm}}$ and $|V_{td}^{\text{ckm}}| \sim \sin \theta_{13}^{\text{ckm}}$.

- We proceed to eqs. (2.29), (2.30). It is impossible to suppress $(Y_A)_{s_L \alpha_L}$, $(Y_A)_{d_L \alpha_L}$, $(Y_A)_{u_L \alpha_L}$ for **all flavors** $\alpha = e, \mu, \tau$ to the order of y_u . Therefore, we do not consider a texture where some of them are $O(y_u)$. For other components, at least one of $(Y_{10})_{u_L s_L}$, $(Y_{10})_{d_L s_L}$, $(Y_{126})_{u_L s_L}$, $(Y_{126})_{d_L s_L}$ is on the order of $|V_{cd}^{\text{ckm}}| \frac{y_t}{y_b} y_s$ because of two equalities below,

$$(Y_{10})_{s_L c_L} + (Y_{126})_{s_L c_L} + (Y_{120})_{s_L c_L} \simeq \frac{y_t}{y_b} y_s \times (c_L - s_R \text{ part of the mixing matrix}), \quad (3.6)$$

$$\begin{aligned} & (Y_{10})_{d_L s_L} + (Y_{126})_{d_L s_L} - V_{ud}^{\text{ckm}} \{ (Y_{10})_{u_L s_L} + (Y_{126})_{u_L s_L} \} \\ & = V_{cd}^{\text{ckm}} \{ (Y_{10})_{c_L s_L} + (Y_{126})_{c_L s_L} \} + V_{td}^{\text{ckm}} \{ (Y_{10})_{t_L s_L} + (Y_{126})_{t_L s_L} \}, \end{aligned} \quad (3.7)$$

and by the facts that $c_L - s_R$ part of the mixing matrix is about 1 because c_L and s_R are 2nd generation flavors, and that $(Y_{120})_{s_L c_L}$ is suppressed compared to $(Y_{10})_{c_L s_L}$, $(Y_{126})_{c_L s_L}$ because it is nearly (2,2)-component of the antisymmetric matrix Y_{120} .² As a result, at least one of $(Y_{10})_{u_L s_L}$, $(Y_{10})_{d_L s_L}$, $(Y_{126})_{u_L s_L}$, $(Y_{126})_{d_L s_L}$ cannot be on the order of y_u . Still, it is possible to make the entire eqs. (2.29), (2.30) proportional to y_u by tuning the colored Higgs mass matrix such that a, b, d, e satisfy

$$a (Y_{10})_{d_L s_L} + d (Y_{126})_{d_L s_L} = 0, \quad b (Y_{10})_{d_L s_L} + e (Y_{126})_{d_L s_L} = 0 \quad (3.8)$$

and at the same time considering the following texture:

$$(Y_{10})_{u_L s_L} = O(y_u), \quad (Y_{126})_{u_L s_L} = O(y_u). \quad (3.9)$$

²The situation that the term $V_{td}^{\text{ckm}} \{ (Y_{10})_{t_L s_L} + (Y_{126})_{t_L s_L} \}$ cancels the term $V_{cd}^{\text{ckm}} \{ (Y_{10})_{c_L s_L} + (Y_{126})_{c_L s_L} \}$ is incompatible with the correct quark Yukawa couplings.

- Finally, we focus on eq. (2.31). The only components that do not appear in eqs. (2.29), (2.30) are $(Y_{10})_{u_L u_L}$, $(Y_{126})_{u_L u_L}$, and there is no obstacle in considering the following texture:

$$(Y_{10})_{u_L u_L} = O(y_u), \quad (Y_{126})_{u_L u_L} = O(y_u). \quad (3.10)$$

To summarize, dimension-five proton decay is suppressed if the components below are all on the order of the up quark Yukawa coupling y_u ,

$$\begin{aligned} & (Y_{10})_{u_R d_R}, \quad (Y_{126})_{u_R d_R}, \quad (Y_{10})_{u_R s_R}, \quad (Y_{126})_{u_R s_R}, \\ & (Y_{10})_{u_L d_L}, \quad (Y_{126})_{u_L d_L}, \quad (Y_{10})_{u_L s_L}, \quad (Y_{126})_{u_L s_L}, \quad (Y_{10})_{u_L u_L}, \quad (Y_{126})_{u_L u_L}, \end{aligned} \quad (3.11)$$

and at the same time the colored Higgs mass matrix is tuned such that $a, b, c, d, e, f, g, h, j$ in eqs. (2.27)–(2.31) satisfy

$$c = f = 0, \quad (3.12)$$

$$a(Y_{10})_{d_L s_L} + d(Y_{126})_{d_L s_L} = 0, \quad b(Y_{10})_{d_L s_L} + e(Y_{126})_{d_L s_L} = 0. \quad (3.13)$$

That the components of eq. (3.11) be on the order of the up quark Yukawa coupling, is the desired conditions for a texture of the Yukawa coupling matrices suppressing dimension-five proton decay.

For reference, below we summarize the estimates on crucial Yukawa coupling components involved in dimension-five proton decay other than eq. (3.11),

$$\begin{aligned} & (Y_{10})_{\tau_R t_R} \sim (Y_{126})_{\tau_R t_R} \sim y_t, & (Y_{120})_{\tau_R t_R} &\ll y_t, \\ & (Y_{120})_{u_R d_R} \sim \frac{y_t}{y_b} y_d, & (Y_{120})_{u_R s_R} &\sim \lambda \frac{y_t}{y_b} y_s, \\ & (Y_A)_{\tau_R u_R} \sim (Y_A)_{t_R d_R} \sim y_t \sin \theta_{13}^{\text{ckm}} & \text{for } A = 10, 126, 120, \\ & (Y_{10})_{d_L s_L} \sim (Y_{126})_{d_L s_L} \sim \lambda \frac{y_t}{y_b} y_s, \end{aligned} \quad (3.14)$$

where $\lambda = 0.22$ and $\sin \theta_{13}^{\text{ckm}} = 0.004$.

We comment on nucleon decay modes other than eqs. (2.14)–(2.16). The partial widths of the $N \rightarrow \pi\beta^+$ and $p \rightarrow \eta\beta^+$ decays ($\beta = e, \mu$) involve the same Yukawa coupling components as the $p \rightarrow K^0\beta^+$ except that s_L is replaced by d_L . Hence, once we consider the texture where $(Y_{10})_{u_L u_L}$, $(Y_{126})_{u_L u_L}$ are on the order of the up quark Yukawa coupling, the $N \rightarrow \pi\beta^+$ and $p \rightarrow \eta\beta^+$ decays are also suppressed. Constraints on the rest of the decay modes are relatively weak [46] and are not in tension with the SUSY SO(10) GUT.

4 Texture of the colored Higgs mass matrix

We present a texture of the colored Higgs mass matrix that gives $c = f = 0$ and $a/d = b/e$. Here $a/d = b/e$ is a necessary condition for eq. (3.13). We utilize the result of ref. [42], and use the same notation of fields, coupling constants and VEVs except that **120** field is written as Σ in our paper. The definitions of the couplings, coupling constants and masses are summarized in appendix A.

The desired texture of the colored Higgs mass matrix is obtained from the following relations of the coupling constants and VEVs:

$$\begin{aligned} \lambda_{18} &= 0, & \lambda_{20} &= 0, & \frac{\lambda_{21}}{\lambda_{19}} &= 3 \frac{\lambda_{17}}{\lambda_{16}}, \\ i A_1 &= -\frac{1}{6} \frac{\lambda_{21}}{\lambda_{19}} \Phi_3, & i A_2 &= -\frac{\sqrt{3}}{6} \frac{\lambda_{21}}{\lambda_{19}} \Phi_2. \end{aligned} \quad (4.1)$$

The above relations are obtained by fine-tuning, which is natural at the quantum level thanks to the non-renormalization theorem. The VEV configuration of eq. (4.1) can be consistent with the F -flatness conditions of the six SM-gauge-singlet components $\Phi_1, \Phi_2, \Phi_3, A_1, A_2, v_R$ when six model parameters, for example $\lambda_5/\lambda_2, \lambda_6/\lambda_2, \lambda_7/\lambda_2, m_1, m_2, m_4$, are tuned appropriately.

When eq. (4.1) holds, the colored Higgs mass matrix has the following texture:

$$W_{\text{colored Higgs}} \supset \left(H^{(\bar{3},1,\frac{1}{3})} \bar{\Delta}^{(\bar{3},1,\frac{1}{3})}_{(6,1,1)} \Delta^{(\bar{3},1,\frac{1}{3})}_{(6,1,1)} \Delta^{(\bar{3},1,\frac{1}{3})}_{(10,1,3)} \Phi^{(\bar{3},1,\frac{1}{3})} \Sigma^{(\bar{3},1,\frac{1}{3})}_{(6,1,3)} \Sigma^{(\bar{3},1,\frac{1}{3})}_{(10,1,1)} \right) \mathcal{M}_{H_C} \begin{pmatrix} H^{(3,1,-\frac{1}{3})} \\ \Delta^{(3,1,-\frac{1}{3})}_{(6,1,1)} \\ \bar{\Delta}^{(3,1,-\frac{1}{3})}_{(6,1,1)} \\ \Delta^{(3,1,-\frac{1}{3})}_{(10,1,3)} \\ \Phi^{(3,1,-\frac{1}{3})} \\ \Sigma^{(3,1,-\frac{1}{3})}_{(6,1,3)} \\ \Sigma^{(3,1,-\frac{1}{3})}_{(10,1,1)} \end{pmatrix}, \quad (4.2)$$

where

$$\mathcal{M}_{H_C} = \begin{pmatrix} m_3 & \frac{\lambda_3 \Phi_2}{\sqrt{30}} - \frac{\lambda_3 \Phi_1}{\sqrt{10}} & -\frac{\lambda_4 \Phi_1}{\sqrt{10}} - \frac{\lambda_4 \Phi_2}{\sqrt{30}} & -\sqrt{\frac{2}{15}} \lambda_4 \Phi_3 & \frac{\lambda_4 \bar{v}_R}{\sqrt{5}} & 0 & 0 \\ \frac{\lambda_4 \Phi_2}{\sqrt{30}} - \frac{\lambda_4 \Phi_1}{\sqrt{10}} & m_2 + i \frac{\lambda_{21}}{\lambda_{19}} \frac{\lambda_6 \Phi_2}{30\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\lambda_3 \Phi_1}{\sqrt{10}} - \frac{\lambda_3 \Phi_2}{\sqrt{30}} & 0 & m_2 - i \frac{\lambda_{21}}{\lambda_{19}} \frac{\lambda_6 \Phi_2}{30\sqrt{2}} & \frac{\lambda_2 \Phi_3}{15\sqrt{2}} & -\frac{\lambda_2 \bar{v}_R}{10\sqrt{3}} & 0 & 0 \\ -\sqrt{\frac{2}{15}} \lambda_3 \Phi_3 & 0 & \frac{\lambda_2 \Phi_3}{15\sqrt{2}} & m_{66} & -\frac{\lambda_2 \bar{v}_R}{5\sqrt{6}} & 0 & 0 \\ \frac{\lambda_3 v_R}{\sqrt{5}} & 0 & -\frac{\lambda_2 v_R}{10\sqrt{3}} & -\frac{\lambda_2 v_R}{5\sqrt{6}} & m_{77} & 0 & 0 \\ -\lambda_{17} \frac{\Phi_3}{\sqrt{3}} & 0 & \lambda_{21} \frac{\Phi_3}{6\sqrt{5}} & \lambda_{21} \frac{\Phi_2}{3\sqrt{5}} & \frac{\lambda_{21} \bar{v}_R}{2\sqrt{15}} & m_{22} & \frac{2\lambda_{15} \Phi_3}{9} \\ -\sqrt{\frac{2}{3}} \lambda_{17} \Phi_2 & 0 & \lambda_{21} \frac{\Phi_2}{3\sqrt{10}} & \lambda_{21} \frac{\Phi_3}{3\sqrt{10}} & \frac{\lambda_{21} \bar{v}_R}{2\sqrt{15}} & \frac{2\lambda_{15} \Phi_3}{9} & m_{33} \end{pmatrix} \quad (4.3)$$

$$m_{66} = m_2 + \lambda_2 \left(\frac{\Phi_1}{10\sqrt{6}} + \frac{\Phi_2}{30\sqrt{2}} \right) - i \frac{\lambda_{21}}{\lambda_{19}} \frac{\lambda_6 \Phi_2}{30\sqrt{2}} \quad (4.4)$$

$$m_{77} = m_1 + \lambda_1 \left(\frac{\Phi_1}{\sqrt{6}} + \frac{\Phi_2}{3\sqrt{2}} + \frac{2\Phi_3}{3} \right) - i \frac{\lambda_{21}}{\lambda_{19}} \frac{\sqrt{2} \lambda_7 \Phi_2}{15} \quad (4.5)$$

$$m_{22} = m_6 + \frac{1}{3} \sqrt{\frac{2}{3}} \lambda_{15} \Phi_1 \quad (4.6)$$

$$m_{33} = m_6 + \frac{\sqrt{2}}{9} \lambda_{15} \Phi_2. \quad (4.7)$$

The dimension-five operators responsible for proton decay eq. (2.11) satisfy at the scale of

the colored Higgs mass,

$$\begin{aligned}
 W_5 = & - \left((\tilde{Y}_{10})_{ij} \ 0 \ (\tilde{Y}_{126})_{ij} \ (\tilde{Y}_{126})_{ij} \ 0 \ (\tilde{Y}_{120})_{ij} \ (\tilde{Y}_{120})_{ij} \right) \mathcal{M}_{H_C}^{-1} \begin{pmatrix} (\tilde{Y}_{10})_{kl} \\ (\tilde{Y}_{126})_{kl} \\ 0 \\ 0 \\ 0 \\ (\tilde{Y}_{120})_{kl} \\ (\tilde{Y}_{120})_{kl} \end{pmatrix} E_i^c U_j^c U_k^c D_l^c \\
 & - \left((\tilde{Y}_{10})_{ij} \ 0 \ (\tilde{Y}_{126})_{ij} \ (\tilde{Y}_{126})_{ij} \ 0 \ 0 \ 0 \right) \mathcal{M}_{H_C}^{-1} \begin{pmatrix} (\tilde{Y}_{10})_{kl} \\ (\tilde{Y}_{126})_{kl} \\ 0 \\ 0 \\ 0 \\ (\tilde{Y}_{120})_{kl} \\ (\tilde{Y}_{120})_{kl} \end{pmatrix} (Q_i Q_j) (Q_k L_l), \quad (4.8)
 \end{aligned}$$

where the inverse of the colored Higgs mass $\mathcal{M}_{H_C}^{-1}$ has the following properties resulting from eq. (4.3):

- The upper-right 5×2 part of $\mathcal{M}_{H_C}^{-1}$ is zero because the upper-right 5×2 part of \mathcal{M}_{H_C} is zero. Thus, the coefficients proportional to $(\tilde{Y}_{10})_{ij}(\tilde{Y}_{120})_{kl}$ or $(\tilde{Y}_{126})_{ij}(\tilde{Y}_{120})_{kl}$ are zero, namely, we get $c = f = 0$ in eqs. (2.27)–(2.31).
- The upper-left 5×5 part of $\mathcal{M}_{H_C}^{-1}$ is given by the inverse of the upper-left 5×5 part of \mathcal{M}_{H_C} , since the upper-right 5×2 part of \mathcal{M}_{H_C} is zero. Then the equalities $(\mathcal{M}_{H_C})_{32} = (\mathcal{M}_{H_C})_{42} = (\mathcal{M}_{H_C})_{52} = 0$ lead to the relation $(\mathcal{M}_{H_C}^{-1})_{11} : (\mathcal{M}_{H_C}^{-1})_{31} : (\mathcal{M}_{H_C}^{-1})_{41} = (\mathcal{M}_{H_C}^{-1})_{12} : (\mathcal{M}_{H_C}^{-1})_{32} : (\mathcal{M}_{H_C}^{-1})_{42}$. Since the coefficients proportional to $(\tilde{Y}_{10})_{ij}(\tilde{Y}_{10})_{kl}$, $(\tilde{Y}_{10})_{ij}(\tilde{Y}_{126})_{kl}$, $(\tilde{Y}_{126})_{ij}(\tilde{Y}_{10})_{kl}$ or $(\tilde{Y}_{126})_{ij}(\tilde{Y}_{126})_{kl}$ are given by

$$\begin{aligned}
 & (\mathcal{M}_{H_C}^{-1})_{11} (\tilde{Y}_{10})_{ij}(\tilde{Y}_{10})_{kl} + \left\{ (\mathcal{M}_{H_C}^{-1})_{31} + (\mathcal{M}_{H_C}^{-1})_{41} \right\} (\tilde{Y}_{126})_{ij}(\tilde{Y}_{10})_{kl} \\
 & + (\mathcal{M}_{H_C}^{-1})_{12} (\tilde{Y}_{10})_{ij}(\tilde{Y}_{126})_{kl} + \left\{ (\mathcal{M}_{H_C}^{-1})_{32} + (\mathcal{M}_{H_C}^{-1})_{42} \right\} (\tilde{Y}_{126})_{ij}(\tilde{Y}_{126})_{kl}, \quad (4.9)
 \end{aligned}$$

the equality $(\mathcal{M}_{H_C}^{-1})_{11} / \left\{ (\mathcal{M}_{H_C}^{-1})_{31} + (\mathcal{M}_{H_C}^{-1})_{41} \right\} = (\mathcal{M}_{H_C}^{-1})_{12} / \left\{ (\mathcal{M}_{H_C}^{-1})_{32} + (\mathcal{M}_{H_C}^{-1})_{42} \right\}$ leads to the desired relation $a/d = b/e$ in eqs. (2.27)–(2.31).

We still have to check that eq. (4.1) is consistent with the situation that a/d satisfies $a(Y_{10})_{d_L s_L} + d(Y_{126})_{d_L s_L} = 0$, that only one pair of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields (corresponding to the MSSM Higgs fields) have almost zero mass compared to the GUT scale, and that r_1, r_2, r_3, r_e, r_D take values that reproduce the realistic quark and lepton masses and flavor mixings. With eq. (4.1), and with the tuning of $\lambda_5/\lambda_2, \lambda_6/\lambda_2, \lambda_7/\lambda_2, m_1, m_2, m_4$ to satisfy the F -flatness conditions, the colored Higgs mass matrix and the mass matrix of the $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components still have free parameters $\Phi_1, \Phi_2, \Phi_3, v_R, m_3, \lambda_{21}/\lambda_{19}, \lambda_{21}/\lambda_2, \lambda_1/\lambda_2, \lambda_3/\lambda_2, \lambda_4/\lambda_2, \lambda_{15}/\lambda_2, \lambda_{17}/\lambda_2$. These free parameters are sufficient to make one pair of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields nearly massless and realize any values of $a/d, r_1, r_2, r_3, r_e, r_D$.

5 Yukawa coupling matrices satisfying the conditions

5.1 Procedures of the analysis

We will obtain specific Yukawa coupling matrices Y_{10} , Y_{126} , Y_{120} which give the realistic quark and lepton masses and flavor mixings and which satisfy the conditions found in section 3 that the components of eq. (3.11) be on the order of the up quark Yukawa coupling y_u . To this end, we fit the experimental values of the quark and lepton masses and flavor mixings with Y_{10} , Y_{126} , Y_{120} and numbers r_1, r_2, r_3, r_e, r_D based on eqs. (2.3)–(2.9) and at the same time minimize the following quantity:

$$\frac{1}{y_u^2} \sum_{A=10,126} |(Y_A)_{u_R d_R}|^2 + |(Y_A)_{u_R s_R}|^2 + |(Y_A)_{u_L d_L}|^2 + |(Y_A)_{u_L s_L}|^2 + |(Y_A)_{u_L u_L}|^2. \quad (5.1)$$

The procedures are as follows: we adopt the following experimental values of the quark and charged lepton masses, quark flavor mixings and gauge coupling constants: we use the results of lattice calculations of the individual up and down quark masses, the strange quark mass, the charm quark mass and the bottom quark mass in $\overline{\text{MS}}$ scheme reviewed in ref. [47], which read $m_u(2 \text{ GeV}) = 2.14(8) \text{ MeV}$, $m_d(2 \text{ GeV}) = 4.70(5) \text{ MeV}$ [48, 49], $m_s(2 \text{ GeV}) = 93.40(57) \text{ MeV}$ [48, 50–52], $m_c(3 \text{ GeV}) = 0.988(11) \text{ GeV}$ [48, 50, 52–54], $m_b(m_b) = 4.203(11) \text{ GeV}$ [48, 52, 55–58]. We use the top quark pole mass measured by CMS in ref. [59], which reads $M_t = 170.5(8) \text{ GeV}$. We calculate the CKM mixing angles and CP phase from the Wolfenstein parameters in ref. [60]. The lepton pole masses and W , Z , Higgs boson pole masses are taken from Particle Data Group [46], and the QCD and QED gauge coupling constants in 5-quark-flavor QCD×QED theory are fixed as $\alpha_s^{(5)}(M_Z) = 0.1181$ and $\alpha^{(5)}(M_Z) = 1/127.95$. The above data are translated into the values of the quark and lepton Yukawa coupling matrices and gauge coupling constants at scale $\mu = M_Z$ in $\overline{\text{MS}}$ scheme with the help of the code [61] based on refs. [62]–[68].

We calculate the two-loop RG equations [69]–[71] of SM from scale $\mu = M_Z$ to the soft SUSY breaking scale μ_{SUSY} . The results are matched to the Yukawa coupling matrices and gauge couplings of MSSM in $\overline{\text{DR}}$ scheme. Here the one-loop threshold corrections of SUSY particles, which are important for the down-type quark and charged lepton Yukawa couplings as $\tan \beta$ is large, are included as ref. [72]. Then we calculate the two-loop RG equations of MSSM from scale μ_{SUSY} to the GUT scale μ_{GUT} . We assume a degenerate SUSY particle mass spectrum where the pole masses of SUSY particles and $\tan \beta$ are given by

$$\begin{aligned} m_{\text{sfermion}} = m_{H^0} = m_{H^\pm} = m_A = 1500 \text{ TeV}, \\ |M_{\tilde{g}}| = |M_{\tilde{W}}| = |\mu_H| = 1500 \text{ TeV}, \quad \tan \beta = 50 \end{aligned} \quad (5.2)$$

and all the A -terms are 0. We set $\mu_{\text{SUSY}} = 1500 \text{ TeV}$ and $\mu_{\text{GUT}} = 2 \cdot 10^{16} \text{ GeV}$. The values of the Yukawa coupling matrices at scale $\mu = \mu_{\text{GUT}}$ are shown in table 1, in the form of the singular values of the matrices and the parameters of the CKM matrix at this scale. The errors of the quark Yukawa couplings, propagated from the experimental errors of the corresponding masses, and the maximal errors of the CKM parameters, obtained by assuming maximal correlation of the experimental errors of the Wolfenstein parameters, are also displayed.

| | Value with eq. (5.2) |
|---|---------------------------|
| y_u | $2.81(11) \times 10^{-6}$ |
| y_c | 0.001433(16) |
| y_t | 0.4722(58) |
| y_d | 0.0003141(33) |
| y_s | 0.006243(38) |
| y_b | 0.3557(16) |
| y_e | 0.0001261 |
| y_μ | 0.02662 |
| y_τ | 0.5095 |
| $\cos \theta_{13}^{\text{ckm}} \sin \theta_{12}^{\text{ckm}}$ | 0.22500(24) |
| $\cos \theta_{13}^{\text{ckm}} \sin \theta_{23}^{\text{ckm}}$ | 0.04171(70) |
| $\sin \theta_{13}^{\text{ckm}}$ | 0.00367(20) |
| δ_{km} (rad) | 1.148(33) |

Table 1. The singular values of the Yukawa coupling matrices and the CKM mixing angles and CP phase in MSSM at $\mu = \mu_{\text{GUT}} = 2 \cdot 10^{16}$ GeV. Also shown are the errors of the quark Yukawa couplings, propagated from the experimental errors of the corresponding masses, and the maximal errors of the CKM parameters, obtained by assuming maximal correlation of the experimental errors of the Wolfenstein parameters.

Also, we evaluate one-loop RG corrections to the Wilson coefficient of the Weinberg operator. We write the Weinberg operator in MSSM as eq. (2.8) and that in SM as $-\mathcal{L} = \frac{1}{2}(C'_\nu)_{ij} \psi_{L_i} \psi_{L_j} H H$ where ψ_{L_i} denote the lepton doublets and H the Higgs field. We express the one-loop RG corrections in MSSM and SM as $C_\nu(\mu) = R(\mu)C_\nu(\mu_{\text{SUSY}})R^T(\mu)$, $C'_\nu(\mu) = R'(\mu)C'_\nu(M_Z)R'^T(\mu)$, respectively, and perform the matching as $C_\nu(\mu_{\text{SUSY}}) = C'_\nu(\mu_{\text{SUSY}})$, since $\tan \beta \gg 1$. We solve the one-loop RG equations and calculate the product of $R(\mu_{\text{GUT}})$ and $R'(\mu_{\text{SUSY}})$. Here we approximate the scale of the Majorana mass $(M_N)_{ij}$ to be μ_{GUT} . The product of $R(\mu_{\text{GUT}})$ and $R'(\mu_{\text{SUSY}})$ in the flavor basis where the lepton doublets have a diagonal Yukawa coupling matrix, is calculated as

$$R(\mu_{\text{GUT}})R'(\mu_{\text{SUSY}}) = \begin{pmatrix} 1.09 & 0 & 0 \\ 0 & 1.09 & 0 \\ 0 & 0 & 1.14 \end{pmatrix}. \tag{5.3}$$

We fit the quark and charged lepton Yukawa couplings and the CKM parameters at $\mu = \mu_{\text{GUT}}$ in table 1 with the Yukawa coupling matrices Y_{10}, Y_{126}, Y_{120} and numbers r_1, r_2, r_3, r_e based on eqs. (2.3)–(2.5). Also, we calculate the neutrino mass matrix up to the overall constant, which is proportional to $C'_\nu(M_Z)$, from Y_{10}, Y_{126}, Y_{120} and r_2, r_D using eqs. (2.6)–(2.9), (5.3), and with it we fit the neutrino oscillation data in NuFIT 5.1 [73, 74]. Meanwhile, we minimize eq. (5.1).

We restrict the parameter space to the region with $r_3 = 0$, since it is easier to minimize eq. (5.1) when $r_3 = 0$. This is because when $r_3 = 0$, $Y_u = Y_{10} + r_2 Y_{126}$ holds and we get $|(Y_{10})_{u_L i} + r_2(Y_{126})_{u_L i}| = |(Y_u)_{u_L i}| \leq y_u$ and $|(Y_{10})_{u_R i} + r_2(Y_{126})_{u_R i}| = |(Y_u)_{u_R i}| \leq y_u$ for

any flavor index i . Under the restriction of $r_3 = 0$, we parametrize the Yukawa coupling matrices as follows: we go to the flavor basis where the isospin-doublet down-type quark components of $\mathbf{16}^i$ matter fields have a diagonal Yukawa coupling matrix. Since $Y_u = Y_{10} + r_2 Y_{126}$ is symmetric, Y_u in this basis can be written as

$$Y_u = V_{\text{CKM}}^T \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c e^{2i d_2} & 0 \\ 0 & 0 & y_t e^{2i d_3} \end{pmatrix} V_{\text{CKM}}, \quad (5.4)$$

where V_{CKM} denotes the CKM matrix and d_2, d_3 are undetermined phases. In the same flavor basis, Y_d can be written as

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} V_{dR}, \quad (5.5)$$

where V_{dR} is an undetermined unitary matrix. From eqs. (2.3)–(2.6), Y_e and Y_D are written in terms of Y_u, Y_d as

$$\frac{1}{r_1} Y_e = Y_u - (3 + r_2) Y_{126} + r_e \frac{1}{r_1} \frac{1}{2} (Y_d - Y_d^T), \quad (5.6)$$

$$Y_D = Y_u - 4r_2 Y_{126} + r_D \frac{1}{r_1} \frac{1}{2} (Y_d - Y_d^T) \quad (5.7)$$

with

$$Y_{126} = \frac{1}{1 - r_2} \left\{ \frac{1}{r_1} \frac{1}{2} (Y_d + Y_d^T) - Y_u \right\}. \quad (5.8)$$

The Majorana mass eq. (2.7) is found to satisfy

$$M_N \propto Y_{126}. \quad (5.9)$$

The analysis of fitting and minimization proceeds with the above parameterization as follows: we fix y_u, y_c, y_t and the parameters of the CKM matrix at the central values in table 1. Then we randomly generate $y_d/r_1, y_s/r_1, y_b/r_1$, phases d_2, d_3 , unitary matrix V_{dR} , and complex numbers r_2, r_e, r_D , and calculate the singular values of $\frac{1}{r_1} Y_e$. We determine r_1 by requiring that the smallest singular value of Y_e equal the value of y_e in table 1. Then we require that the values of y_d, y_s, y_b be within the 3σ ranges in table 1, and the first and second largest singular values of Y_e respectively be within $\pm 0.1\%$ ranges of the values of y_τ, y_μ .³ Because the active neutrino mass matrix M_ν is proportional to $C'_\nu(M_Z)$, we calculate M_ν up to the overall constant as

$$M_\nu \propto (R(\mu_{\text{GUT}})R'(\mu_{\text{SUSY}}))^{-1} Y_D Y_{126}^{-1} Y_D^T (R(\mu_{\text{GUT}})R'(\mu_{\text{SUSY}}))^{T-1}. \quad (5.10)$$

Then we calculate from eq. (5.10) the three neutrino mixing angles $\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13}$ and the ratio of the neutrino mass squared differences $\Delta m_{21}^2 / \Delta m_{31}^2$, and require them to be

³Since the experimental errors of the charged lepton Yukawa couplings are negligibly small, here we loosen the fitting criteria.

within the 3σ ranges of NuFIT5.1 results (with SK atmospheric data). Here we assume the normal mass hierarchy, because it is almost impossible to realize the inverted mass hierarchy from Y_D, Y_{126} as these matrices have hierarchical structures. Finally, we select sets of values of $y_d/r_1, y_s/r_1, y_b/r_1, d_2, d_3, V_{dR}, r_2, r_e, r_D$ that meet the above fitting criteria, calculate eq. (5.1) from the sets, and minimize the value of eq. (5.1).

5.2 Result

From the analysis of section 5.1, we have obtained the values of the Yukawa coupling matrices Y_{10}, Y_{126}, Y_{120} and numbers r_1, r_2, r_e, r_D in appendix B. There, components of Y_{10}, Y_{126} satisfy

$$\begin{aligned} |(Y_{10})_{u_R d_R}|/y_u &= 1.4, & |(Y_{126})_{u_R d_R}|/y_u &= 1.9, & |(Y_{10})_{u_R s_R}|/y_u &= 2.0, & |(Y_{126})_{u_R s_R}|/y_u &= 2.0, \\ |(Y_{10})_{u_L d_L}|/y_u &= 1.9, & |(Y_{126})_{u_L d_L}|/y_u &= 1.8, & |(Y_{10})_{u_L s_L}|/y_u &= 0.33, & |(Y_{126})_{u_L s_L}|/y_u &= 0.34, \\ |(Y_{10})_{u_L u_L}|/y_u &= 0.45, & |(Y_{126})_{u_L u_L}|/y_u &= 0.87. \end{aligned} \tag{5.11}$$

Clearly, the conditions that the components of eq. (3.11) be on the order of the up quark Yukawa coupling are satisfied.

We evaluate how dimension-five proton decay is suppressed when the conditions are met. To this end, we compare “minimal proton partial lifetimes” calculated from the Yukawa coupling matrices of appendix B, with those calculated from results of “fitting without minimizing eq. (5.1)” where we only fit the quark and charged lepton Yukawa couplings, CKM parameters and neutrino oscillation data as section 5.1 but do not minimize eq. (5.1) so that the conditions are not necessarily satisfied. Here the “minimal proton partial lifetimes”, $1/\Gamma_{\max}(p \rightarrow K^+ \bar{\nu})$, $1/\Gamma_{\max}(p \rightarrow K^0 \mu^+)$, $1/\Gamma_{\max}(p \rightarrow K^0 e^+)$, are defined as ($\beta = e, \mu$)

$$\begin{aligned} \Gamma_{\max}(p \rightarrow K^+ \bar{\nu}) &= \frac{m_N}{64\pi} \left(1 - \frac{m_K^2}{m_N^2}\right)^2 \\ &\times \left(|A_{\max}(p \rightarrow K^+ \bar{\nu}_\tau)|^2 + |A_{\max}(p \rightarrow K^+ \bar{\nu}_\mu)|^2 + |A_{\max}(p \rightarrow K^+ \bar{\nu}_e)|^2 \right), \end{aligned} \tag{5.12}$$

$$\Gamma_{\max}(p \rightarrow K^0 \beta^+) = \frac{m_N}{64\pi} \left(1 - \frac{m_K^2}{m_N^2}\right)^2 |A_{\max}(p \rightarrow K^0 \beta^+)|^2, \tag{5.13}$$

where

$$A_{\max}(p \rightarrow K^+ \bar{\nu}_\tau) = |A_{\max}(p \rightarrow K^+ \bar{\nu}_\tau)_{\text{from } C_{5R}}| + |A_{\max}(p \rightarrow K^+ \bar{\nu}_\tau)_{\text{from } C_{5L}}| \tag{5.14}$$

and

$$A_{\max}(p \rightarrow K^+ \bar{\nu}_\tau)_{\text{from } C_{5R}} = \alpha_H(\mu_{\text{had}}) \frac{1}{f_\pi} A_{RL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{\mu_H}{m_{\tilde{t}_R}^2} \frac{1}{16\pi^2} \mathcal{F}' y_t y_\tau A_R^{\tau t}(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \frac{1}{M_{HC}} \\ \times \max_{A,B} \left\{ \left| \left(1 + \frac{D}{3} + F \right) (V_{ts}^{\text{ckm}})^* ((Y_A)_{\tau R t_R} (Y_B)_{u_R d_R} - (Y_A)_{\tau R u_R} (Y_B)_{t_R d_R}) \right. \right. \\ \left. \left. + \frac{2D}{3} (V_{td}^{\text{ckm}})^* ((Y_A)_{\tau R t_R} (Y_B)_{u_R s_R} - (Y_A)_{\tau R u_R} (Y_B)_{t_R s_R}) \right| \right\}, \quad (5.15)$$

$$A_{\max}(p \rightarrow K^+ \bar{\nu}_\tau)_{\text{from } C_{5L}} = \beta_H(\mu_{\text{had}}) \frac{1}{f_\pi} A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \frac{1}{16\pi^2} \mathcal{F} g_2^2 A_L^\tau(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \frac{1}{M_{HC}} \\ \times \max_{A',B'} \left\{ \left| \left(1 + \frac{D}{3} + F \right) (Y_{A'}^{\prime})_{u_L d_L} (Y_{B'}^{\prime})_{s_L \tau_L} + \frac{2D}{3} (Y_{A'}^{\prime})_{u_L s_L} (Y_{B'}^{\prime})_{d_L \tau_L} \right| \right\}, \quad (5.16)$$

$$A_{\max}(p \rightarrow K^+ \bar{\nu}_\beta) = \beta_H(\mu_{\text{had}}) \frac{1}{f_\pi} A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \frac{1}{16\pi^2} \mathcal{F} g_2^2 A_L^\beta(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \frac{1}{M_{HC}} \\ \times \max_{A',B'} \left\{ \left| \left(1 + \frac{D}{3} + F \right) (Y_{A'}^{\prime})_{u_L d_L} (Y_{B'}^{\prime})_{s_L \beta_L} + \frac{2D}{3} (Y_{A'}^{\prime})_{u_L s_L} (Y_{B'}^{\prime})_{d_L \beta_L} \right| \right\}, \quad (5.17)$$

$$A_{\max}(p \rightarrow K^0 \beta^+) = \beta_H(\mu_{\text{had}}) \frac{1}{f_\pi} A_{LL}(\mu_{\text{had}}, \mu_{\text{SUSY}}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \frac{1}{16\pi^2} \mathcal{F} g_2^2 A_L^\beta(\mu_{\text{SUSY}}, \mu_{\text{GUT}}) \frac{1}{M_{HC}} \\ \times (1 - D + F) \max_{A',B'} \{ |(Y_{A'}^{\prime})_{u_L s_L} (Y_{B'}^{\prime})_{u_L \beta_L} - (Y_{A'}^{\prime})_{u_L u_L} (Y_{B'}^{\prime})_{s_L \beta_L}| \}, \quad (5.18)$$

where A, B in eq. (5.15) run as $(A, B) = (10, 10), (10, 126), (126, 10), (126, 126), (120, 10), (120, 126), (120, 120)$, and A', B' in eqs. (5.16)–(5.18) run as $(A', B') = (10, 10), (10, 126), (126, 10), (126, 126)$. We assume the SUSY particle spectrum of eq. (5.2) and take $M_{HC} = 2 \cdot 10^{16}$ GeV in the calculation. Note that eqs. (5.15)–(5.18) take into account the texture of the colored Higgs mass matrix satisfying eqs. (3.12), (3.13). An implication of eqs. (5.14)–(5.18) is that we estimate the maximal values of the amplitudes without specifying the $O(1)$ numbers a, b, d, e, g, h, j in eqs. (2.27)–(2.31) and the relative phase between the Higgsino mass and Wino mass. The “minimal proton partial lifetimes” calculated from the Yukawa coupling matrices of appendix B are⁴

$$1/\Gamma_{\max}(p \rightarrow K^+ \bar{\nu}) = 7.4 \times 10^{33} \text{ years}, \quad (5.19)$$

$$1/\Gamma_{\max}(p \rightarrow K^0 \mu^+) = 1.0 \times 10^{37} \text{ years}, \quad (5.20)$$

$$1/\Gamma_{\max}(p \rightarrow K^0 e^+) = 3.6 \times 10^{39} \text{ years}. \quad (5.21)$$

On the other hand, the “minimal proton partial lifetimes” calculated from multiple results of “fitting without minimizing eq. (5.1)” are distributed as figure 1.

In figure 1, we overlay the values calculated from the Yukawa coupling matrices of appendix B in eqs. (5.19)–(5.21). Also, the current bounds on the $p \rightarrow K^+ \bar{\nu}$ [10], $p \rightarrow K^0 \mu^+$ [78], $p \rightarrow K^0 e^+$ [79] partial lifetimes, and the 3σ discovery reach of 2 years running of Hyper-Kamiokande [80] are shown.

Comparing eqs. (5.19)–(5.21) with figure 1, we see that the proton partial lifetimes calculated from the Yukawa coupling matrices of appendix B are on the upper edge of the distributions of proton partial lifetimes calculated from results of “fitting without minimizing eq. (5.1)”. This confirms that the texture of the Yukawa coupling matrices satisfying the

⁴These values are consistent with the current experimental bounds on the $p \rightarrow K^+ \bar{\nu}$ partial lifetime [10] and on the $p \rightarrow K^0 \mu^+/e^+$ partial lifetimes [78, 79], which justifies our choice of the benchmark SUSY particle mass spectrum eq. (5.2).

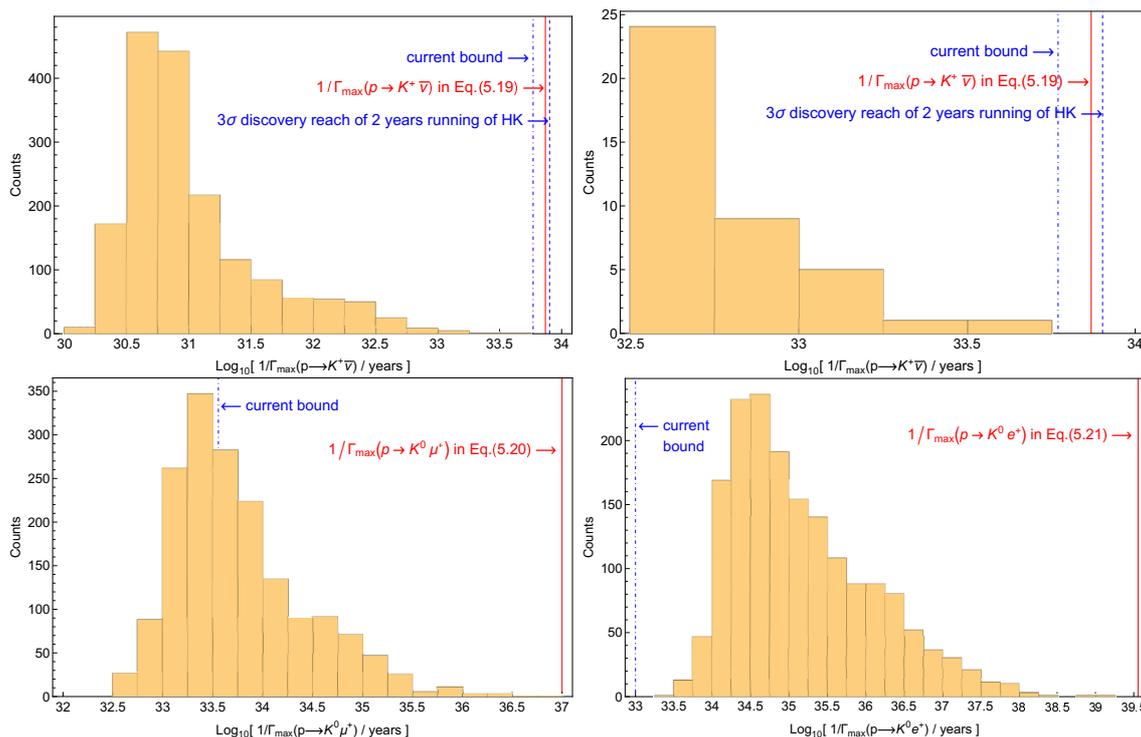


Figure 1. Distributions of the “minimal proton partial lifetimes” calculated from multiple results of “fitting without minimizing eq. (5.1)”. The upper two panels show the distribution of $1/\Gamma_{\max}(p \rightarrow K^+\bar{\nu})$, where the upper-right panel magnifies the right tail of the upper-left one. The blue dot-dashed line indicates the current bound on the $p \rightarrow K^+\bar{\nu}$ partial lifetime [10], the red solid line the value of $1/\Gamma_{\max}(p \rightarrow K^+\bar{\nu})$ calculated from the Yukawa coupling matrices of appendix B in eq. (5.19), and the blue dashed line the 3σ discovery reach of **2 years** running of Hyper-Kamiokande [80]. The lower-left panel shows the distribution of $1/\Gamma_{\max}(p \rightarrow K^0\mu^+)$, where the blue dot-dashed line indicates the current bound on the $p \rightarrow K^0\mu^+$ partial lifetime [78] (part of the line is hidden behind the histogram), and the red solid line the value of $1/\Gamma_{\max}(p \rightarrow K^0\mu^+)$ calculated from the Yukawa coupling matrices of appendix B in eq. (5.20). The lower-right panel shows the distribution of $1/\Gamma_{\max}(p \rightarrow K^0e^+)$, where the blue dot-dashed line indicates the current bound on the $p \rightarrow K^0e^+$ partial lifetime [79], and the red solid line the value of $1/\Gamma_{\max}(p \rightarrow K^0e^+)$ calculated from the Yukawa coupling matrices of appendix B in eq. (5.21).

conditions that *the components of eq. (3.11) be on the order of the up quark Yukawa coupling*, contributes to suppressing dimension-five proton decay. Specifically, the benchmark SUSY particle mass spectrum eq. (5.2), where the SUSY particle masses are all at 1500 TeV and $\tan\beta = 50$, is consistent with all the experimental bounds on proton partial lifetimes if the above conditions are satisfied. On the other hand, if these conditions are not met, this benchmark almost always violates the bound on the $p \rightarrow K^+\bar{\nu}$ partial lifetime. We also see that for this benchmark mass spectrum, when the above conditions are satisfied, we expect to discover the $p \rightarrow K^+\bar{\nu}$ decay with 2 years running of Hyper-Kamiokande.

In addition to the texture of the Yukawa coupling matrices, we have required that the colored Higgs mass matrix be tuned such that eqs. (3.12), (3.13) hold. Now we examine the degree of tuning of the colored Higgs mass matrix necessary to suppress dimension-five proton decay. To this end, we consider non-zero c, f and deviations of a/d and b/e from the relations of eq. (3.13), and evaluate maximum values of $|c|, |f|$ and maximum deviations of a/d and b/e that reduce $1/\Gamma_{\max}(p \rightarrow K^+\bar{\nu})$ from eq. (5.19) by at most 20% (with the same Yukawa coupling matrices). Here the phases of $c, f, a/d, b/e$ are chosen such that they reduce $1/\Gamma_{\max}(p \rightarrow K^+\bar{\nu})$ maximally, and the contributions of $c, f, a/d, b/e$ are studied separately. We numerically find that the maximum values of $|c|, |f|$ are

$$|c| = 0.14, \quad |f| = 0.14, \quad (5.22)$$

and the maximum deviations of a/d and b/e are

$$\left| a + d \frac{(Y_{126})_{d_L s_L}}{(Y_{10})_{d_L s_L}} \right| = 0.0097, \quad \left| b + e \frac{(Y_{126})_{d_L s_L}}{(Y_{10})_{d_L s_L}} \right| = 0.011. \quad (5.23)$$

Interestingly, the requirement of $c = f = 0$ is not so severe, while the conditions of $a(Y_{10})_{d_L s_L} + d(Y_{126})_{d_L s_L} = 0$ and $b(Y_{10})_{d_L s_L} + e(Y_{126})_{d_L s_L} = 0$ must be satisfied with 1% precision. For the other decay modes, the deviations of a/d and b/e do not affect $1/\Gamma_{\max}(p \rightarrow K^0\mu^+)$ and $1/\Gamma_{\max}(p \rightarrow K^0e^+)$. Non-zero c, f whose absolute values are below eq. (5.22) do not alter $1/\Gamma_{\max}(p \rightarrow K^0\mu^+)$ and $1/\Gamma_{\max}(p \rightarrow K^0e^+)$ because the products of Yukawa coupling components associated with c or f in eq. (2.31) are numerically smaller than 0.14 times the largest product of Yukawa coupling components in eq. (2.31).

We comment that the Yukawa coupling matrices and coefficients in appendix B give a prediction on poorly or not measured neutrino parameters, which are the Dirac CP phase of the neutrino mixing matrix, the sum of the neutrino mass, and the effective neutrino mass for neutrinoless double β decay. The prediction is shown in appendix C.

We comment on other nucleon decay modes. The $N \rightarrow \pi\beta^+$ and $p \rightarrow \eta\beta^+$ decays are subdominant compared to the $p \rightarrow K^0\beta^+$ decays, because the amplitudes of $N \rightarrow \pi\beta^+$ and $p \rightarrow \eta\beta^+$ involve the same Yukawa coupling components as those of $p \rightarrow K^0\beta^+$ except that s_L is replaced by d_L . Nevertheless, observation of $N \rightarrow \pi\beta^+$ and $p \rightarrow \eta\beta^+$ along with $p \rightarrow K^0\beta^+$ may provide an experimental clue to the texture of the Yukawa coupling matrices. Hence, we present in appendix D the “minimal partial lifetimes” of these modes calculated from the Yukawa coupling matrices of appendix B through the formulas in ref. [39].

6 Summary

We have pursued the possibility that dimension-five proton decay is suppressed by a texture of the Yukawa coupling matrices in the general renormalizable SUSY SO(10) GUT model where Yukawa coupling matrices of **16** representation matter fields with **10**, **$\overline{126}$** , **120** fields Y_{10}, Y_{126}, Y_{120} give the quark and lepton Yukawa couplings and Majorana mass of the

singlet neutrinos. We have derived conditions for a texture of the Yukawa coupling matrices suppressing dimension-five proton decay, which state that *components* $(Y_{10})_{u_R d_R}$, $(Y_{126})_{u_R d_R}$, $(Y_{10})_{u_R s_R}$, $(Y_{126})_{u_R s_R}$, $(Y_{10})_{u_L d_L}$, $(Y_{126})_{u_L d_L}$, $(Y_{10})_{u_L u_L}$, $(Y_{126})_{u_L u_L}$, $(Y_{10})_{u_L s_L}$, $(Y_{126})_{u_L s_L}$ should be all on the order of the up quark Yukawa coupling y_u . Additionally, the colored Higgs mass matrix should satisfy eqs. (3.12), (3.13). We have obtained the values of the Yukawa coupling matrices that satisfy the above conditions and that are consistent with the experimental data of quark and lepton masses and flavor mixings. By comparing the “minimal proton partial lifetimes” calculated from the Yukawa coupling matrices that meet the conditions and those that do not necessarily so, we have confirmed that the texture of the Yukawa coupling matrices satisfying the conditions contributes to suppressing dimension-five proton decay. Specifically, we have found that a SUSY particle mass spectrum where the SUSY particle masses are all at 1500 TeV and $\tan \beta = 50$ is consistent with all the experimental bounds on proton decay if the above conditions are satisfied. Also, for this mass spectrum, when the conditions are met, we expect to discover the $p \rightarrow K^+ \bar{\nu}$ decay with 2 years running of Hyper-Kamiokande.

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A Superpotential

We review our definition of the coupling constants and masses for H , Δ , $\bar{\Delta}$, Σ , Φ , A fields in **10**, **126**, $\bar{\mathbf{126}}$, **120**, **210**, **45** representations, which follows eq. (2) of ref. [42]. The couplings are defined in the same way as eq. (3) of ref. [42]. Note that **120** representation field is written as D in ref. [42], while we write it as Σ . The coupling constants are defined as

$$\begin{aligned}
 W = & \frac{1}{2}m_1\Phi^2 + m_2\bar{\Delta}\Delta + \frac{1}{2}m_3H^2 \\
 & + \frac{1}{2}m_4A^2 + \frac{1}{2}m_6\Sigma^2 \\
 & + \lambda_1\Phi^3 + \lambda_2\Phi\bar{\Delta}\Delta + (\lambda_3\Delta + \lambda_4\bar{\Delta})H\Phi \\
 & + \lambda_5A^2\Phi - i\lambda_6A\bar{\Delta}\Delta + \frac{\lambda_7}{120}\varepsilon A\Phi^2 \\
 & + \lambda_{15}\Sigma^2\Phi \\
 & + \Sigma\{\lambda_{16}HA + \lambda_{17}H\Phi + (\lambda_{18}\Delta + \lambda_{19}\bar{\Delta})A + (\lambda_{20}\Delta + \lambda_{21}\bar{\Delta})\Phi\} \quad (\text{A.1})
 \end{aligned}$$

where ε denotes the antisymmetric tensor in SO(10) space.

B Values of Y_{10} , Y_{126} , Y_{120} and r_1, r_2, r_e, r_D

We present the values of the Yukawa coupling matrices Y_{10} , Y_{126} , Y_{120} and numbers r_1, r_2, r_e, r_D obtained from the analysis of section 5.1. Y_{10} , Y_{126} , Y_{120} are shown repeatedly

in three different flavor bases. For reference, the central value of the up quark Yukawa coupling at scale $\mu = \mu_{\text{GUT}} = 2 \cdot 10^{16}$ GeV in $\overline{\text{DR}}$ scheme is $y_u = 2.81 \cdot 10^{-6}$.

$$\begin{pmatrix} (Y_{10})_{u_R d_R} & (Y_{10})_{u_R s_R} & (Y_{10})_{u_R b_R} \\ (Y_{10})_{c_R d_R} & (Y_{10})_{c_R s_R} & (Y_{10})_{c_R b_R} \\ (Y_{10})_{t_R d_R} & (Y_{10})_{t_R s_R} & (Y_{10})_{t_R b_R} \end{pmatrix} = \begin{pmatrix} 3.92 \cdot 10^{-6} e^{2.26i} & 5.61 \cdot 10^{-6} e^{-1.26i} & 0.00168 e^{-0.21i} \\ 0.00187 e^{-1.22i} & 0.00810 e^{-1.24i} & 0.0194 e^{-1.04i} \\ 0.00592 e^{-2.61i} & 0.0202 e^{-2.81i} & 0.230 e^{0.46i} \end{pmatrix} \quad (\text{B.1})$$

$$\begin{pmatrix} (Y_{126})_{u_R d_R} & (Y_{126})_{u_R s_R} & (Y_{126})_{u_R b_R} \\ (Y_{126})_{c_R d_R} & (Y_{126})_{c_R s_R} & (Y_{126})_{c_R b_R} \\ (Y_{126})_{t_R d_R} & (Y_{126})_{t_R s_R} & (Y_{126})_{t_R b_R} \end{pmatrix} = \begin{pmatrix} 5.23 \cdot 10^{-6} e^{-2.28i} & 5.70 \cdot 10^{-6} e^{0.89i} & 0.00158 e^{2.01i} \\ 0.00198 e^{1.12i} & 0.00856 e^{1.10i} & 0.0182 e^{1.18i} \\ 0.00618 e^{0.57i} & 0.0197 e^{0.31i} & 0.228 e^{-0.46i} \end{pmatrix} \quad (\text{B.2})$$

$$\begin{pmatrix} (Y_{120})_{u_R d_R} & (Y_{120})_{u_R s_R} & (Y_{120})_{u_R b_R} \\ (Y_{120})_{c_R d_R} & (Y_{120})_{c_R s_R} & (Y_{120})_{c_R b_R} \\ (Y_{120})_{t_R d_R} & (Y_{120})_{t_R s_R} & (Y_{120})_{t_R b_R} \end{pmatrix} = \begin{pmatrix} 0.000365 & 0.00159 & 0.000457 e^{2.44i} \\ 0.00159 e^{-3.13i} & 0.000366 e^{0.06i} & 0.000454 e^{-0.26i} \\ 0.000328 e^{-1.94i} & 0.000539 e^{1.55i} & 1.71 \cdot 10^{-5} e^{2.95i} \end{pmatrix} \quad (\text{B.3})$$

$$\begin{pmatrix} (Y_{10})_{u_L d_L} & (Y_{10})_{u_L s_L} & (Y_{10})_{u_L b_L} \\ (Y_{10})_{c_L d_L} & (Y_{10})_{c_L s_L} & (Y_{10})_{c_L b_L} \\ (Y_{10})_{t_L d_L} & (Y_{10})_{t_L s_L} & (Y_{10})_{t_L b_L} \end{pmatrix} = \begin{pmatrix} 5.45 \cdot 10^{-6} e^{-1.30i} & 0.920 \cdot 10^{-6} e^{1.39i} & 0.00168 e^{0.58i} \\ 0.00187 e^{-2.28i} & 0.00810 e^{0.85i} & 0.0194 e^{-0.25i} \\ 0.00385 e^{1.98i} & 0.0208 e^{-0.72i} & 0.230 e^{1.25i} \end{pmatrix} \quad (\text{B.4})$$

$$\begin{pmatrix} (Y_{126})_{u_L d_L} & (Y_{126})_{u_L s_L} & (Y_{126})_{u_L b_L} \\ (Y_{126})_{c_L d_L} & (Y_{126})_{c_L s_L} & (Y_{126})_{c_L b_L} \\ (Y_{126})_{t_L d_L} & (Y_{126})_{t_L s_L} & (Y_{126})_{t_L b_L} \end{pmatrix} = \begin{pmatrix} 5.10 \cdot 10^{-6} e^{0.41i} & 0.961 \cdot 10^{-6} e^{-2.01i} & 0.00158 e^{2.80i} \\ 0.00198 e^{0.05i} & 0.00856 e^{-3.10i} & 0.0182 e^{1.97i} \\ 0.00395 e^{-1.03i} & 0.0206 e^{2.44i} & 0.228 e^{0.33i} \end{pmatrix} \quad (\text{B.5})$$

$$\begin{pmatrix} (Y_{120})_{u_L d_L} & (Y_{120})_{u_L s_L} & (Y_{120})_{u_L b_L} \\ (Y_{120})_{c_L d_L} & (Y_{120})_{c_L s_L} & (Y_{120})_{c_L b_L} \\ (Y_{120})_{t_L d_L} & (Y_{120})_{t_L s_L} & (Y_{120})_{t_L b_L} \end{pmatrix} = \begin{pmatrix} 0.000365 e^{-1.06i} & 0.00159 e^{2.08i} & 0.000459 e^{-3.06i} \\ 0.00159 e^{-1.05i} & 0.000369 e^{-1.10i} & 0.000452 e^{0.54i} \\ 0.000523 e^{0.29i} & 0.000353 e^{-2.51i} & 1.90 \cdot 10^{-5} e^{-2.51i} \end{pmatrix} \quad (\text{B.6})$$

$$\begin{pmatrix} (Y_{10})_{u_L u_L} & (Y_{10})_{u_L c_L} & (Y_{10})_{u_L t_L} \\ & (Y_{10})_{c_L c_L} & (Y_{10})_{c_L t_L} \\ & & (Y_{10})_{t_L t_L} \end{pmatrix} = \begin{pmatrix} 1.27 \cdot 10^{-6} e^{1.18i} & 7.11 \cdot 10^{-5} e^{0.60i} & 0.00168 e^{0.58i} \\ & 0.00870 e^{0.77i} & 0.0192 e^{-0.27i} \\ & & 0.230 e^{1.25i} \end{pmatrix} \quad (\text{B.7})$$

$$\begin{pmatrix} (Y_{126})_{u_L u_L} & (Y_{126})_{u_L c_L} & (Y_{126})_{u_L t_L} \\ & (Y_{126})_{c_L c_L} & (Y_{126})_{c_L t_L} \\ & & (Y_{126})_{t_L t_L} \end{pmatrix} = \begin{pmatrix} 2.45 \cdot 10^{-6} e^{-1.39i} & 6.69 \cdot 10^{-5} e^{2.83i} & 0.00158 e^{2.80i} \\ & 0.00907 e^{3.11i} & 0.0181 e^{1.96i} \\ & & 0.228 e^{0.32i} \end{pmatrix} \quad (\text{B.8})$$

$$\begin{pmatrix} (Y_{120})_{u_L u_L} & (Y_{120})_{u_L c_L} & (Y_{120})_{u_L t_L} \\ & (Y_{120})_{c_L c_L} & (Y_{120})_{c_L t_L} \\ & & (Y_{120})_{t_L t_L} \end{pmatrix} = \begin{pmatrix} 0 & 0.00164 e^{2.09i} & 0.000435 e^{-2.91i} \\ & 0 & 0.000457 e^{0.54i} \\ & & 0 \end{pmatrix} \quad (\text{B.9})$$

$$r_1 = 0.871, \quad r_2 = 1.06 e^{0.92i}, \quad r_e = 1.01 e^{0.79i}, \quad r_D = 0.482 e^{-3.11i}. \quad (\text{B.10})$$

In eqs. (B.7)–(B.9), we do not display some off-diagonal components because in this flavor basis, Y_{10}, Y_{126} are symmetric and Y_{120} is antisymmetric.

C Prediction on neutrino parameters

The result of the analysis of section 5.1, shown in appendix B, gives the following prediction on the Dirac CP phase of the neutrino mixing matrix, δ_{CP} , the sum of the neutrino mass, $\sum_{i=1}^3 m_i$, and the effective neutrino mass for neutrinoless double β decay, $|m_{ee}|$:

$$\delta_{\text{CP}} = 1.35 \text{ rad}, \quad (\text{C.1})$$

$$\sum_{i=1}^3 m_i = 0.0630 \text{ eV}, \quad (\text{C.2})$$

$$|m_{ee}| = 0.000263 \text{ eV}. \quad (\text{C.3})$$

We caution that there is no clear correlation between the prediction on $\delta_{\text{CP}}, \sum_{i=1}^3 m_i, |m_{ee}|$ and the degree of suppression of dimension-five proton decay, as seen in figure 2 where we plot the results of “fitting without minimizing eq. (5.1)” on the planes of $1/\Gamma_{\text{max}}(p \rightarrow K^+ \bar{\nu})$

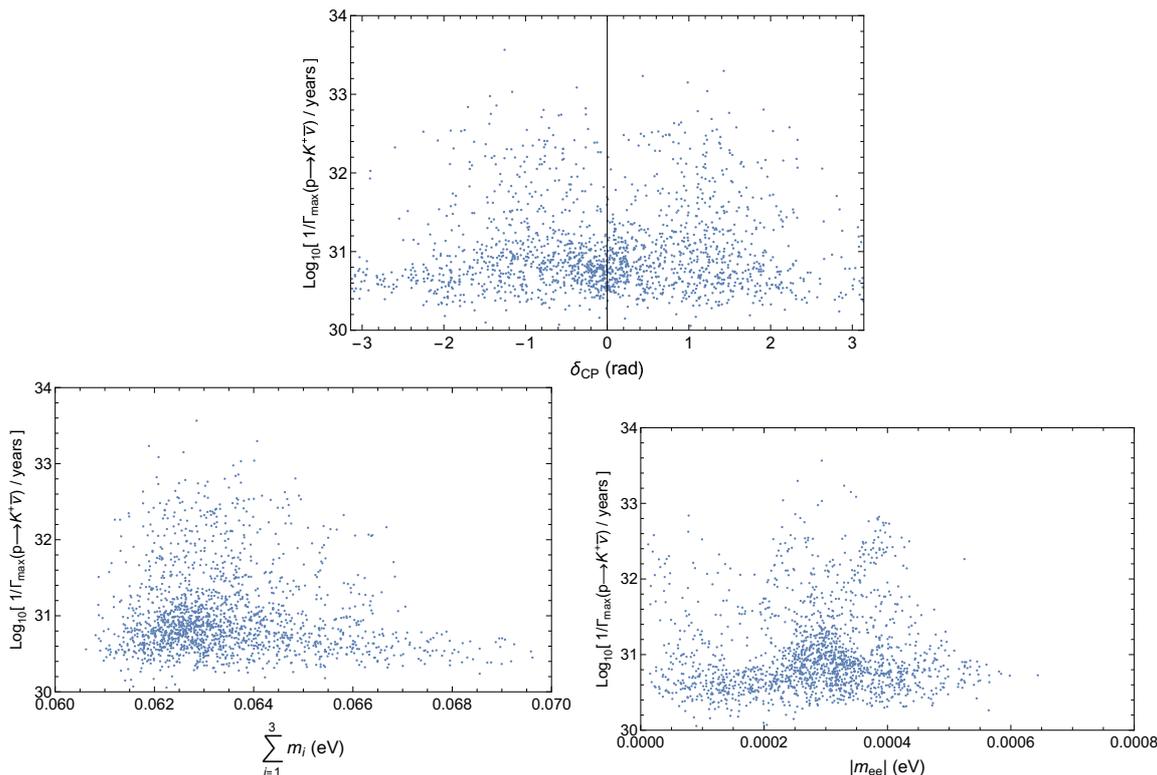


Figure 2. Results of “fitting without minimizing eq. (5.1)” on the planes of $1/\Gamma_{\max}(p \rightarrow K^+\bar{\nu})$ versus δ_{CP} , $\sum_{i=1}^3 m_i$, $|m_{ee}|$ in the upper, lower-left and lower-right panels, respectively.

versus δ_{CP} , $\sum_{i=1}^3 m_i$, $|m_{ee}|$. Similar figures are obtained for $1/\Gamma_{\max}(p \rightarrow K^0\mu^+)$ and $1/\Gamma_{\max}(p \rightarrow K^0e^+)$. Therefore, the prediction of eqs. (C.1)–(C.3) is not a consequence of the texture of the Yukawa coupling matrices suppressing dimension-five proton decay.

D Other nucleon decay modes

The “minimal partial lifetimes” of the $N \rightarrow \pi\beta^+$ and $p \rightarrow \eta\beta^+$ modes ($\beta = e, \mu$) defined analogously to eq. (5.13) and calculated from the Yukawa coupling matrices of appendix B through the formulas in ref. [39] are

$$1/\Gamma_{\max}(p \rightarrow \pi^0\mu^+) = 1.5 \times 10^{37} \text{ years}, \tag{D.1}$$

$$1/\Gamma_{\max}(p \rightarrow \pi^0e^+) = 7.2 \times 10^{39} \text{ years}, \tag{D.2}$$

$$1/\Gamma_{\max}(n \rightarrow \pi^-\mu^+) = 4.9 \times 10^{37} \text{ years}, \tag{D.3}$$

$$1/\Gamma_{\max}(n \rightarrow \pi^-e^+) = 2.4 \times 10^{39} \text{ years}, \tag{D.4}$$

$$1/\Gamma_{\max}(p \rightarrow \eta\mu^+) = 3.9 \times 10^{37} \text{ years}, \tag{D.5}$$

$$1/\Gamma_{\max}(p \rightarrow \eta e^+) = 1.9 \times 10^{40} \text{ years}. \tag{D.6}$$

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