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Letter

Can anti-Unruh effect survive the environment-induced interatomic interaction?

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A R T I C L E I N F O

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A B S T R A C T

The entanglement generated for a uniformly accelerated two-atom system in vacuum during its evolution may increase with acceleration, while that for a static one in a thermal bath always decreases monotonically with temperature. This phenomenon is named as the anti-Unruh effect in terms of the entanglement generated. In this paper, we study the effects of the interatomic interaction induced by the electromagnetic vacuum fluctuations on the entanglement dynamics of two uniformly accelerated atoms. We show that the anti-Unruh phenomenon may exist or disappear depending critically on the configuration of the orientation of the atomic polarization and the directions of the uniform acceleration and the interatomic separation. This is in sharp contrast to the scalar-field case, in which the anti-Unruh phenomenon is always lost when the environment-induced interatomic interaction is considered.

1. Introduction

In quantum field theory, the vacuum state of a quantum field is observer dependent. A well-known example is that the Minkowski vacuum defined by inertial observers is perceived as a thermal bath of Rindler particles by uniformly accelerated observers, i.e. the well-known Unruh effect [1–4]. As a result, a uniformly accelerated atom would spontaneously get excited in the Minkowski vacuum [5–7]. Moreover, in contrast to the fact that the excitation rate of static atoms immersed in a thermal bath increases monotonically with the bath temperature, it may decrease with acceleration for accelerated atoms in certain cases, which is named as the anti-Unruh effect [8,9] in terms of the transition rate.

The situation is more intriguing when two atoms are involved, since there may exist quantum entanglement, which is regarded as one of the most striking features in quantum physics. Recently, a lot of effort has been made to investigate the behaviors of entanglement dynamics [10–19] and entanglement harvesting [20–33] in noninertial frames and in curved spacetimes. In particular, it is of interest to study how the entanglement dynamics of a uniformly accelerated two-atom system is affected by acceleration, and compare the result with that of a static one immersed in a thermal bath at the Unruh temperature. At this point, we note that, in general, the entanglement dynamics of uniformly accelerated atoms shows distinctive features compared with that of static ones in a thermal bath. Remarkably, the maximal concurrence generated for a uniformly accelerated two-atom system during its evolution may increase with acceleration, while that for a static one in a thermal bath always decreases monotonically with temperature [14–16]. This phenomenon is dubbed as the anti-Unruh effect in terms of the entanglement generated in Ref. [16]. Note that this is different from the anti-Unruh effect for two entangled atoms in terms of the collective transition rate investigated in Refs. [34,35]. The Unruh and anti-Unruh effects in terms of the entanglement generated show a unique feature of the vacuum fluctuations viewed by two accelerated observers on the one hand, and suggest in principle a possibility for experimental verification of the acceleration-induced quantum effects by observing the entanglement dynamics of accelerated atoms on the other hand.

For a two-atom system, there are two kinds of environment-induced effects. The first one is the decoherence and dissipation, and the second one is the environment-induced energy shift, including the Lamb shifts of the individual atoms, and an environment-induced interatomic interaction. However, in the works mentioned above concerning the anti-Unruh effect in terms of the entanglement generated [14–16], only the effects of decoherence and dissipation are considered, while the effects of environment-induced interatomic interaction on entanglement dynamics are neglected. In a recent work [36], we examined the en-

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tanglement dynamics of two uniformly accelerated atoms, and found that the anti-Unruh effect in terms of the entanglement generated is lost when the environment-induced interatomic interaction is taken into account. In Ref. [36], the environment is modeled as a fluctuating scalar field in the Minkowski vacuum. A realistic model would be two uniformly accelerated atoms coupled with fluctuating vacuum electromagnetic fields. Since the electromagnetic vacuum fluctuations are anisotropic as seen by accelerated atoms, the entanglement behaviors for atoms polarizable along different directions may be significantly different. In the present paper, we plan to study the entanglement dynamics of two uniformly accelerated atoms coupled with electromagnetic vacuum fluctuations when the environment-induced interatomic interaction is taken into account. As will be shown in detail, in the electromagnetic-field case, the entanglement generation is crucially dependent on the polarization of the atoms. Remarkably, in sharp contrast to the scalar-field case [36], the anti-Unruh phenomenon can survive the environment-induced interatomic interaction.

2. The master equation

We consider an open quantum system composed of two uniformly accelerated atoms coupled with a fluctuating electromagnetic field in the Minkowski vacuum. The Hamiltonian takes the form

$$H = H_S + H_F + H_I. \tag{1}$$

Here H_S is the Hamiltonian of the two-atom system,

$$H_S = \frac{\omega}{2}\sigma_3^{(1)} + \frac{\omega}{2}\sigma_3^{(2)}, \tag{2}$$

where $\sigma_i^{(1)} = \sigma_i \otimes \sigma_0$, $\sigma_i^{(2)} = \sigma_0 \otimes \sigma_i$, with σ_i $(i = 1, 2, 3)$ being the Pauli matrices, σ_0 the 2×2 unit matrix, and ω the energy-level spacing between the ground state $|g\rangle$ and the excited state $|e\rangle$. In Eq. (1), H_F is the Hamiltonian of the electromagnetic fields, and H_I is the interaction Hamiltonian describing the dipole interaction between the atoms and the electromagnetic field, which takes the form

$$H_I = -\mathbf{D}^{(1)}(\tau) \cdot \mathbf{E}[x^{(1)}(\tau)] - \mathbf{D}^{(2)}(\tau) \cdot \mathbf{E}[x^{(2)}(\tau)]. \tag{3}$$

Here $\mathbf{D}^{(\alpha)}(\tau)$ $(\alpha = 1, 2)$ is the electric-dipole moment of the αth atom, and $\mathbf{E}[x^{(\alpha)}(\tau)]$ is the electric-field strength.

In the limit of weak coupling, the Markovian master equation of the two-atom system takes the Gorini-Kossakowski-Lindblad-Sudarshan (GKLS) form [37,38],

$$\frac{\partial \rho(\tau)}{\partial \tau} = -i \sum_{\alpha=1}^{2} \tilde{\omega}_\alpha [\sigma_3^{(\alpha)}, \rho(\tau)] + i \sum_{i,j=1}^{3} \Omega_{ij}^{(12)} [\sigma_i \otimes \sigma_j, \rho(\tau)] + \frac{1}{2} \sum_{\alpha,\beta=1}^{2} \sum_{i,j=1}^{3} C_{ij}^{(\alpha\beta)} [2\sigma_j^{(\beta)} \rho \sigma_i^{(\alpha)} - \sigma_i^{(\alpha)} \sigma_j^{(\beta)} \rho - \rho \sigma_i^{(\alpha)} \sigma_j^{(\beta)}]. \tag{4}$$

Here

$$\tilde{\omega}_\alpha = \omega - \frac{i\mu^2}{2} [\mathcal{K}^{(\alpha\alpha)}(\omega) - \mathcal{K}^{(\alpha\alpha)}(-\omega)], \tag{5}$$

describes a renormalized energy-level spacing. The coefficient matrices $\Omega_{ij}^{(12)}$ and $C_{ij}^{(\alpha\beta)}$ can be written respectively as follows,

$$\Omega_{ij}^{(12)} = \mathcal{A}^{(12)} (\delta_{ij} - \delta_{3i}\delta_{3j}), \tag{6}$$

$$C_{ij}^{(\alpha\beta)} = A^{(\alpha\beta)} \delta_{ij} - i B^{(\alpha\beta)} \epsilon_{ijk} \delta_{3k} - A^{(\alpha\beta)} \delta_{3i} \delta_{3j}, \tag{7}$$

where

$$\mathcal{A}^{(12)} = \frac{i\mu^2}{4} [\mathcal{K}^{(12)}(\omega) + \mathcal{K}^{(12)}(-\omega)], \tag{8}$$

$$A^{(\alpha\beta)} = \frac{\mu^2}{4} [\mathcal{G}^{(\alpha\beta)}(\omega) + \mathcal{G}^{(\alpha\beta)}(-\omega)], \tag{9}$$

$$B^{(\alpha\beta)} = \frac{\mu^2}{4} [\mathcal{G}^{(\alpha\beta)}(\omega) - \mathcal{G}^{(\alpha\beta)}(-\omega)]. \tag{10}$$

In the equations above,

$$\mathcal{K}^{(\alpha\beta)}(\omega) = \sum_{m.n=1}^{3} d_m^{(\alpha)} d_n^{(\beta)*} \mathcal{K}_{mn}^{(\alpha\beta)}(\omega), \tag{11}$$

$$\mathcal{G}^{(\alpha\beta)}(\omega) = \sum_{m.n=1}^{3} d_m^{(\alpha)} d_n^{(\beta)*} \mathcal{G}_{mn}^{(\alpha\beta)}(\omega), \tag{12}$$

where $d_m^{(\alpha)} = \langle g | D_m^{(\alpha)} | e \rangle$ is the transition matrix element of the αth atom, and

$$\mathcal{K}_{mn}^{(\alpha\beta)}(\lambda) = \frac{P}{\pi i} \int_{-\infty}^{\infty} d\omega \frac{\mathcal{G}_{mn}^{(\alpha\beta)}(\omega)}{\omega - \lambda}, \tag{13}$$

$$\mathcal{G}_{mn}^{(\alpha\beta)}(\lambda) = \int_{-\infty}^{\infty} d\Delta\tau \, e^{i\lambda\Delta\tau} G_{mn}^{(\alpha\beta)}(\Delta\tau), \tag{14}$$

are the Hilbert transform and the Fourier transform of the electromagnetic field correlation function $G_{mn}^{(\alpha\beta)}(\Delta\tau) = \langle E_m(\tau, x_\alpha) E_n(\tau', x_\beta) \rangle$ respectively, with P denoting the principal value. Note that the correlation function is invariant under temporal translations, i.e., they dependent on $\Delta\tau = \tau - \tau'$ only.

We assume that the two atoms are accelerating with the same acceleration perpendicular to the separation, so the trajectories of the two atoms can be written as,

$$t_1(\tau) = \frac{1}{a}\sinh(a\tau), \;\; x_1(\tau) = \frac{1}{a}\cosh(a\tau), \;\; y_1(\tau) = 0, \;\; z_1(\tau) = 0,$$

$$t_2(\tau) = \frac{1}{a}\sinh(a\tau), \;\; x_2(\tau) = \frac{1}{a}\cosh(a\tau), \;\; y_2(\tau) = 0, \;\; z_2(\tau) = L, \tag{15}$$

respectively. The Wightman function of the electromagnetic fields in the Minkowski vacuum takes the form

$$\langle E_m(x) E_n(x') \rangle = \frac{1}{4\pi^2} (\partial_0 \partial_0' \delta_{mn} - \partial_m \partial_n') \times \frac{1}{(x - x')^2 + (y - y')^2 + (z - z')^2 - (t - t' - i\epsilon)^2}, \tag{16}$$

where ∂'_μ denotes $\partial/\partial x'^\mu$, and $\epsilon \to 0_+$. In the proper frame of the atoms, the correlation functions are

$$G_{mn}^{(11)}(x, x') = G_{mn}^{(22)}(x, x') = \frac{a^4}{16\pi^2} \frac{1}{\sinh^4(\frac{a\Delta\tau}{2} - i\epsilon)} \delta_{mn}, \tag{17}$$

and

$$G_{mn}^{(\alpha\beta)}(x, x') = \frac{a^4}{16\pi^2} \frac{1}{\left[\sinh^2(\frac{a\Delta\tau}{2} - i\epsilon) - \frac{a^2 L^2}{4}\right]^3} \times \Bigg\{ [\delta_{mn} + a L \epsilon_{\alpha\beta 3} (l_m k_n - l_n k_m)] \sinh^2 \frac{a\Delta\tau}{2} + \frac{a^2 L^2}{4} \left[(\delta_{mn} - 2 l_m l_n) \cosh^2 \frac{a\Delta\tau}{2} + (\delta_{mn} - 2 l_m l_n - 2 k_m k_n) \sinh^2 \frac{a\Delta\tau}{2} \right] \Bigg\}, \tag{18}$$

for $\alpha \neq \beta$, where $k_\mu = (0, 1, 0, 0)$, and $l_\mu = (0, 0, 0, 1)$.

We assume for simplicity that the magnitudes of the electric dipoles of the atoms are the same, i.e., $|\mathbf{d}^{(1)}| = |\mathbf{d}^{(2)}| = |\mathbf{d}|$, but the directions may be different. Then, we obtain

$$C_{ij}^{(11)} = C_{ij}^{(22)} = A_1 \delta_{ij} - i B_1 \epsilon_{ijk} \delta_{3k} - A_1 \delta_{3i} \delta_{3j}, \tag{19}$$

$$C_{ij}^{(12)} = C_{ij}^{(21)} = A_2 \delta_{ij} - i B_2 \epsilon_{ijk} \delta_{3k} - A_2 \delta_{3i} \delta_{3j}, \tag{20}$$

$$\Omega_{ij}^{(12)} = D \delta_{ij} - D \delta_{3i} \delta_{3j}, \tag{21}$$

where

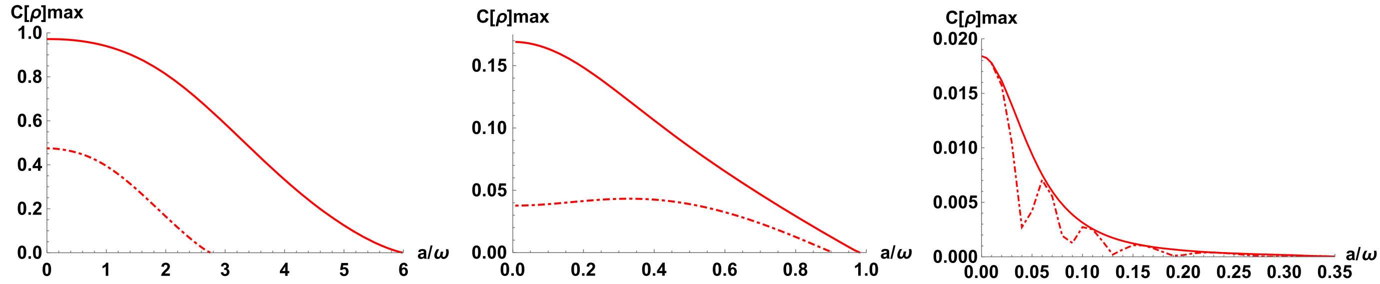

Fig. 1. The maximum of concurrence generated for a uniformly accelerated two-atom system during its evolution with (solid) and without (dot-dashed) the environment-induced interatomic interaction, with $\omega L = 3/10$ (left), $\omega L = 3$ (middle), and $\omega L = 30$ (right). Both of the two atoms are polarizable along the positive x axis.

$$A_1 = \frac{\Gamma_0}{4}\left(1+\frac{a^2}{\omega^2}\right)\coth\frac{\pi\omega}{a},$$
$$A_2 = \frac{\Gamma_0}{4}\sum_{i,j=1}^{3} f_{ij}^{(12)}(\omega,a,L)\hat{d}_i^{(1)}\hat{d}_j^{(2)}\coth\frac{\pi\omega}{a},$$
$$B_1 = \frac{\Gamma_0}{4}\left(1+\frac{a^2}{\omega^2}\right),$$
$$B_2 = \frac{\Gamma_0}{4}\sum_{i,j=1}^{3} f_{ij}^{(12)}(\omega,a,L)\hat{d}_i^{(1)}\hat{d}_j^{(2)},$$
$$D = \frac{\Gamma_0}{4}\sum_{i,j=1}^{3} g_{ij}^{(12)}(\omega,a,L)\hat{d}_i^{(1)}\hat{d}_j^{(2)}, \tag{22}$$

with $\Gamma_0 = \frac{\omega^3|\mathbf{d}|^2}{3\pi}$ being the spontaneous emission rate for inertial atoms in the Minkowski vacuum, and $\hat{d}_i^{(\alpha)}$ a unit vector defined as $\hat{d}_i^{(\alpha)} = d_i^{(\alpha)}/|\mathbf{d}|$. See Eqs. (A.1)-(A.8) in Appendix A for the explicit expressions of $f_{ij}^{(12)}(\omega,a,L)$ and $g_{ij}^{(12)}(\omega,a,L)$.

We assume that the initial density matrix is of the X form, i.e., the nonzero elements are arranged along the diagonal and antidiagonal of the density matrix. To describe the dynamics of the system, we write down the equations of motion of the density matrix elements in the coupled basis $\{|G\rangle = |gg\rangle, |A\rangle = \frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle), |S\rangle = \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle), |E\rangle = |ee\rangle\}$ as,

$$\begin{aligned}
\rho'_{GG} &= -4(A_1-B_1)\rho_{GG} + 2(A_1+B_1-A_2-B_2)\rho_{AA}\\
&\quad +2(A_1+B_1+A_2+B_2)\rho_{SS},\\
\rho'_{EE} &= -4(A_1+B_1)\rho_{EE} + 2(A_1-B_1-A_2+B_2)\rho_{AA}\\
&\quad +2(A_1-B_1+A_2-B_2)\rho_{SS},\\
\rho'_{AA} &= -4(A_1-A_2)\rho_{AA} + 2(A_1-B_1-A_2+B_2)\rho_{GG}\\
&\quad +2(A_1+B_1-A_2-B_2)\rho_{EE},\\
\rho'_{SS} &= -4(A_1+A_2)\rho_{SS} + 2(A_1-B_1+A_2-B_2)\rho_{GG}\\
&\quad +2(A_1+B_1+A_2+B_2)\rho_{EE},\\
\rho'_{AS} &= -4(A_1+iD)\rho_{AS}, \qquad \rho'_{SA} = -4(A_1-iD)\rho_{SA},\\
\rho'_{GE} &= -4A_1\rho_{GE}, \qquad \rho'_{EG} = -4A_1\rho_{EG},
\end{aligned} \tag{23}$$

where $\rho_{IJ} = \langle I|\rho|J\rangle, I,J \in \{G,E,A,S\}$ and $\rho'_{IJ} = \frac{\partial\rho_{IJ}(\tau)}{\partial\tau}$. Since these equations are decoupled from other matrix elements such as ρ_{SG}, the X structure will be retained in the evolution.

3. Anti-Unruh effect in terms of the entanglement generated

In this section, we investigate the entanglement generation for two uniformly accelerated atoms coupled with fluctuating electromagnetic fields, focusing on the effect of the interatomic interaction induced by electromagnetic vacuum fluctuations. We measure entanglement with concurrence [39], which is 0 for separable states, and 1 for maximally entangled states. For X states, the concurrence can be calculated from its general definition [39] as

$$C[\rho(\tau)] = \max\{0, K_1(\tau), K_2(\tau)\}, \tag{24}$$

where

$$K_1(\tau) = \sqrt{[\rho_{AA}(\tau)-\rho_{SS}(\tau)]^2 - [\rho_{AS}(\tau)-\rho_{SA}(\tau)]^2} - 2\sqrt{\rho_{GG}(\tau)\rho_{EE}(\tau)}, \tag{25}$$

$$K_2(\tau) = 2|\rho_{GE}(\tau)| - \sqrt{[\rho_{AA}(\tau)+\rho_{SS}(\tau)]^2 - [\rho_{AS}(\tau)+\rho_{SA}(\tau)]^2}. \tag{26}$$

From Eq. (23), it is clear that for a two-atom system prepared in an X state, the entanglement dynamics is not affected by the environment-induced interatomic interaction unless $\rho_{AS}(0)$ and $\rho_{SA}(0)$ are nonzero. Therefore, to investigate the effects of environment-induced interatomic interaction on entanglement generation, we take the initial state of the two-atom system as $|eg\rangle$, i.e., the nonzero density matrix elements in the coupled basis are $\rho_{AS}(0) = \rho_{SA}(0) = \rho_{AA}(0) = \rho_{SS}(0) = \frac{1}{2}$.

Taking the initial state above into Eq. (23), one obtains $\rho_{GE}(\tau) = \rho_{EG}(\tau) \equiv 0$, and $\rho_{AS}(\tau) = \rho^*_{SA}(\tau) = \frac{1}{2}e^{-4(A_1+iD)\tau}$. Then, $K_2(\tau) \leq 0$ according to Eq. (26), and the concurrence becomes $C[\rho(\tau)] = \max\{0, K_1(\tau)\}$, where

$$K_1(\tau) = \sqrt{[\rho_{AA}(\tau)-\rho_{SS}(\tau)]^2 + \sin^2(4D\tau)e^{-4A_1\tau}} - 2\sqrt{\rho_{GG}(\tau)\rho_{EE}(\tau)}. \tag{27}$$

So, when the environment-induced interatomic interaction is considered, there is an additional positive term $\sin^2(4D\tau)e^{-4A_1\tau}$ oscillating in time, and the oscillation is damped during evolution. Therefore, compared with the case when the environment-induced interatomic interaction is neglected, the entanglement generation is assisted by the environment-induced interatomic interaction. However, the asymptotic state remains unchanged, which is always disentangled for two-atom systems with finite separation [14,15].

In the following, we study explicitly how the maximum of concurrence generated varies with acceleration. In particular, we focus on whether the anti-Unruh effect found in terms of the entanglement generated when the environment-induced interatomic interaction is neglected [14,15] still exists when the environment-induced interatomic interaction is considered. As shown in Figs. 1-3, the relation between the maximum of concurrence and acceleration is related to the polarization directions of the atoms, which we specify as follows.

(1) As shown in Fig. 1, when both of the two atoms are polarizable along the positive x axis, i.e., the direction of acceleration, the maximum of the concurrence generated during evolution decreases monotonically with acceleration for the two-atom systems with a separation much smaller than the transition wavelength (Fig. 1 (left)), no matter the environment-induced interatomic interaction is considered or not. However, for the two-atom systems with a separation comparable to (Fig. 1 (middle)) or much larger than (Fig. 1 (right)) the transition

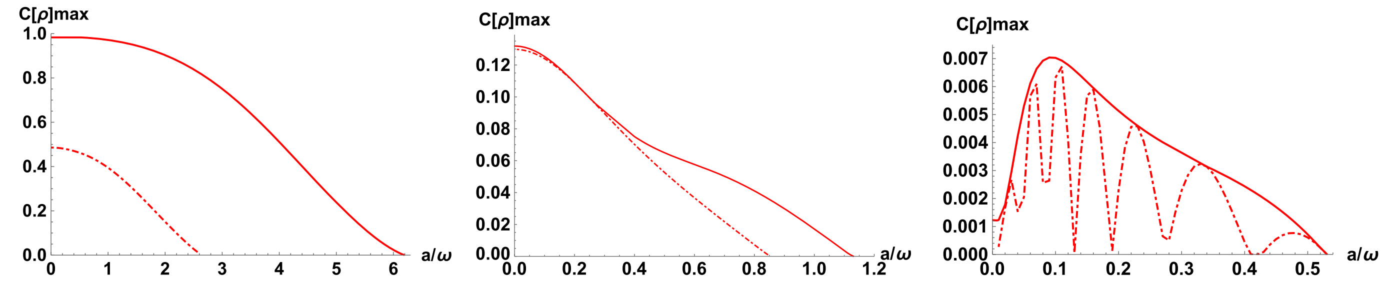

Fig. 2. The maximum of concurrence generated for a uniformly accelerated two-atom system during its evolution with (solid) and without (dot-dashed) the environment-induced interatomic interaction, with $\omega L = 3/10$ (left), $\omega L = 3$ (middle), and $\omega L = 30$ (right). Both of the two atoms are polarizable along the positive z axis.

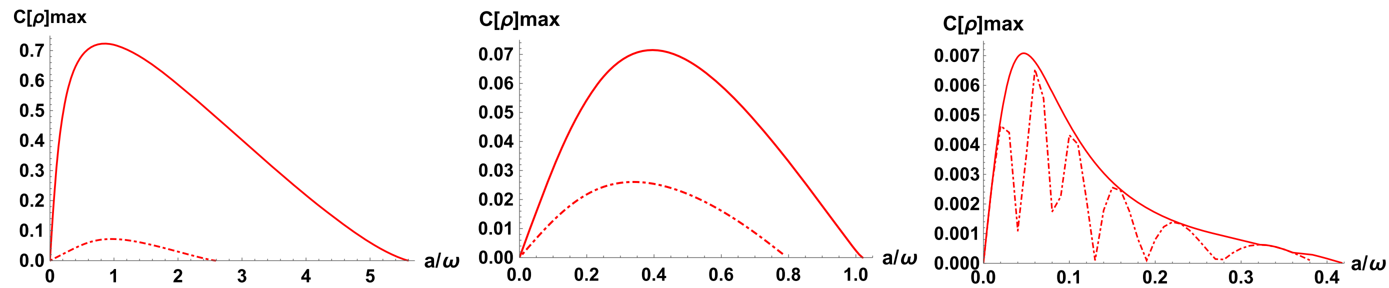

Fig. 3. The maximum of concurrence generated for a uniformly accelerated two-atom system during its evolution with (solid) and without (dot-dashed) the environment-induced interatomic interaction, with $\omega L = 3/10$ (left), $\omega L = 3$ (middle), and $\omega L = 30$ (right). The two atoms are polarizable along the positive z axis and the positive x axis respectively.

wavelength, there exist regions in which the maximum of the concurrence generated during evolution increases with acceleration when the environment-induced interatomic interaction is neglected. However, there is no such region when the environment-induced interatomic interaction is considered. That is, there exists anti-Unruh phenomenon in terms of the entanglement generated when the environment-induced interatomic interaction is neglected. However, the anti-Unruh phenomenon is lost when the environment-induced interatomic interaction is considered. When both of the two atoms are polarizable along the positive y axis, i.e., the direction normal to the plane determined by the acceleration and the interatomic separation, the conclusion is essentially the same. That is, when the two atoms are both polarizable along one of the directions vertical to the interatomic separation, the anti-Unruh phenomenon is lost when the environment-induced interatomic interaction is considered. This agrees with the result in the scalar-field case [36].

(2) When both of the two atoms are polarizable along the positive z axis, i.e., the direction of interatomic separation, the maximum of the concurrence generated during evolution decreases monotonically with acceleration for the two-atom systems with a separation much smaller than (Fig. 2 (left)) or comparable to (Fig. 2 (middle)) the transition wavelength, regardless of whether the environment-induced interatomic interaction is considered or not. When the separation is much larger than the transition wavelength (Fig. 2 (right)), the maximum of the concurrence shows an oscillatory behavior with acceleration if the environment-induced interatomic interaction is neglected, while it increases and then decreases with acceleration when the environment-induced interatomic interaction is considered. That is, when both of the two atoms are polarizable along the direction of interatomic separation, the anti-Unruh phenomenon exists only when the interatomic separation is much larger than the transition wavelength. When one of the two atoms is polarizable along the positive z axis (the direction of interatomic separation), while the other is polarizable along the positive x axis (the direction of acceleration) (Fig. 3), there is no such restriction. For any given separation, the maximal concurrence generated during evolution increases with acceleration when the acceleration is small, regardless of whether the environment-induced interatomic interaction is considered or not. That is, when one of the two atoms is polarizable along the direction of interatomic separation, while the other is polarizable along the direction longitudinal to the plane determined by the acceleration and the interatomic separation, the anti-Unruh phenomenon exists when the environment-induced interatomic interaction is considered. This is dramatically different from that in the toy scalar-field case where the anti-Unruh phenomenon is lost when the environment-induced interatomic interaction is considered [36].

(3) When one of the two atoms is polarizable along the direction normal to the plane determined by the acceleration and the interatomic separation, while the other is polarizable longitudinally (in the xOz plane), the environment-induced interaction vanishes. This can be seen from the expressions of D shown in Eq. (22) and the nonzero components of $g_{ij}^{(12)}(\omega, a, L)$ shown in Eqs. (A.5)-(A.8) in Appendix A. See also Ref. [40], in which the resonance interaction between two uniformly accelerated atoms in an entangled state is studied. Therefore, the anti-Unruh phenomenon in terms of the entanglement generated would not be affected.

4. Conclusion

In this paper, we have investigated the entanglement generation of a uniformly accelerated two-atom system coupled with electromagnetic vacuum fluctuations. In particular, we focus on whether the anti-Unruh effect found in terms of the entanglement generated when the environment-induced interatomic interaction is neglected still exists when the environment-induced interatomic interaction is considered. When the two atoms are polarizable along the same direction vertical to the interatomic separation, the anti-Unruh phenomenon in terms of the entanglement generated disappears when the environment-induced interatomic interaction is considered. However, when one of the two atoms is polarizable along the direction of interatomic separation, while the other is polarizable along the direction longitudinal to the plane determined by the acceleration and the interatomic separation, there exists anti-Unruh phenomenon when the environment-induced interatomic interaction is considered. That is, the anti-Unruh phenomenon in terms of the entanglement generated can survive the interatomic interaction induced by electromagnetic vacuum fluctuations. This is dramatically different from the result in the scalar-field case [36], in which the anti-Unruh phenomenon is always lost when the environment-induced interatomic interaction is considered.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. The expressions of $f_{ij}^{(\alpha\beta)}(\omega,a,L)$ and $g_{ij}^{(\alpha\beta)}(\omega,a,L)$

The explicit expressions of $f_{ij}^{(\alpha\beta)}(\omega,a,L)$ and $g_{ij}^{(\alpha\beta)}(\omega,a,L)$ $(\alpha\neq\beta)$ are shown as follows.

$$\begin{aligned}
&f_{11}^{(12)}(\lambda,a,L)\\
&=f_{11}^{(21)}(\lambda,a,L)\\
&=\frac{12}{\lambda^3L^3(4+a^2L^2)^{5/2}}\\
&\times\left\{2\lambda L(1+a^2L^2)(4+a^2L^2)^{1/2}\cos\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.-[4-4\lambda^2L^2+a^2L^2(2-\lambda^2L^2+a^2L^2)]\sin\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right\},
\end{aligned}\tag{A.1}$$

$$\begin{aligned}
&f_{22}^{(12)}(\lambda,a,L)\\
&=f_{22}^{(21)}(\lambda,a,L)\\
&=\frac{3}{\lambda^3L^3(4+a^2L^2)^{3/2}}\left[\lambda L(2+a^2L^2)(4+a^2L^2)^{1/2}\cos\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.-(4-4\lambda^2L^2-a^2\lambda^2L^4)\sin\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right],
\end{aligned}\tag{A.2}$$

$$\begin{aligned}
&f_{33}^{(12)}(\lambda,a,L)\\
&=f_{33}^{(21)}(\lambda,a,L)\\
&=-\frac{3}{\lambda^3L^3(4+a^2L^2)^{5/2}}\\
&\times\left\{\lambda L(16+2a^2L^2+a^4L^4)(4+a^2L^2)^{1/2}\cos\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.-[32-a^4\lambda^2L^6+4a^2L^2(5-\omega^2L^2)]\sin\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right\},
\end{aligned}\tag{A.3}$$

$$\begin{aligned}
&f_{13}^{(12)}(\lambda,a,L)\\
&=-f_{31}^{(12)}(\lambda,a,L)=-f_{13}^{(21)}(\lambda,a,L)=f_{31}^{(21)}(\lambda,a,L)\\
&=\frac{6a}{\lambda^3L^2(4+a^2L^2)^{5/2}}\left\{\lambda L(2-a^2L^2)(4+a^2L^2)^{1/2}\cos\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.-[4+4\lambda^2L^2+a^2L^2(4+\lambda^2L^2)]\sin\left(\frac{2\lambda}{a}\sinh^{-1}\frac{aL}{2}\right)\right\},
\end{aligned}\tag{A.4}$$

with other components being zero.

$$\begin{aligned}
&g_{11}^{(12)}(\omega,a,L)\\
&=-\frac{12}{\omega^3L^3(4+a^2L^2)^{5/2}}\\
&\times\left\{2\omega L(1+a^2L^2)(4+a^2L^2)^{1/2}\sin\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.+[4-4\omega^2L^2+a^2L^2(2-\omega^2L^2+a^2L^2)]\cos\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right\},
\end{aligned}\tag{A.5}$$

$$\begin{aligned}
&g_{22}^{(12)}(\omega,a,L)\\
&=-\frac{3}{\omega^3L^3(4+a^2L^2)^{3/2}}\\
&\times\left[\omega L(2+a^2L^2)(4+a^2L^2)^{1/2}\sin\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.+(4-4\omega^2L^2-a^2\omega^2L^4)\cos\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right],
\end{aligned}\tag{A.6}$$

$$\begin{aligned}
&g_{33}^{(12)}(\omega,a,L)\\
&=\frac{3}{\omega^3L^3(4+a^2L^2)^{5/2}}\\
&\times\left\{\omega L(16+2a^2L^2+a^4L^4)(4+a^2L^2)^{1/2}\sin\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.+[32-a^4\omega^2L^6+4a^2L^2(5-\omega^2L^2)]\cos\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right\},
\end{aligned}\tag{A.7}$$

$$\begin{aligned}
&g_{13}^{(12)}(\omega,a,L)\\
&=-g_{31}^{(12)}(\omega,a,L)\\
&=-\frac{6a}{\omega^3L^2(4+a^2L^2)^{5/2}}\\
&\times\left\{\omega L(2-a^2L^2)(4+a^2L^2)^{1/2}\sin\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right.\\
&\left.+[4+4\omega^2L^2+a^2L^2(4+\omega^2L^2)]\cos\left(\frac{2\omega}{a}\sinh^{-1}\frac{aL}{2}\right)\right\},
\end{aligned}\tag{A.8}$$

with other components being zero.

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