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AdS$_4$ black holes with nonlinear source in rainbow gravity

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ABSTRACT

We explore new four-dimensional black holes, with the Einstein-Λ theory coupled to nonlinear electrodynamics, in an energy-dependent spacetime. By considering the power-law model of nonlinear electrodynamics and solving the coupled field equations, with properly fixing the parameters of the theory, we obtain two new classes of nonlinearly charged black holes with anti-de Sitter (AdS) asymptote. With the aim of studying the thermodynamic properties of the novel AdS black holes, we calculate the conserved and thermodynamic quantities such as black hole mass, electric charge, electric potential, temperature, and entropy in the presence of rainbow functions. Although some of these quantities are affected by the rainbow functions, interestingly, they satisfy the first law of black hole thermodynamics in its standard form. Based on the canonical ensemble method and by calculating the black hole heat capacity, we analyze the black hole local stability or thermodynamic phase transitions. The horizon radius of those black holes which experience type one or type two phase transition as well as the conditions under which the black holes remain stable are determined. Finally, via consideration of black hole thermal fluctuations, we examine the impacts of quantum gravitational corrections on the thermodynamic properties of our new AdS rainbow black holes.

1. Introduction

Combining the gravity with quantum mechanics to establish the quantum theory of gravity is still one of the most important problems in the context of theoretical physics [1–3]. Gravity's rainbow, in which the quantum gravitational effects are taken into account, is an attempt in this direction. This theory is constructed out by extending the doubly special relativity to the curved spacetimes, and can be considered as the Planck-scale generalization of the usual gravity theory [4,5]. Besides, doubly special relativity is the deformed version of usual special relativity, in which the usual dispersion relation is promoted to the well-known modified dispersion relation. Although, the modified dispersion relation is expected to play an important role in constructing the quantum theory of gravity, it is no longer Lorentz invariant. Doubly or deformed special relativity, which is accomplished based on a class of nonlinear Lorentz transformations, preserves Lorentz invariance of the modified dispersion relation. In this theory, light speed and Planck energy are two invariant quantities which determine the upper bounds of the speed and energy that a particle can attain, respectively [6–9].

However, doubly special relativity, as the Planck-scale formalism of the special relativity, has now been extended to the curved space times, and it is named conventionally as doubly general relativity or gravity's rainbow. In rainbow gravity, the geometry of space time depends on the energy of the test particle, which probes the geometry. Thus, the geometry of space time seems different for the particles having different amounts of energy. Therefore, instead of a single metric, there is a family of energy-dependent metrics. This is why the doubly general relativity is named as gravity's rainbow [10,11].

On the other hand, Maxwell's classical electromagnetic theory, as one of the most successful theories, seems to have some shortcomings. The related failures are attempted to be addressed by modifying the original action to that of the so-called nonlinear electrodynamics. Among the various theories of nonlinear electrodynamics, which are nonlinear functions of Maxwell invariant $\mathcal{F} = F^{\alpha\beta}F_{\alpha\beta}$, the Born-Infeld, power-law, exponential and logarithmic models are of most interest [12–16]. All of the models of nonlinear electrodynamics reduce to Maxwell's electrodynamics as a special case [16]. Indeed, nonlinear theories of electrodynamics are more suitable when the electromagnetic fields are of high strength. They contain higher powers of $\mathcal{F}$, which are believed that indicate the photon-photon interactions. For

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the case of enough weak electromagnetic fields the photon self-interactions are negligible, and Maxwell's/or linear electromagnetic theory can be used successfully [17,18]. One of the challenges of the linear or Maxwell's theory of electrodynamics is that it violates the conformal invariant symmetry in the spacetimes with the dimensions other than four. The power-law theory of nonlinear electrodynamics is the only model of proposed nonlinear electrodynamics that preserves its conformal invariance in all spacetimes with arbitrary dimensions [19–21].

Although, the various models of nonlinear electrodynamics are motivated for solving the problems of the appearance of the infinite field and self-energy for the pointlike charged particles in Maxwell's theory, this issue has now provided an interesting subject area for studying the charged exact solutions of the alternative theories of gravity. Now, there are a variety of novel and interesting works in which Maxwell's theory is promoted to nonlinear electrodynamics. Motivated by the interesting results provided through the study of physics in the energy-dependent spacetimes such as black hole remnant [22,23], nonsingular universe [24], this work is devoted to the study of nonlinearly charged AdS black holes in the presence of rainbow functions. Three-dimensional exact black hole solutions have been obtained, and the related thermodynamic properties have been studied in my last work [25]. Here, this idea is extended to the case of four-dimensional AdS spacetimes. The main object of this paper is to obtain new exact solutions of the four-dimensional Einstein-nonlinear electrodynamics with the cosmological constant in energy-dependent spherically symmetric spacetime. Also, the study of the thermodynamic properties and especially thermodynamic stability or phase transition of the black holes are of interest. Besides, quantum gravitational effects on the black hole thermodynamics are examined by consideration of the black hole thermal fluctuations.

The paper is outlined as follows: In Sec.2, by varying the suitable action of four-dimensional Einstein-Λ gravity theory, in the presence of the power-law nonlinear electrodynamics, the coupled field equations have been solved in an energy-dependent and spherically symmetric geometry. As a result, two novel classes of nonlinearly charged black holes have been found with the AdS asymptotic behavior. Sec.3 is devoted to the study of thermodynamic properties of the black hole solutions we just obtained. The temperature, entropy, electric potential, conserved mass, and electric charge of the new AdS black holes have been calculated in the presence of rainbow functions. It has been shown that, although some of these quantities get modified by the rainbow functions, the first law of black hole thermodynamics is still valid for either of the AdS black hole solutions. Then, from the viewpoint of the canonical ensemble method and regarding the signature of the black hole heat capacity, a thermodynamic stability analysis has been performed. The location of the type one and type two phase transition points and the conditions under which the black holes are stable have been characterized. Finally, the corrections arisen from the consideration of quantum gravity effects have been investigated by studying the black hole thermal fluctuations. Sec.4 is dedicated to summarizing and discussing the results.

2. AdS rainbow black hole solutions

The action of nonlinearly charged four-dimensional black holes in the Einstein-Λ gravity theory is written as [26]

$$I=-\frac{1}{16\pi}\int\sqrt{-g}\,[R-2\Lambda+\mathcal{L}(\mathcal{F})]\,d^4x. \tag{2.1}$$

Note that R is the Ricci scalar, Λ is the cosmological constant which is written as $\Lambda=-3\ell^{-2}$ in AdS spacetimes. The last term denotes the electromagnetic Lagrangian density as a function of Maxwell's invariant $\mathcal{F}=F^{\mu\nu}F_{\mu\nu}$ with $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and A_μ is the electromagnetic potential. Here, we are interested on the power-law nonlinear electrodynamics in the following form [20,26]

$$\mathcal{L}(\mathcal{F})=(-\mathcal{F})^p, \tag{2.2}$$

which states the electromagnetic Lagrangian as the power of Maxwell invariant, with p, as the power, sometimes is named as the nonlinearity parameter. By varying the action (2.1) with respect to gravitational field we get the Einstein's field equations as

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\Lambda g_{\mu\nu}=\frac{1}{2}g_{\mu\nu}(-\mathcal{F})^p+2p(-\mathcal{F})^{p-1}F_{\mu\alpha}F_\nu^{\ \alpha}. \tag{2.3}$$

Also, varying the action (2.1) with respect to the electromagnetic field one obtains

$$\partial_\mu\left[\sqrt{-g}\mathcal{L}'(\mathcal{F})F^{\mu\nu}\right]=0, \tag{2.4}$$

where prime means derivative with respect to the argument. Note that F_{tr} is the only non-vanishing element of the Faraday's tensor. Now we explore the solutions of the gravitational and electromagnetic field equations as given in Eqs. (2.3) and (2.4). In this regard, we start with a four-dimensional static and spherically symmetric energy-dependent line element as an ansatz. It takes the following general form [27]

$$ds^2=-\frac{V(r)}{f^2(\varepsilon)}dt^2+\frac{1}{g^2(\varepsilon)}\left[\frac{dr^2}{V(r)}+r^2(d\theta^2+\sin^2\theta d\varphi^2)\right]. \tag{2.5}$$

Here, $f(\varepsilon)$ and $g(\varepsilon)$ are known as the so-called temporal and spatial rainbow functions, respectively. The argument ε is a dimensionless quantity defined by $\varepsilon=E/E_P$, in which E_P is the Planck-scale energy and E denotes the test particle's energy. They are restricted to fulfill following requirements

$$\lim_{\varepsilon\to0}f(\varepsilon)=1,\quad \text{and}\quad \lim_{\varepsilon\to0}g(\varepsilon)=1. \tag{2.6}$$

With the help of above conditions one can recover the infrared limit of the results letting $\varepsilon\to0$. There are some proposed forms for $f(\varepsilon)$ and $g(\varepsilon)$ as functions of ε, among them are [28,29]

$$f(\varepsilon)=1, \quad \text{and} \quad g(\varepsilon)=\sqrt{1-\eta\varepsilon^n}, \tag{2.7}$$

$$f(\varepsilon)=\frac{e^{\xi\varepsilon}-1}{\xi\varepsilon}, \quad \text{and} \quad g(\varepsilon)=1, \tag{2.8}$$

$$f(\varepsilon)=g(\varepsilon)=\frac{1}{1-\lambda\varepsilon}, \tag{2.9}$$

where, the dimensionless constants η, n, ξ and λ, are named as the rainbow parameters. The order of magnitudes of $f(\varepsilon)$ and $g(\varepsilon)$ are approximated to or slightly different from unity.

Now, by considering the A_μ as a function of r, we have $F_{tr}=-\partial_r A_t(r)$. As the result one can show that

$$\mathcal{F}=-2f^2(\varepsilon)g^2(\varepsilon)(F_{tr}(r))^2=-2f^2(\varepsilon)g^2(\varepsilon)(A_t'(r))^2, \tag{2.10}$$

and making use of Eqs. (2.5) and (2.6) in the electromagnetic field equations (2.4) we have

$$r(A_t'(r))^{2p-2}\left[2A_t'(r)+(2p-1)rA_t''(r)\right]=0, \quad \text{for} \quad p\neq\frac{1}{2}. \tag{2.11}$$

This differential equation has a full solution which can be identified as

$$A_t(r)=\begin{cases} -q\,\ln\left(\frac{r}{\ell}\right) & \text{for} \quad p=\frac{3}{2}, \\ -q\left(\frac{2p-1}{2p-3}\right)r^{\frac{2p-3}{2p-1}} & \text{for} \quad \frac{1}{2}<p<\frac{3}{2}. \end{cases} \tag{2.12}$$

Having the temporal component of the vector potential the nonzero component of electromagnetic tensor is obtained as

$$F_{tr}=\begin{cases} \frac{q}{r} & \text{for} \quad p=\frac{3}{2}, \\ q\,r^{\frac{-2}{2p-1}} & \text{for} \quad \frac{1}{2}<p<\frac{3}{2}. \end{cases} \tag{2.13}$$

It must be noted that q as the integration constant is related to black hole charge. Also, F_{tr} recovers the corresponding quantity for the Reissner-Nordström-anti-de Sitter (R-N-AdS) black holes if the value $p=1$ is chosen. It is worth mentioning that the restriction $\frac{1}{2}<p\leq\frac{3}{2}$ makes $A_t(r)$ to converge at infinity, and to be physically reasonable.

The other unknown function which must be calculated is the metric function $V(r)$. By use of Eqs. (2.3) and (2.5) one leads to the following differential equations for different components of the gravitational field equation

$$e_{tt}=e_{rr}=\begin{cases} g^2(\varepsilon)\left(V''(r)+\frac{2V'(r)}{r}\right)+2\Lambda-2\sqrt{2}\frac{q_\epsilon^3}{r^3}=0 & \text{for} \quad p=\frac{3}{2}, \\ g^2(\varepsilon)\left(V''(r)+\frac{2V'(r)}{r}\right)+2\Lambda-2^p q_\epsilon^{2p}r^{-\frac{4p}{2p-1}}=0, & \text{for} \quad \frac{1}{2}<p<\frac{3}{2}, \end{cases} \tag{2.14}$$

$$e_{\theta\theta}=e_{\varphi\varphi}=\begin{cases} 2g^2(\varepsilon)\left(\frac{V'(r)}{r}+\frac{V'(r)-1}{r^2}\right)+2\Lambda+4\sqrt{2}\frac{q_\epsilon^3}{r^3}=0, & \text{for} \quad p=\frac{3}{2}, \\ 2g^2(\varepsilon)\left(\frac{V'(r)}{r}+\frac{V'(r)-1}{r^2}\right)+2\Lambda+(2p-1)2^p q_\epsilon^{2p}r^{-\frac{4p}{2p-1}}=0, & \text{for} \quad \frac{1}{2}<p<\frac{3}{2}. \end{cases} \tag{2.15}$$

Here, e_{tt}, e_{rr}, $e_{\theta\theta}$ and $e_{\varphi\varphi}$ represent the tt, rr, $\theta\theta$ and $\varphi\varphi$ components of the gravitational field equation (2.3). Also, we have used the definition $q_\epsilon=f(\varepsilon)g(\varepsilon)q$. Now, we solve the differential equations (2.14) and (2.15) for obtaining the only unknown function $V(r)$. It seems that the number of equations is not consistent with that of unknown quantities. For more investigation, by use of Eqs. (2.14) and (2.15), it is easily shown that

$$e_{tt}=\left(1+\frac{r}{2}\frac{d}{dr}\right)e_{\theta\theta}, \quad \text{for} \quad \frac{1}{2}<p\leq\frac{3}{2}, \tag{2.16}$$

from which one can conclude that differential equations (2.14) and (2.15) are not independent. Therefore, we can solve the first order differential equation (2.14) and ensure that the solution fulfils the second order differential equation (2.15).

The solution to the differential equations (2.14) and (2.15) can be obtained easily in the following forms

$$V(r)=\begin{cases} 1-\frac{m}{r}-\frac{\Lambda r^2}{3g^2(\varepsilon)}-\frac{2\sqrt{2}q_\varepsilon^3}{rg^2(\varepsilon)}\ln\left(\frac{r}{\ell}\right), & \text{for} \quad p=\frac{3}{2}, \\ 1-\frac{m}{r}-\frac{\Lambda r^2}{3g^2(\varepsilon)}-\frac{(2p-1)^2(2)^{p-1}q_\varepsilon^{2p}}{(2p-3)g^2(\varepsilon)}r^{\frac{-2}{2p-1}}, & \text{for} \quad \frac{1}{2}<p<\frac{3}{2} \end{cases} \tag{2.17}$$

where m is another integration constant. In the following, we show that it is related to the black hole mass. We must note that in the case of $p=1$ the power-law nonlinear electrodynamics (2.2) reduces to the Maxwell's electromagnetic theory and the metric function (2.17) is written as

$$V(r)=1-\frac{m}{r}-\frac{\Lambda}{3g^2(\varepsilon)}r^2+\frac{f^2(\varepsilon)q^2}{r^2}, \quad \text{for} \quad p=1, \tag{2.18}$$

which is nothing but the metric function of the R-N-A(dS) black holes in gravity's rainbow [30].

In order to consider the asymptotic behavior of the solutions, we focus on the metric function $V(r)$ in the limiting case $r\to\infty$. It is clear that the p dependent power of r (i.e. $\frac{-2}{2p-1}$) is negative for $p>\frac{1}{2}$, positive for $p<\frac{1}{2}$ and $\lim_{r\to\infty}\frac{\ln r}{r}=0$. Thus, noting Eq. (2.17) we have

$$\lim_{r\to\infty} V(r) = 1 - \frac{\Lambda}{3g^2(\varepsilon)}r^2 \quad \text{for} \quad \frac{1}{2} < p \leq \frac{3}{2}. \tag{2.19}$$

Therefore, the metric function $V(r)$ describes an asymptotically AdS spacetime for the allowed p values with the cosmological constant is affected by the spacial rainbow function. Also, for the case $p = 0$, the spacetime is described by the following metric function

$$V(r) = 1 - \frac{m}{r} - \frac{1}{3}\left[\frac{\Lambda}{g^2(\varepsilon)} - \frac{1}{2g^2(\varepsilon)}\right]r^2, \tag{2.20}$$

which is a pure AdS one with the following effective cosmological constant

$$\Lambda_{eff} = \frac{2\Lambda - 1}{2g^2(\varepsilon)}. \tag{2.21}$$

Now, we discuss the curvature singularities related to the spacetime geometry described by the metric function $V(r)$. The Ricci and Kretschmann scalars are two important ones which can help us to distinguish the coordinate and intrinsic singularities. As a matter of calculation One can show that the Ricci and Kretschmann scalars take the following explicit forms [31]

$$R = \begin{cases} 4\Lambda + \frac{2\sqrt{2}}{r^3}q_\varepsilon^3, & \text{for} \quad p = \frac{3}{2}, \\ 4\Lambda + (p-1)2^{p+1}q_\varepsilon^{2p}r^{\frac{-4p}{2p-1}}, & \text{for} \quad \frac{1}{2} < p < \frac{3}{2} \end{cases} \tag{2.22}$$

$$R^{\mu\nu\rho\lambda}R_{\mu\nu\rho\lambda} = \begin{cases} \frac{8q_\varepsilon^3}{r^6}\left\{13q_\varepsilon^3 - 5\sqrt{2}m + \ln\left(\frac{r}{\ell}\right)\left[6q_\varepsilon^3\ln\left(\frac{r}{\ell}\right) + 3\sqrt{2}m - 10q_\varepsilon^3\right]\right\} + \frac{8}{3}\Lambda^2 + \frac{12m^2g^4(\varepsilon)}{r^6} + \frac{8\sqrt{2}\Lambda}{r^3}q_\varepsilon^3, & \text{for} \quad p = \frac{3}{2}, \\ \frac{8\Lambda^2}{3} + \frac{12m^2g^4(\varepsilon)}{r^6} + A_0 r^{-\frac{2}{2p-1}-4} + A_1 r^{-\frac{4p}{2p-1}} + A_2 r^{-\frac{8p}{2p-1}} - A_0 r^{-\frac{4p}{2p-1}-2} + A_3 r^{-\frac{4p}{2p-1}-3}, & \text{for} \quad \frac{1}{2} < p < \frac{3}{2}, \end{cases} \tag{2.23}$$

where

$$A_0 = \frac{(2p-1)^2 2^{p+2}}{2p-3}g^2(\varepsilon)q_\varepsilon^{2p}, \qquad A_1 = \frac{\Lambda}{3}(p-1)2^{p+3}q_\varepsilon^{2p}, \tag{2.24}$$

$$A_2 = \frac{2^{2p+1}q_\varepsilon^{4p}}{(2p-3)^2}(8p^4 - 16p^3 + 22p^2 - 10p + 3), \qquad A_3 = \frac{mp(2p+1)2^{p+3}}{2p-3}g^2(\varepsilon)q_\varepsilon^{2p}. \tag{2.25}$$

Note that the Ricci and Kretschmann scalars, obtained here, reduce to those of R-N-AdS black hole in the special case $p = 1$ by taking the infrared limit.

The results show that, the divergent point of the Ricci and Kretschmann scalars is located at the origin $r = 0$. They lead to finite value for finite values of r. Therefore, $r = 0$ is an essential singularity for the asymptotically AdS black holes found here. We have plotted the metric function $V(r)$ versus r in Fig. 1. The plots show that our novel AdS exact solutions with permitted p-values in the range $\frac{1}{2} < p \leq \frac{3}{2}$ can present black holes with two horizons, extreme black holes and naked singularity black holes for different values of parameters p, $f(\varepsilon)$ and $g(\varepsilon)$.

Regarding the existence of the event horizons (Fig. 1) and appearance of the curvature singularities [Eqs. (2.22) and (2.23)] our exact solutions, in the presence of nonlinear electrodynamics, are really AdS black holes. In what comes later, we study the thermodynamic properties of these novel black hole solutions.

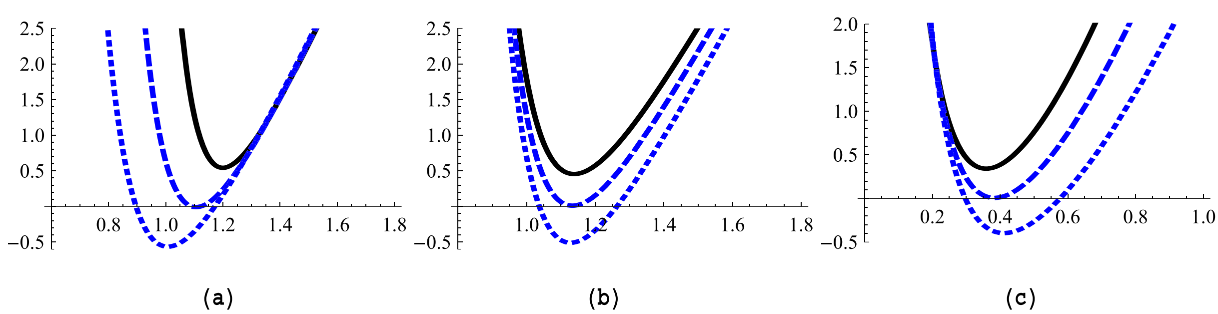

Fig. 1. $V(r)$ versus r for $\Lambda = -3$, $M = 3$, $Q = 2$, $\ell = 1$ Eqs. (2.13) and (2.14). (a) $f(\varepsilon) = 1.2$, $g(\varepsilon) = 0.7$ and $p = 0.58$ (continues), 0.61 (dashed), 0.65 (dotted). (b) $p = 0.6$, $g(\varepsilon) = 0.7$ and $f(\varepsilon) = 1.12$ (continues), 1.24 (dashed), 1.38 (dotted). (c) $f(\varepsilon) = 0.65$, $p = \frac{3}{2}$ and $g(\varepsilon) = 0.41$ (continues), 0.485 (dashed), 0.52 (dotted).

3. Thermodynamics

Here, we study the thermodynamic properties of the novel charged AdS black hole solutions found in the previous section. Through the calculation of the conserved and thermodynamic quantities, we check the validity of the first law of black hole thermodynamics. Next, we explore the black hole stability or phase transitions making use of the canonical ensemble method. By calculating the black hole heat capacity, with the black hole charge as a constant, we determine the location of the type one and type two phase transition points. Also, we find the conditions under which the black holes remain stable for either of the $p = \frac{3}{2}$ and $\frac{1}{2} < p < \frac{3}{2}$ cases. Then, by considering the effects of thermal fluctuations, we investigate the thermodynamic properties and analyze the thermodynamic stability or phase transition of the new AdS black holes.

3.1. Thermodynamical first law

Now, we seek for the satisfaction of the first law of thermodynamics for our four-dimensional AdS black hole solutions. So, we need to calculate the black hole conserved and thermodynamic quantities. Let's start with the calculation of the black hole electric charge Q, as a conserved quantity, in terms of the integration constant q. Making use of the Gauss's electric law, the electric charge can be found by calculating the flux of the electric field at infinity (i.e. $r \to \infty$), that is [19,21]

$$Q = \frac{p}{4\pi} \int \frac{r^2}{f(\varepsilon) g^3(\varepsilon)} (-\mathcal{F})^{p-1} F_{\mu\nu} d\Omega. \tag{3.1}$$

Making use of Eq. (2.13) in Eq. (3.1), after some simple calculations, we arrive at

$$Q = \begin{cases} \frac{3}{\sqrt{2} g^2(\epsilon)} q_\varepsilon^2 & \text{for} \quad p = \frac{3}{2}, \\ \frac{p 2^{p-1}}{g^2(\epsilon)} q_\varepsilon^{2p-1} & \text{for} \quad \frac{1}{2} < p < \frac{3}{2}, \end{cases} \tag{3.2}$$

which is compatible with the result of ref. [30]. Also, it is just the charge of the R-N-AdS black holes provided that the conditions $p = 1$ and $f(\epsilon) = 1 = g(\epsilon)$ are imposed. Besides, noting Eq. (3.2) one can argue that the black hole total charge gets modified in the presence of gravity's rainbow.

The other conserved quantity to be calculated is the black hole mass M, which is related to the other integration constant m. Noting the fact that the spacetime under consideration is asymptotically AdS, the counterterm method can be used [32,33] for calculating the conserved mass. It is matter of calculation to show that [34]

$$M = \frac{m}{2 f(\varepsilon) g(\varepsilon)}. \tag{3.3}$$

One can obtain the Hawking temperature, associated with the black hole horizon $r = r_+$ which is the real root(s) of $V(r_+) = 0$, in terms of the black hole surface gravity $\kappa = \sqrt{(\nabla_\mu \chi_\nu)(\nabla^\mu \chi^\nu)}$. Taking $\chi^\mu = (-1, 0, 0, 0)$ one is able to show that $\kappa = \frac{g(\varepsilon)}{2 f(\varepsilon)} V'(r = r_+)$. Therefore

$$T = \frac{\kappa}{2\pi} = \frac{g(\varepsilon)}{4\pi f(\varepsilon)} \frac{d}{dr} V(r)|_{r=r_+} = \frac{g(\varepsilon)}{4\pi f(\varepsilon) r_+} \begin{cases} 1 - \frac{\Lambda r_+^2}{g^2(\varepsilon)} - \frac{2\sqrt{2} q_\varepsilon^3}{g^2(\varepsilon) r_+} & \text{for} \quad p = \frac{3}{2}, \\ 1 - \frac{\Lambda r_+^2}{g^2(\varepsilon)} - \frac{2p-1}{g^2(\varepsilon)} \frac{2^{p-1} q_\varepsilon^{2p}}{r_+^{\frac{2}{2p-1}}} & \text{for} \quad \frac{1}{2} < p < \frac{3}{2}, \end{cases} \tag{3.4}$$

which reduces to the temperature of R-N-AdS rainbow black holes in the case $p = 1$. Note that the relation $V(r_+) = 0$ has been used for eliminating the mass parameter m from Eq. (3.4). In addition, it is worth mentioning that the extreme black holes (i.e. black holes with zero temperature) occur if q and r_+ be chosen such that $T(r_{ext}, q_{ext}) = 0$. With this issue in mind, making use of Eq. (3.4) we have

$$q_{ext}^{2p} = \frac{g^2(\varepsilon) - \Lambda r_+^2}{2^{p-1}(2p-1)[f(\varepsilon) g(\varepsilon)]^{2p}} r_{ext}^{\frac{2}{2p-1}} \quad \text{for} \quad \frac{1}{2} < p \leq \frac{3}{2}. \tag{3.5}$$

As shown in Fig. 2, our solutions produce extreme black holes if $r_+ = r_{ext}$, physical black holes with positive temperature for $r_+ >_{ext}$ and unphysical black holes, having negative temperature, provided that $r_+ > r_{ext}$.

Next, we calculate the entropy of the black hole. It can be obtained by use of the Hawking-Bekenstein entropy-area law, that is

$$S = \frac{A}{4} = \frac{\pi r_+^2}{g^2(\varepsilon)}. \tag{3.6}$$

Also, the black hole's electric potential, relative to a reference point located at infinite distance from the black hole horizon, can be obtained by use of the following standard relation [20,35,36]

$$\phi(r_+) = A_\mu \chi^\mu|_{\text{reference}} - A_\mu \chi^\mu|_{r=r_+}. \tag{3.7}$$

Thus, noting Eq. (2.12) we have

$$U(r_+) = \begin{cases} -q \ln\left(\frac{r_+}{\ell}\right) & \text{for} \quad p = \frac{3}{2}, \\ -q \left(\frac{2p-1}{2p-3}\right) (r_+)^{\frac{2p-3}{2p-1}} & \text{for} \quad \frac{1}{2} < p < \frac{3}{2}, \end{cases} \tag{3.8}$$

which is consistent with the electric potential of the R-N-AdS black hole in the case $p = 1$.

Here, we check the validity of the first law of black hole thermodynamics for the conserved and thermodynamic quantities obtained. To do so, we obtain the black hole mass M as a function of black hole entropy S and charge Q as the complete set of extensive quantities. We use Eqs. (3.2), (3.3) and (3.6) with the help of relation $V(r_+) = 0$ and find the following Smarr-type mass formula

$$M(Q, S) = \begin{cases} \frac{1}{2f(\varepsilon)} \left(\frac{S}{\pi}\right)^{\frac{1}{2}} \left\{1 - \frac{\Lambda S}{3\pi} - 4\sqrt[4]{2} \left(\frac{Q}{3}\right)^{\frac{3}{2}} \left(\frac{\pi}{S}\right)^{\frac{1}{2}} \ln\left[\frac{g(\varepsilon)}{\ell} \left(\frac{S}{\pi}\right)^{\frac{1}{2}}\right]\right\} & \text{for} \quad p = \frac{3}{2}, \\ \frac{1}{2f(\varepsilon)} \left(\frac{S}{\pi}\right)^{\frac{1}{2}} \left\{1 - \frac{\Lambda S}{3\pi} - \frac{(2p-1)^2}{2p-3} \left(\frac{\pi}{S}\right)^{\frac{-1}{2p-1}} \left(\frac{Q}{p\, 2^{p-1}}\right)^{\frac{2p}{2p-1}}\right\} & \text{for} \quad \frac{1}{2} < p < \frac{3}{2}. \end{cases} \tag{3.9}$$

By treating Q and S as a complete set of extensive parameters for the mass $M(S,Q)$ and define the intensive parameters conjugate to them as temperature T and electric potential U, we obtain

$$T=\left(\frac{\partial M}{\partial S}\right)_Q, \qquad U=\left(\frac{\partial M}{\partial Q}\right)_S, \tag{3.10}$$

which are compatible with the temperature and electric potential given in Eqs. (3.4) and (3.8). It means that the thermodynamics quantities, we obtained in this section, satisfy the first law of black hole thermodynamics in the form

$$dM=TdS+UdQ, \tag{3.11}$$

for either of the new black hole solutions we just found.

3.2. Thermal stability analysis

At this stage, we study the local stability or phase transitions of the introduced black holes in the canonical ensemble method. It is well-known that black hole, as the thermodynamic systems, are locally stable if their heat capacity is positive. An unstable black hole experiences the thermodynamic phase transition to be stabilized. The phase transition points are the vanishing and divergent points of the black hole heat capacity. The vanishing points (real roots) of the heat capacity are named conventionally as the type one phase transition points. The points at which the black hole heat capacity diverges are known as the type two phase transition points. The black hole heat capacity, with constant black hole charge, is defined as $\mathcal{H}_Q=T\,(\partial S/\partial T)_Q=T/\left(\partial^2 M/\partial S^2\right)_Q$ [37,38]. Now, it is a matter of calculation to show that

$$\mathcal{H}_Q=-\frac{2\pi r_+^2}{g^2(\varepsilon)}\begin{cases}\dfrac{g^2(\varepsilon)r_+-\Lambda r_+^3-2\sqrt{2}q_\varepsilon^3}{g^2(\varepsilon)r_++\Lambda r_+^3-4\sqrt{2}q_\varepsilon^3}, & \text{for}\quad p=\frac{3}{2},\\[2ex] \dfrac{g^2(\varepsilon)r_+^{\frac{2}{2p-1}}-\Lambda r_+^{\frac{4p}{2p-1}}-(2p-1)2^{p-1}q_\varepsilon^{2p}}{g^2(\varepsilon)r_+^{\frac{2}{2p-1}}+\Lambda r_+^{\frac{4p}{2p-1}}-(2p+1)2^{p-1}q_\varepsilon^{2p}}, & \text{for}\quad \frac{1}{2}<p<\frac{3}{2}.\end{cases} \tag{3.12}$$

In order to analyze the thermodynamic stability or phase transition of the physical black holes, we need to have the vanishing and divergence points of the black hole heat capacity. Since one can not obtain these points analytically, we have plotted T and $\mathcal{H}_Q$ versus r_+ in Fig. 3 for both of $\frac{1}{2}<p<\frac{3}{2}$ and $p=\frac{3}{2}$ cases, separately. Noting the curves, one can see that $\mathcal{H}_Q$ does not diverge, and there is no point of type two phase transition. There is only one point at which the black hole heat capacity vanishes, and it coincides with the vanishing point of the black hole temperature named as r_{ext}. Thus, the black holes with the horizon radius $r_+=r_{ext}$ undergo type one phase transition. The physical black holes with the horizon radii greater than r_{ext} are locally stable.

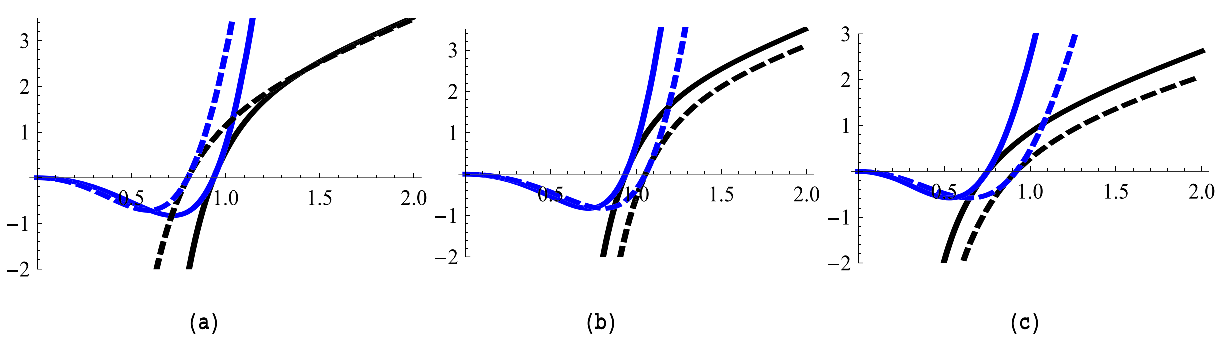

Fig. 2. $4T$ (black) and $\mathcal{H}_Q$ (blue) versus r_+ for $\Lambda=-3$, $Q=2$ Eqs. (3.8) and (3.16). (a) $f(\varepsilon)=0.7$, $g(\varepsilon)=0.8$ and $p=0.8$ (continues), 1 (dashed). (b) $p=0.8$, $f(\varepsilon)=0.7$ and $g(\varepsilon)=0.8$ (continues), 0.9 (dashed). (c) $f(\varepsilon)=0.8$, $p=\frac{3}{2}$ and $g(\varepsilon)=0.9$ (continues), 1.1 (dashed).

3.3. Consideration of thermal fluctuations

Up to now, the thermodynamic quantities have been calculated by neglecting the impacts of thermal fluctuations (TF). However, as the size of the black holes decreases, we have to take into account the effects of TF. The TF are small enough, and they can be analyzed as a perturbation around the equilibrium state. In the presence of TF the black hole entropy is expressed in the following form

$$S^{(TF)}=S-\frac{\eta}{2}\ln\left(ST^2\right)+\frac{\eta_1}{S}+\frac{\eta_2}{S^2}+\cdots, \tag{3.13}$$

where, η, η_1 and η_2 are named as the correction parameters and depend on the details of the model under consideration [39]. Here, we consider the first-order corrections only. Note that investigation of surface gravity confirms that black hole temperature is not affected when the first-order corrections are considered [40,41]. Therefore, T and S are the uncorrected black hole entropy and temperature given in Eqs. (3.4) and (3.6), respectively.

By considering the first-order corrections, the black hole entropy is the only thermodynamic quantity that is affected by TF. It gets logarithmic correction with the explicit form of

$$S^{(TF)}=\begin{cases}\frac{\pi r_+^2}{g^2(\varepsilon)}-\frac{\eta}{2}\ln\left[\frac{1}{16\pi f^2(\varepsilon)}\left(1-\frac{\Lambda r_+^2}{g^2(\varepsilon)}-\frac{2\sqrt{2}q_\varepsilon^3}{g^2(\varepsilon)r_+}\right)^2\right], & \text{for} \quad p=\frac{3}{2},\\ \frac{\pi r_+^2}{g^2(\varepsilon)}-\frac{\eta}{2}\ln\left[\frac{1}{16\pi f^2(\varepsilon)}\left(1-\frac{\Lambda r_+^2}{g^2(\varepsilon)}-\frac{2p-1}{g^2(\varepsilon)}\frac{2^{p-1}q_\varepsilon^{2p}}{r_+^{\frac{2}{2p-1}}}\right)^2\right], & \text{for} \quad \frac{1}{2}<p<\frac{3}{2}.\end{cases} \tag{3.14}$$

Lets to check the validity of the first law of black hole thermodynamics when the impacts of TF are considered. There are two possibilities to investigate the effects of TF on the first law of black hole thermodynamics, which we proceed separately.

Starting from the black hole mass presented in Eq. (3.9) and noting the $S^{(TF)}$ given by Eq. (3.13), we obtain

$$\frac{\partial M}{\partial Q}=U, \quad \text{and} \quad \frac{\partial M}{\partial S^{(TF)}}=\frac{T}{1-\frac{\eta}{2S}\left(1-\frac{2S}{T}\frac{\partial T}{\partial S}\right)}. \tag{3.15}$$

It shows that the first law of black hole thermodynamics remains valid if the following relation is fulfilled

$$\frac{2S}{T}\frac{\partial T}{\partial S}=1. \tag{3.16}$$

Now, one can show that Eq. (3.16) results in

$$r_+=\begin{cases}\frac{3\sqrt{2}q_\varepsilon^3}{g^2(\varepsilon)}, & \text{for} \quad p=\frac{3}{2},\\ \left(\frac{p2^p q_\varepsilon^{2p}}{g^2(\varepsilon)}\right)^{\frac{2p-1}{2}}, & \text{for} \quad \frac{1}{2}<p<\frac{3}{2}.\end{cases} \tag{3.17}$$

It means that, the first law of black hole thermodynamics is valid in the form of Eq. (3.11), only for the black holes with the horizon radius given in Eq. (3.17) [42]. But it can be shown that, the horizon radius presented in Eq. (3.16) does not satisfy the relation $V(r=r_+)=0$. As a result, the first law of black hole thermodynamics is no longer valid.

There is another approach which defines the black hole mass in terms of the Helmholtz free energy $F=-\int S^{(TF)}dT$ as [40,43]

$$M^{(TF)}=F+TS^{(TF)}. \tag{3.18}$$

Noting Eq. (3.18) one is able to show that

$$\frac{\partial M^{(TF)}}{\partial S^{(TF)}}=T, \quad \text{and} \quad \frac{\partial M^{(TF)}}{\partial Q}=U+\frac{\eta g(\varepsilon)q_\varepsilon}{\pi(2p+1)f(\varepsilon)}r_+^{-\frac{2p+1}{2p-1}}. \tag{3.19}$$

Thus, the first law of black hole thermodynamics is valid only for the case $\eta=0$.

Now, we continue by performing a thermal stability analysis by considering the effects of TF. It is matter of calculations to show that

$$\mathcal{H}_Q^{(TF)}=T\left(\frac{\partial S^{(TF)}}{\partial T}\right)_Q=\mathcal{H}_Q+\frac{2\eta(2^{p-1}q_\varepsilon^{2p}-\Lambda r_+^{\frac{4p}{2p-1}})}{g^2(\varepsilon)r_+^{\frac{2}{2p-1}}+\Lambda r_+^{\frac{4p}{2p-1}}-(2p-1)2^{p-1}q_\varepsilon^{2p}}, \quad \text{for} \quad \frac{1}{2}<p\leq\frac{3}{2}, \tag{3.20}$$

where, $\mathcal{H}_Q$ is the black hole heat capacity when the black hole TF have been neglected [Eq. (3.12)]. The plots of T and $\mathcal{H}_Q^{(TF)}$ versus r_+ have been shown in Fig. 3 for the cases $p=\frac{3}{2}$ and $\frac{1}{2}<p<\frac{3}{2}$, separately. The plots show that, $\mathcal{H}_Q^{(TF)}$ has no divergence point, and no type two phase transition takes place. There is only one point of type one phase transition located at the vanishing point of black hole heat capacity named as $r_+=r_0$. But, due to the impacts of TF, it no longer coincides with the vanishing point of the black hole temperature. The physical black holes with the horizon radii in the range $r_+>r_0$ are locally stable.

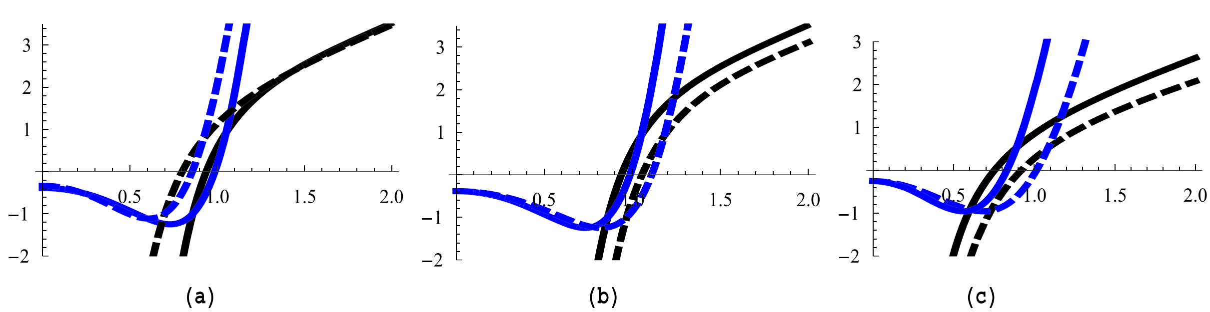

Fig. 3. $4T$ (black) and $\mathcal{H}_Q^{(TF)}$ (blue) versus r_+ for $\Lambda=-3$, $Q=2$, $\eta=0.5$, Eqs. (3.8) and (3.16). (a) $f(\varepsilon)=0.7$, $g(\varepsilon)=0.8$ and $p=0.8$ (continues), 1 (dashed). (b) $p=0.8$, $f(\varepsilon)=0.7$ and $g(\varepsilon)=0.8$ (continues), 0.9 (dashed). (c) $f(\varepsilon)=0.8$, $p=\frac{3}{2}$ and $g(\varepsilon)=0.9$ (continues), 1.1 (dashed).

4. Conclusion

We investigated the four-dimensional rainbow black hole solutions with the power-law nonlinear electrodynamics in an energy-dependent spacetime. We obtained the exact solution of the coupled field equations, and by imposing the physical constraints on the nonlinearity parameter p, we introduced two classes of new exact solutions. Noting the existence of the horizon radii, and appearance of the curvature singularities, we showed that our exact solutions are asymptotically AdS black holes. The solutions can present the two-horizon, naked singularity, and extreme black holes if the parameters are fixed, properly (Fig. 1).

Next, we obtained the electric charge and mass of the black hole, as the conserved quantities, by applying the Gauss's electric law and counterterm method, respectively. Also, we calculated the entropy, temperature, and electric potential by use of geometric methods. Through a Smarr-type mass formula, we obtained the black hole mass as a function of the black hole charge and entropy. We proved that the conserved and thermodynamic quantities satisfy the first law of black hole thermodynamics in its standard form [Eq. (3.11)].

Then, we performed a thermal stability analysis by implying the canonical ensemble method. By calculating the black hole heat capacity, with the black hole charge as a constant, we found that the black hole heat capacity does not diverge, and there is no point of type two phase transition. There is only one point of type one phase transition located at the vanishing point of the black hole temperature, labeled by $r_+ = r_{ext}$. Also, the black holes with the horizon radii in the interval $r_+ > r_{ext}$, have positive temperature and heat capacity, are locally stable (Fig. 2).

Finally, we investigated the quantum gravitational effects on the thermodynamic properties and thermal stability of the black holes. It was proved that, when the black hole thermal fluctuations are taken into account, the first law of black hole thermodynamics is no longer valid. By calculating the black hole heat capacity, we found that the black hole stability is affected by the quantum corrections. Although the number of thermodynamic phase transition points remains unchanged, the location of these points no longer coincides with that of the horizon radius of the extreme black holes (Fig. 3).

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References

[1] V.A. Kostelecky, S. Samuel, Phys. Rev. D 39 (1989) 683.
[2] R. Gambini, J. Pullin, Phys. Rev. D 59 (1999) 124021.
[3] G. Amelino-Camelia, J.R. Ellis, N.E. Mavromatos, D.V. Nanopoulos, S. Sarkar, Nature 393 (1998) 763.
[4] M.R. Douglas, N.A. Nekrasov, Rev. Mod. Phys. 73 (2001) 977.
[5] F. Girelli, E.R. Livine, D. Oriti, Nucl. Phys. B 708 (2005) 411.
[6] J. Magueijo, L. Smolin, Phys. Rev. D 67 (2003) 044017.
[7] R. Garattini, E.N. Saridakis, Int. J. Mod. Phys. D 25 (2016) 1650101.
[8] A.F. Ali, Phys. Rev. D 89 (2014) 104040.
[9] E.J. Son, W. Kim, Mod. Phys. Lett. A 30 (2015) 1550178.
[10] J. Magueijo, L. Smolin, Class. Quantum Gravity 21 (2004) 1725.
[11] J. Magueijo, L. Smolin, Phys. Rev. D 71 (2005) 026010.
[12] M. Dehghani, Phys. Rev. D 99 (2019) 024001.
[13] A. Sheykhi, F. Naeimipour, S.M. Zebarjad, Phys. Rev. D 91 (2015) 124057.
[14] M. Dehghani, S.F. Hamidi, Phys. Rev. D 96 (2017) 104017.
[15] M. Dehghani, Phys. Rev. D 98 (2018) 044008.
[16] S.H. Hendi, J. High Energy Phys. 03 (2012) 065.
[17] M. Dehghani, S.F. Hamidi, Phys. Rev. D 96 (2017) 044025.
[18] S.I. Kruglov, Phys. Rev. D 92 (2015) 123523.
[19] M. Dehghani, Phys. Rev. D 97 (2018) 044030.
[20] M. Kord Zangeneh, M.H. Dehghani, A. Sheykhi, Phys. Rev. D 92 (2015) 104035.
[21] M. Dehghani, Phys. Rev. D 99 (2019) 104036.
[22] A.F. Ali, M. Faizal, M.M. Khalil, Nucl. Phys. B 894 (2015) 341.
[23] A.F. Ali, Phys. Rev. D 89 (2014) 104040.
[24] S.H. Hendi, M. Momennia, B. Eslam Panah, M. Faizal, Astrophys. J. 827 (2016) 153.
[25] M. Dehghani, Phys. Lett. B 793 (2019) 234.
[26] M. Dehghani, Phys. Rev. D 94 (2016) 104071.
[27] M. Dehghani, Phys. Lett. B 781 (2018) 553.
[28] U. Jacob, F. Mercati, G. Amelino-Camelia, T. Piran, Phys. Rev. D 82 (2010) 084021.
[29] G. Amelino-Camelia, Living Rev. Relativ. 16 (2013) 5.
[30] M. Dehghani, Phys. Lett. B 785 (2018) 274.
[31] M. Dehghani, Eur. Phys. J. Plus 133 (2018) 474.
[32] V. Balasubramanian, P. Kraus, Commun. Math. Phys. 28 (1999) 413.
[33] J. Brown, J. York, Phys. Rev. D 47 (1993) 1407.
[34] S.H. Hendi, M. Faizal, B. Eslam Panah, S. Panahiyan, M. Momennia, Eur. Phys. J. C 76 (2016) 296.
[35] M. Dehghani, Phys. Rev. D 96 (2017) 044014.
[36] M. Dehghani, Phys. Lett. B 773 (2017) 105.
[37] S.H. Hendi, A. Sheykhi, S. Panahiyan, B. Eslam Panah, Phys. Rev. D 92 (2015) 064028.
[38] M. Dehghani, Int. J. Mod. Phys. D 27 (2018) 1850073.
[39] B. Pourhassan, M. Faizal, S. Upadhyay, L.A. Asfar, Eur. Phys. J. C 77 (2017) 555.
[40] S.H. Hendi, B. Eslam Panah, S. Panahiyan, M. Momennia, Eur. Phys. J. C 77 (2017) 647.
[41] M. Dehghani, Phys. Lett. B 777 (2018) 351.
[42] B. Pourhassan, M. Faizal, Z. Zaz, A. Bhat, Phys. Lett. B 773 (2017) 325.
[43] M. Dehghani, Phys. Lett. B 799 (2019) 135037.