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Minimal texture of quark mass matrices and precision CKM measurements

Shivali Kaundal, Aakriti Bagai, Gulsheen Ahuja*, and Manmohan Gupta*

Department of Physics, Centre of Advanced Study, Panjab University, Chandigarh, India
*E-mail: gulsheen@pu.ac.in, mmgupta@pu.ac.in

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We have carried out an extensive analysis of all possible minimal texture quark mass matrices implying 169 texture-6 zero combinations. One finds that all these combinations are ruled out: a good number of these analytically, the other possibilities being excluded by the present quark mixing data. Interestingly, even if there are future changes in the ranges of the light quark masses, these conclusions remain valid.

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Over the last couple of decades, noticeable progress has been made in measuring the quark mixing parameters. Most of the Cabibbo–Kobayashi–Maskawa (CKM) [1,2] parameters are now known within an accuracy of around 5%; this can be considered as being at the level of "precision measurements" in the context of CKM phenomenology. Similarly, a good deal of progress has been made in the measurement of the quark masses; in particular, the light quark masses, m_u, m_d and m_s, have registered remarkable progress in their measurements in the last decade. This has been possible due to the lattice simulations providing the most reliable determination of the strange quark mass and of the average of the up and down quark masses, as emphasized by Flavour Lattice Averaging Group (FLAG) [3]. In view of the relationship of the CKM matrix with the mass matrices, precision measurements of CKM parameters would undoubtedly have implications for the quark mass matrices. Similarly, considerable narrowing of the ranges of the light quark masses would allow us to determine the nature and structure of the quark mass matrices which are compatible with the CKM phenomenology.

It is well known that the mass matrices, having their origin in the Higgs fermion couplings, are arbitrary in the Standard Model (SM), therefore the number of free parameters available with a general mass matrix is 36 which is much larger than the number of physical observables; e.g., in the quark sector, the ten observables include six quark masses, three mixing angles, and one CP-violating phase. Therefore, to develop viable phenomenological fermion mass matrices one has to limit the number of free parameters in these matrices. It may be noted that in the SM, as well as its extensions, wherein the right-handed quarks are singlets, without loss of generality one can always consider the mass matrices to be Hermitian. In this context, the idea of texture zero mass matrices [4–6] consisting of finding the phenomenological quark mass matrices which are in tune with the low-energy data, i.e. observables like quark masses, quark mixing angles, angles of the unitarity triangle in the quark sector, etc., has proved to be quite successful in explaining the fermion mixing data. A particular square matrix is considered to be texture-n zero if the sum of the number of diagonal zeros and half the number of the symmetrically placed off-diagonal zeros is n.

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In the quark sector, the concept of texture zeros was introduced implicitly by Weinberg [7] and explicitly by Fritzsch [8,9], the original Fritzsch ansatz being given by

$$M_{\rm U} = \begin{pmatrix} 0 & A_{\rm U} & 0 \\ A_{\rm U}^* & 0 & B_{\rm U} \\ 0 & B_{\rm U}^* & C_{\rm U} \end{pmatrix}, \quad M_{\rm D} = \begin{pmatrix} 0 & A_{\rm D} & 0 \\ A_{\rm D}^* & 0 & B_{\rm D} \\ 0 & B_{\rm D}^* & C_{\rm D} \end{pmatrix}, \tag{1}$$

where $M_{\rm U}$ and $M_{\rm D}$ correspond to the mass matrices in the up and down sectors, respectively, with complex off-diagonal elements, i.e. $A_i = |A_i|e^{\iota\alpha}$ and $B_i = |B_i|e^{\iota\beta}$, $i = {\rm U, D}$, $\iota = \sqrt{-1}$, whereas C_i is the real element of the matrix. Using the above definition of texture zero mass matrices, each of the above matrices is said to be texture-3 zero type, abd together these are referred to as texture-6 zero quark mass matrices. The abovementioned original Fritzsch ansatz, as well as several other texture-5 and -4 zero versions, have been examined in Refs. [10–16]. In particular, Refs. [10] and [15] have discussed texture-6 zero quark mass matrices and have arrived at the conclusion that these look to be incompatible with the quark mixing data. However, in these references, a detailed and comprehensive analysis indicating how and to what extent these matrices are ruled out has not been discussed. In particular, neither reference considers all possible texture-6 zero quark mass matrices, nor do they relate the various possibilities through permutation symmetry, which has gained significance in the context of quark–lepton symmetry due to which the emphasis has now shifted to formulating the texture structure of fermion mass matrices incorporating permutation symmetry and Abelian symmetries.

In the absence of any firm theoretical foundation for choosing a particular texture for the mass matrices, it becomes interesting to analyze all possible mass matrix texture structures to check their viability with the present refined data. It may be noted that the maximum number of texture zeros which can be introduced in the quark mass matrices is three in each sector, resulting in a minimal number of parameters or elements of the mass matrices. In view of this, we refer to texture-6 zero quark mass matrices as the minimal texture of quark mass matrices. It is interesting to note that it is not only the matrices mentioned in Eq. (1) that correspond to texture-3 zero mass matrices; along with these, there are several other possible structures which can be considered to be texture-3 zero matrices. The purpose of the present work is to first enumerate all possible minimal texture quark mass matrices, i.e. texture-6 zero mass matrices, and to relate these possibilities using permutation symmetry. As a next step, we have examined in a detailed and comprehensive manner the viability of all these possible mass matrices, keeping in mind the improvements in the measurements of the light quark masses, m_u, m_d, and m_s, as well as “precision measurements” of the CKM parameters.

One can check that the total number of structures for a texture-n zero mass matrix comes out to be

$$^6C_n = \frac{6!}{n!(6-n)!}, \tag{2}$$

where there are six ways to enter zeros in the mass matrices. Using this, for $n = 3$ one can arrive at the following 20 possible structures (S_0 to S_{19}) for texture-3 zero mass matrices:

(1) Placing all three zeros along diagonal positions:

$$S_0 = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$

where an $\times$ represents non-vanishing entries.

(2) Placing two zeros along diagonal positions:

$$S_1 = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & \times \end{pmatrix},$$
$$S_4 = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad S_5 = \begin{pmatrix} 0 & \times & \times \\ \times & \times & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad S_6 = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix},$$
$$S_7 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad S_8 = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}, \quad S_9 = \begin{pmatrix} \times & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}.$$

(3) Placing one zero along diagonal positions:

$$S_{10} = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad S_{11} = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}, \quad S_{12} = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix},$$
$$S_{13} = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & 0 \end{pmatrix}, \quad S_{14} = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad S_{15} = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & \times \end{pmatrix},$$
$$S_{16} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad S_{17} = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & 0 \\ \times & 0 & \times \end{pmatrix}, \quad S_{18} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}.$$

(4) Placing all zeros in off-diagonal positions:

$$S_{19} = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}.$$

In general, one has the freedom to consider the mass matrices in the up and down sectors, i.e. $M_{\rm U}$ and $M_{\rm D}$, to be any of the above 20 patterns, resulting in 400 combinations corresponding to texture-6 zero mass matrices. However, since these matrices have to yield physical quark masses as their eigenvalues, the trace as well as the determinant should be non-zero, i.e.

$$\text{Trace}\, M_{\rm U,D} \neq 0 \quad \text{and} \quad \text{Det}\, M_{\rm U,D} \neq 0. \tag{3}$$

Imposing these constraints, one can immediately see that either the trace or the determinant of the structures S_0, S_7, S_8, S_9, S_{16}, S_{17}, and S_{18} vanishes, and hence out of 20 possible patterns, we are left with 13 structures to be considered either as $M_{\rm U}$ or $M_{\rm D}$, leading to 169 possible texture-6 zero combinations.

It may be noted that structure S_1 is in fact the Fritzsch ansatz mentioned in Eq. (1). Interestingly, one finds that the structures S_1, S_2, S_3, S_4, S_5, and S_6 are related as

$$S_j = p_j^{\rm T} S_1 p_j, \qquad j = 1, \ldots, 6, \tag{4}$$

Table 1. Possible texture-3 zero mass matrices belonging to classes I and II.

	Class I	Class II
a	$\begin{pmatrix} 0 & Ae^{\iota\alpha} & 0 \\ Ae^{-\iota\alpha} & 0 & Be^{\iota\beta} \\ 0 & Be^{-\iota\beta} & C \end{pmatrix}$	$\begin{pmatrix} 0 & Ae^{\iota\alpha} & 0 \\ Ae^{-\iota\alpha} & D & 0 \\ 0 & 0 & C \end{pmatrix}$
b	$\begin{pmatrix} 0 & 0 & Ae^{\iota\alpha} \\ 0 & C & Be^{-\iota\beta} \\ Ae^{-\iota\alpha} & Be^{\iota\beta} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & Ae^{\iota\alpha} \\ 0 & C & 0 \\ Ae^{-\iota\alpha} & 0 & D \end{pmatrix}$
c	$\begin{pmatrix} 0 & Ae^{-\iota\alpha} & Be^{\iota\beta} \\ Ae^{\iota\alpha} & 0 & 0 \\ Be^{-\iota\beta} & 0 & C \end{pmatrix}$	$\begin{pmatrix} D & Ae^{-\iota\alpha} & 0 \\ Ae^{\iota\alpha} & 0 & 0 \\ 0 & 0 & C \end{pmatrix}$
d	$\begin{pmatrix} C & Be^{-\iota\beta} & 0 \\ Be^{\iota\beta} & 0 & Ae^{-\iota\alpha} \\ 0 & Ae^{\iota\alpha} & 0 \end{pmatrix}$	$\begin{pmatrix} C & 0 & 0 \\ 0 & D & Ae^{-\iota\alpha} \\ 0 & Ae^{\iota\alpha} & 0 \end{pmatrix}$
e	$\begin{pmatrix} 0 & Be^{\iota\beta} & Ae^{-\iota\alpha} \\ Be^{-\iota\beta} & C & 0 \\ Ae^{\iota\alpha} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} D & 0 & Ae^{-\iota\alpha} \\ 0 & C & 0 \\ Ae^{\iota\alpha} & 0 & 0 \end{pmatrix}$
f	$\begin{pmatrix} C & 0 & Be^{-\iota\beta} \\ 0 & 0 & Ae^{\iota\alpha} \\ Be^{\iota\beta} & Ae^{-\iota\alpha} & 0 \end{pmatrix}$	$\begin{pmatrix} C & 0 & 0 \\ 0 & 0 & Ae^{\iota\alpha} \\ 0 & Ae^{-\iota\alpha} & D \end{pmatrix}$

where p_j are the following six permutation matrices:

$$p_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad p_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad p_3 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

$$p_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad p_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad p_6 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{6}$$

These six matrices, S_1 to S_6, have been placed in class I of Table 1, and for further discussions will be referred to as I_a, I_b, etc. Further, interestingly, the six structures $S_{10}, S_{11}, S_{12}, S_{13}, S_{14}$, and S_{15} are also related through permutations and have been placed in class II of Table 1; they will henceforth be referred to as II_a, II_b, etc. The remaining structure, S_{19}, will be discussed separately. Therefore, instead of discussing 169 possible texture-6 zero combinations, we will first discuss 144 possibilities of Hermitian mass matrices which can be arrived at by considering M_U and M_D to be from class I and/or class II of Table 1.

Coming to the methodology, it essentially involves considering a possible texture-6 zero combination, i.e. M_U and M_D being one of the above patterns. The viability of the considered combination is ensured by examining the compatibility of the CKM matrix constructed from the given combination of mass matrices with the recent one given by Particle Data Group (PDG) [18]. To this end, as a first step, the Hermitian matrix M_i ($i = \mathrm{U}, \mathrm{D}$) can be expressed as

$$M_i = P_i^\dagger M_i^{\mathrm{r}} P_i, \tag{7}$$

where M_i^{r} corresponds to the real matrix and P_i denotes the phase matrix. The real matrix M_i^{r} can then be diagonalized by the orthogonal transformations O_i, i.e.

$$M_i^{\mathrm{diag}} = O_i^{\mathrm{T}} M_i^{\mathrm{r}} O_i = O_i^{\mathrm{T}} P_i M_i P_i^\dagger O_i, \tag{8}$$

where $M_i^{\mathrm{diag}} = \mathrm{diag}(m_1, -m_2, m_3)$, the subscripts 1, 2, and 3 referring respectively to u, c, and t for the up sector and d, s, and b for the down sector. In order to examine the viability of the considered combination, one needs to obtain the CKM matrix using the relation

$$V_{\mathrm{CKM}} = O_{\mathrm{U}}^{\mathrm{T}} P_{\mathrm{U}} P_{\mathrm{D}}^\dagger O_{\mathrm{D}} = V_{\mathrm{U}}^\dagger V_{\mathrm{D}}, \tag{9}$$

where the unitary matrices V_{U} ($= P_{\mathrm{U}}^\dagger O_{\mathrm{U}}$) and V_{D} ($= P_{\mathrm{D}}^\dagger O_{\mathrm{D}}$) are the diagonalizing transformations for the matrices M_{U} and M_{D}, respectively.

To begin with, let us consider the matrix I_a of class I. The corresponding real matrix M_i^{r} can be expressed as

$$M_i^{\mathrm{r}} = \begin{pmatrix} 0 & |A_i| & 0 \\ |A_i| & 0 & |B_i| \\ 0 & |B_i| & C_i \end{pmatrix} \tag{10}$$

and P_i, the phase matrix, is given by

$$P_i = \begin{pmatrix} e^{-\iota\alpha_i} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{\iota\beta_i} \end{pmatrix}. \tag{11}$$

It may be mentioned that the remaining five matrices belonging to class I can be similarly expressed in terms of a real matrix M_i^{r} and the corresponding phase matrix P_i. An essential step for the construction of the diagonalization transformation is to consider the invariants trace M_i^{r}, trace $(M_i^{\mathrm{r}})^2$, and det M_i^{r} to yield relations involving elements of mass matrices. For all six matrices belonging to class I, using these invariants, the relations of the matrix elements in terms of quark masses can be expressed as

$$C_i = (m_1 - m_2 + m_3), \quad |A_i|^2 + |B_i|^2 = (m_1 m_2 + m_2 m_3 - m_1 m_3), \quad |A_i|^2 C_i = (m_1 m_2 m_3). \tag{12}$$

Corresponding to the matrix I_a, the diagonalizing transformation O_i is given as

$$O_i = \begin{pmatrix} \left(\frac{m_2 m_3 (m_3 - m_2)}{C_i (m_1 + m_2)(m_3 - m_1)}\right)^{\frac{1}{2}} & \left(\frac{m_1 m_3 (m_1 + m_3)}{C_i (m_1 + m_2)(m_3 + m_2)}\right)^{\frac{1}{2}} & \left(\frac{m_1 m_2 (m_2 - m_1)}{C_i (m_3 + m_2)(m_3 - m_1)}\right)^{\frac{1}{2}} \\ \left(\frac{m_1 (m_3 - m_2)}{(m_1 + m_2)(m_3 - m_1)}\right)^{\frac{1}{2}} & -\left(\frac{m_2 (m_1 + m_3)}{(m_1 + m_2)(m_3 + m_2)}\right)^{\frac{1}{2}} & \left(\frac{m_3 (m_2 - m_1)}{(m_3 - m_1)(m_2 + m_3)}\right)^{\frac{1}{2}} \\ -\left(\frac{m_1 (m_1 + m_3)(m_2 - m_1)}{C_i (m_1 + m_2)(m_3 - m_1)}\right)^{\frac{1}{2}} & \left(\frac{m_2 (m_3 - m_2)(m_2 - m_1)}{C_i (m_1 + m_2)(m_3 + m_2)}\right)^{\frac{1}{2}} & \left(\frac{m_3 (m_3 - m_2)(m_3 + m_1)}{C_i (m_3 + m_2)(m_3 - m_1)}\right)^{\frac{1}{2}} \end{pmatrix}. \tag{13}$$

For the other matrices belonging to class I, one can obtain the corresponding diagonalizing transformations O_i in a similar manner.

Similarly, for all the matrices belonging to class II, the relations of the mass matrix elements in terms of the quark masses are given by

$$\begin{aligned} C_i + D_i &= (m_1 - m_2 + m_3), \\ |A_i|^2 - C_i D_i &= (m_1 m_2 + m_2 m_3 - m_1 m_3), \\ |A_i|^2 C_i &= (m_1 m_2 m_3). \end{aligned} \tag{14}$$

For the matrix II_a, the corresponding real matrix M_i^{r} can be expressed as

$$M_i^{\mathrm{r}} = \begin{pmatrix} 0 & |A_i| & 0 \\ |A_i| & D_i & 0 \\ 0 & 0 & C_i \end{pmatrix}, \tag{15}$$

with the phase matrix P_i being

$$P_i = \begin{pmatrix} e^{-\iota\alpha_i} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{16}$$

Further, the corresponding diagonalizing transformation can be expressed as

$$O_i = \begin{pmatrix} \left(\frac{m_2}{m_1+m_2}\right)^{\frac{1}{2}} & \left(\frac{m_1}{m_1+m_2}\right)^{\frac{1}{2}} & 0 \\ \left(\frac{m_1}{m_1+m_2}\right)^{\frac{1}{2}} & -\left(\frac{m_2}{m_1+m_2}\right)^{\frac{1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

One can obtain similar matrices for the other five class II matrices.

As the next step of our analysis, we present all possible texture-6 combinations, wherein M_{U} and M_{D} can be considered from class I and/or class II, leading to the following:

Category 1: M_{U} and M_{D} both from class I.
Category 2: M_{U} from class I and M_{D} from class II.
Category 3: M_{D} from class I and M_{U} from class II.
Category 4: M_{U} and M_{D} both from class II.

We first discuss Category 1, wherein the matrices M_{U} and M_{D} can each be any of the six possible structures, namely $\mathrm{I}_{a\text{–}f}$. This results in a total of 36 combinations of texture-6 zero mass matrices. From these, we first consider the six combinations wherein both M_{U} and M_{D} have the same structure, i.e. $\mathrm{I}_a\mathrm{I}_a$, $\mathrm{I}_b\mathrm{I}_b$, etc. Constructing the corresponding CKM matrices, one finds that all six are the same, i.e. they have the same expressions for each if the nine CKM matrix elements. These matrix elements are given as follows:

$$\begin{aligned} V_{ud} = &\left(\frac{m_d\,(m_b - m_s)}{(m_b - m_d)\,(m_d + m_s)}\right)^{\frac{1}{2}} \left(\frac{(-m_c + m_t)\,m_u}{(m_t - m_u)\,(m_c + m_u)}\right)^{\frac{1}{2}} \\ &+ e^{-\iota\phi_1} \left(\frac{m_b\,(m_b - m_s)\,m_s}{(m_b - m_d)\,(m_b + m_d - m_s)\,(m_d + m_s)}\right)^{\frac{1}{2}} \left(\frac{m_c m_t\,(-m_c + m_t)}{(m_t - m_u)\,(m_c + m_u)\,(-m_c + m_t + m_u)}\right)^{\frac{1}{2}} \\ &+ e^{\iota\phi_2} \left(\frac{m_d\,(m_b + m_d)\,(-m_d + m_s)}{(m_b - m_d)\,(m_b + m_d - m_s)\,(m_d + m_s)}\right)^{\frac{1}{2}} \left(\frac{(m_c - m_u)\,m_u\,(m_t + m_u)}{(m_t - m_u)\,(m_c + m_u)\,(-m_c + m_t + m_u)}\right)^{\frac{1}{2}}, \end{aligned}$$

$$V_{us} = -\left(\frac{(m_b+m_d)\,m_s}{(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{(-m_c+m_t)\,m_u}{(m_t-m_u)\,(m_c+m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{-\iota\phi_1}\left(\frac{m_b m_d\,(m_b+m_d)}{(m_b+m_d-m_s)\,(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c m_t\,(-m_c+m_t)}{(m_t-m_u)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$- e^{\iota\phi_2}\left(\frac{(m_b-m_s)\,m_s\,(-m_d+m_s)}{(m_b+m_d-m_s)\,(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{(m_c-m_u)\,m_u\,(m_t+m_u)}{(m_t-m_u)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

$$V_{ub} = \left(\frac{m_b\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{(-m_c+m_t)\,m_u}{(m_t-m_u)\,(m_c+m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{-\iota\phi_1}\left(\frac{m_d m_s\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c m_t\,(-m_c+m_t)}{(m_t-m_u)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$- e^{\iota\phi_2}\left(\frac{m_b\,(m_b+m_d)\,(m_b-m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{(m_c-m_u)\,m_u\,(m_t+m_u)}{(m_t-m_u)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

$$V_{cd} = -\left(\frac{m_d\,(m_b-m_s)}{(m_b-m_d)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(m_t+m_u)}{(m_c+m_t)\,(m_c+m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{-\iota\phi_1}\left(\frac{m_b\,(m_b-m_s)\,m_s}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t m_u\,(m_t+m_u)}{(m_c+m_t)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$- e^{\iota\phi_2}\left(\frac{m_d\,(m_b+m_d)\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(-m_c+m_t)\,(m_c-m_u)}{(m_c+m_t)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

$$V_{cs} = \left(\frac{(m_b+m_d)\,m_s}{(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(m_t+m_u)}{(m_c+m_t)\,(m_c+m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{-\iota\phi_1}\left(\frac{m_b m_d\,(m_b+m_d)}{(m_b+m_d-m_s)\,(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t m_u\,(m_t+m_u)}{(m_c+m_t)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{\iota\phi_2}\left(\frac{(m_b-m_s)\,m_s\,(-m_d+m_s)}{(m_b+m_d-m_s)\,(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(-m_c+m_t)\,(m_c-m_u)}{(m_c+m_t)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

$$V_{cb} = -\left(\frac{m_b\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(m_t+m_u)}{(m_c+m_t)\,(m_c+m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{-\iota\phi_1}\left(\frac{m_d m_s\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t m_u\,(m_t+m_u)}{(m_c+m_t)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{\iota\phi_2}\left(\frac{m_b\,(m_b+m_d)\,(m_b-m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(-m_c+m_t)\,(m_c-m_u)}{(m_c+m_t)\,(m_c+m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

$$V_{td} = \left(\frac{m_d\,(m_b-m_s)}{(m_b-m_d)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t\,(m_c-m_u)}{(m_c+m_t)\,(m_t-m_u)}\right)^{\frac{1}{2}}$$
$$+ e^{-\iota\phi_1}\left(\frac{m_b\,(m_b-m_s)\,m_s}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(m_c-m_u)\,m_u}{(m_c+m_t)\,(m_t-m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$- e^{\iota\phi_2}\left(\frac{m_d\,(m_b+m_d)\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t\,(-m_c+m_t)\,(m_t+m_u)}{(m_c+m_t)\,(m_t-m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

$$V_{ts} = -\left(\frac{(m_b+m_d)\,m_s}{(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t\,(m_c-m_u)}{(m_c+m_t)\,(m_t-m_u)}\right)^{\frac{1}{2}}$$
$$+\,e^{-\iota\phi_1}\left(\frac{m_b m_d\,(m_b+m_d)}{(m_b+m_d-m_s)\,(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(m_c-m_u)\,m_u}{(m_c+m_t)\,(m_t-m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$+\,e^{\iota\phi_2}\left(\frac{(m_b-m_s)\,m_s\,(-m_d+m_s)}{(m_b+m_d-m_s)\,(m_b+m_s)\,(m_d+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t\,(-m_c+m_t)\,(m_t+m_u)}{(m_c+m_t)\,(m_t-m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

$$V_{tb} = \left(\frac{m_b\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t\,(m_c-m_u)}{(m_c+m_t)\,(m_t-m_u)}\right)^{\frac{1}{2}}$$
$$+\,e^{-\iota\phi_1}\left(\frac{m_d m_s\,(-m_d+m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_c\,(m_c-m_u)\,m_u}{(m_c+m_t)\,(m_t-m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}}$$
$$+\,e^{\iota\phi_2}\left(\frac{m_b\,(m_b+m_d)\,(m_b-m_s)}{(m_b-m_d)\,(m_b+m_d-m_s)\,(m_b+m_s)}\right)^{\frac{1}{2}}\left(\frac{m_t\,(-m_c+m_t)\,(m_t+m_u)}{(m_c+m_t)\,(m_t-m_u)\,(-m_c+m_t+m_u)}\right)^{\frac{1}{2}},$$

where the phases $\phi_1 = \alpha_{\rm U} - \alpha_{\rm D}$ and $\phi_2 = \beta_{\rm U} - \beta_{\rm D}$ are related to the phases associated with the elements of the mass matrices.

For the purpose of numerical analysis, we first consider the texture combination $\mathrm{I}_a\mathrm{I}_a$, implying $M_{\rm U}$ and $M_{\rm D}$ both being of the form I_a. As mentioned earlier, this particular combination corresponds to the Fritzsch ansatz mentioned in Eq. (1). It has been shown [10,15], without getting into details, that this texture combination is ruled out due to the CKM matrix element V_{cb}. As a first step, it would be interesting to present the details regarding the ruling out of this combination, keeping in mind refinements in the measurements of the light quark masses. To this end, we have first investigated the dependence of the matrix element V_{cb} with respect to the light quark masses. As can be seen from the analytic expressions corresponding to the CKM matrix elements above, in order to construct the CKM matrix elements one needs to provide values of quark masses as well as phases ϕ_1 and ϕ_2 as inputs.

The "current" quark masses at M_Z energy scale [17] are given by

$$\begin{aligned} &m_u = 1.45^{+0.56}_{-0.45}\,\text{MeV}, && m_d = 2.9^{+0.5}_{-0.4}\,\text{MeV}, && m_s = 57.7^{+16.8}_{-15.7}\,\text{MeV},\\ &m_c = 0.635 \pm 0.086\,\text{GeV}, && m_b = 2.82^{+0.09}_{-0.04}\,\text{GeV}, && m_t = 172.1 \pm 0.6 \pm 0.9\,\text{GeV}. \end{aligned} \tag{18}$$

The most recent lattice values [3] of the quark mass ratios $\frac{m_u}{m_d}$ and $\frac{m_s}{m_{ud}}$, wherein m_{ud} is defined as $\frac{1}{2(m_u+m_d)}$, are

$$\frac{m_u}{m_d} = 0.45\,(3) \quad \text{and} \quad \frac{m_s}{m_{ud}} = 27.30\,(34). \tag{19}$$

For the purpose of our calculations, in order to investigate to what extent the texture combination $\mathrm{I}_a\mathrm{I}_a$ remains ruled out, we have assumed a relatively wider range of mass m_u, i.e. 0–3.0 MeV, and then, using the mass ratios in Eq. (19), we have obtained the corresponding wider ranges of masses m_d and m_s. Further, in the absence of any information regarding the values of the phases ϕ_1 and ϕ_2, these have been given full variation from 0° to 360°. Along with these inputs, we have imposed the following recent value as per PDG 2018 [18] of the precisely known CKM matrix element V_{us} as a constraint:

$$V_{us} = 0.2243 \pm 0.0005. \tag{20}$$

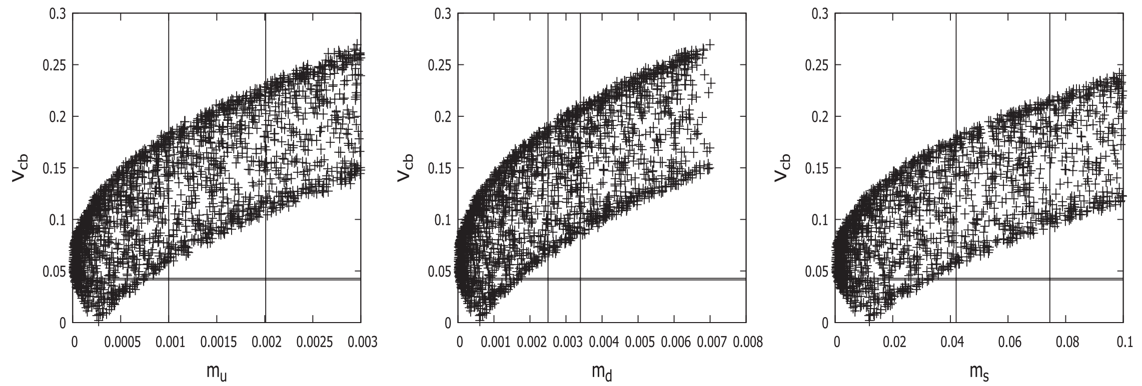

Fig. 1. Allowed range of V_{cb} with respect to the light quark masses obtained by imposing V_{us} as a constraint.

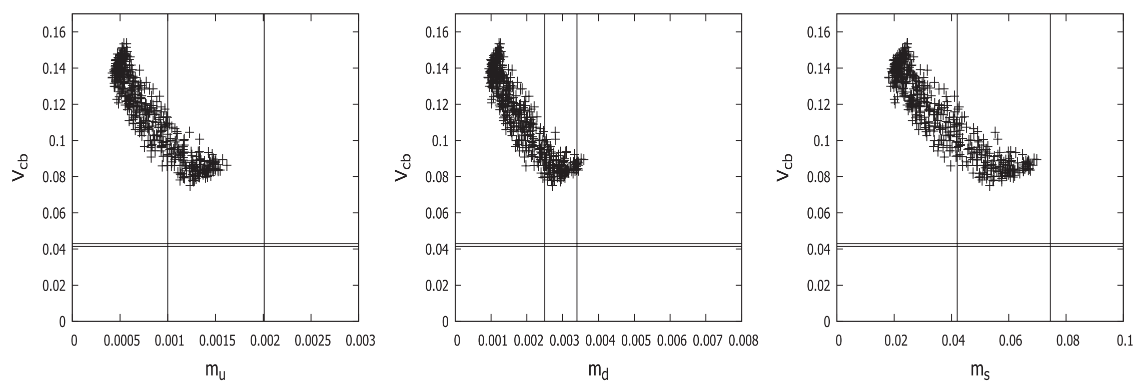

Fig. 2. Allowed range of V_{cb} with respect to the light quark masses obtained by imposing both V_{us} and V_{ub} as constraints.

Using the abovementioned inputs and constraint, in Fig. 1 we have shown the dependence of the CKM matrix element V_{cb} with respect to the light quark masses m_u, m_d, and m_s. The vertical lines in these plots depict the ranges of these masses given in Eq. (18), whereas the narrow experimental range [18] of the element V_{cb}, i.e. $(42.2 \pm 0.8) \times 10^{-3}$, is shown by very closely spaced horizontal lines. From these plots, one can note that for the ranges of m_u, m_d, and m_s given in Eq. (18), the allowed range of V_{cb} obtained here has no overlap with its experimentally determined range, thereby ruling out this combination of mass matrices. However, interestingly, it appears that if the lower limits of the light quark masses get pushed slightly lower, the CKM matrix element V_{cb} obtained here would show an overlap with its experimentally determined range.

To investigate this further, in addition to considering the matrix element V_{us} as a constraint we have also imposed the following value [18],

$$V_{ub} = (3.94 \pm 0.36) \times 10^{-3}, \tag{21}$$

as another constraint and have again plotted the dependence of V_{cb} on the light quark masses, shown in Fig. 2. These plots clearly indicate that when both V_{us} and V_{ub} are imposed as constraints then the allowed range of V_{cb} obtained here lies much outside its experimental range, completely ruling out the texture combination $\mathrm{I}_a\mathrm{I}_a$. This conclusion remains valid even if there are considerable changes in the input parameters.

In order to have a better understanding of the abovementioned results, as well as for the sake of completeness, we present the magnitudes of the CKM matrix elements, obtained by considering the

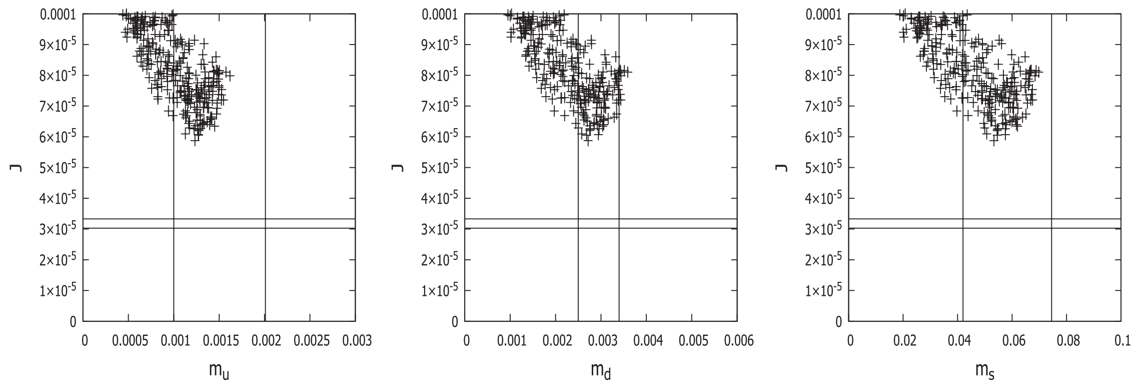

Fig. 3. Allowed range of J with respect to the light quark masses obtained by imposing both V_{us} and V_{ub} as constraints.

masses mentioned in Eq. (18) as inputs and both V_{us} and V_{ub} values as constraints:

$$V_{\text{CKM}} = \begin{pmatrix} 0.9743\text{–}0.9746 & 0.2238\text{–}0.2247 & 0.0036\text{–}0.0042 \\ 0.2228\text{–}0.2241 & 0.9694\text{–}0.9718 & 0.0749\text{–}0.1018 \\ 0.0168\text{–}0.0229 & 0.0731\text{–}0.0993 & 0.9947\text{–}0.9971 \end{pmatrix}. \tag{22}$$

A look at this matrix immediately reveals that the ranges of the CKM elements V_{cb}, V_{td}, V_{ts}, and V_{tb} show no overlap with those obtained by recent global analysis as per PDG 2018 [18]:

$$V_{\text{CKM}} = \begin{pmatrix} 0.9744\text{–}0.9746 & 0.2241\text{–}0.2250 & 0.0035\text{–}0.0038 \\ 0.2239\text{–}0.2248 & 0.9735\text{–}0.9737 & 0.0414\text{–}0.0429 \\ 0.0087\text{–}0.0092 & 0.0406\text{–}0.0421 & 0.9990\text{–}0.9991 \end{pmatrix}. \tag{23}$$

Further, besides determining the quark mixing matrix elements, we have also evaluated the CP-violating phase δ, Jarlskog's rephasing invariant parameter J, and the CP asymmetry parameter $\sin 2\beta$:

$$\delta = 79.2^\circ\text{–}90^\circ, \quad J = (5.84\text{–}9.49) \times 10^{-5}, \quad \sin 2\beta = 0.354\text{–}0.430. \tag{24}$$

Again, we find that the above ranges show absolutely no overlap with the experimentally determined ranges [18] of these quantities:

$$\delta = 68.4^\circ\text{–}77.7^\circ, \quad J = (3.03\text{–}3.33) \times 10^{-5}, \quad \sin 2\beta = 0.674\text{–}0.708. \tag{25}$$

For the sake of completeness, Figs. 3 and 4 show plots of Jarlskog's rephasing invariant parameter J as well as the CP asymmetry parameter $\sin 2\beta$, respectively, with respect to the light quark masses m_u, m_d, and m_s. The inputs for these plots are the same as for the earlier V_{cb} versus the light quark masses plots, along with both V_{us} and V_{ub} as constraints. Again, these plots reveal that the allowed ranges of these parameters have no overlap with their experimental ranges, presented as solid horizontal lines.

The above discussion, therefore, leads to the conclusion that not only does the texture combination I_aI_a get ruled out, but also this conclusion remains valid even if there are considerable changes in the input parameters. Similarly, as emphasized earlier, the other such combinations wherein both M_{U} and M_{D} have the same structure are also not compatible with the recent quark mixing data. This can also be checked using permutation symmetry.

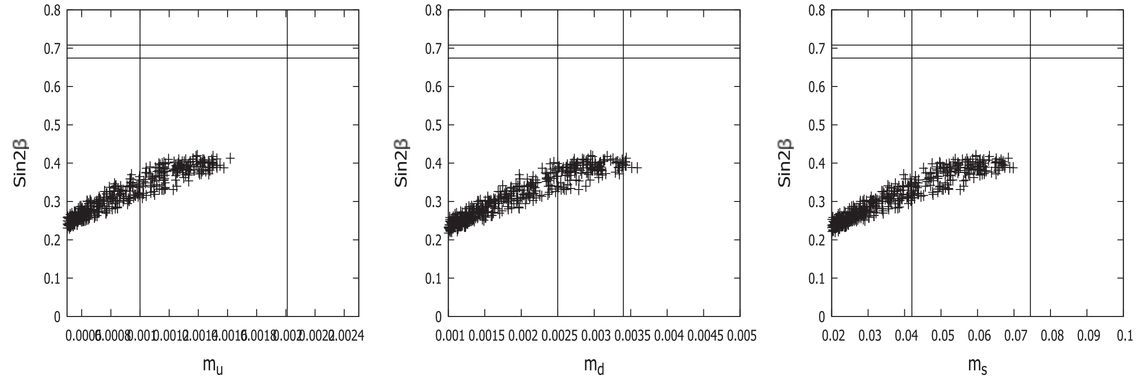

Fig. 4. Allowed range of $\sin 2\beta$ with respect to the light quark masses obtained by imposing both V_{us} and V_{ub} as constraints.

For the remaining 30 combinations of Category 1, wherein $M_{\rm U}$ and $M_{\rm D}$ can have different structures, i.e. of type $\mathrm{I}_a\mathrm{I}_b$, $\mathrm{I}_c\mathrm{I}_d$, etc., the above methodology can be repeated in order to check the viability of the various combinations. Interestingly, for these 30 combinations one finds that the CKM matrix so obtained does not have the usual structure wherein the diagonal elements are almost unity whereas the off-diagonal elements are much smaller than these. For example, considering the combination $\mathrm{I}_c\mathrm{I}_d$, i.e. $M_{\rm U}$ having structure I_c and $M_{\rm D}$ having structure I_d, the CKM matrix obtained, wherein we have used the hierarchy of quark masses and presented only the leading-order terms, is given by

$$V_{\rm CKM} = \begin{pmatrix} e^{-\iota\alpha_{\rm U}}\left(\frac{m_d}{m_s}\right)^{\frac{1}{2}} & -e^{-\iota\alpha_{\rm U}} & e^{-\iota\alpha_{\rm U}}\left(\frac{m_s}{m_b}\right)^{\frac{1}{2}} + e^{-\iota\beta_{\rm D}}\left(\frac{m_u}{m_c}\right)^{\frac{1}{2}} \\ e^{-\iota\beta_{\rm D}}\left(\frac{m_d}{m_b}\right)^{\frac{1}{2}} + e^{\iota(\alpha_{\rm D}+\beta_{\rm U})}\left(\frac{m_c}{m_t}\right)^{\frac{1}{2}} & -e^{-\iota\alpha_{\rm U}}\left(\frac{m_u}{m_c}\right)^{\frac{1}{2}} - e^{-\iota\beta_{\rm D}}\left(\frac{m_s}{m_b}\right)^{\frac{1}{2}} & -e^{-\iota\beta_{\rm D}} \\ e^{\iota(\alpha_{\rm D}+\beta_{\rm U})} & e^{\iota(\alpha_{\rm D}+\beta_{\rm U})}\left(\frac{m_d}{m_s}\right)^{\frac{1}{2}} & e^{-\iota\beta_{\rm D}}\left(\frac{m_c}{m_t}\right)^{\frac{1}{2}} \end{pmatrix}!.$$

From this matrix structure one can easily find out that the off-diagonal elements, e.g. V_{us}, V_{cb}, and V_{td}, are of the order of unity, whereas diagonal elements, V_{ud}, V_{cs}, and V_{tb}, are smaller than unity, which is in complete contrast to the usual structure of the CKM matix. This can also be seen by carrying out a numerical analysis for this case using the inputs mentioned in Eq. (18). The CKM matrix obtained comes out to be

$$V_{\rm CKM} = \begin{pmatrix} 0.2105\text{–}0.2234 & 0.9565\text{–}0.9723 & 0.0844\text{–}0.1973 \\ 0.0167\text{–}0.1021 & 0.0743\text{–}0.1968 & 0.9784\text{–}0.9948 \\ 0.9709\text{–}0.9763 & 0.2072\text{–}0.2308 & 0.0552\text{–}0.0644 \end{pmatrix}!. \quad (26)$$

This matrix clearly does not have the usual structure of the CKM matrix since the diagonal elements, V_{ud}, V_{cs}, and V_{tb}, are much smaller than unity whereas the off-diagonal elements, V_{us}, V_{cb}, and V_{td}, are approximately 1. Also, this matrix is again not at all compatible with the one given by PDG 2018. Therefore, one can conclude that the 30 combinations of Category 1 wherein $M_{\rm U}$ and $M_{\rm D}$ have different structures are also ruled out. It may be interesting to note that even if there are changes in the ranges of the light quark masses in the future, these 30 combinations would still be ruled out since the structure of the CKM matrices obtained for these are not the usual ones. This, therefore, has implications for models being built using the "top down" approach.

Considering the combinations pertaining to Categories 2 and 3, wherein $M_{\rm U}$ is a matrix from class I and $M_{\rm D}$ is a matrix from class II, and vice versa, respectively, interestingly, a detailed analysis of all these 36 combinations for each category show results similar to those for Category 1. In particular, if we consider the six cases $\mathrm{I}_a\mathrm{II}_a$, $\mathrm{I}_b\mathrm{II}_b$, etc. belonging to Category 2 or the six cases $\mathrm{II}_a\mathrm{I}_a$, $\mathrm{II}_b\mathrm{I}_b$, etc. pertaining to Category 3, constructing the corresponding CKM matrices, one finds that all six are same for each category, i.e. they have the same expressions for all nine CKM matrix elements. This can also be checked using permutation symmetry. Corresponding to the combination $\mathrm{I}_a\mathrm{II}_a$, the matrix arrived at by using the earlier mentioned inputs is

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.9744\text{–}0.9746 & 0.2238\text{–}0.2247 & 0.0025\text{–}0.0031 \\ 0.2234\text{–}0.2245 & 0.9724\text{–}0.9731 & 0.0561\text{–}0.0647 \\ 0.0122\text{–}0.0143 & 0.0547\text{–}0.0632 & 0.9979\text{–}0.9984 \end{pmatrix}, \tag{27}$$

whereas for the case $\mathrm{II}_a\mathrm{I}_a$, we get

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.9744\text{–}0.9746 & 0.2238\text{–}0.2248 & 0.0049\text{–}0.0080 \\ 0.2214\text{–}0.2230 & 0.9633\text{–}0.9670 & 0.1248\text{–}0.1508 \\ 0.0282\text{–}0.0364 & 0.1218\text{–}0.1472 & 0.9885\text{–}0.9922 \end{pmatrix}. \tag{28}$$

A look at these matrices reveals that although they have the usual CKM matrix structure, i.e. the diagonal elements are nearly unity whereas the off-diagonal elements are much smaller, one may note that none of them are compatible with the recent one given by PDG 2018, thereby ruling these out. Similar to the Category 1 results, if one considers the remaining 30 combinations belonging to each category, one finds that the CKM matrices now obtained do not have the usual structure. For example, for Category 2, considering the case $\mathrm{I}_c\mathrm{II}_d$, the leading-order CKM matrix obtained is

$$V_{\mathrm{CKM}} = \begin{pmatrix} \left(\frac{m_u}{m_c}\right)^{\frac{1}{2}} & -e^{-\iota\alpha_{\mathrm{U}}}\left(\frac{m_s}{m_b}\right)^{\frac{1}{2}} - e^{\iota\alpha_{\mathrm{D}}+\iota\beta_{\mathrm{U}}}\left(\frac{m_u}{m_t}\right)^{\frac{1}{2}} & e^{-\iota\alpha_{\mathrm{U}}} \\ -1 & \sim 0 & e^{-\iota\alpha_{\mathrm{U}}}\left(\frac{m_u}{m_c}\right)^{\frac{1}{2}} \\ \left(\frac{m_c}{m_t}\right)^{\frac{1}{2}} & e^{\iota\alpha_{\mathrm{D}}+\iota\beta_{\mathrm{U}}} & e^{\iota\alpha_{\mathrm{D}}+\iota\beta_{\mathrm{U}}}\left(\frac{m_s}{m_b}\right)^{\frac{1}{2}} \end{pmatrix}.$$

Similarly, for Category 3, for the combination $\mathrm{II}_c\mathrm{I}_d$, we obtain the following leading-order CKM matrix:

$$V_{\mathrm{CKM}} = \begin{pmatrix} e^{-\iota\alpha_{\mathrm{U}}}\left(\frac{m_d}{m_s}\right)^{\frac{1}{2}} & -e^{-\iota\alpha_{\mathrm{U}}} & e^{-\iota\alpha_{\mathrm{U}}}\left(\frac{m_s}{m_b}\right)^{\frac{1}{2}} + e^{-\iota\beta_{\mathrm{D}}}\left(\frac{m_u}{m_c}\right)^{\frac{1}{2}} \\ e^{-\iota\beta_{\mathrm{D}}}\left(\frac{m_d}{m_b}\right)^{\frac{1}{2}} & -e^{-\iota\beta_{\mathrm{D}}}\left(\frac{m_s}{m_b}\right)^{\frac{1}{2}} - e^{-\iota\alpha_{\mathrm{U}}}\left(\frac{m_u}{m_c}\right)^{\frac{1}{2}} & -e^{-\iota\beta_{\mathrm{D}}} \\ e^{\iota\alpha_{\mathrm{D}}} & e^{\iota\alpha_{\mathrm{D}}}\left(\frac{m_d}{m_s}\right)^{\frac{1}{2}} & \sim 0 \end{pmatrix}.$$

A look at these two matrices clearly shows that in each, one diagonal matrix element becomes nearly zero indicating that they do not have the usual CKM matrix structure, hence ruling out all such possibilities.

Coming to the combinations pertaining to Category 4, wherein both M_{U} and M_{D} are matrices mentioned in class II, detailed analysis of all these cases shows that unlike the matrices considered in Category 1, the CKM matrices obtained for the six possibilities with both the mass matrices having the same structure, i.e. of type $\mathrm{II}_a\mathrm{II}_a$, $\mathrm{II}_b\mathrm{II}_b$, etc., do not have the usual CKM matrix structure. For example, for the case $\mathrm{II}_a\mathrm{II}_a$, the CKM matrix is given by

$$\begin{pmatrix} e^{\iota(\alpha_{\mathrm{D}}-\alpha_{\mathrm{U}})}\left(\frac{m_s}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_c}{m_c+m_u}\right)^{\frac{1}{2}} + \left(\frac{m_d}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_u}{m_c+m_u}\right)^{\frac{1}{2}} & e^{\iota(\alpha_{\mathrm{D}}-\alpha_{\mathrm{U}})}\left(\frac{m_d}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_c}{m_c+m_u}\right)^{\frac{1}{2}} - \left(\frac{m_s}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_u}{m_c+m_u}\right)^{\frac{1}{2}} & 0 \\ -\left(\frac{m_d}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_c}{m_c+m_u}\right)^{\frac{1}{2}} + e^{\iota(\alpha_{\mathrm{D}}-\alpha_{\mathrm{U}})}\left(\frac{m_s}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_u}{m_c+m_u}\right)^{\frac{1}{2}} & \left(\frac{m_s}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_c}{m_c+m_u}\right)^{\frac{1}{2}} + e^{\iota(\alpha_{\mathrm{D}}-\alpha_{\mathrm{U}})}\left(\frac{m_d}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_u}{m_c+m_u}\right)^{\frac{1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

As is evident, this matrix is clearly ruled out since the elements V_{ub}, V_{cb}, V_{td}, and V_{ts} come out to be 0, whereas the value of the element V_{tb} is 1. The other five such combinations yield similar matrices. Further, all the remaining 30 combinations with $M_{\rm U}$ and $M_{\rm D}$ not having the same structure result in CKM matrices with the element V_{tb} being 0, thereby ruling out all of these:

$$\begin{pmatrix} \left(\frac{m_d}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_c}{m_c+m_u}\right)^{\frac{1}{2}} & -\left(\frac{m_s}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_c}{m_c+m_u}\right)^{\frac{1}{2}} & e^{\iota(\alpha_{\rm D}-\alpha_{\rm U})}\left(\frac{m_u}{m_c+m_u}\right)^{\frac{1}{2}} \\ \left(\frac{m_d}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_u}{m_c+m_u}\right)^{\frac{1}{2}} & -\left(\frac{m_s}{m_d+m_s}\right)^{\frac{1}{2}}\left(\frac{m_u}{m_c+m_u}\right)^{\frac{1}{2}} & -e^{\iota(\alpha_{\rm D}-\alpha_{\rm U})}\left(\frac{m_c}{m_c+{\rm m}_u}\right)^{\frac{1}{2}} \\ \left(\frac{m_s}{m_d+m_s}\right)^{\frac{1}{2}} & \left(\frac{m_d}{m_d+m_s}\right)^{\frac{1}{2}} & 0 \end{pmatrix}.$$

Finally, we come to the last possibility, wherein one can consider $M_{\rm U}$ and/or $M_{\rm D}$ both having structure S_{19}, mentioned earlier while discussing all possible texture-3 zero mass matrices. It is trivial to note that the case wherein both $M_{\rm U}$ and $M_{\rm D}$ have structure S_{19} is ruled out as the corresponding CKM matrix obtained would be a unit matrix. Next, we consider $M_{\rm U}$ being S_{19}, whereas $M_{\rm D}$ belongs to Class I. Pertaining to this combination, out of the six possible cases, if $M_{\rm D}$ is considered to have structure I_a, one arrives at a CKM matrix having the usual structure,

$$V_{\rm CKM} = \begin{pmatrix} 0.9748\text{–}0.9763 & 0.2160\text{–}0.2227 & 0.0004\text{–}0.0008 \\ 0.2134\text{–}0.2209 & 0.9644\text{–}0.9682 & 0.1243\text{–}0.1518 \\ 0.0281\text{–}0.0336 & 0.1211\text{–}0.1480 & 0.9884\text{–}0.9922 \end{pmatrix}, \tag{29}$$

though this matrix is ruled out by comparing it with the one given by PDG 2018. For the remaining five cases, the structure of the CKM matrix is not the usual one, hence ruling these out. Further, one can also consider $M_{\rm D}$ having the form S_{19}, whereas $M_{\rm U}$ can have any of the six structures belonging to Class I. Out of all the six CKM matrices obtained pertaining to these combinations, only one case, wherein $M_{\rm U}$ has the structure I_a, yields a CKM matrix having the usual structure:

$$V_{\rm CKM} = \begin{pmatrix} 0.9985\text{–}0.9992 & 0.0387\text{–}0.0537 & 0.0024\text{–}0.0031 \\ 0.0388\text{–}0.0539 & 0.9967\text{–}0.9974 & 0.0560\text{–}0.0647 \\ 0.0000\text{–}0.0001 & 0.0561\text{–}0.0647 & 0.9979\text{–}0.9984 \end{pmatrix}, \tag{30}$$

but these matrix elements do not lie within the range given by PDG 2018, and hence this is also ruled out. For the other five cases, the CKM matrices arrived at do not have the usual structure.

One can also consider the possibilities wherein either $M_{\rm U}$ or $M_{\rm D}$ has structure S_{19}, whereas correspondingly $M_{\rm D}$ or $M_{\rm U}$ respectively belongs to Class II. It can be easily checked that for all 12 such cases, the CKM matrices thus constructed are found to have four vanishing elements, hence ruling out all these possibilities.

To summarize, in view of the refinements in the measurements of small quark masses m_u, m_d, and m_s, as well as in the CKM matrix elements, we have carried out an extensive analysis of all possible quark mass matrices having minimal texture, implying texture-6 zero quark mass matrices. In all, we have examined 169 possible texture-6 zero combinations; interestingly, many of these combinations can be ruled out analytically. For the remainder, corresponding CKM matrices have been constructed and compared with the latest mixing data. Again, one finds that all these possibilities are excluded by the present quark mixing data. These conclusions remain valid even if, in future, there are changes in the ranges of the light quark masses or if there are small perturbations in the structures of these texture-6 zero mass matrices. In conclusion, all the 169 possible quark mass matrices with minimal texture are ruled out in the present era of precision measurements, with important implications for model building.

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