Scotogenic S_3 symmetric generation of realistic neutrino mixing

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(Received 24 April 2019; published 12 August 2019)

Realistic neutrino mixing is achieved at a one-loop level radiatively using $S_3 \times Z_2$ symmetry. The model is comprised of two right-handed neutrinos, maximally mixed to produce the structure of the left-handed Majorana neutrino mass matrix characterized by $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and any value of θ_{12}^0 particular to the tribimaximal (TBM), bimaximal (BM) and golden ratio (GR) or other mixings. A small deviation from this maximal mixing between the two right-handed neutrinos could generate nonzero θ_{13}, shifts of the atmospheric mixing angle θ_{23} from $\pi/4$, and also could correct the solar mixing angle θ_{12} by a small amount altogether in a single step. In this scotogenic mechanism of generating a nonzero θ_{13} by shifting from the maximal mixing in the right-handed neutrino sector, two Z_2 odd inert scalar $SU(2)_L$ doublets were used, the lightest of which can serve as a dark matter candidate.

DOI: 10.1103/PhysRevD.100.035009

I. INTRODUCTION

Neutrinos oscillate owing to their massive nature as established by the oscillation experiments. The mass eigenstates and flavor eigenstates are different and are related by the Pontecorvo, Maki, Nakagawa, Sakata—PMNS—matrix,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta} \\ -c_{23}s_{12} + s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} + s_{23}s_{13}s_{12}e^{i\delta} & -s_{23}c_{13} \\ -s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} + c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{1}$$

Here, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Needless to mention that the mass eigenstates are nondegenerate.

Nonzero θ_{13}, though small in comparison to the other mixing angles, was discovered in 2012 by the short-baseline reactor antineutrino experiments [1]. Before these nonzero θ_{13} results, models were studied in literature that correspond to the tribimaximal (TBM), bimaximal (BM), and golden ratio (GR) mixings (that we now onwards collectively refer to as popular lepton mixings). All these mixings have $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and tuning θ_{12}^0 to the specific values as shown in Table I produced the different mixing patterns viz. TBM, BM, and GR.

Setting $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ in Eq. (1) will yield a general structure for all popular mixing as

$$U^0 = \begin{pmatrix} \cos\theta_{12}^0 & \sin\theta_{12}^0 & 0 \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

The current 3σ global fit [2,3] for θ_{13}, θ_{23}, and θ_{12} as from NuFIT3.2 of 2018 [2] are

$$\begin{aligned} \theta_{12} &= (31.42 - 36.05)^\circ, \\ \theta_{23} &= (40.3 - 51.5)^\circ, \\ \theta_{13} &= (8.09 - 8.98)^\circ. \end{aligned} \tag{3}$$

So popular mixing and nonzero θ_{13} observations are not in harmony. Several model-building exercises have been taking place since the observation of the nonzero θ_{13} to include it in the popular mixing framework. In [4], the possibility of the smallness of θ_{13} and $\Delta m^2_{\rm solar}$ to have a common origin was explored. In some effort [5], a dominant component was characterized by larger oscillation parameters such as $\Delta m^2_{\rm atmos}$ and $\theta_{23} = \pi/4$, whereas the smaller mixing parameters viz. nonzero θ_{13}, θ_{12}, solar

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TABLE I. The values θ_{12}^0 corresponding to various popular lepton mixings namely, TBM, BM, and GR patterns.

Model	TBM	BM	GR
θ_{12}^0	35.3°	45.0°	31.7°

splitting, and deviation of atmospheric mixing from maximality were produced by a smaller seesaw [6] component as a perturbation to the dominant one.[1] In [8,9], the mixing angle $\theta_{13} = 0$ was produced using various symmetries, and nonvanishing θ_{13} was produced by a perturbation to these symmetric forms.

The popular mixings were amended at tree level using a two-component Lagrangian with the discrete symmetries $A4$, S_3 in [10,11]. In these models, type II seesaw yielded the dominant component that gave the popular mixing; corrections to which were offered by type I seesaw subdominant component. A similar enterprise just for the no solar mixing (NSM) case, i.e., $\theta_{12}^0 = 0$ using $A4$ was pursued[2] in [12]. In [13], TBM was obtained radiatively using $A4$. Recent works with realistic neutrino mixings can be found in [14,15].

Here, we discuss a radiative $S_3 \times Z_2$ model.[3] Some earlier works on S_3 in the context of neutrino mass are [16,17]. A neutrino mass with $S_3 \times Z_2$ within left-right symmetry was studied in [18]. A common practice [19] was to find a symmetry among the three neutrinos that can produce a mass matrix that can be expressed as a linear combination of a democratic matrix $M_{\rm dem}$ and an identity matrix I, like $c_1 I + c_2 M_{\rm dem}$ with c_1 and c_2 being two complex numbers. This could serve as a reasonable scenario to start with from which some models obtained realistic mixing through a perturbation to such initial structures [19], whereas in some models [20], various GUT symmetries or extradimensional theories were considered to generate these initial structures, and renormalization group effects at high energies were explored to obtain realistic mixing. Another way [21] of constructing S_3 models is to have a 3-3-1 local gauge symmetry and later on, associate it to a $(B - L)$ extension or use soft breaking of S_3. Since S_3 has irreducible representations of one-dimension and two-dimension, the latter can be used to obtain maximal mixing in the $\nu_\mu - \nu_\tau$ block [22]. Collider signatures of S_3 flavor symmetry was vividly studied in [23]. S_3 models are also studied in the quark sector [24]. Some earlier studies on scotogenic models can be found in [25].

[1]For some earlier models with similar goals, see [7].

[2]The dominant type II seesaw had a vanishing solar splitting; thus, one can make use of degenerate perturbation theory to get a large solar mixing.

[3]A brief account on discrete group S_3 in presented in Appendix A of the paper.

In this work, our objective is to use S_3 to radiatively[4] obtain

1. The structure of the mixing matrix of a popular mixing kind as shown in Eq. (2) that is characterized by $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and θ_{12}^0 of any of the alternatives displayed in Table I.
2. Realistic neutrino mixings, i.e., precisely nonzero θ_{13}, shifts of the atmospheric mixing angle θ_{23} from maximality and tiny corrections to the solar mixing angle θ_{12}.

In this radiative $S_3 \times Z_2$ model, neutrino masses and mixings are generated at one loop. The model has two right-handed neutrinos comprising an S_3 doublet, that are maximally mixed to obtain the structure as required by popular mixings as in Eq. (2). A small deviation from this maximal mixing in the right-handed neutrino sector could produce in a single step nonzero θ_{13}, shifts of θ_{23} from $\pi/4$, and small corrections to θ_{12} as is required by the mixing to be realistic. To achieve this, two Z_2 odd scalars η_i, $(i = 1, 2)$ were required; the lightest among them can be a good dark matter candidate. A similar analysis based on $A4$ was performed, where instead of using deviations from maximal mixing between the two right-handed neutrino states to generate nonzero θ_{13}, small mass splittings between two right-handed neutrinos were used as in [27].

II. THE $S_3 \times Z_2$ MODEL

In the mass basis, the left-handed neutrino Majorana mass matrix is $M_{\nu L}^{\rm mass} = {\rm diag}(m_1, m_2, m_3)$. One can transport this in its flavor basis with the help of the common form of the popular lepton mixing matrix U^0 in Eq. (2) as

$$M_{\nu L}^{\rm flavor} = U^0 M_{\nu L}^{\rm mass} U^{0T} = \begin{pmatrix} a & c & c \\ c & b & d \\ c & d & b \end{pmatrix}. \tag{4}$$

The a, b, c, and d used here are given by

$$\begin{aligned} a &= m_1 \cos^2\theta_{12}^0 + m_2 \sin^2\theta_{12}^0 \\ b &= \frac{1}{2}(m_1 \sin^2\theta_{12}^0 + m_2 \cos^2\theta_{12}^0 + m_3) \\ c &= \frac{1}{2\sqrt{2}} \sin 2\theta_{12}^0 (m_2 - m_1) \\ d &= \frac{1}{2}(m_1 \sin^2\theta_{12}^0 + m_2 \cos^2\theta_{12}^0 - m_3). \end{aligned} \tag{5}$$

Thus,

$$\tan 2\theta_{12}^0 = \frac{2\sqrt{2}c}{b + d - a}. \tag{6}$$

[4]A systematic analysis of radiative neutrino mass models can be found in [26].

TABLE II. All fields along with their respective charges. We confine this model to the neutrino sector only.

	$SU(2)_L$	S_3	Z_2
Leptons			
$L_{e_L} \equiv (\nu_e \quad e^-)_L$	2	1	1
$L_{\zeta_L} \equiv \begin{pmatrix} \nu_\mu & \mu^- \\ \nu_\tau & \tau^- \end{pmatrix}_L$	2	2	1
$N_{\alpha R} \equiv \begin{pmatrix} N_{1R} \\ N_{2R} \end{pmatrix}$	1	2	−1
Scalars			
$\Phi \equiv \begin{pmatrix} \phi_1^+ & \phi_1^0 \\ \phi_2^+ & \phi_2^0 \end{pmatrix}$	2	2	1
$\eta \equiv \begin{pmatrix} \eta_1^+ & \eta_1^0 \\ \eta_2^+ & \eta_2^0 \end{pmatrix}$	2	2	−1

It is essential for a, b, c, and d to be nonzero for the neutrino masses to be realistic and nondegenerate.

Our prime intent is to generate the form of $M_{\nu L}^{\text{flavor}}$ in Eq. (4) radiatively with one loop. Thus, one has to designate each of the fields in our model with particular $S_3 \times Z_2$ quantum numbers. There are two right-handed neutrinos present in the model. Maximal mixing between these two right-handed neutrino fields can produce the desired form of the left-handed Majorana neutrino mass matrix in Eq. (4) that corresponds to $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and θ_{12}^0 of the popular lepton mixing scenarios. After obtaining the form in Eq. (4), we will see, in due course, that a slight shift from this maximal mixing between the right-handed neutrino states is capable of yielding realistic neutrino mixings, viz. a nonzero θ_{13}, deviation of atmospheric mixing θ_{23} from $\pi/4$ as well as small corrections to solar mixing θ_{12}.

The model has the three left-handed lepton $SU(2)_L$ doublets $L_{\zeta_L} \equiv (\nu_\zeta \zeta^-)_L^T$, where $\zeta = e, \mu, \tau$, out of which L_{μ_L} and L_{τ_L} comprise a doublet of S_3 whereas L_{eL} remains a singlet under S_3. Apart from these, there are two Standard Model (SM) gauge singlet right-handed neutrinos $N_{\alpha R}$, ($\alpha = 1, 2$) that transform as a doublet under S_3. The scalar spectrum of the model has a couple of inert $SU(2)_L$ doublet scalars, $\eta_i \equiv (\eta_i^+, \eta_i^0)^T$, ($i = 1, 2$), forming an S_3 doublet (η). We also have two other $SU(2)_L$ doublet scalars, namely $\Phi_j \equiv (\phi_j^+, \phi_j^0)^T$, ($j = 1, 2$), that are combined to form an S_3 doublet (Φ). Besides the S_3, the model also has an unbroken Z_2 symmetry under which all other fields except the right-handed neutrinos and the scalar η are even. After spontaneous symmetry breaking (SSB), ϕ_j get a vacuum expectation value (vev), but η_i do not. Let v_j be the vevs of ϕ_j^0, i.e., $\langle \Phi_j \rangle \equiv v_j$, ($j = 1, 2$). Fields and their specific charges are shown in Table II. We deal with the neutrino sector only in this model. The charged lepton mass matrix is diagonal in the basis in which we perform the analysis, and the entire mixing comes from the neutrino sector.

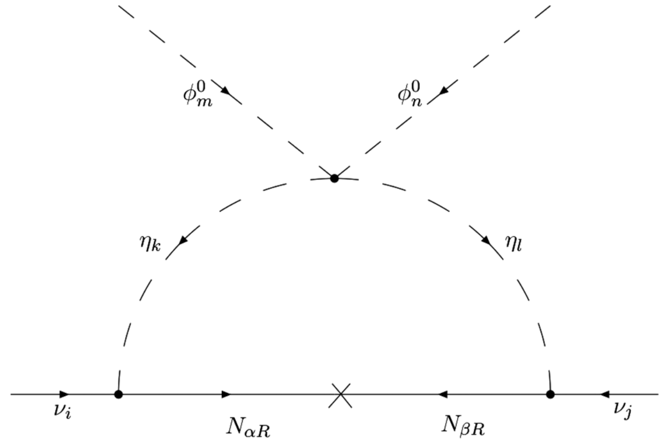

FIG. 1. One-loop scotogenic neutrino mass generation using $S_3 \times Z_2$ symmetry.

Neutrino mass can be generated radiatively at the one-loop level from Fig. 1. The neutrino mass matrix will receive contributions from the following terms of the $S_3 \times Z_2$ invariant scalar potential from the scalar four-point vertex[5]:

$$\begin{aligned} V_{\text{relevant}} \supset\ & \lambda_1[\{(\eta_2^\dagger\phi_2 + \eta_1^\dagger\phi_1)^2\} + \text{H.c.}] \\ & + \lambda_2[\{(\eta_2^\dagger\phi_2 - \eta_1^\dagger\phi_1)^2\} + \text{H.c.}] \\ & + \lambda_3[\{(\eta_1^\dagger\phi_2)(\eta_2^\dagger\phi_1) + (\eta_2^\dagger\phi_1)(\eta_1^\dagger\phi_2)\} + \text{H.c.}]. \end{aligned} \tag{7}$$

Here, all the quartic couplings λ_j ($j = 1, 2, 3$) are taken real.

At all the three vertices of Fig. 1, all symmetries are conserved. The Dirac vertices conserving $S_3 \times Z_2$ can be written as

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = {} & y_1[(\bar{N}_{2R}\eta_2^0 + \bar{N}_{1R}\eta_1^0)\nu_e] \\ & + y_2[(\bar{N}_{1R}\eta_2^0)\nu_\tau + (\bar{N}_{2R}\eta_1^0)\nu_\mu] + \text{H.c.} \end{aligned} \tag{8}$$

Since the left-handed neutrinos ν_{ζ_L} transform as a doublet of S_3 for ($\zeta = \mu, \tau$) and is invariant under S_3 if $\zeta = e$, the Yukawa couplings involved are different for ($\zeta = \mu, \tau$) and $\zeta = e$, namely, y_1 for $\zeta = e$ and y_2 for ($\zeta = \mu, \tau$), respectively.

Let us now have a look at the right-handed neutrino sector. Recall we have two SM gauge singlet right-handed neutrinos, N_{1R} and N_{2R}, that transform as a doublet of S_3. Thus, the $S_3 \times Z_2$ invariant direct mass term for the right-handed neutrinos will look like

[5]Two η are created and two ϕ are destroyed at the scalar four point vertex causing terms of $(\eta^\dagger\phi)(\eta^\dagger\phi)$ nature to be pertinent among other terms in the scalar potential. The complete scalar potential containing all the terms can be found in Appendix B.

$$\mathcal{L}_{\text{right-handed neutrinos}} = \frac{1}{2} m_{R_{12}} [N_{1R}^T C^{-1} N_{2R} + N_{2R}^T C^{-1} N_{1R}]. \tag{9}$$

Thus, the S_3 symmetry allows a symmetric mass matrix with only nonzero off diagonal terms for the right-handed neutrinos. If one allows the soft breaking of S_3 at the scale where the right-handed neutrinos get mass by introducing terms like

$$\mathcal{L}_{\text{soft}} = \frac{1}{2}[m_{R_{11}} N_{1R}^T C^{-1} N_{1R} + m_{R_{22}} N_{2R}^T C^{-1} N_{2R}] \tag{10}$$

to get nonzero diagonal entries, then one can write the right-handed neutrino mass matrix as

$$M_{\nu_R} = \frac{1}{2} \begin{pmatrix} m_{R_{11}} & m_{R_{12}} \\ m_{R_{12}} & m_{R_{22}} \end{pmatrix}. \tag{11}$$

The symmetric structure of the matrix in Eq. (11) also reflects its Majorana nature.

Before moving on, let us have a brief discussion about the dark matter candidates in the model. It is a common practice in literature to stabilize dark matter candidate with discrete symmetries like Z_2. Thus, the Z_2 symmetry is an indication that this model can provide dark matter candidate. Both the right-handed neutrinos and the scalar fields η are odd under Z_2, among which η are chosen lighter than the right-handed neutrinos $N_{\alpha R}$, $(\alpha = 1, 2)$. Although from the m_η^2 term in Eq. (B1), the η_i $(i = 1, 2)$ appear to be degenerate in mass; since the S_3 symmetry is softly broken in the right-handed neutrino sector, it can lead to small mass splitting between the two η_i, $(i = 1, 2)$. The lightest among the two η_i, $(i = 1, 2)$ can be the dark matter candidate.

With the model ingredients ready, at this stage, we are in a position to present a basic description of the left-handed Majorana neutrino mass matrix arising from Fig. 1; the detailed expressions for which will be provided at a later stage of our analysis. To set the stage of the discussion, let us first sketchily indicate how the elements of the left-handed neutrino mass matrix will receive contributions from this one-loop diagram [28] in Fig. 1. Let us make a few simplifying assumptions to make the expressions look less complicated at the moment. For this purpose, let λ commonly represent some combinations of the three quartic couplings given in Eq. (B), i.e., λ_1, λ_2, and λ_3. Also the splitting between the masses of η_1 and η_2 comprising the S_3 doublet is neglected, and m_0 is assumed to be the common mass of them. Further, if the real part of η_j^0 is denoted by η_{Rj} and η_{Ij} be the imaginary part of η_j^0, then difference between the masses of η_{Rj} and η_{Ij} can be taken to be proportional to λv_j and can be small, in general.

It is imperative to note that under S_3, ν_e is invariant, whereas ν_ζ $(\zeta = \mu, \tau)$ transform as a doublet. This feature will manifest through the Yukawa couplings [see Eq. (8)] at the two Dirac vertices, which in its turn will dictate the structure of the left-handed neutrino mass matrix. Let $z \equiv \frac{m_R^2}{m_0^2}$, where m_R is the average mass of the heavy right-handed neutrino states. Since z always appears only in the logarithm, we do not distinguish between the masses of the different right-handed neutrinos for the purpose of defining z throughout. Under this assumption, the second diagonal entry, for example, will have the form,

$$(M_{\nu_L}^{\text{flavor}})_{22} = \lambda \frac{v_m v_n}{8\pi^2} \frac{y_2^2}{m_{R_{22}}} [\ln z - 1]. \tag{12}$$

It is noteworthy that Eq. (12) is valid in the limit $m_R^2 \gg m_0^2$. For $(M_{\nu_L}^{\text{flavor}})_{22}$, as noted earlier in Eq. (8), ν_μ couples only to N_{2R}; thus, at both the Dirac vertices, N_{2R} will couple with ν_μ. Hence, the (2,2) element of the left-handed neutrino mass matrix will get contribution from $m_{R_{22}}$ only. Also, y_2 is the only Yukawa coupling that will appear since we are dealing with ν_μ at both the Dirac vertices for $(M_{\nu_L}^{\text{flavor}})_{22}$. From similar arguments, one can obtain an expression for $(M_{\nu_L}^{\text{flavor}})_{33}$ just by replacing $m_{R_{22}}$ by $m_{R_{11}}$ in Eq. (12).

Let us now concentrate on the off diagonal (2,3) entry. Thus, one has to consider ν_μ at one of the Dirac vertices and ν_τ at the other. From Eq. (8), one can note that ν_μ couples to N_{2R} only, whereas ν_τ does so with N_{1R}. Thus, at one of the Dirac vertices, we will have N_{1R} and N_{2R} at the other. Therefore, off diagonal entries from right-handed neutrino mass matrix will come into play, and $(M_{\nu_L}^{\text{flavor}})_{23}$ will get contributions from $m_{R_{12}}$ in addition to that from $m_{R_{11}}$ and $m_{R_{22}}$. Needless to mention that the Yukawa coupling involved will be y_2 as can be seen from Eq. (8). Thus, one can write

$$(M_{\nu_L}^{\text{flavor}})_{23} = \lambda \frac{v_m v_n}{8\pi^2} \frac{y_2^2 m_{R_{12}}}{m_{R_{11}} m_{R_{22}}} [\ln z - 1]. \tag{13}$$

While writing down Eq. (13), we are taking into account the mass insertion approximation. In a similar spirit, one can write down expressions for (1,1), (1,2), and the (1,3) entries of the left-handed Majorana neutrino mass matrix.

For notational ease, let us absorb everything else present in the rhs of expressions for the elements of the left-handed Majorana neutrino mass matrix, as in Eq. (12) and Eq. (13) except for the Yukawa couplings, quartic couplings, and the vevs in loop contributing factors say $r_{\alpha\beta}$ given by

$$\begin{aligned} r_{11} &\equiv \frac{1}{8\pi^2 m_{R_{11}}} [\ln z - 1], \\ r_{22} &\equiv \frac{1}{8\pi^2 m_{R_{22}}} [\ln z - 1], \\ r_{12} &\equiv \frac{m_{R_{12}}}{8\pi^2 m_{R_{11}} m_{R_{22}}} [\ln z - 1]. \end{aligned} \tag{14}$$

From Eqs. (12), (13), (14), and (7), the left-handed neutrino Majorana mass matrix radiatively generated at one loop as shown in Fig. 1 is

$$M_{\nu_L}^{\text{flavor}} = \begin{pmatrix} \chi_1 & \chi_4 & \chi_5 \\ \chi_4 & \chi_2 & \chi_6 \\ \chi_5 & \chi_6 & \chi_3 \end{pmatrix}, \tag{15}$$

where

$$\begin{aligned}
\chi_1 &\equiv y_1^2[4r_{12}v_1v_2(\lambda_3+\lambda_1-\lambda_2)+(r_{11}v_1^2+r_{22}v_2^2)(\lambda_1+\lambda_2)] \\
\chi_2 &\equiv y_2^2[r_{22}(\lambda_1+\lambda_2)v_1^2] \\
\chi_3 &\equiv y_2^2[r_{11}(\lambda_1+\lambda_2)v_2^2] \\
\chi_4 &\equiv y_1y_2[r_{12}(\lambda_1+\lambda_2)v_1^2+2r_{22}(\lambda_3+\lambda_1-\lambda_2)v_1v_2] \\
\chi_5 &\equiv y_1y_2[r_{12}(\lambda_1+\lambda_2)v_2^2+2r_{11}(\lambda_3+\lambda_1-\lambda_2)v_1v_2] \\
\chi_6 &\equiv y_2^2[2r_{12}(\lambda_3+\lambda_1-\lambda_2)v_1v_2].
\end{aligned} \tag{16}$$

Here, $\langle\Phi_j\rangle \equiv v_j$ with ($j = 1, 2$).

For the left-handed neutrino mass matrix in Eq. (15) to be of the form of Eq. (4), i.e., the structure needed for $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and θ_{12}^0 of the popular mixing kind, we have to set $\chi_1 \neq \chi_2 = \chi_3$ as well as $\chi_4 = \chi_5$. This is achieved when $v_1 = v_2 = v$ and $r_{11} = r_{22} = r$. The condition $r_{11} = r_{22} = r$ when translated in terms of the right-handed neutrino mass matrix in Eq. (11) using Eq. (14) will lead to

$$M_{\nu_R} = \frac{1}{2}\begin{pmatrix} m_{R_{11}} & m_{R_{12}} \\ m_{R_{12}} & m_{R_{11}} \end{pmatrix}. \tag{17}$$

The matrix in Eq. (17) corresponds to maximal mixing in the right-handed neutrino sector. Thus, to get the form of left-handed neutrino mass matrix as in Eq. (4), it is necessary to have $v_1 = v_2 = v$ as well as maximal mixing between N_{1R} and N_{2R}; i.e., we have to set $r_{11} = r_{22} = r$. Implementing these constraints to the general form of the mass matrix in Eq. (15), we get

$$M_{\nu_L}^{\text{flavor}} = v^2\begin{pmatrix} y_1^2[4r_{12}\lambda_{123}+2r\lambda_{12}] & y_1y_2[r_{12}\lambda_{12}+2r\lambda_{123}] & y_1y_2[r_{12}\lambda_{12}+2r\lambda_{123}] \\ y_1y_2[r_{12}\lambda_{12}+2r\lambda_{123}] & y_2^2r\lambda_{12} & y_2^2(2r_{12}\lambda_{123}) \\ y_1y_2[r_{12}\lambda_{12}+2r\lambda_{123}] & y_2^2(2r_{12}\lambda_{123}) & y_2^2r\lambda_{12} \end{pmatrix}. \tag{18}$$

Here, $\lambda_{12} \equiv \lambda_1 + \lambda_2$ and $\lambda_{123} \equiv \lambda_3 + \lambda_1 - \lambda_2$. To get the form of $M_{\nu_L}^{\text{flavor}}$ in Eq. (4), one has to identify

$$\begin{aligned}
a &\equiv y_1^2v^2[4r_{12}\lambda_{123}+2r\lambda_{12}] \\
&= y_1^2v^2[4r_{12}(\lambda_3+\lambda_1-\lambda_2)+2r(\lambda_1+\lambda_2)] \\
b &\equiv y_2^2v^2r\lambda_{12} = y_2^2v^2r(\lambda_1+\lambda_2) \\
c &\equiv y_1y_2v^2[r_{12}\lambda_{12}+2r\lambda_{123}] \\
&= y_1y_2v^2[r_{12}(\lambda_1+\lambda_2)+2r(\lambda_3+\lambda_1-\lambda_2)] \\
d &\equiv y_2^2v^2(2r_{12}\lambda_{123}) = y_2^2v^2[2r_{12}(\lambda_3+\lambda_1-\lambda_2)].
\end{aligned} \tag{19}$$

So far, we are able to obtain the form of left-handed neutrino mass matrix required for $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and θ_{12}^0 of the popular mixing varieties. With this in hand, the obvious follow-up enterprise, as mentioned earlier, will be to obtain realistic mixing viz. nonzero θ_{13}, deviations of the atmospheric mixing angle θ_{23} from $\pi/4$ as well as tiny corrections to θ_{12} also. To get such realistic neutrino mixing, we have to shift from the choice of $r_{11} = r_{22} = r$, i.e., allow the two diagonal entries of the right-handed neutrino mass matrix to slightly differ from each other. In other words, let $r_{22} = r_{11} + \epsilon$, where ϵ is a small quantity. Therefore, one gets back the general form of $M_{\nu R}$ in Eq. (11) characterized by nonmaximal mixing between N_{1R} and N_{2R}. Thus, setting $r_{22} = r_{11} + \epsilon$ is precisely shifting from the maximal mixing between the two right-handed neutrino states. With $v_1 = v_2 = v$ still valid, we can get a dominant component of $M_{\nu_L}^{\text{flavor}}$ as in Eq. (18) denoted M^0 and a smaller contribution M' proportional to ϵ. Hence,

$$M_{\nu_L}^{\text{flavor}} = M^0 + M' \tag{20}$$

with

$$M^0 = v^2\begin{pmatrix} y_1^2[4r_{12}\lambda_{123}+2r_{11}\lambda_{12}] & y_1y_2[r_{12}\lambda_{12}+2r_{11}\lambda_{123}] & y_1y_2[r_{12}\lambda_{12}+2r_{11}\lambda_{123}] \\ y_1y_2[r_{12}\lambda_{12}+2r_{11}\lambda_{123}] & y_2^2r_{11}\lambda_{12} & y_2^2(2r_{12}\lambda_{123}) \\ y_1y_2[r_{12}\lambda_{12}+2r_{11}\lambda_{123}] & y_2^2(2r_{12}\lambda_{123}) & y_2^2r_{11}\lambda_{12} \end{pmatrix}, \tag{21}$$

and

$$M' = \epsilon \begin{pmatrix} x & y & 0 \\ y & x' & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{22}$$

where

$$\begin{aligned} x &\equiv y_1^2 v^2 \lambda_{12} = y_1^2 v^2 (\lambda_1 + \lambda_2) \\ x' &\equiv y_2^2 v^2 \lambda_{12} = y_2^2 v^2 (\lambda_1 + \lambda_2) \\ y &\equiv y_1 y_2 v^2 \lambda_{123} = y_1 y_2 v^2 (\lambda_3 + \lambda_1 - \lambda_2). \end{aligned} \tag{23}$$

M^0 in Eq. (21) will represent the form of left-handed neutrino mass matrix needed for $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and θ_{12}^0 of the popular mixing types as in Eq. (4) when we identify

$$\begin{aligned} a' &\equiv y_1^2 v^2 [4 r_{12} \lambda_{123} + 2 r_{11} \lambda_{12}] \\ &= y_1^2 v^2 [4 r_{12} (\lambda_3 + \lambda_1 - \lambda_2) + 2 r_{11} (\lambda_1 + \lambda_2)] \\ b' &\equiv y_2^2 v^2 r_{11} \lambda_{12} = y_2^2 v^2 r_{11} (\lambda_1 + \lambda_2) \\ c' &\equiv y_1 y_2 v^2 [r_{12} \lambda_{12} + 2 r_{11} \lambda_{123}] \\ &= y_1 y_2 v^2 [r_{12} (\lambda_1 + \lambda_2) + 2 r_{11} (\lambda_3 + \lambda_1 - \lambda_2)] \\ d' &\equiv y_2^2 v^2 (2 r_{12} \lambda_{123}) = y_2^2 v^2 [2 r_{12} (\lambda_3 + \lambda_1 - \lambda_2)] \end{aligned} \tag{24}$$

in the same spirit[6] as was done in case of Eq. (19).

With the help of a nondegenerate perturbation theory, we can calculate the corrections to eigenvalues and eigenvectors of M^0 from M'. The unperturbed flavor basis is given by the columns of the mixing matrix U^0 as shown in Eq. (2). For ease of presentation, it is useful to define

$$\begin{aligned} \gamma &\equiv (b' - 3d' - a') \quad \text{and} \\ \rho &\equiv \sqrt{a'^2 + b'^2 + 8c'^2 + d'^2 - 2a'b' - 2a'd' + 2b'd'}. \end{aligned} \tag{25}$$

Thus, the third ket after receiving first order corrections will take the form,

$$|\psi_3\rangle = \begin{pmatrix} \frac{\epsilon}{\gamma^2 - \rho^2} [\rho(\sqrt{2} y \cos 2\theta_{12}^0 - x' \sin 2\theta_{12}^0) - \gamma \sqrt{2} y] \\ -\frac{1}{\sqrt{2}} [1 + \xi\epsilon] \\ \frac{1}{\sqrt{2}} [1 - \xi\epsilon] \end{pmatrix}. \tag{26}$$

Here, we have used

$$\xi \equiv [\gamma x' + \rho(x' \cos 2\theta_{12}^0 + \sqrt{2} y \sin 2\theta_{12}^0)]/(\gamma^2 - \rho^2). \tag{27}$$

If we consider a CP-conserving scenario then,

$$\sin\theta_{13} = \frac{\epsilon}{\gamma^2 - \rho^2} [\rho(\sqrt{2} y \cos 2\theta_{12}^0 - x' \sin 2\theta_{12}^0) - \gamma \sqrt{2} y]. \tag{28}$$

The expression for a nonzero θ_{13} in terms of the parameters of our model viz. ϵ, the vacuum expectation values v and the quartic couplings λ_i, $(i = 1, 2, 3)$, can be obtained with help of Eqs. (24), (25), and (28).

The shift of θ_{23} from $\pi/4$ can be found from Eq. (26) as

$$\tan\varphi \equiv \tan(\theta_{23} - \pi/4) = \xi\epsilon. \tag{29}$$

The first-order corrections to the first and second ket will contribute to changes in θ_{12}. Defining

$$\beta \equiv \frac{[\frac{y}{\sqrt{2}} \cos 2\theta_{12}^0 + \frac{1}{2}(x - \frac{x'}{2}) \sin 2\theta_{12}^0]}{\rho} \tag{30}$$

will lead to corrected solar mixing angle given by,

$$\tan\theta_{12} = \frac{\sin\theta_{12}^0 + \epsilon\beta \cos\theta_{12}^0}{\cos\theta_{12}^0 - \epsilon\beta \sin\theta_{12}^0}. \tag{31}$$

Needless to mention, expressions for the corrected θ_{12} in Eq. (31) and deviations of θ_{23} from maximal mixing in Eq. (29) can be translated in terms of parameters of this $S_3 \times Z_2$ symmetric model by applying Eqs. (24), (25), (27), and (30).

In our entire analysis, we have taken $r_{\alpha\beta}$, $(\alpha, \beta = 1, 2)$, to be real therefore allowing no CP violation. But one can associate Majorana phases to masses of the right-handed neutrinos; thus, $r_{\alpha\beta}$ can be complex quantities. Therefore, ϵ can also be complex that can give rise to CP violation from Eq. (26).

Finally, we want to make a remark on the flavor changing decays of the charged leptons. For a charged lepton flavor violation (LFV), one requires the part of the Yukawa Lagrangian similar to Eq. (8),

$$\begin{aligned} \mathcal{L}_{\rm LFV} = y_1 [(\bar{N}_{2R} \eta_2^+ + \bar{N}_{1R} \eta_1^+) e^-] \\ + y_2 [(\bar{N}_{1R} \eta_2^+) \tau^- + (\bar{N}_{2R} \eta_1^+) \mu^-] + \text{H.c.} \end{aligned} \tag{32}$$

At a one-loop level, LFV processes can take place through diagrams as shown in Fig. 2. From Eq. (32), it is readily seen that the $\mu^- \to e^-\gamma$, $\tau^- \to e^-\gamma$, and $\tau^- \to \mu^-\gamma$ processes in Fig. 2 are disallowed in the model. Specifically, the η_i and N_α fields needed at the two Yukawa vertices in Fig. 2 for these LFV processes to occur can never be matched, taking into account Eq. (32). Thus, these LFV processes are

[6] We are introducing the primed notation to differentiate from the $r_{11} = r_{22} = r$ case.

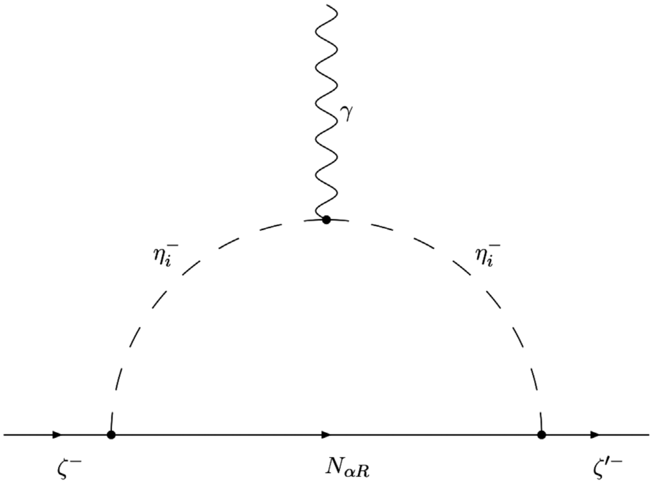

FIG. 2. Decays of the charged leptons at one loop. Here, ζ^- and ζ'^- stands for (e^-, μ^-, τ^-). For charged lepton flavor violating (LFV) processes, $\zeta^- \neq \zeta'^-$. Kinematically, only $\mu^- \to e^-\gamma$, $\tau^- \to e^-\gamma$ and $\tau^- \to \mu^-\gamma$ are allowed and are therefore searched for. S_3 symmetry forbids LFV processes at a one-loop level in this model.

identically zero at the one-loop level as long as the S_3 symmetry is conserved[7].

III. CONCLUSION

In a nutshell, a radiative $S_3 \times Z_2$ symmetric scheme of scotogenic generation of realistic neutrino mixing is put forward. The model has two right-handed neutrinos, N_{1R} and N_{2R}, which when maximally mixed can radiatively yield the form of a left-handed Majorana neutrino mass matrix at one loop characterized by $\theta_{13} = 0$, $\theta_{23} = \pi/4$, and θ_{12}^0 of any of the values specific to the tribimaximal (TBM), bimaximal (BM), and golden ratio (GR) mixing, collectively termed as popular lepton mixings. A small deviation from maximal mixing between the two right-handed neutrino states can produce realistic mixing angles, i.e., nonzero θ_{13}, shifts of the atmospheric mixing angle θ_{23} from $\pi/4$ and small corrections to θ_{12}. There are two inert $SU(2)_L$ doublet scalar fields η_i, $(i = 1, 2)$ in the model. Since the η_i are odd under the action of the unbroken Z_2, the lightest among these two scalars can serve as dark matter.

ACKNOWLEDGMENTS

My sincere thanks to Professor Amitava Raychaudhuri for discussions and valuable suggestions.

APPENDIX A: THE GROUP S_3

It is the permutation group of three objects [29] and therefore, has $3! = 6$ elements. S_3 has two generators A and B that satisfy $A^2 = I = B^3$ and $(AB)(AB) = I$. The group properties can be clearly understood from the group table shown in Table III.

TABLE III. The group table of the discrete symmetry S_3.

	I	A	B	C	D	F
I	I	A	B	C	D	F
A	A	I	C	B	F	D
F	F	C	I	D	A	B
C	C	F	D	I	B	A
D	D	B	A	F	I	C
B	B	D	F	A	C	I

It has two one-dimensional representations 1 and $1'$, as well as one two-dimensional representation 2. The one-dimensional representation 1 is immune to both A and B, whereas $1'$ flips sign when acted by A. In two-dimensions, the group can be represented by the following matrices that obey all the properties discussed so far:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}. \tag{A1}$$

Here, $\omega = e^{2\pi i/3}$ is a cube root of one. With the generators in Eq. (A1), we can construct the rest of the members of the group as

$$C = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \qquad D = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}, \qquad F = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}. \tag{A2}$$

S_3 is characterized by the following product rules:

$$1 \times 1' = 1', \qquad 1' \times 1' = 1, \quad \text{and} \quad 2 \times 2 = 2 + 1 + 1'. \tag{A3}$$

All the matrices M_{ij} in Eqs. (A1) and (A2) obey

$$\sum_{j,l=1,2} \alpha_{jl} M_{ij} M_{kl} = \alpha_{ik}. \tag{A4}$$

Here, $\alpha_{ij} = 0$ if $i = j$ and $\alpha_{ij} = 1$ if $i \neq j$.

Let $\Phi \equiv \binom{\phi_1}{\phi_2}$ and $\Psi \equiv \binom{\psi_1}{\psi_2}$ be two doublets of S_3 which when combined according to Eq. (A3) will yield

$$\phi_1\psi_2 + \phi_2\psi_1 \equiv 1, \qquad \phi_1\psi_2 - \phi_2\psi_1 \equiv 1', \quad \text{and} \quad \begin{pmatrix} \phi_2\psi_2 \\ \phi_1\psi_1 \end{pmatrix} \equiv 2. \tag{A5}$$

Often, we have to work with Hermitian conjugate of the fields. Owing to the properties of the complex

[7]Since neutrino masses are small, one can neglect the LFV processes mediated by the W boson.

representations of S_3, [say, as for B displayed in Eq. (A1)], the Hermitian conjugate of Φ is given by $\Phi^\dagger \equiv \binom{\phi_2^\dagger}{\phi_1^\dagger}$. This $\Phi^\dagger$ when combined with Ψ, keeping Eq. (A3) in mind, we get

$$\phi_2^\dagger\psi_2 + \phi_1^\dagger\psi_1 \equiv 1, \qquad \phi_2^\dagger\psi_2 - \phi_1^\dagger\psi_1 \equiv 1' \quad \text{and} \quad \begin{pmatrix} \phi_1^\dagger\psi_2 \\ \phi_2^\dagger\psi_1 \end{pmatrix} \equiv 2. \tag{A6}$$

Equations (A5) and (A6) play a pivotal role in determining the structure of the mass matrices in the model.

APPENDIX B: THE SCALAR POTENTIAL

The scalar sector of the model, as can be seen from Table II, is comprised of two inert $SU(2)_L$ doublets, $\eta_i \equiv (\eta_i^+ \eta_i^0)^T$, $(i = 1,\ 2)$, forming a doublet under S_3 denoted by η and two other $SU(2)_L$ doublet scalar fields $\Phi_j \equiv (\phi_j^+ \phi_j^0)^T$, $(j = 1, 2)$, represented by Φ, transforming as a doublet under S_3. Under the unbroken Z_2, η is odd, whereas Φ is even. Thus, after SSB, ϕ_j^0 can acquire vevs v_j, $(j = 1, 2)$, but the η_i^0 cannot. The complete scalar potential consisting of all the terms allowed by the SM gauge symmetry and $S_3 \times Z_2$ is given by

$$\begin{aligned} V_{\text{total}} = {} & m_\eta^2(\eta_2^\dagger\eta_2 + \eta_1^\dagger\eta_1) + m_\phi^2(\phi_2^\dagger\phi_2 + \phi_1^\dagger\phi_1) + \tilde\lambda_1(\eta_2^\dagger\eta_2 + \eta_1^\dagger\eta_1)^2 + \tilde\lambda_2(\eta_2^\dagger\eta_2 - \eta_1^\dagger\eta_1)^2 + \tilde\lambda_3(\phi_2^\dagger\phi_2 + \phi_1^\dagger\phi_1)^2 \\ & + \tilde\lambda_4(\phi_2^\dagger\phi_2 - \phi_1^\dagger\phi_1)^2 + \tilde\lambda_5[(\eta_2^\dagger\eta_2 + \eta_1^\dagger\eta_1)(\phi_2^\dagger\phi_2 + \phi_1^\dagger\phi_1)] + \tilde\lambda_6[(\eta_2^\dagger\eta_2 - \eta_1^\dagger\eta_1)(\phi_2^\dagger\phi_2 - \phi_1^\dagger\phi_1)] \\ & + \tilde\lambda_7[(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)] + \tilde\lambda_8[(\eta_1^\dagger\eta_2)(\eta_2^\dagger\eta_1)] + \tilde\lambda_9[\{(\phi_1^\dagger\phi_2)(\eta_2^\dagger\eta_1)\} + \{(\phi_2^\dagger\phi_1)(\eta_1^\dagger\eta_2)\}] + V_{\text{relevant}}, \end{aligned} \tag{B1}$$

where

$$\begin{aligned} V_{\text{relevant}} = {} & \lambda_1[\{(\eta_2^\dagger\phi_2 + \eta_1^\dagger\phi_1)^2\} + \text{H.c.}] \\ & + \lambda_2[\{(\eta_2^\dagger\phi_2 - \eta_1^\dagger\phi_1)^2\} + \text{H.c.}] \\ & + \lambda_3[\{(\eta_1^\dagger\phi_2)(\eta_2^\dagger\phi_1) + (\eta_2^\dagger\phi_1)(\eta_1^\dagger\phi_2)\} + \text{H.c.}]. \end{aligned} \tag{B2}$$

Since at the four-point scalar vertex in Fig. 1, two ϕ are destroyed and two η are created, the terms only of $(\eta^\dagger\phi)(\eta^\dagger\phi)$ type play a crucial role in determining the neutrino mass matrix. Thus, we call these terms as the relevant part of the scalar potential, represented by V_{relevant} in Eq. (B2). The quartic couplings λ_j $(j = 1,\ 2,\ 3)$ appearing in Eq. (B2) were taken to be real for the analysis.

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