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Weak cosmic censorship conjecture and black hole shadow for black hole with generalized uncertainty principle

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Abstract Based on string theory, loop quantum gravity, black hole physics, and other theories of quantum gravity, physicists have proposed generalized uncertainty principle (GUP) modifications. In this work, within the framework of GUP gravity theory, we successfully derive an exact solution to Einstein's field equation, and discuss the possibility of using EHT to test GUP and how GUP changes the weak cosmic censorship conjecture for black holes. We analyze two different ways of constructing GUP rotating black holes (model I and model II). Model I takes into account the modification of mass by GUP, i.e., the change in mass by quantization of space, and the resulting GUP rotating black hole metric (18) is similar in form to the Kerr black hole metric. Model II takes into account the modification of the rotating black hole when GUP is an external field, where GUP acts like an electric charge, and the resulting GUP rotating black hole metric (19) is similar in form to the Kerr–Newman black hole metric. The difference between (18) and (19) in the spacetime linear structure provides a basis for us to examine the physical nature of GUP rotating black holes from observation. By analyzing the shadow shape of the GUP rotating black hole, we discover intriguing characteristics regarding the impact of first-order and second-order momentum correction coefficients on the black hole's shadow shape. These findings will be instrumental in future GUP testing using EHT. Additionally, by incident test particle and scalar field with a rotating GUP black hole, the weak cosmic censorship conjecture is not violated in either extreme black holes or near-extreme black holes.

1 Introduction

In 1905, Einstein proposed his theory of special relativity, which organically combined space and time. In 1915, Einstein proposed his theory of general relativity, which extended special relativity by linking the effects of gravity to the curvature of spacetime [1]. These insights revolutionized our understanding of the universe. General relativity has been highly successful where spacetime is a continuous quantity. For example, the length of an object can be infinitely small, and the measure of time is also a continuous quantity. However, the continuity of spacetime contradicts the Heisenberg uncertainty principle in quantum mechanics. In quantum mechanics, the mathematical form of the Heisenberg uncertainty principle is $\Delta x \Delta p \geq \frac{\hbar}{2}$, which translates into the uncertainty of energy; similarly, time is $\Delta E \Delta t \geq \frac{\hbar}{2}$. We will see that in quantum mechanics, quantities are basically quantized, but Δx and Δt are continuous. In order to introduce the quantum nature of spacetime into quantum mechanics, Snyder et al. generalized the uncertainty principle, whose initial form is $\Delta x \Delta p \geq \frac{\hbar}{2}(1+\beta(\Delta p)^2+\beta\langle\hat{p}\rangle^2)$ [2]. This generalization introduces the notion of minimal observable length, which their work shows is not inconsistent with Lorentz invariance and is therefore a self-consistent physical theory. Since then, physicists have conducted extensive and profound studies of the generalized uncertainty principle (GUP) [3].

After discussing the GUP and its contributions to theoretical physics, we realize that although GUP provides an important theoretical framework for understanding quantum gravity and the minimal measurable length, it encounters certain challenges in preserving Lorentz invariance. As an advanced improvement to this issue, the introduction of the relativistic generalized uncertainty principle (RGUP) has successfully resolved the issues related to Lorentz covariance, frame dependence, and the linearity of momentum superposition that were present in GUP. A series of recent outstanding papers [4–10] have cleverly shown how adopting RGUP effectively maintains Lorentz invariance and has opened new avenues for introducing the concept of a minimal measur-

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able length in theoretical physics, unaffected by the choice of reference frame. The authors of these papers, with their profound insights and innovative methods, have not only propelled the application of RGUP in quantum field theory but also provided new theoretical perspectives for dealing with physical phenomena at high energies. Their work represents a significant breakthrough in the field. GUP can be considered the non-relativistic approximation of RGUP, thereby making the study of GUP modifications to black holes foundational for understanding RGUP modifications to black holes. In our future work, we will prepare to extensively investigate the modifications of black holes by RGUP and its related physics.

The physical discussion of GUP is important, but cannot be tested by current high-energy physics experiments. Here, we only introduce the content of studying GUP using black hole physics. The main problem is to construct a black hole metric that takes GUP into account, and then to calculate the effect of GUP in black hole physics. The specific behaviors and properties of the GUP have been studied extensively when it is considered in the spacetime of the Schwarzschild black hole, where the radius of the event horizon of the black hole increases [11–16]. It is worth mentioning that in the field of exploring the impact of the GUP on black hole physics, Hayam Yassin and Abdel Nasser Tawfik, among others, have provided significant theoretical support and profound insights [17–22]. Their research not only thoroughly analyzes the dual role of the GUP and the modified dispersion relations (MDR) in the phenomenology of quantum gravity, but also delves into the importance of modifications to black hole thermodynamics and Friedmann equations by GUP. In particular, their work has reproduced the horizon areas of different types of black holes and conducted detailed studies on the quantum corrections to the Bekenstein–Hawking entropy, revealing how GUP affects the microscopic structure of black hole entropy and predicting how these modifications could influence the thermodynamic behavior of black holes. These findings not only provide a theoretical foundation for constructing black hole metrics considering GUP and analyzing the role of GUP in black hole physics in this research, but also enrich our understanding of the complexity of black hole thermodynamic systems. In loop quantum gravity theory, physicist obtained the modified Schwarzschild black hole in the GUP case [23,24]. In addition, physicists also try to use GUP to solve the information problem of black holes, and find that if GUP is taken into account, the Hawking radiation of the modified Schwarzschild black hole or Kerr black hole will show a new property, that is, the black hole will leave a residue in the final stage of evaporation, which can resolve the information problem of black holes to some extent [3]. Currently, there are two main approaches to constructing black holes in GUP. First, the GUP does not change the equivalent metric structure of the Schwarzschild black hole, but only the total mass of the system, which is an essential modification of profound physical significance [25]. Second, we consider GUP as some outer field, similar to electric charge, so the modification in the Schwarzschild black hole metric will differ from the previous case by adding an $\propto \frac{1}{r^2}$ term to the spacetime metric [26]. These two different constructions will deepen the understanding of the nature of GUP.

In this paper, we study solutions of Einstein's field equations for rotating black holes under the GUP effect. In Sect. 2, the GUP and Schwarzschild black hole corrections are introduced. In Sect. 3, we study the GUP corrections to rotating black holes and obtain exact solutions of the Einstein field equations taking the GUP into account. In Sect. 4, we investigate the possibility of using black hole shadows to test GUP. In Sect. 5, we calculate the effect of GUP on the weak cosmic censorship conjecture. Lastly, Sect. 6 is devoted to a summary.

2 Nonrotating black hole and GUP

The introduction of GUP is closely related to the minimal observable length, and the introduction of GUP does not violate Lorentz invariance [25]. Throughout this paper, we will use the following form of GUP:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1-\frac{\alpha l_p}{\hbar}\Delta p+\frac{\beta l_p^2}{\hbar^2}(\Delta p)^2\right], \tag{1}$$

where α and β are dimensionless model parameters. When both α and β are zero, Eq. (1) reduces to the Heisenberg uncertainty principle. Here, l_p is the Planck length. According to Eq. (1), there exists a minimal observable length $\Delta x \geq (\Delta x)_{min} \approx \left(\sqrt{\beta}-\frac{\alpha}{2}\right) l_p$ and a maximal observable momentum $\Delta p \leq (\Delta p)_{max} \approx \frac{\alpha\hbar}{\beta l_p}$. These results show that it is impossible to measure lengths below $(\Delta x)_{min}$, setting a minimum limit to the detection capabilities of physics. In reference [25], they arrive at the modified black hole metric, whose spacetime line elements are

$$ds^2=\left[1-\frac{2M_1}{r}\right]dt^2+\left[1-\frac{2M_1}{r}\right]^{-1}dr^2 \\ +r^2 d\Omega^2, \qquad (Model\ I) \tag{2}$$

where $d\Omega^2=d\theta^2+\sin^2\theta d\phi^2$, and the total mass of the modified black hole, is

$$M_1=M\left[1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right]. \tag{3}$$

In this approach to the construction of a black hole, the GUP has a major effect on the mass, and the value of M deviates from the usual definition due to the presence of α and β. This leads us to a renewed understanding of the concept of mass. Due to the equivalence principle, gravitational mass is equivalent to inertial mass, so result (3) will redefine our understanding of inertial mass.

For the second construction of the GUP black hole, we shall give a brief introduction. According to Eq. (1)

$$\Delta x \approx \frac{1}{2E}\left[1-\alpha E+\beta E^2\right]. \tag{4}$$

In the process of obtaining equation (4), we use the standard dispersion relation $E = p$. On the other hand, the uncertainty in the photon wavelength depends on the Schwarzschild black hole radius

$$\Delta x = 2\pi R_s = 4\pi M. \tag{5}$$

Considering $E \approx T$, and T is the Bekenstein–Hawking classical temperature, the mathematical expression of T can be obtained by combining (4) and (5)

$$T = \frac{1}{2\beta}\left[\alpha + 4\pi M - \sqrt{(\alpha+4\pi M)^2 - 4\beta}\right]. \tag{6}$$

In this construction method, the spacetime line elements of the spherically symmetric GUP black hole are assumed to be [26]

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + g^{-1}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2. \quad (Model\ II) \tag{7}$$

Its metric coefficients are set to

$$f(r) = g(r) = 1 - \frac{2M}{r} + \varepsilon\frac{M^2}{r^2}. \tag{8}$$

Here, ε is a dimensionless parameter. The Bekenstein–Hawking classical temperature corresponding to the metric (7) is

$$T(\varepsilon) = \frac{1}{4\pi}\frac{\mathrm{d}f(r)}{\mathrm{d}r}|_{r=r_H} = \frac{1}{2\pi M}\frac{\sqrt{1-\varepsilon}}{(1+\sqrt{1-\varepsilon})^2}. \tag{9}$$

Therefore, $T(\varepsilon) = T$, and the relation between the GUP parameter β and the dimensionless parameter ε is

$$\begin{aligned}
&\frac{1}{\pi M}\frac{\sqrt{1-\varepsilon}}{(1+\sqrt{1-\varepsilon})^2} \\
&\quad = \frac{1}{\beta}\left[\alpha + 4\pi M - \sqrt{(\alpha+4\pi M)^2 - 4\beta}\right], \\
&\varepsilon = 1 - \left[\frac{\beta}{2\pi M\left[\alpha + 4\pi M - \sqrt{(\alpha+4\pi M)^2-4\beta}\right]}\right. \\
&\quad \left.\times\left(1\pm\sqrt{1-\frac{4\pi M\left[\alpha+4\pi M-\sqrt{(\alpha+4\pi M)^2-4\beta}\right]}{\beta}}\right)-1\right]^2.
\end{aligned} \tag{10}$$

By the construction of a spherically symmetric metric for the GUP black hole, it can be found that this construction treats the GUP as an external field rather than as a change in spacetime itself. This is different from the first construction of the GUP black hole metric. Different structural pathways reveal different physical implications of the GUP.

Herein, we briefly introduce the spherically symmetric metric of GUP black holes, which will facilitate our subsequent generalization to the rotating case. For spherically symmetric metrics, the GUP approach has been employed to study their thermodynamic properties, entropy, and the equations of state derived under various types of spherically symmetric black holes in several seminal papers (see Refs. [19–22]). These studies play a key role in further understanding the thermodynamic systems of black holes.

3 Kerr-like black hole spacetime

Here we will generalize the spherically symmetric spacetime metric to the axisymmetric case using a transformation (i.e., the Newman–Janis method) [27–29]. In this method, the black hole spin a and angular coordinate θ are introduced by a complex coordinate transformation in the light cone coordinate system. By generalizing the various metric coefficients of the spherically symmetric case to the case involving a and θ, the general form of the spacetime metric in the light cone coordinate system can be obtained. Then, using the standard coordinate transformation, we can obtain the spacetime metric in the Boyer–Lindquist (BL) coordinate system, whose general expression is

$$\begin{aligned}
\mathrm{d}s^2 = &-\frac{\Psi}{\rho^2}\left(1-\frac{2\bar{f}}{\rho^2}\right)\mathrm{d}t^2 + \frac{\Psi}{\Delta}\mathrm{d}r^2 \\
&-\frac{4a\bar{f}\Psi\sin^2\theta}{\rho^4}\mathrm{d}t\mathrm{d}\phi + \Psi\mathrm{d}\theta^2 + \frac{\Psi\Sigma\sin^2\theta}{\rho^4}\mathrm{d}\phi^2.
\end{aligned} \tag{11}$$

The meanings of the symbols in this metric are as follows:

$$\begin{aligned}
&\rho^2 = k(r) + a^2\cos^2\theta, \quad k(r) = h(r)\sqrt{\frac{f(r)}{g(r)}} = r^2, \\
&\bar{f} = \frac{1}{2}k(r) - \frac{1}{2}h(r)f(r) = \frac{1}{2}r^2 - \frac{1}{2}r^2 f(r), \\
&\Sigma = \left(k(r)+a^2\right)^2 - a^2\Delta(r)\sin^2\theta \\
&\quad = \left(r^2+a^2\right)^2 - a^2\Delta(r)\sin^2\theta, \\
&\Delta(r) = r^2 f(r) + a^2.
\end{aligned} \tag{12}$$

In these expressions, once the metric of spherically symmetric spacetime is determined, all functions except the unknown function Ψ in Eq. (11) can be determined. So far, two physical conditions have not been treated, namely the axisymmetric condition $G_{r\theta} = 0$ and Einstein's gravitational field equations $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$. The spacetime metric (11) must satisfy both conditions. In this case, the unknown function in Eq. (11) will satisfy the following equations:

$$\left(k(r)+a^2y^2\right)^2\left(3\frac{\partial\Psi}{\partial r}\frac{\partial\Psi}{\partial y^2}-2\Psi\frac{\partial^2\Psi}{\partial r\partial y^2}\right)$$
$$=3a^2\frac{\partial k}{\partial r}\Psi^2, \tag{13}$$
$$\Psi\left(\left(\frac{\partial k}{\partial r}\right)^2+k\left(2-\frac{\partial^2 k}{\partial r^2}\right)-a^2y^2\left(2+\frac{\partial^2 k}{\partial r^2}\right)\right)$$
$$+\left(k+a^2y^2\right)\left(4y^2\frac{\partial\Psi}{\partial y^2}-\frac{\partial k}{\partial r}\frac{\partial\Psi}{\partial r}\right)=0. \tag{14}$$

For the spacetime metrics (2) and (7), $f(r)=g(r)$ and $k(r)=r^2$, and then Eqs. (13) and (14) are simplified as

$$\left(r^2+a^2y^2\right)^2\left[3\frac{\partial\Psi}{\partial r}\frac{\partial\Psi}{\partial y^2}-2\Psi\frac{\partial^2\Psi}{\partial r\partial y^2}\right]=6a^2r\Psi^2, \tag{15}$$
$$\Psi\left[4r^2-4a^2y\right]+\left(r^2+a^2y^2\right)\left(4y^2\frac{\partial\Psi}{\partial y^2}-2r\frac{\partial\Psi}{\partial r}\right)=0. \tag{16}$$

By solving Eqs. (15) and (16), it is found that $\Psi(r,\theta,a)$ can be written in the following form:

$$\Psi(r,\theta,a)=r^2+a^2\cos^2\theta. \tag{17}$$

Now, we have obtained the axisymmetric form corresponding to a given spherically symmetric metric, so that once a spherically symmetric black hole metric is given, a rotating black hole solution is obtained.

(a) Model I

$$\mathrm{d}s^2=-\left[1-\frac{2Mr-4\alpha r+\frac{8\beta}{M}r}{\rho^2}\right]\mathrm{d}t^2+\frac{\rho^2}{\Delta}\mathrm{d}r^2$$
$$-\frac{2a\sin^2\theta\left[2Mr-4\alpha r+\frac{8\beta}{M}r\right]}{\rho^2}\mathrm{d}t\mathrm{d}\phi+\rho^2\mathrm{d}\theta^2$$
$$+\frac{\Sigma\sin^2\theta}{\rho^2}\mathrm{d}\phi^2, \tag{18}$$

$\rho^2=r^2+a^2\cos^2\theta$, $\Sigma=\left(r^2+a^2\right)^2-a^2\Delta\sin^2\theta$, $\Delta=r^2-2\,M\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)r+a^2$.

This spacetime metric describes the modification of the Kerr black hole when the GUP is considered as an internal property of spacetime. When GUP is not taken into account, that is, $\alpha=\beta=0$, the spacetime metric degenerates to a Kerr black hole.

Next, we analyze the fundamental properties of the black hole solution. When we generalize spherically symmetric GUP black holes to the axisymmetric case, the metric (18) exists in at least one event horizon. According to the algebraic equation $\Delta=r^2-2\,M\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)r+a^2=0$ satisfied by the event horizon of the black hole, the solution of this equation can be obtained as $r_\pm=M\left[1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right]\pm\sqrt{M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)^2-a^2}$, where r_+ is the event horizon and r_- is the causal horizon. If both r_+ and r_- have physical meaning, then $M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)^2-a^2\geq 0$ is required, which is a strong constraint on the GUP parameters α and β.

According to $r_\pm=M\left[1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right]\pm\sqrt{M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)^2-a^2}$, when $\alpha=\beta=0$, $r_\pm=M\pm\sqrt{M^2-a^2}$, then $r_\pm$ degenerates to the Kerr black hole case. When $\frac{4\beta}{M^2}-\frac{2\alpha}{M}=0$, that is $2\beta=\alpha M$, $r_\pm=M\pm\sqrt{M^2-a^2}$, this is also the case for Kerr black holes, so this is a very interesting phenomenon; when the first and second terms of GUP satisfy certain conditions (i.e., $2\beta=\alpha M$), model I will be reduced to the vacuum solution case. It is easy to see from the expression of Δ that the GUP parameters α and β have opposite changes on the properties of Kerr black hole.

(b) Model II

$$\mathrm{d}s^2=-\left[1-\frac{2Mr-\varepsilon M^2}{\rho^2}\right]\mathrm{d}t^2+\frac{\rho^2}{\Delta}\mathrm{d}r^2$$
$$-\frac{2a\sin^2\theta\left[2Mr-\varepsilon M^2\right]}{\rho^2}\mathrm{d}t\mathrm{d}\phi+\rho^2\mathrm{d}\theta^2$$
$$+\frac{\Sigma\sin^2\theta}{\rho^2}\mathrm{d}\phi^2, \tag{19}$$

$\rho^2=r^2+a^2\cos^2\theta$, $\Sigma=\left(r^2+a^2\right)^2-a^2\Delta\sin^2\theta$, $\Delta=r^2-2Mr+\varepsilon M^2+a^2$.

This black hole metric describes the modification of the Kerr black hole when the GUP is taken to be an external matter field. When GUP is not considered (i.e., there is no matter field), $\varepsilon=0$, then the black hole metric degenerates to the Kerr black hole.

Next, we analyze the constraints on the model parameters in the black hole solution. In order for the spacetime metric (19) to really describe a black hole, the metric must have an event horizon. Mathematically, this condition can be expressed as if $\Delta=r^2-2Mr+\varepsilon M^2+a^2=0$ has at least one real root, that is, $r_\pm=M\pm\sqrt{(1-\varepsilon)M^2-a^2}$, then $\varepsilon\leq 1-\frac{a^2}{M^2}$. This is a strong constraint on the value of the GUP parameter ε.

From the spherically symmetric GUP black hole metrics (2) and (7) and the axisymmetric GUP black hole metrics (18) and (19), it can be seen that there are obvious differences in the black hole metric constructed in different ways. For (18) to have some common properties with the black hole described by (19), we need to make them equivalent in some properties, and then find the relation between the two. The relationship between the parameters of model I and model II can be derived through the equivalence of Hawking temperatures in the case of spherical symmetry. However, when it comes to rotating black holes, such thermodynamic equivalence does not exist, which is a noteworthy aspect for discussion.

By comparing the black hole metrics (18) and (19), we find the following: For the black hole metric (18), the spacetime linear element ds^2 is consistent with the Kerr black hole, except that M is multiplied by a modification term $(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2})$ determined by the GUP parameters. For the black hole metric (19), the spacetime linear element ds^2 is consistent with the Kerr–Newman black hole, and the metric (19) can be obtained by replacing Q with $\sqrt{\varepsilon}M$ in the Kerr–Newman black hole metric. Thus, we can obtain the following results: if GUP is considered as a modification to the Kerr black hole when it is an internal property of spacetime, then the properties of the black hole (described by the metric (18)) are consistent with those of the Kerr black hole; if GUP is considered as a modification to the Kerr black hole when it is an external field, then the properties of the black hole (described by the metric (19)) are consistent with those of the Kerr–Newman black hole.

4 Black hole shadow

4.1 Geodesics and photon spheres

Photons move along geodesics in the spacetime background of a rotating black hole. In order to study the orbital motion of a photon, it is necessary to study the geodesic equation of the photon in the spacetime of a rotating black hole. Here we will write down the main calculation process and the analysis ideas [30,31]. In general, the Laplacian of the spacetime of a rotating black hole can be expressed as

$$\mathcal{L}=\frac{1}{2}g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}, \tag{20}$$

where $g_{\mu\nu}$ is the metric coefficient matrix determined by the black hole metrics (18) and (19), x^{μ} is the four-dimensional spacetime coordinates, and the symbol "·" represents the derivative of affine parameter τ on the geodesic. Through the Laplacian (20), the four-dimensional momentum of the photon can be calculated as follows:

$$p_t=\frac{\partial\mathcal{L}}{\partial\dot{t}}=\left[\frac{a^2\sin^2\theta}{\rho^2}-\frac{\Delta}{\rho^2}\right]\dot{t}+\left[\frac{a\Delta\sin^2\theta}{\rho^2}-\frac{a\left(a^2+r^2\right)\sin^2\theta}{\rho^2}\right]\dot{\phi}, \tag{21}$$

$$p_r=\frac{\partial\mathcal{L}}{\partial\dot{r}}=\frac{\rho^2}{\Delta}\dot{r}, \tag{22}$$

$$p_\theta=\frac{\partial\mathcal{L}}{\partial\dot{\theta}}=\rho^2\dot{\theta}, \tag{23}$$

$$p_\phi=\frac{\partial\mathcal{L}}{\partial\dot{\phi}}=\left[\frac{a\Delta\sin^2\theta}{\rho^2}-\frac{a\left(a^2+r^2\right)\sin^2\theta}{\rho^2}\right]\dot{t}+\left[\frac{\left(a^2+r^2\right)^2\sin^2\theta}{\rho^2}-\frac{a^2\Delta\sin^4\theta}{\rho^2}\right]\dot{\phi}. \tag{24}$$

Since the black hole in question is steady-state and axisymmetric, $\mathcal{L}$ is not an explicit function of time t and angular coordinate ϕ, which makes p_t and p_ϕ constant functions, denoted as $-E$ and L_ϕ, respectively. To obtain the geodesic equation for the photon, we start with the Hamilton–Jacobi equation satisfied by the photon, which has the form

$$-\frac{\partial S}{\partial\tau}=\frac{1}{2}g^{\mu\nu}\frac{\partial S}{\partial x^{\mu}}\frac{\partial S}{\partial x^{\nu}}. \tag{25}$$

Here, S is the principal function of the Hamiltonian. If S can be solved by separating variables, then the basic form of S reduces to $S=\frac{1}{2}m^2\sigma-Et+L_\phi\phi+S_\theta(\theta)+S_r(r)$. For photon geodesics, S can be reduced to $S=-Et+L_\phi\phi+S_\theta(\theta)+S_r(r)$. By introducing a Cartesian integration constant $\mathcal{K}$ and the functions R and H corresponding to S_r and S_θ, we can obtain the separated variable solutions of the photon geodesics, which are

$$\rho^2\frac{dt}{d\tau}=E\left[\frac{\left(r^2+a^2\right)\left(r^2+a^2-a\lambda\right)}{\Delta}+a\left(\lambda-a\sin^2\theta\right)\right], \tag{26}$$

$$\left(\rho^2\frac{dr}{d\tau}\right)^2=R, \tag{27}$$

$$\left(\rho^2\frac{d\theta}{d\tau}\right)^2=H, \tag{28}$$

$$\rho^2\frac{d\phi}{d\tau}=E\left[\frac{a\left(r^2+a^2\right)-a^2\lambda}{\Delta}+\frac{\lambda-a\sin^2\theta}{\sin^2\theta}\right]. \tag{29}$$

The functions R, H, λ, and η are as follows:

$$R=E^2\left[\left(r^2+a^2-a\lambda\right)^2-\eta\Delta\right], \tag{30}$$

$$H=E^2\left[\eta-\left(\frac{\lambda}{\sin\theta}-a\sin\theta\right)^2\right], \tag{31}$$

$$\lambda=\frac{L_\phi}{E}, \tag{32}$$

$$\eta=\frac{\mathcal{K}}{E^2}. \tag{33}$$

Due to the strong gravitational field and extreme properties of the black hole, there is a minimal stable orbit r_c in its vicinity. By examining the basic properties of the function R, the mathematical conditions satisfied by the minimal stable orbit are

$$R(r_c)=0,\qquad \frac{dR(r)}{dr}|_{r=r_c}=0. \tag{34}$$

By substituting Eq. (30) into the above equations, the expression of functions λ and η can be obtained.

For model I, the expression is

$$\lambda=\frac{1}{a}\left[2+r-\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right]^{-1}$$

$$\times\left(\left(r^2+a^2\right)\left(2+r-\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right)\right.$$
$$\left.-4\left(r^2-2Mr\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)+a^2\right)\right), \quad (35)$$

$$\eta=\frac{r^3}{a^2}\left[2+r-\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right]^{-2}$$
$$\times\left(8a^2+\frac{16Ma^2}{r^2}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right.$$
$$\left.-r\left(r-2+\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right)^2\right). \quad (36)$$

For model II, the expression is

$$\lambda=\frac{1}{a}\left[2+r-\frac{2M}{r}\right]^{-1}\left(\left(r^2+a^2\right)\right.$$
$$\left.\times\left(2+r-\frac{2M}{r}\right)-4\left(r^2-2Mr+\varepsilon M^2+a^2\right)\right), \quad (37)$$

$$\eta=\frac{r^3}{a^2}\left[2+r-\frac{2M}{r}\right]^{-2}$$
$$\times\left(8a^2+\frac{16Ma^2}{r^2}-\frac{16\varepsilon M^2a^2}{r^3}\right.$$
$$\left.-r\left(r-2+\frac{6M}{r}-\frac{4\varepsilon M^2}{r^2}\right)^2\right). \quad (38)$$

By substituting Eq. (34) into Eq. (30), we can get the second derivative of the function R

$$\frac{d^2R}{dr^2}|_{r_c}=8E^2\left[r^2+\frac{2r\Delta\left(\Delta'-r\Delta''\right)}{\Delta'^2}\right]|_{r=r_c}. \quad (39)$$

The stability of the photon orbit can be judged by the positive or negative of $\frac{d^2R}{dr^2}|_{r_c}$. When $\frac{d^2R}{dr^2}|_{r_c}>0$, the geodesic motion of the photon is unstable, and vice versa. For $\frac{d^2R}{dr^2}|_{r_c}>0$, the set of photon orbits is the shape of the black hole shadow. The photon orbit can form a photon sphere only when function $H\geq 0$. This condition can be simplified as follows:

For model I

$$\left(4r^3-8Mr^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)+4ra^2\right.$$
$$\left.-\left(r^2+a^2\cos^2\theta\right)\left(2r-2M\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right)\right)^2|_{r=r_c}$$
$$\leq 16a^2r^2\left(r^2-2Mr\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)+a^2\right)\sin^2\theta|_{r=r_c}. \quad (40)$$

For model II

$$\left(4r^3-8Mr^2+4\varepsilon M^2r+4ra^2-\left(r^2+a^2\cos^2\theta\right)\right.$$
$$\left.\times\ (2r-2M)\right)^2|_{r=r_c}$$
$$\leq 16a^2r^2\left(r^2-2Mr+\varepsilon M^2+a^2\right)\sin^2\theta|_{r=r_c}. \quad (41)$$

Equations (40) and (41) both have two positive real roots (denoted as r_{c_+} and r_{c_-}, respectively), and the value range of the photon sphere is $r_{c_-}\leq r_c\leq r_{c_+}$.

4.2 Black hole shadow shape

The calculation and discussion in Sect. 4.1 reveal that the motion of photons near the GUP black hole gives rise to the formation of a photon shape, which can be attributed to the presence of a minimum stable orbit within the spacetime of the GUP black hole. Because the generalized uncertainty principle is taken into account, the calculation results for minimum stable orbital radius and photon sphere are different from those of Kerr black holes. Therefore, it is possible to test the correctness of the GUP by using black hole shadows. The research progress in black hole shadow and related calculation methods can be found in various works in the literature (e.g. [32–39]). Next, we will calculate the image formed by photons emitted from the photon sphere measured by observers on Earth, and examine the influence of different GUP model parameters on the shape of the black hole shadow; then we will propose the possibility of using the observation of the black hole shadow and its photon ring to test it.

When we measure the black hole photon sphere on Earth, we generally need to approximate the model by assuming that the observer is a stationary observer at infinity, and that the observer can be approximated to a point. In the calculation of the black hole shadow, the celestial coordinate system is generally chosen, which uses two coordinates $(\bar{\alpha},\bar{\beta})$ to describe the position of the black hole shadow in the celestial coordinate system, similar to a projection. According to physical understanding, the coordinates $\bar{\alpha},\bar{\beta}$ in the celestial coordinate system should be related to the angular coordinates in the BL coordinate system, and the specific relationship is as follows:

$$\bar{\alpha}=\lim_{r_0\to\infty}\left[-r_0^2\sin\theta_0\frac{d\phi}{dr}\right],$$
$$\bar{\beta}=\lim_{r_0\to\infty}\left[r_0^2\frac{d\theta}{dr}\right]. \quad (42)$$

If the observer is on the equatorial plane, the condition $\theta=\pi/2$ is satisfied. The coordinates in the celestial coordinate system will then take the following form:

$$\bar{\alpha}=-\lambda,$$
$$\bar{\beta}=\pm\sqrt{\eta}. \quad (43)$$

For the GUP black holes discussed here,specific expressions

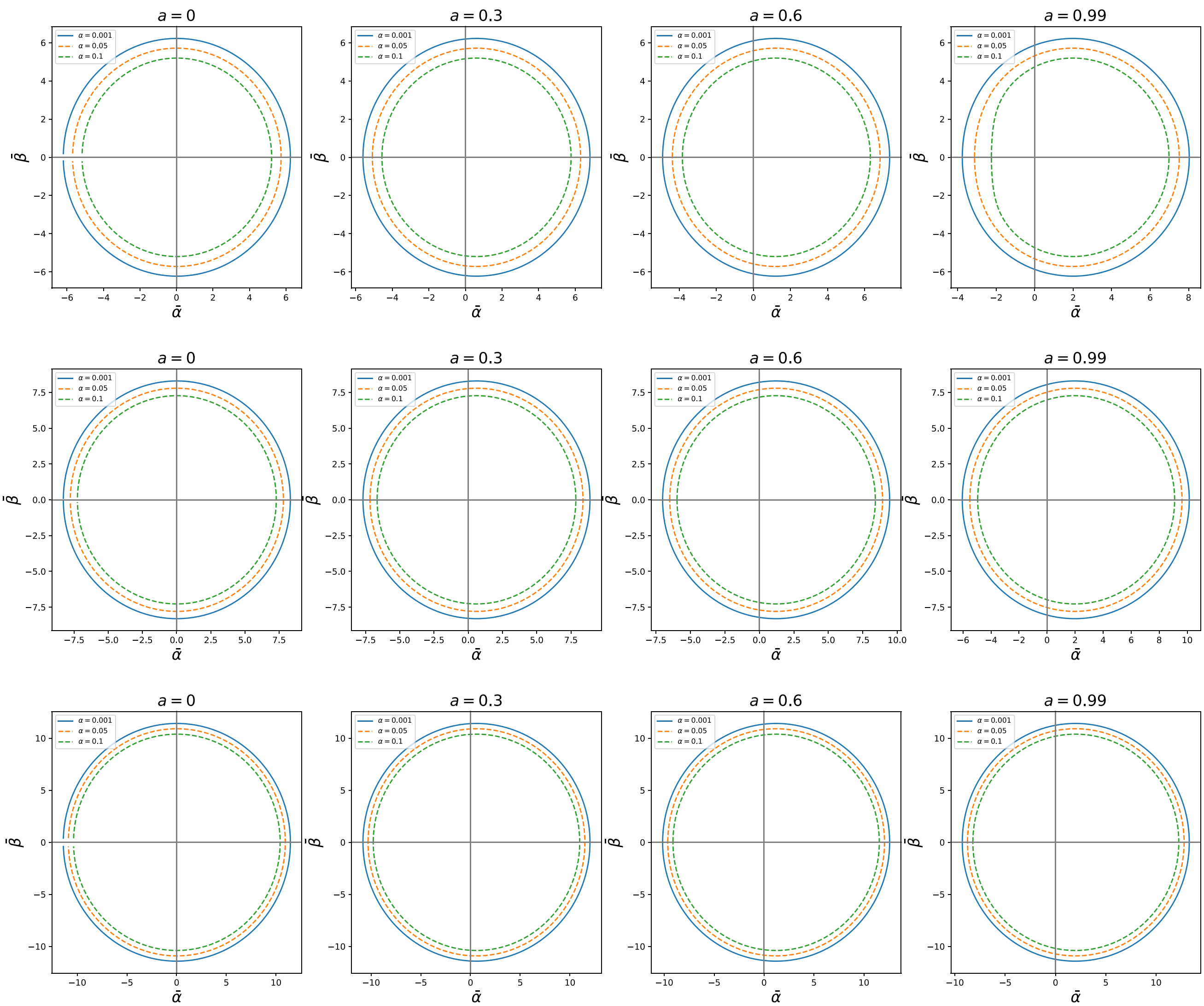

Fig. 1 The shadow shape of the GUP rotating black hole under different model parameters (model I). The panels from left to right represent the process of increasing the spin of the black hole, which is $a = 0, 0.3, 0.6, and 0.99$, respectively. The panels from top to bottom represent the process of increasing the first-order momentum correction coefficient, which is $\beta = 0.05, 0.15, and 0.3$, respectively. Curves of various colors correspond to different α values

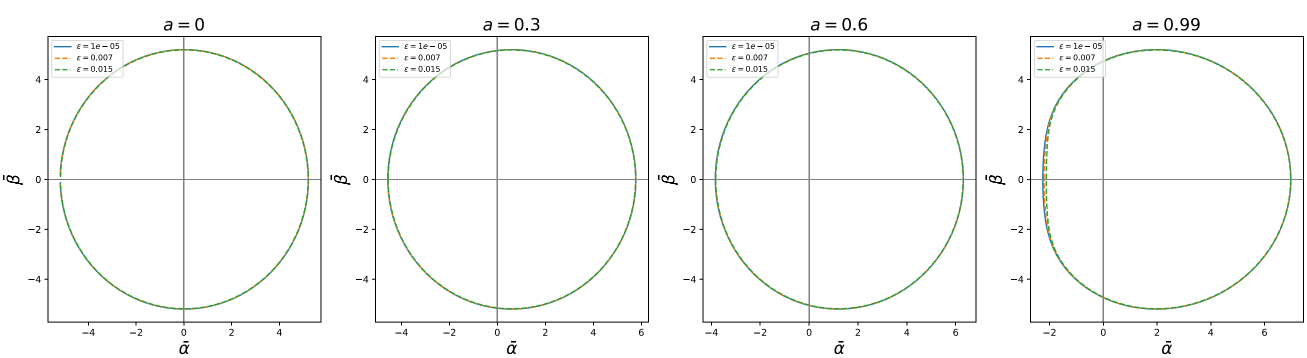

Fig. 2 The shadow shape of the GUP rotating black hole under different model parameters (model II). The panels from left to right represent the process of increasing the spin of the black hole, which is $a = 0, 0.3, 0.6, and 0.99$, respectively. The panels from top to bottom represent the process of increasing the mixing correction coefficient, which is $\varepsilon = 1e-05, 0.007, and 0.015$, respectively. Curves of various colors correspond to different ε values

can be obtained. For model I, its celestial coordinate expression is

$$\bar{\alpha} = \frac{-1}{a}\left[2+r-\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right]^{-1}\left(\left(r^2+a^2\right)\left(2+r-\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right)\right.$$
$$\left.-4\left(r^2-2Mr\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)+a^2\right)\right),$$
$$\bar{\beta} = \pm\sqrt{\frac{r^3}{a^2}\left[2+r-\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right]^{-2}\left(8a^2+\frac{16Ma^2}{r^2}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)-r\left(r-2+\frac{2M}{r}\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\right)^2\right)}. \quad (44)$$

For model II, its celestial coordinate expression is

$$\bar{\alpha} = \frac{-1}{a}\left[2+r-\frac{2M}{r}\right]^{-1}\left(\left(r^2+a^2\right)\left(2+r-\frac{2M}{r}\right)-4\left(r^2-2Mr+\varepsilon M^2+a^2\right)\right),$$
$$\bar{\beta} = \pm\sqrt{\frac{r^3}{a^2}\left[2+r-\frac{2M}{r}\right]^{-2}\left(8a^2+\frac{16Ma^2}{r^2}-\frac{16\varepsilon M^2a^2}{r^3}-r\left(r-2+\frac{6M}{r}-\frac{4\varepsilon M^2}{r^2}\right)^2\right)}. \quad (45)$$

According to Eq. (45), it can be found that the celestial coordinates $(\bar{\alpha}, \bar{\beta})$ are functions of the GUP parameter and the spin of the black hole. By using $\bar{\alpha}$as a function of $\bar{\beta}$, we can draw a two-dimensional image of the black hole shadow. The details of the GUP black hole shadow are found by numerical calculation of model I and model II (see Figs. 1, 2).

For model I, when the first-order momentum correction coefficients α and second-order momentum correction coefficients β of the GUP are zero, the black hole shadow shape degenerates to the Kerr black hole case. When the first-order momentum correction coefficients α and second-order momentum correction coefficients β of the GUP are both nonzero, the black hole shadow is determined by the model parameters α and β. When the black hole spin and second-order momentum correction coefficient β are fixed, the black hole shadow scale decreases with the increase in the first-order momentum correction coefficient α. When the black hole spin and the first-order momentum correction coefficient α are fixed, the black hole shadow scale increases with the increase in the second-order momentum correction coefficient β. That is to say, the first-order momentum correction coefficient α and the second-order momentum correction coefficient β change the shadow size of the black hole in the opposite direction.

When the black hole spin $a = 0.99$, with the increase in the first-order momentum correction coefficient α, the black hole shadow scale is increasingly distorted, so there is a certain degree of devolution between the first-order momentum correction coefficient α and the influence of the black hole spin on the black hole shadow shape. When considering the larger value of the second-order momentum correction coefficient β, the black hole shadow shape is very close to the spherically symmetric black hole case even if the black hole spin is close to 1. The significant influence of the first-order momentum correction coefficients $\alpha$and the second-order momentum correction coefficients β on the shape of the black hole shadow makes it possible to test it with Event Horizon Telescope (EHT) observations.

For model II, with the increase in the value of the mixing correction coefficient ε, the black hole shadow scale gradually decreases, but its value changes very slowly, which makes it possible to test it with EHT unless the mixing correction coefficient ε changes sharply.

In fact, since both model I and model II must satisfy the generalized uncertainty principle (1), the Hawking temperature corresponding to the black hole metric (2) and the black hole metric (7) should be consistent, which makes the parameters of model I and model II satisfy the relation (10). Even if the two models can be coordinated by adjusting the values of the parameters, the variation rules of the black hole shadow shape are different because the black hole spacetime metric corresponding to model I is different from that corresponding to model II. In view of this, we believe that future EHT observations can be used to test it in order to better understand the quantum gravitational effects of black holes.

5 Weak cosmic censorship conjecture in GUP-BH and co-constraints

In Sect. 4 we discussed the possibility of using future EHT observations to examine GUP black holes. Next we are interested in how GUP affects the nature of black holes, such as whether the weak cosmic censorship conjecture of black holes is violated when considering the GUP model. The work and methods for testing the weak cosmic censorship conjecture by incident particles can be found in the literature (e.g. [40–43]). The work and methods for testing the weak cosmic censorship conjecture by incident scalar field can be found in various works in the literature (e.g. [44–50]). Here, we will examine whether the event horizon of the GUP black hole can be destroyed by test particles and scalar fields incident to the GUP black hole, and discuss the influence of the GUP on the weak cosmic censorship conjecture.

5.1 Weak cosmic censorship conjecture (WCCC) was tested via particle incidence

According to the discussion of model I and model II in Sect. 3, the analytical expression of the change in GUP model parameters (model I, model parameters are α and β; model II, the model parameter is ε) on the event horizon of the black hole can be obtained, and the basic results are summarized by Eq. (46):

$$r_H = \begin{cases} M\left[1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right]+\sqrt{M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)^2-a^2} \\ \quad model\ I \quad Kerr-like \\ M+\sqrt{(1-\varepsilon)M^2-a^2}, \\ \quad model\ II \quad Kerr-Newman-like \end{cases} . \tag{46}$$

For model I, if the spacetime line element (18) describes a black hole, the condition $M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)^2 \geq a^2$ must be satisfied. For model II, if the spacetime line element (19) describes a black hole, the condition $(1-\varepsilon)M^2 \geq a^2$ must be satisfied. The conditions for destroying the event horizon of a black hole are as follows. For model I we need to satisfy the inequality

$$M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)^2 < a^2. \tag{47}$$

For model II we need to satisfy the inequality

$$(1-\varepsilon)M^2 < a^2. \tag{48}$$

The inequality equations (51) and (52) can be rewritten as the following expressions

$$J > M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right) \tag{49}$$

and

$$J > M^2\sqrt{1-\varepsilon}. \tag{50}$$

Wen the rotating GUP black hole in the equatorial plane encounters an incident test particle with mass, the GUP black hole and the test particle will have a complex interaction. When the test particle crosses the event horizon of the GUP black hole, both the mass and angular momentum of the GUP black hole increase (labeled δE and δJ). By examining whether the mass and angular momentum of the composite system satisfy the event horizon destruction condition (inequality equations (49) or (50)), we can analyze how the GUP parameters affect the testing of the weak cosmic censorship conjecture. The calculation procedure employed here adheres to the standard methodology. We will only present the outcomes along with their corresponding analysis.

The results show that the event horizon of the GUP black hole can be destroyed when the mass increment (δE) and angular momentum increment (δJ) of the test particle meet certain conditions. For model I, this condition is

$$\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)\delta E^2+2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)M\delta E \\ +M^2\left(1-\frac{2\alpha}{M}+\frac{4\beta}{M^2}\right)-J < \delta J < \frac{1}{\Omega_H}\delta E. \tag{51}$$

For model II, the condition becomes

$$\sqrt{1-\varepsilon}\delta E^2+2\sqrt{1-\varepsilon}M\delta E \\ +M^2\sqrt{1-\varepsilon}-J < \delta J < \frac{1}{\Omega_H}\delta E. \tag{52}$$

If the GUP black hole is an extremely rotating black hole, when considering the first approximation of δE, there is no new term in the black hole event horizon angular velocity Ω_H, which makes it possible for no test particle to satisfy the event horizon destruction condition (51) and (52). This shows that for both model I and model II, the introduction of GUP does not violate the weak cosmic censorship conjecture of rotating black holes. For the near-extreme black hole case, a parameter is introduced to describe the degree of closeness to the extreme black hole, and then the destroy conditions (51) and (52) are simplified. The results show that the black hole event horizon cannot be destroyed for model I and model II. The weak cosmic censorship conjecture is not violated.

5.2 WCCC was tested via scalar field scatter

In addition to testing the weak cosmic censorship conjecture by incident test particles on a rotating GUP black hole, another method can be used to test the weak cosmic censorship conjecture of a black hole, that is, incident scalar fields on an extreme or near-extreme black hole to destroy the event horizon of the black hole. If the event horizon can be destroyed within the range of optional GUP model parameters, then the weak cosmic censorship conjecture is violated. Otherwise, there is no violation.

If there is a scalar field near the GUP rotating black hole, then the scalar field will interact with the black hole, and in the semiclassical case, the equation of motion of the scalar field can be calculated in the black hole spacetime background. If the scalar field is labeled with ψ and the mass of the corresponding particle is labeled with μ, then the equation of motion of the scalar field in the spacetime background of the GUP rotating black hole satisfies the Klein–Gordon equation, whose basic form is

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi) - \mu^2\psi = 0, \tag{53}$$

where g is the determinant of the GUP rotating black hole metric, and $g^{\mu\nu}$ is the inverse form of the spacetime metric, both of which can be calculated by the spacetime metric (18) and (19). By substituting the GUP black hole metric (18) and (19) into the equation of motion (53), it can be simplified to the following form:

$$\begin{aligned}
&-\frac{(r^2+a^2)^2 - a^2\Delta\sin^2\theta}{\Delta\Sigma^2}\frac{\partial^2\psi}{\partial t^2}\\
&-\frac{4aM}{\Delta\Sigma^2}\frac{\partial^2\psi}{\partial t\partial\phi} + \frac{1}{\Sigma^2}\frac{\partial}{\partial r}\left(\Delta\frac{\partial\psi}{\partial r}\right)\\
&+\frac{1}{\Sigma^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)\\
&+\frac{\Delta - a^2\sin^2\theta}{\Delta\Sigma^2\sin\theta}\frac{\partial^2\psi}{\partial\phi^2} - \mu^2\psi = 0.
\end{aligned} \tag{54}$$

This equation can be processed by variable separation, which decomposes ψ into $\psi(t, r, \theta, \phi) = e^{-i\omega t}R(r)S_{lm}(\theta)e^{im\phi}$, where $S_{lm}(\theta)$ is a spherical function, and the integers l and m are quantum numbers. By separating variables in this way, Eq. (54) can be simplified into two equations, one of which is an angular equation, whose solution is a spherical harmonic function, and the other is a radial equation, whose specific expression is

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \left[\frac{(r^2+a^2)^2}{\Delta}\omega^2 - \frac{4aM}{\Delta}m\omega\right.\\
&\left.+\frac{m^2a^2}{\Delta} - \mu^2r^2 - \lambda_{lm}\right]R(r) = 0.
\end{aligned} \tag{55}$$

By examining Eq. (55), we can find that if we want to obtain an analytical expression of the radial function $R(r)$ near the event horizon, since $\Delta = 0$ near the event horizon, Eq. (55) will lose its physical meaning (this is because Eq. (55) corresponds to the BL coordinate system). Therefore, Eq. (55) needs to be converted to other coordinate systems in order to solve it; in general, the Eddington–Finkelstein coordinate system (also known as the turtle coordinate) is selected, and for radial coordinates, the following relationship is satisfied:

$$\frac{\mathrm{d}r}{\mathrm{d}r_*} = \frac{\Delta}{r^2+a^2}. \tag{56}$$

The turtle coordinate r_* at the GUP rotating black hole event horizon does not make Eq. (55) singular. Substituting the coordinate transform (56) into (55) transforms the radial equation (55) into

$$\begin{aligned}
&\frac{\Delta}{(r^2+a^2)^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2+a^2)\frac{\mathrm{d}R}{\mathrm{d}r_*} + \frac{\mathrm{d}^2R}{\mathrm{d}r_*^2}\\
&+\left(\omega - \frac{ma}{r^2+a^2}\right)^2R(r) + \left[\frac{\Delta}{(r^2+a^2)^2}2am\omega\right.\\
&\left.-\frac{\Delta}{(r^2+a^2)^2}(\mu^2r^2+\lambda_{lm})\right]R(r) = 0.
\end{aligned} \tag{57}$$

Near the event horizon of a GUP rotating black hole(for model I and model II), $\Delta = 0$ simplifies Eq. (57) to the following equation:

$$\frac{\mathrm{d}^2R}{\mathrm{d}r_*^2} + (\omega - m\Omega_H)^2R = 0. \tag{58}$$

This equation is a second-order ordinary differential equation, and its analytical solution is $R(r) = \exp[\pm i(\omega - m\Omega_H)r_*]$. Plug it into the variable separation expression of ψ, whose final solution is

$$\psi(t, r, \theta, \phi) = \exp[-i(\omega - m\Omega_H)r_*]e^{-i\omega t}S_{lm}(\theta)e^{im\phi}. \tag{59}$$

With this solution, we have obtained all the information of the scalar field near the event horizon, and can use it to calculate various physical quantities.

Suppose that a scalar field is incident to the GUP rotating black hole spacetime background. Due to the maximum effective potential of the black hole spacetime in the radial direction, the spacetime will absorb and reflect the scalar field. These physical processes change the energy and angular momentum of the black hole. The corresponding energy increment and angular momentum increment can be calculated from the scalar field (59). For the scalar field considered in this section, the expression of the energy–momentum tensor can be obtained for the scalar field moving in the spacetime background of a GUP rotating black hole

$$T_{\mu\nu} = \partial_\mu\psi\partial_\nu\psi^* - \frac{1}{2}g_{\mu\nu}(\partial_\mu\psi\partial^\nu\psi^* + \mu^2\psi^*\psi). \tag{60}$$

By combining the expression of the energy momentum tensor (60) with the scalar field (59), all the energy–momentum nonzero components of the scalar field can be obtained. Using these nonzero components, we can calculate the energy flow and angular momentum flux of the scalar field incident on the black hole. The energy flow of a scalar field through the event horizon is

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \int_H T^r{}_t\sqrt{-g}\mathrm{d}\theta\mathrm{d}\phi = \omega(\omega - m\Omega_H)(r_H^2 + a^2), \tag{61}$$

the angular momentum flux of a scalar field through the event horizon is

$$\frac{\mathrm{d}J}{\mathrm{d}t} = \int_H T^r{}_\phi\sqrt{-g}\mathrm{d}\theta\mathrm{d}\phi = m(\omega - m\Omega_H)(r_H^2 + a^2). \tag{62}$$

From these two expressions, it can be seen that whether the incident scalar field transfers energy or angular momentum to the black hole depends on the relative size of ω and $m\Omega_H$. In a very short time, the energy increment and angular momentum increment of the scalar field transferred to the black hole are

$$\mathrm{d}E = \omega(\omega - m\Omega_H)(r_H^2 + a^2)\mathrm{d}t \tag{63}$$

and

$$\mathrm{d}J = m(\omega - m\Omega_H)(r_H^2 + a^2)\mathrm{d}t. \tag{64}$$

A scalar field incident on a black hole can be divided into an infinite number of small segments ($\mathrm{d}t$) for segmentation discussion. Assume that the mass and angular momentum of the GUP rotating black hole before the scalar field incident on the black hole are M and J, respectively. The mass and angular momentum of the GUP rotating black hole–scalar field complex system are $M^{'}$ and $J^{'}$, respectively. They satisfy the relation $M^{'} = M + \mathrm{d}E$ and $J^{'} = J + \mathrm{d}J$. Then, by examining the signs of $M^{'2}\omega_0 - J^{'}$ of the composite system, we can determine whether the event horizon of the composite system exists, and whether the weak cosmic supervision conjecture is violated when the scalar field is incident on the GUP rotating black hole. Next, we will only give the calculation results, and the calculation process can be referred to [47,49].

By calculation, for a scalar field of mode (l, m), the expression $M^{'2}\omega_0 - J^{'}$ for a composite system is

$$M^{'2}\omega_0 - J^{'} = (M^2\omega_0 - J) + 2M\omega_0 m^2\left(\frac{\omega}{m} - \frac{1}{2M\omega_0}\right) \times \left(\frac{\omega}{m} - \Omega_H\right)(r_H^2 + a^2)\mathrm{d}t. \tag{65}$$

As long as we can judge the sign of (65), we can discuss whether the scalar field can destroy the event horizon. When considering extreme black holes, since extreme black holes satisfy the condition $J = M^2\omega_0$, then $M^{'2}\omega_0 - J^{'}$ is reduced to the following form:

$$M^{'2}\omega_0 - J^{'} = 2M\omega_0 m^2\left(\frac{\omega}{m} - \frac{1}{2M\omega_0}\right) \times \left(\frac{\omega}{m} - \Omega_H\right)(r_H^2 + a^2)\mathrm{d}t. \tag{66}$$

For model I and model II, for extreme black holes, since Ω_H does not have any shift, that is, $\Omega_H = \frac{1}{2\,M\omega_0}$, then $M^{'2}\omega_0 - J^{'}$ is always greater than zero, which indicates that for a scalar field incident extreme GUP black hole, the event horizon of the black hole cannot be destroyed, and the weak cosmic censorship conjecture is not violated. For the case of near-extreme GUP black holes, a parameter describing the degree of near-extreme black holes can also be introduced for discussion (similar to the analysis techniques in Sect. 5.1). The calculation results show that $M^{'2}\omega_0 - J^{'}$ is also always greater than zero for near-extreme black holes, so the black hole event horizon cannot be destroyed, and the weak cosmic censorship conjecture is still not violated.

6 Summary

In this study, we obtain the rotating black hole solutions of the Einstein gravitational field equations when the GUP modifications are taken into account, and we discuss the physical properties of these solutions. On the physical picture, we construct the GUP modifications to the Kerr black hole in two different ways. Model I mainly considers GUP only for mass modification, which changes our understanding of gravitational mass and inertial mass, and this construction is closer to physical nature. Model II considers the GUP as an effect of an external field, so the energy–momentum tensor corresponding to the black hole system is not zero. In this case, the physical meaning of GUP for black hole modification is completely different from that of model I. From the perspective of black hole thermodynamics, for the spherically symmetric GUP black hole case, model I and model II should be equivalent, so we get the relationship between model parameters α, β and ε (10). Although the GUP black hole modifications constructed by these two methods are consistent in black hole thermodynamics, their spacetime properties are different. For the GUP spinning black hole case, the thermodynamics are not equivalent (the Hawking temperatures corresponding to model I and model II are not equal). The black hole metric (18) obtained by model I is similar to that of the Kerr black hole, and the black hole metric (19) obtained by model II is similar to that of the Kerr–Newman black hole.

The potential of black hole shadows in testing GUP is explored based on the spacetime of a rotating black hole (18) and (19), leading to the following findings. On one hand, when both the first-order momentum correction coefficient α and the second-order momentum correction coefficient β of GUP are nonzero, the determination of the black hole shadow relies on the model parameters α and β. Furthermore, these coefficients exhibit an opposite effect on altering the size of the black hole shadow, which is a significant characteristic that may potentially be confirmed in future EHT observations. On the other hand, when considering the higher magnitude of the second-order momentum correction coefficient β, even in cases where the black hole spin approaches 1, the shape of the black hole shadow closely resembles that of a spherically symmetric black hole. This observation suggests a potential degenerate effect between the second-order momentum correction coefficient β and the black hole spin.

The weak cosmic censorship conjecture for GUP rotating black holes is also investigated herein. By introducing test particles and scalar fields into the GUP rotating black hole, we find no violation of the weak cosmic censorship conjecture in either extreme or near-extreme black hole scenarios. Hence, even modification of black hole spacetime due to GUP does not alter the discourse on the nature of black holes. The weak cosmic censorship conjecture remains unaffected by GUP; however, it cannot be guaranteed that the strong cosmic censorship conjecture will not be influenced. In future research, we will investigate the applicability of the strong cosmic censorship conjecture to GUP rotating black holes.

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Code Availability Statement The manuscript has no associated code/software. [Author's comment: This article is purely theoretical research and does not involve generating or analyzing any code/software.]

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