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Erratum

Erratum to: A theoretical analysis of the semileptonic decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$

Rafel Escribano[1,2,a], **Emilio Royo**[1,2,b]

[1] Grup de Física Teòrica, Departament de Física, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona, Spain
[2] Institut de Física d'Altes Energies (IFAE), The Barcelona Institute of Science and Technology, Campus UAB, Bellaterra, 08193 Barcelona, Spain

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Abstract This erratum corrects Eqs. (6) and (15), Table 1, and Fig. 2 of the original article. An additional footnote is added at the end of the first paragraph in Section III. Finally, the paragraph discussing the use of the energy-dependent width for the ρ^0 propagator is modified.

First, the sign of the regulator in Eq. (6) is incorrect. It should read $-i\varepsilon$, i.e.,

$$\begin{aligned}
\alpha_V &= e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \\
&\quad \times \left[\frac{2A_1}{\Delta_{1V} - i\varepsilon} - \frac{B_1}{(\Delta_{1V} - i\varepsilon)^2} \right], \\
\beta_V &= e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \\
&\quad \times \left[\frac{2C_1}{\Delta_{1V} - i\varepsilon} - \frac{D_1}{(\Delta_{1V} - i\varepsilon)^2} \right], \\
\sigma_V &= e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \\
&\quad \times \left[\frac{2A_2}{\Delta_{2V} - i\varepsilon} - \frac{B_2}{(\Delta_{2V} - i\varepsilon)^2} \right], \\
\tau_V &= e^2 \frac{g_{V\eta^{(\prime)}\gamma}\, g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \\
&\quad \times \left[\frac{2C_2}{\Delta_{2V} - i\varepsilon} - \frac{D_2}{(\Delta_{2V} - i\varepsilon)^2} \right]. \qquad (6)
\end{aligned}$$

Next, an additional footnote has be added at the end of the first paragraph in Section III which reads "It is worth mentioning that comparison between the numerical results for Ω and Σ in Eq. (8) using the approach presented in this work and Passarino–Veltman reduction techniques implemented in software packages such as, e.g., LoopTools [1] was carried out for different points of phase space to assess the performance of our method. It was found that our results were in agreement with those from the above package for points far from the edge of phase space, which provides a level of confidence in our approach, but in sharp disagreement for points near the edge of phase space. This is, however, a well-known drawback of the Passarino–Veltman reduction and variants due to the appearance of Gram determinants in the denominator, which spoils the numerical stability when they become small or even zero giving rise to spurious singularities (see, e.g., Refs. [2–4]). For processes with up to four external particles, this usually happens near the edge of phase space [2], which is consistent with our findings."

The paragraph in page 5 starting with "Given the very wide decay width…" is replaced by "Given the very wide decay width of the ρ^0 resonance, which, in turn, is associated to its very short lifetime, the use of the usual Breit–Wigner approximation for the ρ^0 propagator is not justified. Instead, an energy-dependent width for the vector propagator ought to be considered, which may be written for a generic $\hat{q}^2$ as follows

$$\Gamma_{\rho^0}(\hat{q}^2) = \Gamma_{\rho^0} \times \left(\frac{\hat{q}^2 - 4m_{\pi^\pm}^2}{m_{\rho^0}^2 - 4m_{\pi^\pm}^2} \right)^{3/2} \times \theta(\hat{q}^2 - 4m_{\pi^\pm}^2), \qquad (14)$$

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[a] e-mail: rescriba@ifae.es
[b] e-mail: eroyo@ifae.es (corresponding author)

Table 1 Decay widths and branching ratios for the six C-conserving decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ). First error is experimental, second is down to numerical integration and third is due to model dependency

Decay	Γ_{th}	BR_{th}	BR_{exp}
$\eta \to \pi^0 e^+ e^-$	$2.7(1)(1)(2) \times 10^{-6}$ eV	$2.0(1)(1)(1) \times 10^{-9}$	$< 7.5 \times 10^{-6}$ (CL = 90%) [5]
$\eta \to \pi^0 \mu^+ \mu^-$	$1.4(1)(1)(1) \times 10^{-6}$ eV	$1.1(1)(1)(1) \times 10^{-9}$	$< 5 \times 10^{-6}$ (CL = 90%) [6]
$\eta' \to \pi^0 e^+ e^-$	$8.7(5)(6)(6) \times 10^{-4}$ eV	$4.5(3)(4)(4) \times 10^{-9}$	$< 1.4 \times 10^{-3}$ (CL = 90%) [6]
$\eta' \to \pi^0 \mu^+ \mu^-$	$3.3(2)(4)(3) \times 10^{-4}$ eV	$1.7(1)(2)(2) \times 10^{-9}$	$< 6.0 \times 10^{-5}$ (CL = 90%) [6]
$\eta' \to \eta e^+ e^-$	$8.3(0.5)(0.1)(3.5) \times 10^{-5}$ eV	$4.3(0.3)(0.2)(1.8) \times 10^{-10}$	$< 2.4 \times 10^{-3}$ (CL = 90%) [6]
$\eta' \to \eta \mu^+ \mu^-$	$3.0(0.2)(0.1)(1.1) \times 10^{-5}$ eV	$1.5(1)(1)(5) \times 10^{-10}$	$< 1.5 \times 10^{-5}$ (CL = 90%) [6]

where $\theta(x)$ is the Heaviside step function. Strictly, one would now need to plug Eq. (14) into Eq. (2) and perform the loop integral, which represents a computation challenge in its own right and is outside of the scope of the present work.[1]

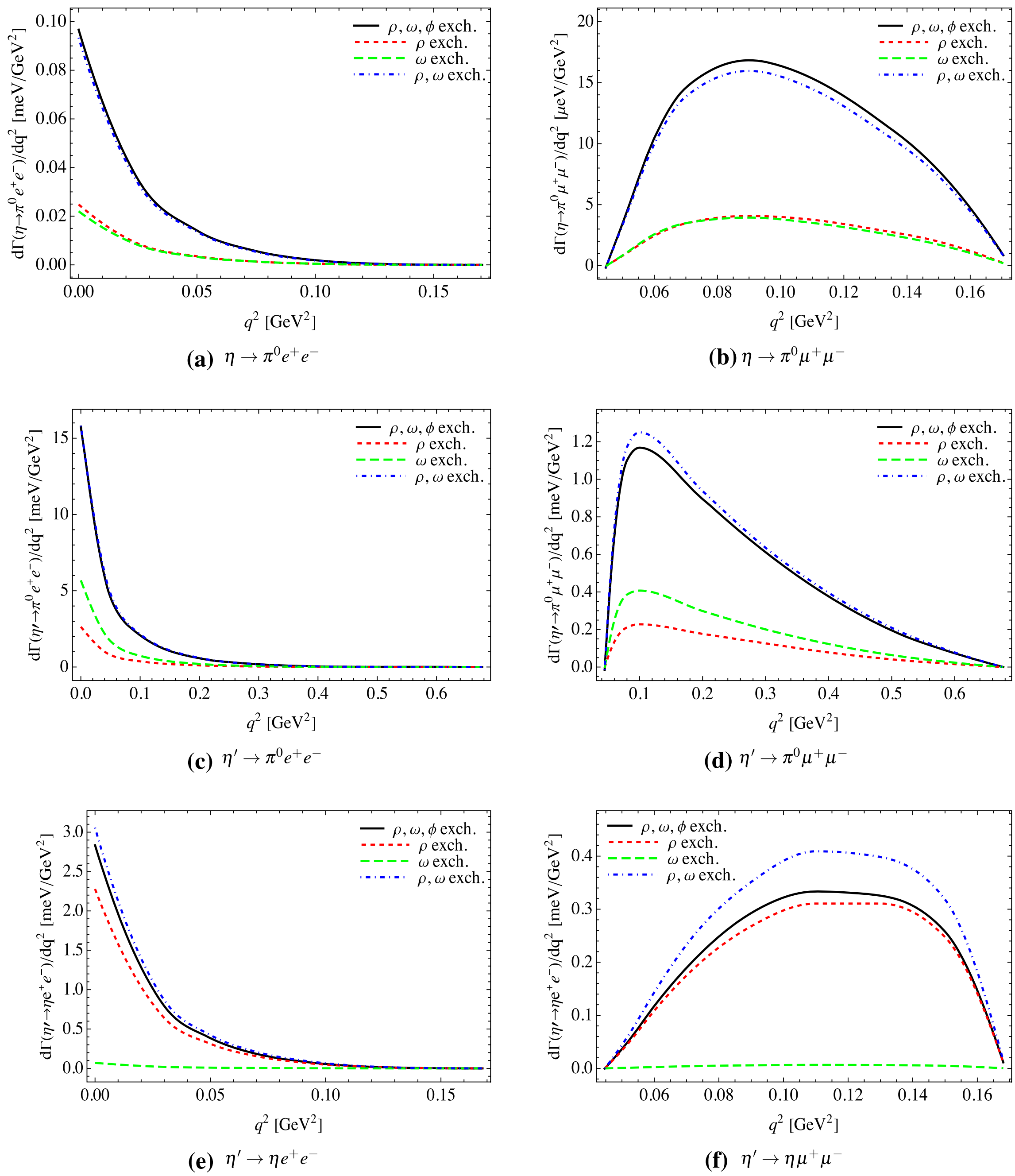
With this in mind, and for the sake of simplicity, we resolve to stick with the Breit–Wigner approximation for the ρ^0 propagator despite being a potential source of error. The energy-dependent propagator is not needed, though, for the ω and ϕ resonances, as their associated decay widths are narrow and, therefore, use of the usual Breit–Wigner approximation suffices."

Finally, in Eq. (15) there is a missing $|e|$ factor. It should read

$$g_{VP\gamma}\,\hat{F}_{VP\gamma}(q^2) = C_{VP\gamma}\,|e|\frac{4\sqrt{2}\,h_V}{f_\pi} \times \left(1 + \frac{\sigma_V f_V}{\sqrt{2}\,h_V}\frac{q^2}{M_{V'}^2 - q^2}\right). \tag{15}$$

The above corrections lead to a new set of results and dilepton energy spectra which are summarised in Table 1 and Fig. 1 of this erratum, respectively.

[1] One could write, for example, the ρ^0 energy-dependent propagator $f(s) = \frac{m_\rho^2}{m_\rho^2 - s - i m_\rho \Gamma_\rho(s)}$ as a once-subtracted dispersion relation, $f(s) = f(s_0) + \frac{s - s_0}{\pi}\int_{s_{\text{th}}}^{\infty}\frac{\text{Im}\, f(s')\, \text{d}s'}{(s' - s_0)(s' - s - i\epsilon)}$, where s_{th} is the particle production threshold, in the case at hand $s_{\text{th}} = 4m_\pi^2$, and s_0 is the subtraction point such that $s_0 < s_{\text{th}}$, e.g. $s_0 = 0$. One would then perform the loop integral in the usual way, leaving the dispersion integral to the end of the computation.



(a) $\eta \to \pi^0 e^+ e^-$ **(b)** $\eta \to \pi^0 \mu^+ \mu^-$

(c) $\eta' \to \pi^0 e^+ e^-$ **(d)** $\eta' \to \pi^0 \mu^+ \mu^-$

(e) $\eta' \to \eta e^+ e^-$ **(f)** $\eta' \to \eta \mu^+ \mu^-$

Fig. 1 Dilepton energy spectra corresponding to the six C-conserving semileptonic decay processes $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ) as a function of the dilepton invariant mass q^2

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