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Critical behavior and microscopic structure of charged AdS black holes via an alternative phase space

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ABSTRACT

It has been argued that charged Anti-de Sitter (AdS) black holes have similar thermodynamic behavior as the Van der Waals fluid system, provided one treats the cosmological constant as a thermodynamic variable (pressure) in an extended phase space. In this paper, we disclose the deep connection between charged AdS black holes and Van der Waals fluid system from an alternative point of view. We consider the mass of an AdS black hole as a function of square of the charge Q^2 instead of the standard Q, i.e. $M = M(S, Q^2, P)$. We first justify such a change of view mathematically and then ask if a phase transition can occur as a function of Q^2 for fixed P. Therefore, we write the equation of state as $Q^2 = Q^2(T, \Psi)$ where Ψ (conjugate of Q^2) is the inverse of the specific volume, $\Psi = 1/v$. This allows us to complete the analogy of charged AdS black holes with Van der Waals fluid system and derive the phase transition as well as critical exponents of the system. We identify a thermodynamic instability in this new picture with real analogy to Van der Waals fluid with physically relevant Maxwell construction. We therefore study the critical behavior of isotherms in Q^2–Ψ diagram and deduce all the critical exponents of the system and determine that the system exhibits a small–large black hole phase transition at the critical point (T_c, Q_c^2, Ψ_c). This alternative view is important as one can imagine such a change for a given single black hole i.e. acquiring charge which induces the phase transition. Finally, we disclose the microscopic properties of charged AdS black holes by using thermodynamic geometry. Interestingly, we find that scalar curvature has a gap between small and large black holes, and this gap becomes exceedingly large as one moves away from the critical point along the transition line. Therefore, we are able to attribute the sudden enlargement of the black hole to the strong repulsive nature of the internal constituents at the phase transition.

1. Introduction

Inspired by the black hole physics a profound connection between the laws of thermodynamics and the gravitational systems has been argued to exist. A pioneering work in this respect was done by Bekenstein and Hawking [1,2] who disclosed that the entropy (S) and temperature (T) of a black hole satisfy the first law of thermodynamics, $dM = TdS$, where M is the mass of the black hole. Later, the thermodynamic phase space of black hole was extended by considering the charge Q and the cosmological constant Λ (pressure P) [3–6] as the thermodynamic variables. By consideration the energy formation of the thermodynamic system, the authors of Ref. [7] showed that the mass of AdS black hole M is indeed the enthalpy H. Therefore, the first law of black hole thermodynamics was written in the form $dM \equiv dH = TdS + VdP + \Phi dQ$, where V and Φ are volume and electrical potential, respectively.

Phase transition has gained attention as a thermodynamic property of AdS black holes ever since gravity correspondence was discovered with thermal field theory. The first study on black hole phase transitions was done by Hawking and Page [8] who demonstrated a certain phase transition in Schwarzschild AdS black hole. This transition can be interpreted as a confinement–deconfinement phase transition in the dual quark gluon plasma [9]. Recently, authors of [10] have shown that Hawking and Page phase transition can be found by Ruppeiner geometry. One of the important topics in phase transition is the critical point (continuous phase transition) because thermodynamic properties of the system exhibit non-analytic behavior. Such non-analytic behavior is described in terms of power-laws whose exponents define the universality class

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of various systems. Variation of the electric charge affects the thermodynamic behavior of black hole and consequently it can lead to critical phenomena. Authors of Refs. [3,4] reportedly showed that a phase transition occurs between large and small black holes in Q–Φ plane. They claimed that this behavior is similar to Van der Waals phase transition. However, as we will show in this paper, the phase transition they studied in [3,4] when Q is considered as a thermodynamic variable, is mathematically problematic and physically unconventional. Similar studies were also carried out by treating the cosmological constant as the thermodynamic pressure in an extended phase space, with its conjugate variable as volume [11–16]. By exploring the behavior of the pressure P versus specific volume v (with fixed charge Q and $v = 2r_+$), the authors of Ref. [5] showed the existence of a continuous and discontinuous phase transition between small and large charged AdS black holes. This transition is analogous to the Van der Waals liquid–gas phase transition and belongs to the same universality class. In this view, the cosmological constant is treated as thermodynamically equivalent to the pressure of the system. However, in general relativity (GR) the cosmological constant is usually assumed as a constant related to the background of AdS geometry. Indeed, the cosmological constant, which is usually assumed as the zero point energy of the field theory, defines the background of the spacetime. Therefore, from a physical point of view, it is difficult to consider the cosmological constant as a pressure of a system which can take on arbitrary values.

It seems natural to think of variation of charge Q of a black hole and keep the cosmological constant as a fixed parameter, since the charge of a black hole is a natural external variable which can vary. However, previous works [3,4] have considered the energy differential as ΦdQ with $\Phi = Q/r_+$, with r_+ the event horizon radius. They have identified a phase transition and have studied its associated thermodynamic behavior. As we will show shortly, such a view of thermodynamic conjugate variables (Q and $\Phi = Q/r_+$) which are not mathematically independent can lead to physically irrelevant quantities such as $(\partial Q/\partial \Phi)_T$ which is supposed to be a thermodynamic response function, but mathematically ill-defined. In the present work, we offer an alternative view of such an phase space and definition of new response function which naturally leads to physically relevant quantity. As we will show, the critical behavior indeed occurs in Q^2–Ψ plane, where $\Psi = 1/2r_+$. Thus, the first law of black hole thermodynamics as well as the Smarr relation are subsequently modified. We identify a small–large black hole phase transition and obtain the critical point as well as the critical exponents. Perhaps more interestingly, by calculating the scalar curvature, we are able to establish a direct link between microscopic interactions and the resulting macroscopic phase transition.

The outline of our paper is as follows: in the next section, we study thermodynamics of charged AdS black holes by replacing term ΦdQ with ΨdQ^2 in the first law of thermodynamics. In section 3, we investigate the critical behavior and phase transition of charged AdS black holes by treating the charge as the thermodynamic variable and keeping the cosmological constant (pressure) as a fixed parameter. In section 4, we explore microscopic properties of charged AdS black holes by applying thermodynamic geometry towards thermodynamics of the system. In particular, we focus on studying the kind of intermolecular interaction along the transition curve for small and large black holes by using the Ruppeiner geometry. We finish with concluding remarks in the last section.

2. Thermodynamics of charged AdS black holes

Our starting point is the action of Einstein–Maxwell theory in the background of AdS spacetime [5]

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\mathcal{R} - 2\Lambda - F_{\mu\nu}F^{\mu\nu}\right), \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electrodynamics field tensor with gauge potential A_μ, $\mathcal{R}$ is the Ricci scalar, $\Lambda = -3/l^2$ is the negative cosmological constant and l is the AdS radius. The line element of Reissner–Nordström (RN)-AdS black holes can be written as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{2}$$

where $d\Omega^2$ is the metric of 2-sphere and $f(r)$ is given by

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}, \tag{3}$$

where M and Q are, respectively, the Arnowitt–Deser–Misner (ADM) mass and charge of the black hole. The Maxwell equation also yields the electric field as $F_{tr} = Q/r^2$. The event horizon r_+, is the largest root of $f(r_+) = 0$, and hence the mass of black hole is written as

$$M(r_+) = r_+ + \frac{Q^2}{r_+} + \frac{r_+^3}{l^2}. \tag{4}$$

The Hawking temperature of the RN-AdS black hole on event horizon r_+ can be calculated as [5]:

$$T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right). \tag{5}$$

The entropy of the charged black hole which is a quarter of the event horizon area is $S = \pi r_+^2$. Now, we explore thermodynamics of RN-AdS black hole in a new phase space. We consider the entropy S, square of charge Q^2 and negative cosmological constant Λ ($P = -\Lambda/(8\pi)$), as independent variables. Thus, the ADM mass of black hole is enthalpy [7], ($H = M$) and is obtained as

$$M(S, Q^2, P) = \frac{1}{6\sqrt{\pi S}}\left(3S + 8PS^2 + 3\pi Q^2\right). \tag{6}$$

The intensive parameters conjugate to S, Q^2 and P are defined by $T \equiv (\partial M/\partial S)_{P,\,Q^2}$, $\Psi \equiv \left(\partial M/\partial Q^2\right)_{S,\,P}$, $V \equiv (\partial M/\partial P)_{S,\,Q^2}$, where T is the temperature, $\Psi = 1/(2r_+)$, and the thermodynamics volume V is obtained via $V = \int 4S dr_+ = 4\pi r_+^3/3$. The new variable Ψ is the inverse of the specific volume $v = 2r_+$ in the natural unit where $l_p = 1$ [5]. Therefore, the above thermodynamic relation satisfies the first law as:

$$dM = TdS + \Psi dQ^2 + VdP. \tag{7}$$

From scaling argument, we arrive at the Smarr formula as [7]:

$$M = 2\left(TS + \Psi Q^2 - VP\right). \tag{8}$$

Let us compare the first law of thermodynamics we have proposed here in Eq. (7) with the usual first law that has been studied in the literature. For example, the authors of [5] proposed the first law of thermodynamics in an extended phase space in the from

$$dM = TdS + \Phi dQ + VdP, \tag{9}$$

where $\Phi = Q/r_+$ is the electric potential, measured at infinity with respect to the horizon. The corresponding Smarr formula is [5]

$$M = 2(TS - VP) + \Phi Q. \tag{10}$$

It is important to note that we have replaced the usual ΦdQ term in the first law with ΨdQ^2. The extended phase space associated with $P = -\Lambda/(8\pi)$ is still the same. Also, the associated Smarr formula given in Eqs. (8) and (10) is the same, since $2\Psi Q^2 = Q^2/r_+ = \Phi Q$.

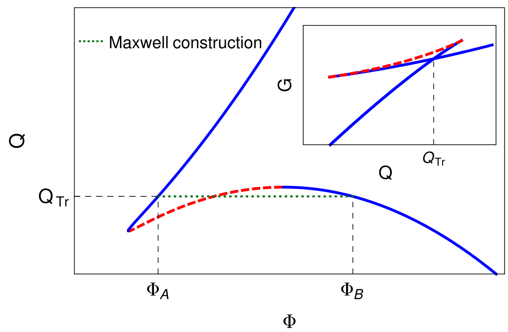

Fig. 1. (Color online.) The behaviors of isothermal Q–Φ diagram and the corresponding G–Q diagram (inset) of charged AdS black holes for the case of $l = 1$. Note that, after Maxwell construction both regions of positive and negative slope still remain. Compare with Figs. 2 and 3.

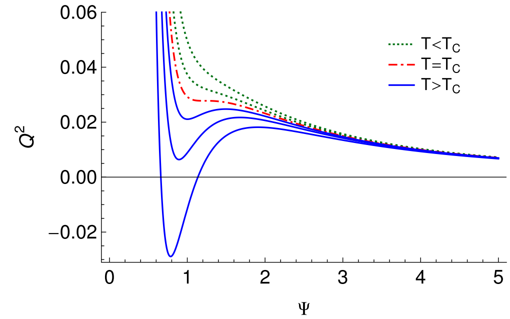

Fig. 2. (Color online.) The behavior of isothermal Q^2–Ψ diagram of charged AdS black holes for the case of $l = 1$. Note that in this approach the isotherms look essentially the same as Van der Waals isotherms, compare to Fig. 1 (conventional approach) which exhibits unusual isotherms.

Now, the question we ask is which one of the two sets of equations, i.e. Eqs. (7) and (8) or Eqs. (9) and (10) is more appropriate for investigation of AdS black hole thermodynamics? We have already argued that the use of extended phase space ($V dP$) is physically difficult to justify as it implies arbitrary values of cosmological constant. However, and more important to our propose here, our question boils down to what is the appropriate thermodynamic variable representing the charge of a AdS black hole, Q^2 or Q? A look at Eqs. (3), (4), (5) and (6) shows that the charge of AdS black hole is never represented as Q but always as Q^2. However, there is far more fundamental reason for choosing Q^2 instead of Q. The reason is the corresponding conjugate variable, Φ vs. Ψ. The conjugate thermodynamic variables are supposed to be mathematically independent variables. For example change of volume leads to a change of pressure through some physical process ($V dP$), or change of temperature leads to change of entropy ($T dS$). It is through this independence that physically relevant response functions are defined like compressibility $(-1/V(\partial V/\partial P))$ and heat capacity $(T(\partial S/\partial T))$, which are required to be positive for a stable thermodynamic system. If one looks at the Q–Φ space, one immediately realizes such independence is violated as the conjugate variable $\Phi \equiv Q/r_+$ explicitly depends on Q itself! This makes the definition of a response function $(\partial Q/\partial \Phi)$ physically and mathematically problematic. This is important, as phase transitions are driven by thermodynamic instabilities which are manifested in unusual behavior in the corresponding response functions, thus making appropriate definition of them of fundamental importance.

Therefore a change in Φ might automatically bring a change in Q without any physical response from the system, making $\partial Q/\partial \Phi$ meaningless as a physical response function. In order to see how this can lead to problems (see Refs. [3] and [4] for example), we have plotted the Gibbs free energy as a function of Q in the inset of Fig. 1 and the corresponding "unstable" isotherm in Q–Φ diagram. The instability associated with multivaluedness of G is not removed by the standard Maxwell construction as is clearly seen in the figure. This is so because both regions of positive and negative slope $(\partial Q/\partial \Phi)_T$ still remain even after Maxwell construction. Therefore Maxwell construction does not lead to stable isotherms. Since the entire thermodynamics of phase transition is based on identification of stable and unstable regimes, one can conclude that previous studies which have been based on such a view lead to suspicious results.

Here, we propose to consider isotherms in the Q^2–Ψ diagrams and use the physically and mathematically relevant response function $(\partial Q^2/\partial \Psi)$ in order to distinguish regions of stable and unstable system. Note that our proposed alternative view leads to a natural response function which measures how the size of a black hole ($\Psi \sim r_+^{-1}$) changes with changes in its charge (Q^2). We will see that this alternative view remedies the problems seen Fig. 1 and more importantly leads to a natural correspondence with the Van der Waals fluid and the associated small–large black hole phase transition without a need for extended phase space.

3. Phase transition and critical exponents

In this section, based on the first law of thermodynamics given in Eq. (7) and the enthalpy (mass) of system given in Eq. (8), we propose an alternative approach towards critical behavior of black holes by considering the pressure $P = 3/(8\pi l^2)$ as a fixed external parameter and allow the charge of the black hole to vary. Therefore, by using Eq. (5), one may write the equation of state $Q^2(T, \Psi)$ as,

$$Q^2 = r_+^2 + \frac{3r_+^4}{l^2} - 4\pi r_+^3 T. \tag{11}$$

The Q^2–Ψ isothermal diagram is shown in Fig. 2. In this figure there are some parts of the isotherms which correspond to a negative Q^2. Clearly these parts of diagram are physically not acceptable. Note that this also occurs in the usual Van der Waals fluid where the pressure can become negative for certain values of T [17]. However, more importantly, oscillating part of the isotherm indicates instability region $\left(\left(\partial Q^2/\partial \Psi\right)_T > 0\right)$. Both of these physically unstable features are remedied by the usual Maxwell equal area construction [17],

$$\oint \Psi dQ^2 = 0, \tag{12}$$

as depicted in Fig. 3. Isothermal diagrams show that, for l constant and $T = T_c$, there is an inflection point which is the critical point where a continuous phase transition occurs. Therefore, the critical point can be characterized by

$$\left.\frac{\partial Q^2}{\partial \Psi}\right|_{T_c} = 0, \qquad \left.\frac{\partial^2 Q^2}{\partial \Psi^2}\right|_{T_c} = 0. \tag{13}$$

One obtains:

$$T_c = \frac{1}{\pi l}\sqrt{\frac{2}{3}}, \qquad Q_c^2 = \frac{l^2}{36}, \qquad \Psi_c = \sqrt{\frac{3}{2l^2}}, \tag{14}$$

which leads to a universal (l independent) constant,

$$\rho_c \equiv Q_c^2 T_c \Psi_c = \frac{1}{36\pi}. \tag{15}$$

The behavior of a thermodynamic system can also be characterized by the Gibbs free energy, $G = M - TS$. In a fixed pressure regime, the Gibbs free energy reduces to

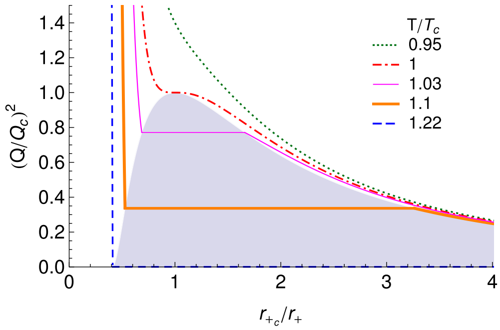

Fig. 3. (Color online.) The behavior of isothermal Q^2–Ψ diagram of charged AdS black holes constructed by Maxwell equal area law. Here we have set $l = 1$ and rescaled the axes.

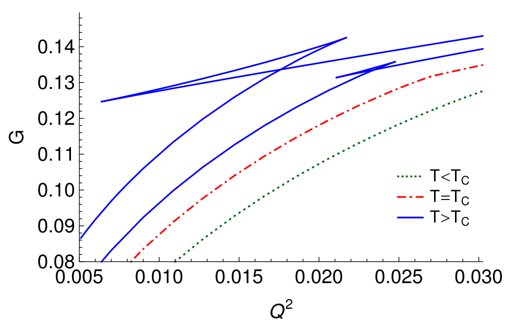

Fig. 4. (Color online.) Gibbs free energy of charged AdS black holes with $l = 1$. Curves are shifted for clarity.

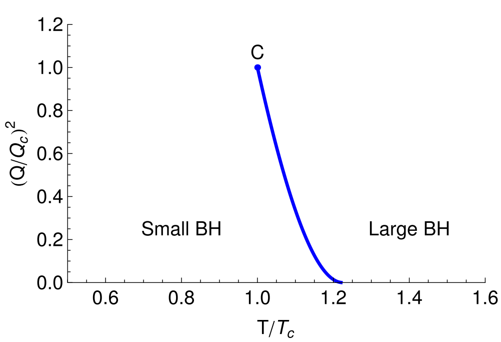

Fig. 5. (Color online.) Transition line of small-large BH phase transition of charged AdS black hole in the Q^2–T plane. The critical point marks the end of the transition line.

$$G = G\left(T, Q^2\right) = \left(\frac{r_+}{4} + \frac{3Q^2}{4r_+} - \frac{r_+^3}{4l^2}\right)\omega, \tag{16}$$

where $\omega = 4\pi$ is the area of unit 2-sphere, and $r_+ = r_+\left(T, Q^2\right)$, see Eq. (11). The behavior of the Gibbs free energy in term of Q^2 is depicted in Fig. 4. Here, the multi-valued behavior of Gibbs free energy indicates that the system has a discontinuous (first order) phase transition from small black hole to large black hole. Evidently, for $T > T_c$ the square of charge and the temperature of charged AdS black hole are constant during the phase transition. The transition line can be obtained from Maxwell's equal area law and Gibbs free energy, which shows a small-large black hole phase transition. Such a phase diagram is shown in Fig. 5 where one can see that the extremal large black hole does not exist.

Next, we turn to calculate the critical exponents in this new *phase space* approach. The behavior of thermodynamic functions in the vicinity of the critical point is characterized by the critical exponents. Let us define the reduced thermodynamic variables

$$\Psi_r \equiv \frac{\Psi}{\Psi_c}, \qquad Q_r^2 \equiv \frac{Q^2}{Q_c^2}, \qquad T_r \equiv \frac{T}{T_c}. \tag{17}$$

To find the critical exponents, we write the reduced variables in the form $T_r = 1 + t$, $\Psi_r = 1 + \psi$, $Q_r^2 = 1 + \varrho$, where, t, ψ and ϱ show the deviation from the critical point. First, we consider the entropy as a function of temperature T and $\Psi = 1/(2r_+)$ as

$$S = S(T, \Psi) = \frac{\pi}{4\Psi^2}, \tag{18}$$

which is independent of temperature T. Therefore the specific heat at fixed Ψ reads,

$$C_\Psi = T\frac{\partial S}{\partial T}\bigg|_\Psi = 0.$$

Since, the exponent α describes the behavior of C_Ψ near the critical point as $C_\Psi \propto |t|^\alpha$, one finds $\alpha = 0$. By using Eq. (17), equation of state (11) translates into the law of corresponding states,

$$Q_r^2 = \frac{6}{\Psi_r^2} + \frac{3}{\Psi_r^4} - \frac{8T_r}{\Psi_r^3}, \tag{19}$$

which is the equation of state in an l-independent form. One can expand Eq. (19) near the critical points as

$$\varrho = -8t + 24t\psi - 4\psi^3 + O\left(t\psi^2, \psi^4\right). \tag{20}$$

Applying the Maxwell's equal area law and differentiating Eq. (20) with respect to ψ at a fixed $t > 0$, leads to

$$\varrho = -8t + 24t\psi_l - 4\psi_l^3 = -8t + 24t\psi_s - 4\psi_s^3,$$
$$0 = \Psi_c \int_{\psi_l}^{\psi_s} \psi\left(24t - 12\psi^2\right) d\psi, \tag{21}$$

where ψ_s and ψ_l denote the event horizon of small and large black holes, respectively. Eqs. (21) have the nontrivial solution

$$\psi_s = -\psi_l = \sqrt{6t}. \tag{22}$$

So, the behavior of the order parameter near the critical point can be calculated as

$$|\psi_s - \psi_l| = 2\psi_s = 2\sqrt{6}t^{1/2} \Longrightarrow \beta = 1/2. \tag{23}$$

To obtain the critical exponent γ, we may determine the behavior of the function

$$\chi_T = \frac{\partial \Psi}{\partial Q^2}\bigg|_T,$$

near the critical point as $\chi_T \propto |t|^{-\gamma}$. Using Eq. (20), one obtains

$$\chi_T \propto \frac{\Psi_c}{24Q_c^2 t} \Longrightarrow \gamma = 1. \tag{24}$$

The shape of the critical isotherm $t = 0$ is calculated by $\varrho = -4\psi^3 \Longrightarrow \delta = 3$. In this way, we have obtained the set of critical exponents for charged AdS black holes by treating the charge as a thermodynamic variable and keeping fixed the cosmological constant (pressure). Note that the obtained critical exponents in this section coincide with those obtained for Van der Waals fluid [5,17]. Also, the authors of Ref. [5] obtained the same critical exponents for charged AdS black holes by keeping the charge as a fixed parameter and treating the cosmological constant as a thermodynamic pressure and its conjugate quantity as a thermodynamic volume in an extended phase space. Our thermodynamic approach to phase space is more natural and potentially more realistic. For example, one can easily imagine increasing Q while keeping T

Table 1
The allowed ranges of Ψ/Ψ_c.

	$\Psi/\Psi_c < \sqrt{\Psi_0}$	$\Psi/\Psi_c > \sqrt{\Psi_0}$
R	Positive	Negative
T	Positive	Negative
Validity	Allowed	Not allowed

constant and observing the resulting continuous increase in black hole size (r_+). More importantly, however, our phase transition diagrams (e.g. Fig. 3) indicate that there is an instability region where intermediate-size black holes should never be observed for $T_c < T < 1.22T_c$. This is precisely the Maxwell constructed region where an abrupt change from a small to large black hole occurs. In the extended phase space approach this transition occurs as a function of cosmological constant while in the present approach it occurs as a function of Q^2.

4. Thermodynamic geometry and microscopic structure

In this section we intend to study microscopic properties of charged AdS black holes by applying thermodynamic geometry towards thermodynamics of the system. In particular, we focus on studying the kind of intermolecular interaction along the transition curve for small and large black holes by using the Ruppeiner geometry obtained from the thermodynamic fluctuation theory [18]. Since Ricci scalar is a thermodynamic invariant, the Ruppeiner geometry defined in (M, Q^2) space can be rewritten in the Weinhold energy form [19]:

$$g_{\mu\nu} = \frac{1}{T}\frac{\partial^2 M}{\partial X^\mu \partial X^\nu}, \tag{25}$$

where $X^\mu = (S, Q^2)$. We can calculate thermodynamic Ricci scalar (scalar curvature R) that is a thermodynamic invariant analogous to that of GR. The sign of R gives information about intermolecular interaction in a thermodynamic system i.e. positivity (negativity) of R refers to dominance of repulsive (attractive) interaction in thermodynamic system [20,21]. $R = 0$ shows there is no interaction in the system [22]. Using the above, one can calculate the Ruppeiner scalar curvature with fixed pressure $l = 1$. Then

$$R = \frac{36\,(\Psi/\Psi_c)^2\left[(\Psi/\Psi_c)^2 + 1\right]}{\pi\left[3 + 6\,(\Psi/\Psi_c)^2 - (Q/Q_c)^2\,(\Psi/\Psi_c)^4\right]}. \tag{26}$$

As shown in Table 1, the sign of R depends on the value of Q/Q_c, i.e. the scalar curvature is positive (repulsive intermolecular interaction) for $\Psi/\Psi_c < \sqrt{\Psi_0}$ where

$$\Psi_0 = \frac{3 + \sqrt{9 + 3\,(Q/Q_c)^2}}{(Q/Q_c)^2}, \tag{27}$$

and the region of $\Psi/\Psi_c > \sqrt{\Psi_0}$ is not physically allowed due to negative absolute temperature. Fig. 6 depicts the behavior of scalar curvature R versus temperature T/T_c along the transition curve (Fig. 4) for small and large black holes ($T_c \le T < 1.22T_c$). This figure shows that R is positive (repulsive intermolecular interaction) and is the same value at critical point for small and large black holes. Moreover, the Ruppeiner scalar has a gap between small and large black holes for $T_c < T < 1.22T_c$. Furthermore, one can note that R becomes increasingly large as T increases for a small BH (upper branch). This behavior is very interesting as diverging R indicates a very strong repulsive force. For example, our results show that a small BH at $T \approx 1.22T_c$ behaves much like a fermi gas at $T \approx 0$ [23], where fermi exclusion principle dominates the thermodynamic behavior of the system with strong degenerate pressure.

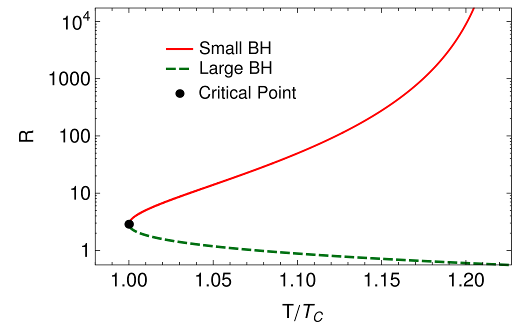

Fig. 6. (Color online.) The Ruppeiner scalar curvature R along the transition curve for small and large black holes. Note the logarithmic scale on the R axis.

However, it is also interesting to speculate on the possible microscopic nature of the phase transition at hand here. As one moves away from the critical point (continuous transition) along the transition line, one can see that the corresponding phase transition becomes discontinuous and more sudden in nature. The resulting large black hole is larger the further away one moves away from the critical point along the transition line (see Fig. 2, e.g. $T/T_c = 1.1$). Now, one can see from Fig. 5 that the associated R value for such a transition has a larger and larger gap. All possible stable large back holes have nearly R value of zero, indicating weakly-interacting constituents. However, the repulsive nature of these constituents on the small BH side of the transition line can be very large which is indicative of a very large outward pressure and a tendency to expand. The picture that emerges here is that phase transition from small to large black holes is driven by a repulsive nature of the interacting constituents whose tendencies are to expand the black hole. The larger this tendency at the transition (i.e. larger R) the larger the resulting black hole (smaller r_+). We are therefore able to draw certain conclusions about internal structure and interactions of a black hole simply by looking at its possible phase transitions.

5. Concluding remarks

In this paper we have studied the small-large black hole phase transition which has been intensively studied in previous years. We have not considered the extended phase space view where cosmological constant is thought to take arbitrary values, although such an approach could be taken within our proposed view. On the other hand, we have pointed out the problems associated with the traditional Q–Φ view in the non-extended phase space. Subsequently, we have offered an alternative view in the Q^2–Ψ plane where $\Psi = 1/2r_+$. Unlike previous studies, this change of view naturally leads to physically and mathematically meaningful response function $(\partial Q^2/\partial\Psi)_T$ whose sign clearly signifies stable and unstable regimes. We have characterized this instability as a small–large black hole phase transition and have characterized such a transition, including critical point, critical exponents, and a universal constant. As the name implies, cosmological constant does not offer a natural variable. However, as we have shown, for a given cosmological constant, one can imagine black holes taking on various amount of charge, which can subsequently lead to a phase transition. We have also studied microscopic properties of charged AdS black holes by considering thermodynamic geometry. This provides important insights into the nature of interactions among the black hole's constituents. Our results indicate that the transition from small black hole to large black hole is caused by a strongly repulsive interaction amongst the constituents, which upon expansion and the subsequent phase transition to a large black hole, lead to a relaxation of such internal forces ($R \approx 0$). This is in contrast with the usual liquid–gas transition where the *attractive* forces among the constituents lead to *condensation* and a subsequent large specific volume. We end by observing that no stable small black hole is possible for $T > 1.22T_c$, as the unstable regime extends to arbitrary large Ψ, e.g. see Fig. 2. Such isotherms only exhibit stable large black hole regime with nearly vertical isotherms. This behavior indicates that, for $T > 1.22T_c$, the size of the black hole is essentially a function of temperature only, where addition of charge does not significantly alter the size of the large black hole as the constituents have reached a non-interacting regime of $R = 0$ and thus no further expansion is possible. Therefore, in this new alternative view of the phase space, the attractive force between the constituents is the deriving force that not only determines the size of the black hole but is the essential mechanism causing the instability and the subsequent phase transition.

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