Common origin of the strong CP and CKM phases in string compactifications

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A R T I C L E   I N F O

A B S T R A C T

We explore the scenario where both the strong CP and Cabbibo-Kobayashi-Maskawa (CKM) phases are determined by the same axion field. Such a scenario is naturally realized in string compactifications. We find that there exists parameter region to realize the tiny strong CP phase and observed CKM phase in magnetized D-brane models.

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Contents

1. Introduction

In the standard model, the strong CP phase $\theta_{\rm eff} = \theta_{\rm QCD} + {\rm Arg}\{{\rm Det}(y_u y_d)\}$ is severely constrained by experiments to be smaller than 10^{-10} [1–3], but the CP violating CKM phase is of $\mathcal{O}(1)$. This strong CP problem indicates a new mechanism to solve not accommodated in the standard model. The most promising scenario solving the strong CP problem is to introduce the QCD axion which is a pseudo Nambu-Goldstone boson associated with the Peccei-Quinn (PQ) symmetry [4]. The strong CP phase $\theta_{\rm eff}$ can be determined by a dynamics of the axion field.

When we consider the string theory as well as higher-dimensional theory as an ultraviolet completion of the standard model, promoting the CP phase to the axion is naturally realized. Indeed, in the string theory, all the couplings are functions of moduli fields whose vacuum expectation values determine the size of the couplings in the standard model. In particular, both the QCD phase $\theta_{\rm QCD}$ as well as ${\rm Arg}\{{\rm Det}(y_u y_d)\}$ are functions of axion fields in general. It is then interesting to ask whether the QCD and CKM phases have a common origin in string compactifications and at the same time the tiny strong CP phase $\theta_{\rm eff} \ll 1$ is compatible with the $\mathcal{O}(1)$ CKM phase.

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In the low-energy effective action of string theory, the gauge kinetic function determining the QCD phase is linearly dependent of the moduli fields, whereas the axion dependence of the Yukawa couplings is model dependent in general. If the Yukawa couplings have the Froggatt-Nielsen (FN) type structure [5] to realize the hierarchical structure of the fermion masses, $y_u \propto e^{2\pi i (k_{Q_i} + k_{U_j})a}$ $y_d \propto e^{2\pi i (k_{Q_i} + k_{D_j})a}$ with $k_{Q_i}, k_{U_j}, k_{D_j}$ being the FN charges of the quarks and a the axion, the CKM phase is induced by the nonvanishing field value of the axion a. Furthermore, $\mathrm{Arg}\{\mathrm{Det}(y_u y_d)\}$ is also linearly dependent of the axion, namely $\mathrm{Arg}\{\mathrm{Det}(y_u y_d)\} \propto a$. It indicates that when the axion has a common origin of the QCD and CKM phases, $\theta_{\mathrm{eff}} = 0$ requires $\langle a \rangle = 0$. Then, the CKM phase cannot be generated. In this way, a certain non-trivial axion dependence to the Yukawa couplings is required to realize $\theta_{\mathrm{eff}} \ll 1$ compatible with the $\mathcal{O}(1)$ CKM phase. In this paper, we resolve this issue in a specific string compactification and explore the parameter region in the moduli space of the axion field to realize such a scenario. It gives a new insight on the strong CP problem from the view point of string theory.

This paper is organized as follows. In section 2, we show the origin of the QCD and CKM phases in Type IIB string on toroidal orientifold with D3/D7-branes. In particular, in Type IIB flux vacua, it is possible to have a relation between the QCD and CKM phases. In section 3, we explicitly evaluate both phases in a specific three-generation model realized in magnetized D7-branes and explore the parameter region leading to the observed CKM phase consistent with the tiny strong CP phase. It turns out that the axion controlling the magnitude of both phases provides semi-realistic observed values, namely mass ratios for the quarks, the elements of the CKM matrix and the Jarlskog invariant, thanks to the non-trivial axion-dependent Yukawa couplings. Section 4 is devoted to the conclusions.

2. The model

In this section, we show how to relate the QCD phase with the CKM phase in the effective action of superstring theory with an emphasis on Type IIB string theory on toroidal orientifold $\Pi_{i=1}^{3}(T^2)_i/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D3/D7-branes.

2.1. Origin of the QCD and CKM phases

Let us first consider the origin of the QCD phase on the magnetized $D7_a$-brane wrapping the 4-cycle $(T^2)_j \times (T^2)_k$ with $j \neq k$, where the U(1) magnetic fluxes F_a are introduced as

$$\frac{m_a^j}{l_s^2} \int\limits_{(T^2)_j} F_a^j = n_a^j, \tag{1}$$

with $l_s = 2\pi\sqrt{\alpha'}$ being the string length. Here, m_a^j and n_a^j are the wrapping number of $D7_a$-brane and the quantized flux, respectively. The gauge kinetic function on the magnetized $D7_a$-brane is given by [6–8]

$$f_{D7_a} = |m_a^k m_a^j| \left(T^i - \frac{n_a^k}{m_a^k} \frac{n_a^j}{m_a^j} \tau \right), \qquad (i \neq j \neq k) \tag{2}$$

from which the CP phase is determined by two axions, originating from the Ramond-Ramond 4-form $\mathrm{Re}(T^i) = \int_{(T^2)_j \times (T^2)_k} C_4$ and Ramond-Ramond 0-form $\mathrm{Re}(\tau) = C_0$, respectively. The imaginary part of the Kähler moduli T_i now denotes the volume of four-cycle wrapped by the D7-brane, following [8].

Next, we discuss the origin of the CKM phase in Yukawa couplings of matter fields living on magnetized D7-branes. Let us consider $U(N)$ magnetic flux on N stacks of D7-branes such that $U(N)$ gauge symmetry on D7-branes is broken to $U(N_a) \times U(N_b) \times U(N_c)$ with $N = N_a + N_b + N_c$. Thanks to the magnetic fluxes, bifundamental zero-modes for $(N_\alpha, \bar{N}_\beta)$, $\alpha, \beta = a, b, c$ have the net number of index labeled by $p = 0, 1, \cdots, |I_{\alpha\beta}^j| - 1$ with $I_{\alpha\beta}^j = n_\alpha^j/m_\alpha^j - n_\beta^j/m_\beta^j$ on each 2-torus $(T^2)_j$ wrapped by D7-branes and these degenerate chiral zero-modes can be identified with the quarks and/or leptons. From the analysis in the low-energy effective action of magnetized D7-branes, Yukawa couplings of such chiral zero-modes are found by calculating the overlap integral of zero-mode wavefunctions. On each 2-torus $(T^2)_j$ inside the 4-cycle wrapped by magnetized D7-branes, holomorphic Yukawa couplings are provided by [9]

$$Y_{pqs} = \vartheta \begin{bmatrix} -\frac{1}{I_{ab}^j} \left(\frac{q}{I_{ca}^j} + \frac{s}{I_{bc}^j} \right) \\ 0 \end{bmatrix} \left(0, \tau_j \left| I_{ab}^j I_{bc}^j I_{ca}^j \right| \right), \tag{3}$$

up to the normalization factor, where $q = 0, 1, \cdots, |I_{ca}^j| - 1$, $s = 0, 1, \cdots, |I_{bc}^j| - 1$[1] and ϑ denotes the Jacobi theta function as a function of the complex structure modulus τ_j

$$\vartheta \begin{bmatrix} c \\ 0 \end{bmatrix} (0, \tau_j) \equiv \sum_{l \in \mathbb{Z}} e^{\pi i (c+l)^2 \tau_j}. \tag{4}$$

Note that a higher-dimensional gauge coupling as well as the Kähler metric of the matter field are involved in the physical Yukawa couplings, but they are real. In this way, the CKM phase is determined by the real part of complex structure moduli and the dependence of the axion field is not a Froggatt-Nielsen type as explicitly analyzed in a concrete model in section 3. The above calculation can be extended to the $T^2/\mathbb{Z}_N$ orbifold, where the Yukawa couplings are provided by the linear combination of Yukawa couplings. (For more details, see Ref. [10].) In the next section, we propose the mechanism to correlate the QCD phase to the CKM phase.

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[1] Here, we employ $p = s - q \mod I_{ab}$. (For more details, see, Ref. [9].)

2.2. Relation between the QCD and CKM phases

From now on, we show one of the mechanisms to relate the QCD phase with the CKM phase in Type IIB flux compactification on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with hodge numbers $(h_{1,1}, h_{2,1}) = (3, 51)$. The closed string moduli in this setup are the axio-dilaton τ, three Kähler moduli T_i and untwisted complex structure moduli τ_j with $i, j = 1, 2, 3$. The Kähler potential of moduli fields is described by[2]

$$K = -\ln(-i(\tau - \bar{\tau})) - \sum_i \ln(-i(T_i - \bar{T}_i)) - \sum_j \ln(-i(\tau_j - \bar{\tau}_j)). \tag{5}$$

The superpotential of complex structure moduli and axio-dilaton can be generated by an existence of three-form fluxes [11],

$$W = (\tau - f\tau_3)g(\tau_1, \tau_2), \tag{6}$$

where we consider a particular form of the three-form fluxes including f and $g(\tau_1, \tau_2)$ is the proper function stabilizing $\tau_{1,2}$. The reason why we choose the above specific superpotential is that it leads to the massless direction in the (τ, τ_3) moduli space at the supersymmetric Minkowski minimum [12,13],

$$\partial_\tau W = \partial_{\tau_1} W = \partial_{\tau_2} W = \partial_{\tau_3} W = W = 0, \tag{7}$$

where $g(\tau_1, \tau_2)$ is supposed to satisfy the above stabilization conditions, for instance, $g(\tau_1, \tau_2) = (a^1\tau_1 + a^2\tau_2)$ with $a^{1,2}$ being three-form flux quanta. As discussed later, the flat direction in the (τ, τ_3) moduli space plays a crucial role of relating the QCD phase with the CKM phase. From the minimum $\tau = f\tau_3$, we define the flat direction (τ_f) and the stabilized direction (τ_h):

$$\tau_f \equiv \mathcal{N}^{-1/2}(f\tau + \tau_3),$$
$$\tau_h \equiv \mathcal{N}^{-1/2}(\tau - f\tau_3), \tag{8}$$

with $\mathcal{N} = 1 + f^2$. Below the mass scale of stabilized τ_h with $\langle \tau_h \rangle = 0$, the axio-dilaton and the complex structure moduli are described by the same modulus τ_f

$$\tau = \mathcal{N}^{-1/2} f\tau_f,$$
$$\tau_3 = \mathcal{N}^{-1/2} \tau_f. \tag{9}$$

When magnetized D7-branes wrap the third torus $(T^2)_3$ and flavor structure of the quark sector is determined by the magnetic flux on $(T^2)_3$, the CKM phase is determined by τ_3, namely τ_f below the mass scale of stabilized τ_h. In this way, τ_f controls the magnitude of not only the QCD phase through Eq. (2), but also the CKM phase through Eq. (3). Note that Kähler axion $\mathrm{Re}(T)$ contributes to the QCD phase as in Eq. (2), but in the following analysis, we assume that $\mathrm{Re}(T)$ is stabilized at the origin $\langle \mathrm{Re}(T) \rangle = 0$ due to the non-perturbative effects for the Kähler moduli which enjoys a certain discrete symmetry for the axion. Such a simplification is useful to study the contribution from τ_f in the effective CP phase. Furthermore, we assume that the effective CP phase does not have the contributions from soft supersymmetry-breaking terms like gaugino masses and $B\mu$ terms to simplify our analysis. For that reason, we focus on the QCD and CKM phases, namely

$$\theta_{\mathrm{eff}} = \theta_{\mathrm{QCD}} + \mathrm{Arg}\{\mathrm{Det}(y_u y_d)\}, \tag{10}$$

and both are determined by the common axion τ_f. In the following analysis, we explore the magnitudes of both CP phases on the basis of a concrete magnetized D-brane model.

Finally, we comment on another possible scenario to entangle the QCD phase with the CKM phase. When there exist one-loop threshold corrections to the gauge kinetic function of D7-branes, the gauge kinetic function has a modular invariant function with respect to the complex structure moduli [14–17]. Then, both CP phases are related with each other. In this paper, we concentrate on the three-form flux scenario leading to a common origin of the QCD and CKM phases.

3. Concrete magnetized D-brane model

To analyze the behavior of both the QCD and CKM phases in the moduli space of axion field τ_f, we choose the specific magnetized D7-brane configuration wrapping the first and third torus and we assume that the flavor structure is only determined by the third torus $(T^2)_3$ on which the U(1) magnetic fluxes are inserted. In particular, we consider the toroidal orbifold $(T^2)_3/\mathbb{Z}_2$. Our purpose is to reveal whether there exists a moduli space of the axion field leading to the observed CKM phase consistent with the tiny strong CP phase or not. Therefore, we have not considered the global consistency conditions like tadpole cancellation conditions in this paper, since they depend on the existence of hidden sector as well as an amount of three-form fluxes.

As discussed in Ref. [18], we start from U(8) super Yang-Mills action which can be regarded as the low-energy effective action of stacks of D7-branes. The magnetic fluxes are introduced to break U(8) to the standard-model gauge groups plus extra U(1)s. As displayed in Table 1, we assign the magnetic flux n of quarks and Higgs fields and $\mathbb{Z}_2$ parity such that there exist three generations of quarks and five pairs of Higgs. Here, we choose the wrapping number of magnetized D7-branes $m^3 = 1$.

The Yukawa couplings in the quark sector

$$Y_{IJK} H_K Q_{LI} Q_{RJ} = (Y_{IJ0}H_0 + Y_{IJ1}H_1 + Y_{IJ2}H_2 + Y_{IJ3}H_3 + Y_{IJ4}H_4)Q_{LI}Q_{RJ}, \tag{11}$$

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[2] Here and in what follows, we adopt the reduced Planck mass unit, unless we specify it.

Table 1

Magnetic fluxes for three generations of left-handed quarks Q_L, right-handed quarks Q_R and five pairs of Higgs H.

	Q_L	Q_R	H
n ($\mathbb{Z}_2$ parity)	-5 (even)	-7 (odd)	12 (odd)

are given by [18]

$$Y_{IJ0} = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_5 - \eta_{65} & \eta_{185} - \eta_{115} & \sqrt{2}(\eta_{55} + \eta_{125}) \\ \eta_{173} - \eta_{103} - \eta_{187} + \eta_{163} & \eta_{67} - \eta_{137} - \eta_{53} + \eta_{17} & \eta_{113} - \eta_{43} - \eta_{127} + \eta_{197} \\ \eta_{79} - \eta_{149} - \eta_{19} + \eta_{89} & \eta_{101} - \eta_{31} - \eta_{199} + \eta_{151} & \eta_{139} - \eta_{209} - \eta_{41} + \eta_{29} \end{pmatrix},$$

$$Y_{IJ1} = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_{170} - \eta_{110} & \eta_{10} - \eta_{130} & \sqrt{2}(\eta_{50} + \eta_{190}) \\ \eta_2 - \eta_{142} - \eta_{58} + \eta_{82} & \eta_{178} - \eta_{38} - \eta_{122} + \eta_{158} & \eta_{62} - \eta_{202} - \eta_{118} + \eta_{22} \\ \eta_{166} - \eta_{26} - \eta_{194} + \eta_{94} & \eta_{74} - \eta_{206} - \eta_{46} + \eta_{94} & \eta_{106} - \eta_{34} - \eta_{134} + \eta_{146} \end{pmatrix},$$

$$Y_{IJ2} = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_{75} - \eta_{135} & \eta_{165} - \eta_{45} & \eta_{15} - \eta_{195} \\ \eta_{173} - \eta_{33} - \eta_{117} + \eta_{93} & \eta_3 - \eta_{207} - \eta_{123} + \eta_{87} & \eta_{183} - \eta_{27} - \eta_{57} + \eta_{153} \\ \eta_9 - \eta_{201} - \eta_{51} + \eta_{81} & \eta_{171} - \eta_{39} - \eta_{129} + \eta_{81} & \eta_{69} - \eta_{141} - \eta_{111} + \eta_{99} \end{pmatrix},$$

$$Y_{IJ3} = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_{100} - \eta_{140} & \eta_{80} - \eta_{200} & \eta_{160} - \eta_{20} \\ \eta_{68} - \eta_{208} - \eta_{128} + \eta_{152} & \eta_{172} - \eta_{32} - \eta_{52} + \eta_{88} & \eta_8 - \eta_{148} - \eta_{188} + \eta_{92} \\ \eta_{184} - \eta_{44} - \eta_{124} + \eta_{164} & \eta_4 - \eta_{136} - \eta_{116} + \eta_{164} & \eta_{176} - \eta_{104} - \eta_{64} + \eta_{76} \end{pmatrix},$$

$$Y_{IJ4} = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_{145} - \eta_{205} & \eta_{95} - \eta_{25} & \eta_{85} - \eta_{155} \\ \eta_{107} - \eta_{37} - \eta_{47} + \eta_{23} & \eta_{73} - \eta_{143} - \eta_{193} + \eta_{157} & \eta_{167} - \eta_{97} - \eta_{13} + \eta_{83} \\ \eta_{61} - \eta_{131} - \eta_{121} + \eta_{11} & \eta_{179} - \eta_{109} - \eta_{59} + \eta_{11} & \eta_1 - \eta_{71} - \eta_{181} + \eta_{169} \end{pmatrix},$$

up to the normalization factor.[3] Now, we define

$$\eta_N \equiv \vartheta \begin{bmatrix} \frac{N}{M} \\ 0 \end{bmatrix} (0, \mathcal{N}^{-1/2} \tau_f M) \tag{12}$$

with $M = 420$ and $\tau_3 = \mathcal{N}^{-1/2} \tau_f$.

Before searching for the CKM phase compatible with the tiny strong CP phase, we discuss the axion-dependence of the Yukawa couplings in the next section.

3.1. CP phase from the Yukawa couplings

To reveal the functional behavior of the Yukawa couplings with respect to the axion τ_f, we approximate the Jacobi-theta function with [19]

$$\eta_N = \vartheta \begin{bmatrix} \frac{N}{M} \\ 0 \end{bmatrix} (0, \mathcal{N}^{-1/2} \tau_f M) \sim e^{\frac{i \tau_f}{420} \mathcal{N}^{-1/2} N^2} \tag{13}$$

which is valid in the large complex structure limit $\text{Im}(\tau_3) = \mathcal{N}^{-1/2} \text{Im}(\tau_f) \gg 1$. Under this approximation, Yukawa couplings are expanded as

$$Y_{IJ0} = \begin{pmatrix} \sqrt{2}\eta_5 & -\sqrt{2}\eta_{115} & \sqrt{2}\eta_{55} \\ -\eta_{103} & \eta_{17} & -\eta_{43} \\ -\eta_{19} & -\eta_{31} & \eta_{29} \end{pmatrix},$$

$$Y_{IJ1} = \begin{pmatrix} -\sqrt{2}\eta_{110} & \sqrt{2}\eta_{10} & \sqrt{2}\eta_{50} \\ \eta_2 & -\eta_{38} & \eta_{22} \\ -\eta_{26} & -\eta_{46} & -\eta_{34} \end{pmatrix},$$

$$Y_{IJ2} = \begin{pmatrix} \sqrt{2}\eta_{75} & -\sqrt{2}\eta_{45} & -\sqrt{2}\eta_{15} \\ -\eta_{33} & -\eta_3 & -\eta_{27} \\ \eta_9 & -\eta_{39} & -\eta_{69} \end{pmatrix}, \tag{14}$$

$$Y_{IJ3} = \begin{pmatrix} \sqrt{2}\eta_{100} & \sqrt{2}\eta_{80} & -\sqrt{2}\eta_{20} \\ \eta_{68} & -\eta_{32} & \eta_8 \\ -\eta_{44} & \eta_4 & -\eta_{64} \end{pmatrix},$$

$$Y_{IJ4} = \begin{pmatrix} \sqrt{2}\eta_{145} & -\sqrt{2}\eta_{25} & \sqrt{2}\eta_{85} \\ \eta_{23} & \eta_{73} & -\eta_{13} \\ \eta_{11} & \eta_{11} & \eta_1 \end{pmatrix}.$$

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[3] In the following analysis, we focus on the mass ratios of quarks, elements of the CKM matrix and the Jarlskog invariant. It is then enough to omit the overall factors in Yukawa couplings, because the flavor structure is governed by the holomorphic Yukawa couplings.
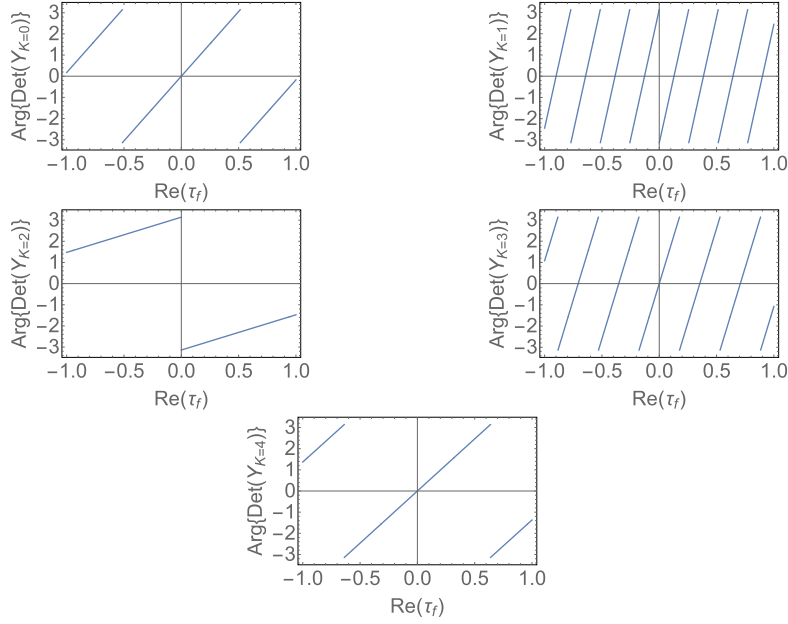
Fig. 1. Plots of $\text{Arg}\{\text{Det}(Y_{IJK})\}$ with $K = 0, 1, 2, 3, 4$ by setting $f = 1$ and $\text{Im}(\tau_f) = 2$.

Recalling that $\text{Arg}\{\text{Det}(y_u y_d)\}$ has the following property

$$\text{Arg}\{\text{Det}(y_u y_d)\} = \text{Arg}\{\text{Det}(y_u)\} + \text{Arg}\{\text{Det}(y_d)\}, \tag{15}$$

for non-zero complex numbers $\text{Det}(y_u), \text{Det}(y_d)$, the functional behavior of the CKM phase with respect to the axion can be understood by evaluating the $\text{Det}(Y_K)$ with $K = 0, 1, 2, 3, 4$. In the large complex structure limit $\text{Im}(\tau_3) = \mathcal{N}^{-1/2}\text{Im}(\tau_f) \gg 1$, the approximate form of $\text{Det}(Y_K)$ is given by

$$\text{Det}(Y_{IJ0}) \sim e^{11i\pi\tau_3/4}\left(1 - e^{4i\pi\tau_3}\right), \qquad \text{Det}(Y_{IJ1}) \sim -e^{11i\pi\tau_3}\left(1 + e^{24i\pi\tau_3}\right),$$

$$\text{Det}(Y_{IJ2}) \sim e^{3i\pi\tau_3/4}\left(-1 + 2e^{6i\pi\tau_3}\right), \qquad \text{Det}(Y_{IJ3}) \sim e^{8i\pi\tau_3}\left(1 - e^{4i\pi\tau_3}\right),$$

$$\text{Det}(Y_{IJ4}) \sim e^{61i\pi\tau_3/28}\left(1 + e^{4i\pi\tau_3/7}\right), \tag{16}$$

from which $\text{Arg}\{\text{Det}(Y_{IJK})\}$ is a non-linear function of the axion $\text{Re}(\tau_3)$, rather than the linear function.

Indeed, Fig. 1 shows that $\text{Arg}\{\text{Det}(Y_{IJK})\}$ employing Eq. (14) is a complicated function of $\text{Re}(\tau_f)$, where we set $\text{Im}(\tau_f) = 2$ and $f = 1$. The functional behavior is the same even when we set the other $\text{Im}(\tau_f)$ and f. Such an axion dependence is a consequence of the definition of the argument $\text{Arg}\{\text{Det}(z)\}$ with z being a complex number:

$$\text{Arg}\{\text{Det}(z)\} = \begin{cases} \text{Arctan}\left(\dfrac{\text{Im}(\text{Det}(z))}{\text{Re}(\text{Det}(z))}\right) & (\text{Re}(\text{Det}(z)) > 0, \text{Im}(\text{Det}(z)) \gtrless 0) \\[2mm] \text{Arctan}\left(\dfrac{\text{Im}(\text{Det}(z))}{\text{Re}(\text{Det}(z))}\right) + \pi & (\text{Re}(\text{Det}(z)) < 0, \text{Im}(\text{Det}(z)) > 0) \\[2mm] \text{Arctan}\left(\dfrac{\text{Im}(\text{Det}(z))}{\text{Re}(\text{Det}(z))}\right) - \pi & (\text{Re}(\text{Det}(z)) < 0, \text{Im}(\text{Det}(z)) < 0) \end{cases} . \tag{17}$$

3.2. Suppressed CP phase

In this section, we take into account the QCD phase θ_{QCD} in addition to the CP phase from Yukawa couplings treated in the previous section and check whether there exists the axionic moduli space to realize the tiny strong CP phase and the $\mathcal{O}(1)$ CKM phase. The QCD phase is now determined by

$$\theta_{\text{QCD}} = M_{\text{QCD}}\tau = M_{\text{QCD}}\mathcal{N}^{-1/2}f\tau_f, \tag{18}$$

originating from Eq. (2) with Eq. (9) by assuming $\text{Re}(T) = 0$ for the Kähler axion. Here, we denote the magnetic flux contributions by M_{QCD}. To evaluate the magnitude of the CKM phase, we examine the Jarlskog invariant (J)

$$J\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln} = \text{Im}\left[V_{ij}V_{kl}V_{il}^*V_{kj}^*\right], \tag{19}$$
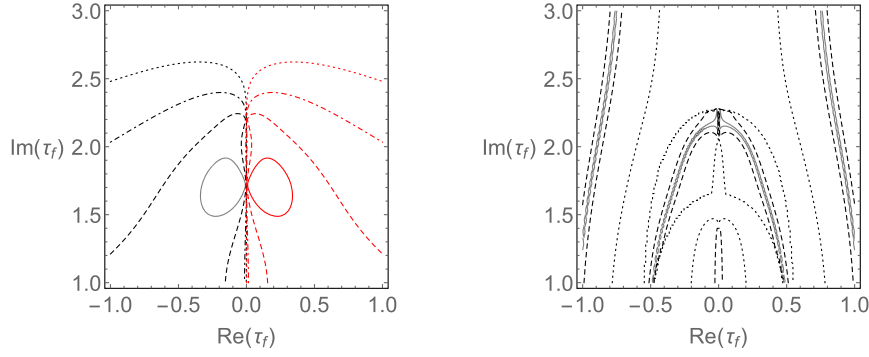
Fig. 2. The functional behavior of the Jarlskog invariant J in the left panel and the effective CP phase $\text{Log}(|\theta_{\text{eff}}|)$ in the right panel with respect to $\text{Re}(\tau_f)$ and $\text{Im}(\tau_f)$, where the parameters are set as in Eq. (23). In the left panel, black (red) dotted, dotdashed, dashed and solid curves correspond to $J = 10^{-7}, 10^{-6}, 10^{-5}, 3 \times 10^{-5}$ ($J = -10^{-7}, -10^{-6}, -10^{-5}, -3 \times 10^{-5}$), respectively. In the right panel, black dotted, dashed and solid curves correspond to $\text{Log}(|\theta_{\text{eff}}|) = 1, -1, -3$, respectively.
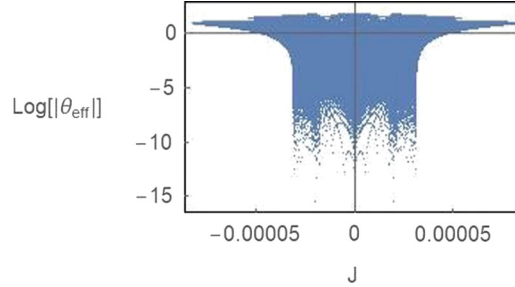
Fig. 3. The functional behavior of the Jarlskog invariant J versus the effective CP phase $\text{Log}(|\theta_{\text{eff}}|)$ within $-1/2 \leq \text{Re}(\tau_f) \leq 1/2$ and $1 \leq \text{Im}(\tau_f) \leq 2.5$ with the step size 5×10^{-4}, where the parameters are the same with Fig. 2. When the step size is narrower and narrower, the effective CP phase θ_{eff} is close to 0 at the specific value of the Jarlskog invariant.

where V_{ij} is the element of the CKM matrix and ϵ_{ikm} is the Levi-Civita symbol.

For an illustrative purpose, we adopt the specific mass matrices for up- and down-type quarks:

$$M_u = Y_{IJ4}\langle H_{u,4}\rangle + Y_{IJ3}\langle H_{u,3}\rangle = \langle H_{u,4}\rangle \left(Y_{IJ4} + Y_{IJ3}\rho_u\right),$$
$$M_d = Y_{IJ4}\langle H_{d,4}\rangle + Y_{IJ3}\langle H_{d,3}\rangle = \langle H_{d,4}\rangle \left(Y_{IJ4} + Y_{IJ3}\rho_d\right), \tag{20}$$

meaning that up- and down-type Higgs $H_{u,d}$ are linear combinations of H_4 and H_3. Here and in what follows, we assume that both Higgs fields are assumed to be nonvanishing real field values and for our purpose, the vacuum expectation values of Higgs fields are parametrized by

$$\rho_u = \frac{\langle H_{u,3}\rangle}{\langle H_{u,4}\rangle}, \qquad \rho_d = \frac{\langle H_{d,3}\rangle}{\langle H_{d,4}\rangle}. \tag{21}$$

The overall factors $\langle H_{u(d),4}\rangle$ in Eq. (20) are assumed to realize the scale of quark masses. The reason why we adopt Y_{IJ4} and Y_{IJ3} for the quark mass matrices is that they have a hierarchical structure among three generations of quarks as analytically discussed in Ref. [19]. The above form of quark mass matrices leads to the following CP phase in the large complex structure limit $\tau_3 \gg 1$,

$$\text{Arg}\{\text{Det}(y_u y_d)\} \simeq \sum_{i=u,d} \sqrt{2}e^{27i\pi\tau_3/14}\left(e^{i\pi\tau_3/4} + e^{23i\pi\tau_3/28} - \rho_i - e^{4i\pi\tau_3/7}\rho_i - e^{9i\pi\tau_3/28}\rho_i^2\right), \tag{22}$$

where $\rho_{u,d}$ is assumed to be $\mathcal{O}(1)$.

Following the above setup, we numerically estimate the Jarlskog invariant J and the effective CP phase θ_{eff} as functions of $\text{Re}(\tau_f)$ and $\text{Im}(\tau_f)$ in Fig. 2, where we choose the following parameters

$$f = 1, \qquad M_{\text{QCD}} = 1, \qquad \rho_u = 0.3, \qquad \rho_d = 0.4. \tag{23}$$

From Fig. 2, the effective CP phase vanishes periodically in the axionic direction due to the property of the arctangent function in Eq. (17) and at the same time, a small but finite Jarlskog invariant J can be realized at the minimum with $\theta_{\text{eff}} = 0$ as shown in Fig. 3. Indeed, semi-realistic values of the mass ratios for quarks, elements of the CKM matrix and the Jarlskog invariant J are obtained at the benchmark point in Table 2. This result is a consequence of the non-trivial axion-dependent function of $\text{Arg}\{\text{Det}(y_u y_d)\}$. For instance, when the Yukawa couplings have a FN-type, θ_{eff} is a linear function of the axion, indicating that vanishing θ_{eff} is occurred at $\text{Re}(\tau_f) = 0$. Since the nonvanishing CKM phase is induced by a nonzero value of $\text{Re}(\tau_f)$, we cannot obtain a nonzero Jarlskog invariant in a FN-type scenario. As a result, the important point to realize a small but finite J at $\theta_{\text{eff}} = 0$ is the non-trivial axion dependent function of the Yukawa couplings. In this paper, we assume a proper mechanism to realize $\theta_{\text{eff}} = 0$ by non-perturbative effects in a hidden sector at a scale larger than the electroweak scale. If the hidden sector also involves the axion-dependent CP phase from the Yukawa couplings in addition to the CP phase from the gauge kinetic function, it would lead to the observed value of the Jarlskog invariant at $\theta_{\text{eff}} = 0$. We hope to report on this in a

Table 2
The mass ratios for quarks, elements of the CKM matrix and the Jarlskog invariant J at the benchmark point, where we set $\mathrm{Re}(\tau_f) \simeq -0.2188$ and $\mathrm{Im}(\tau_f) = 2$ leading to $\theta_{\mathrm{eff}} \simeq 0$ and parameters are chosen as in Eq. (23). Here, we use the GUT scale running masses for the observed values [20] and the value of the Jarlskog invariant in Ref. [21].

	Benchmark values	Observed values
$(m_u, m_c, m_t)/m_t$	$(5.7 \times 10^{-4}, 1.2 \times 10^{-2}, 1)$	$(6.5 \times 10^{-6}, 3.2 \times 10^{-3}, 1)$
$(m_d, m_s, m_b)/m_b$	$(9.8 \times 10^{-4}, 2.0 \times 10^{-2}, 1)$	$(1.1 \times 10^{-3}, 2.2 \times 10^{-2}, 1)$
$\lvert V_{CKM} \rvert$	$\begin{pmatrix} 0.98 & 0.19 & 0.0054 \\ 0.19 & 0.98 & 0.035 \\ 0.0038 & 0.036 & 1.0 \end{pmatrix}$	$\begin{pmatrix} 0.97 & 0.22 & 0.0037 \\ 0.22 & 0.97 & 0.042 \\ 0.0090 & 0.041 & 1.0 \end{pmatrix}$
J	1.98×10^{-5}	3.18×10^{-5}

future work. Furthermore, we focus on the Yukawa couplings of quarks living on magnetized D-branes wrapping tori for technical reason, but it is interesting to explore more general background like Calabi-Yau orientifolds.

4. Conclusions

From the view point of string theory, the strong CP and CKM phases are not constants, but they are determined by the axion fields originated from the higher-dimensional gauge fields. It is then natural to ask whether both phases have a common origin and at the same time, the observed value of the CKM phase is compatible with almost vanishing strong CP phase or not.

In this paper, we first proposed the mechanism to relate the strong CP phase with the CKM phase in Type IIB flux vacua with magnetized D7-branes. We demonstrated that the axio-dilaton appearing in the gauge kinetic function and the complex structure moduli in Yukawa couplings on magnetized D7-branes are entangled by certain three-form fluxes which lead to the massless direction in the moduli spaces of the axion-dilaton and one of the complex structure moduli. Note that it is possible to have a common axion field associated with the complex structure moduli in the gauge kinetic function and Yukawa couplings through one-loop threshold corrections to the gauge kinetic function [14–17].

To estimate the value of the CKM phase, we examine the Yukawa couplings on magnetized D-branes wrapping tori on which analytical calculation has been performed in Ref. [9]. It is known that the CP phase is induced by the nonvanishing axion field. If the CP phase $\mathrm{Arg}\{\mathrm{Det}(y_u y_d)\}$ is linearly dependent of the axion as in the Froggatt-Nielsen model [5], the strong CP phase θ_{eff} becomes zero at the origin of the axion field. However, thanks to the non-trivial axion-dependent function of the CP phase $\mathrm{Arg}\{\mathrm{Det}(y_u y_d)\}$ on toroidal background with magnetic fluxes, we find that observed value of the Jarlskog invariant is consistent with the vanishing strong CP phase. In this paper, we focus on the bare CP phases, but radiative corrections as well as the supersymmetry-breaking effects give rise to nonvanishing CP phases in general, which will be one of the important future work. Furthermore, we assume the stabilization of axion field by certain non-perturbative dynamics in a hidden sector. We will leave the detailed axion stabilization for a future work. The relation between the strong CP and CKM phases would be possible for not only the toroidal orientifold background in Type IIB string context, but also more general Calabi-Yau orientifolds in other superstring theory, for instance, Type IIA intersecting D6-brane system and heterotic line bundle models. This is because three-form fluxes give rise to the massless direction in the moduli spaces of the axio-dilaton and the complex structure moduli. Both play an important role of determining the gauge kinetic function as well as the Yukawa couplings. We will report on this interesting work in the future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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