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Mass generation from a non-perturbative correction: Massive Neveu–Schwarz field and graviton in (3+1) dimensions

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We show that the massless form fields in (4+1)-dimensional non-perturbation theory of emergent gravity become massive in a perturbative phase without the Higgs mechanism. In particular, an axionic scalar sourced by a non-perturbative dynamical correction is absorbed by the form fields to describe a massive Neveu–Schwarz (NS) field theory on an emergent gravitational $(3\bar{3})$-brane pair. Arguably the novel idea of the Higgs mechanism is naturally invoked in an emergent gravity underlying a CFT_6. Analysis reveals "gravito-weak" and "electro-weak" phases respectively on a vacuum pair in $(4 + 1)$ and $(3 + 1)$ dimensions. It is argued that the massive NS field quanta may govern an emergent graviton on a gravitational 3-brane.

Subject Index B00, B02, B23, B25, B33

1. Introduction

The general theory of relativity (GTR) is governed by a metric tensor dynamics in 4D underlying a pseudo-Riemannian manifold. The GTR is a second-order formulation and is geometric. It describes an interacting classical theory and hence it rules out the possibility of a perturbation quantum theory. Furthermore, the coupled nature of differential field equations in GTR ensures non-linear solutions, which are believed to be sourced by an appropriate energy–momentum tensor possibly underlying a non-linear gauge field. Thus, the quantum field dynamical correction to GTR demands a non-perturbation (NP) formulation at second order.

Einstein gravity is believed to be an emergent phenomenon that is not fundamental but rather a low-energy limit of some theory [1]. Interestingly, the idea of gravity dynamics emerging from gauge theory has been known for quite some time. It has been shown that a non-commutative gauge theory with symmetry group $U(n)$ describes gravity coupled to $SU(n)$ gauge fields in 4 dimensions [2]. The emergent gravity in such a theory was shown to be non-commutative since it encodes the $U(1)$ gauge field degrees of freedom.

It has also been shown that gravity can emerge from a gauge theory in non-commutative (NC) spacetime, which further implies that gravity is a composite picture that emerges from gauge fields in a fuzzy spacetime [3]. This NC field theory / gravity correspondence was further explored for a constant $B_{\mu\nu}$ background, where the emergent gravity picture corresponds to NC gravity in the perturbative phase [4]. Also, electromagnetism (NC) can be realized as a geometrical property of spacetime just like gravity [5]. Self-dual electromagnetism in non-commutative spacetime was also shown to be equivalent to self-dual emergent Einstein gravity, and therefore can shed some light on the instantons scenario in the emergent gravity picture [6].

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Interestingly, the theoretical requirement has been attempted with a dynamical geometric torsion $\mathcal{H}_3$ to second order while keeping the Neveu–Schwarz (NS) form on-shell at an emergent first order [7–9]. The non-perturbative formulation in a gauge choice has led to an emergent metric which turns out to be dynamical. Generically, a geometric torsion in an emergent gravity is a dynamical formulation at order 1.5, where the metric dynamics can gain significance at the expense of the non-perturbative dynamical correction [10]. The idea has led to a non-supersymmetric formulation for an NP theory of quantum gravity in $(4+1)$ dimensions which may be identified with a stabilized string vacuum on a gravitational $(3\bar{3})$-brane pair. In addition, the need for an extra dimension to the GTR in an NP theory of gravity is consistent with the fact that a 10-dimensional type IIA superstring provides a hint towards a supersymmetric non-perturbation M-theory in 11 dimensions [11,12].

In this article we present an elegant tool to generate mass for a gauge field by a geometric torsion in a $(4+1)$-dimensional NP theory. Generically the NP tool has been shown to generate a mass for the NS two-form on a gravitational $(3\bar{3})$-brane pair. In particular, a $(3+1)$-dimensional massive NS field quantum dynamics is argued to describe an emergent graviton in the same spacetime dimension. It is shown that the local degree of the NP correction is absorbed by the NS field and hence the axionic scalar in the NP sector may formally be identified with a Goldstone boson established in the Higgs mechanism [13]. Furthermore, the emergent NP theory, underlying a CFT_6, is revisited with renewed interest to reveal the Higgs mechanism naturally on a gravitational $(4\bar{4})$-brane pair. The emergent $(4+1)$-dimensional curvatures are argued to describe the "gravito-weak" phase of the NP theory underlying the gravitational and weak interactions respectively on a $\bar{4}$-brane and on a 4-brane. The NP correction is exploited to realize a duality between a strongly coupled weak interactions and weakly coupled gravity with a cosmological constant.

2. Glimpse at non-perturbative physics

In the context, a Dirichlet (D)-brane in 10-dimensional type IIA or IIB superstring theory is believed to be a potential candidate to describe a non-perturbative world due to their Ramond–Ramond (RR) charges [14]. The D-brane dynamics is precisely governed by an open string boundary fluctuations, and the Einstein gravity underlying a closed string is known to decouple from a D-brane. Interestingly, for a constant NS background in an open string theory, the $U(1)$ gauge field turns out to be non-linear on a D-brane and has been shown to describe an open string metric [15]. The non-linear gauge dynamics on a D-brane is approximated by the Dirac–Born–Infeld (DBI) action. Various near-horizon black holes have been explored using the open string metric on a D-brane in the recent past [16–23].

However, the mathematical difficulties do not allow an arbitrary NS field to couple to an open string boundary, though it is known to describe a torsion in 10 dimensions. A torsion is shown to modify the covariant derivative and hence the effective curvatures in a superstring theory [24,25]. In the recent past a constant NS field on a D_4-brane has been exploited for its gauge dynamics in an emergent theory [7–9]. In particular, the Kalb–Ramond (KR) field dynamics are used to define a modified derivative $\mathcal{D}_\mu$ uniquely. This has been shown to govern an emergent curvature on a gravitational $(3\bar{3})$-brane pair.

The stringy pair production by the KR form primarily generalizes the established Schwinger pair production mechanism [26]. The non-perturbation tool was vital to explain the Hawking radiation phenomenon [27] at the event horizon of a black hole. The novel idea was applied to the open strings pair production [28] by an electromagnetic field. Furthermore, the mechanism was explored to argue for the M-theory underlying a vacuum creation of a $(D\bar{D})_9$ pair at the cosmological horizon [29].

In particular, the stringy pair production by the KR quanta has been explored in diversified contexts to obtain: (i) a degenerate Kerr [30,31], (ii) a natural explanation of quintessential cosmology [32–35], (iii) an emergent Schwarzschild/topological de Sitter, i.e. a mass pair on $(4\bar{4})$-brane [36,37], and (iv) a fundamental theory in 12 dimensions and an emergent M-theory in 11 dimensions [10]. Recently, higher-dimensional charged black hole solutions to Einstein's equation were found underlying non-commutative geometry, which resembles Reissner–Nordström at large distances and de Sitter spacetime at short distances [38]. A Kerr–de Sitter and Kerr–anti-de Sitter black hole solution has also been found underlying dark matter background assumed to be acting like a perfect fluid [39]. These solutions were also generalized to include a cosmological constant. Generically, the *stringy* nature and the *pair production tool* respectively ensure a quantum gravity phase and a non-perturbative phenomenon. Thus an emergent stringy pair is believed to describe an NP theory of emergent gravity in a 1.5-order formulation. Preliminary investigation has revealed that the NP theory sourced by a CFT_6 may lead to a unified description of all four fundamental forces in nature. Analysis is in progress and is beyond the scope of this article.

3. Two-form (KR↔NS) dynamics

We begin with the KR form $U(1)$ dynamics on a D_4-brane in the presence of a background (open string) metric $G^{(\text{NS})}_{\mu\nu}$ which is known to be sourced by a constant NS form [15]. The gauge-theoretic action is given by

$$S = \frac{-1}{(8\pi^3 g_s)\alpha'^{3/2}} \int d^5x\sqrt{-G^{(\text{NS})}}\, H_{\mu\nu\lambda}H^{\mu\nu\lambda}, \tag{1}$$

where $G^{(\text{NS})} = \det G^{(\text{NS})}_{\mu\nu}$. The KR field dynamics H_3 is absorbed, as a torsion connection, and modifies $\nabla_\mu \to \mathcal{D}_\mu$. The modified derivative leads to an emergent description where the NS field becomes dynamical [7]. It defines a geometric torsion:

$$\begin{aligned} \mathcal{H}_{\mu\nu\lambda} &= \mathcal{D}_\mu B^{(\text{NS})}_{\nu\lambda} + \text{ cyclic in } (\mu,\nu,\lambda) \\ &= H_{\mu\nu\rho}B_\lambda^{(\text{NS})\rho} + H_{\mu\nu\alpha}B_\rho^{(\text{NS})\alpha}B_\lambda^{(\text{NS})\rho} + \cdots \end{aligned} \tag{2}$$

The $U(1)$ gauge invariance of $\mathcal{H}_3^2$ under NS field transformation incorporates a symmetric $f_{\mu\nu} = \bar{\mathcal{H}}_{\mu\alpha\beta}\mathcal{H}^{\alpha\beta}{}_\nu$ correction which in turn defines an emergent metric: $G^{\text{EG}}_{\mu\nu} = G^{(\text{NS})} \pm f_{\mu\nu}$. The generic curvature tensors are worked out using the commutator of the modified derivative operator:

$$\begin{aligned} \left[\mathcal{D}_\mu, \mathcal{D}_\nu\right] A_\lambda &= \left(\mathcal{R}_{\mu\nu\lambda}{}^\rho + \mathcal{K}_{\mu\nu\lambda}{}^\rho\right) A_\rho - 2\mathcal{H}_{\mu\nu}{}^\rho\, \mathcal{D}_\rho A_\lambda, \\ \left[\mathcal{D}_\mu, \mathcal{D}_\nu\right] \psi &= -2\, \mathcal{H}_{\mu\nu}{}^\rho\, \mathcal{D}_\rho \psi, \end{aligned} \tag{3}$$

where $\mathcal{R}_{\mu\nu\lambda}{}^\rho$ denotes the Riemann tensor. For a constant metric the Riemann tensor becomes trivial. $\mathcal{H}_3$ ensures an NS field dynamics in an emergent metric scenario. The fourth-order curvature tensor $\mathcal{K}_{\mu\nu\lambda\rho}$ can be split into a symmetric pair and a non-symmetric pair under an interchange of the first and second pair of indices. The irreducible curvatures have been worked out [10] to obtain an emergent NP theory of gravity for an on-shell NS field.

4. Mass generation as a non-perturbation effect

We begin with an NP theory of emergent gravity in $(4 + 1)$ dimensions underlying a geometric torsion $\mathcal{H}_3$ in a 1.5-order formulation [10]. The effective action has been shown to govern an NS field dynamics in an emergent first-order (perturbation) gauge theory and a local geometric torsion $\mathcal{H}_3$ in a second-order NP theory. It is given by

$$S_{\rm NP} = \frac{1}{\kappa'^3}\int d^5x\,\sqrt{-g}\,\left(\mathcal{K} - \frac{1}{48}\,\mathcal{F}_4^2\right),$$
$$\mathcal{F}_4 = \sqrt{2\pi\alpha'}\big(d\mathcal{H}_3 - \mathcal{H}_3 \wedge \mathcal{F}_1\big). \tag{4}$$

Equivalently, the emergent theory may be described by the geometric form(s). We set $\kappa'^2 = (2\pi\alpha') = 1$ in this article. Then the effective actions are:

$$S_{\rm NP} = -\frac{1}{12}\int \sqrt{-g}\,\left(\mathcal{H}_{\mu\nu\lambda}\mathcal{H}^{\mu\nu\lambda} + 6\big(\mathcal{D}_\mu\psi\big)\big(\mathcal{D}^\mu\psi\big)\right),$$
$$= -\frac{1}{4}\int \sqrt{-g}\,\left(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + 2\big(\mathcal{D}_\mu\psi\big)\big(\mathcal{D}^\mu\psi\big)\right). \tag{5}$$

The first term in all three actions of Eqs. (4)–(5) sources an emergent metric and hence a torsion-free geometry in the absence of the second term there. A propagating geometric torsion is described by the second term, which is indeed a dynamical NP correction. The emergent curvature scalar $\mathcal{K}$ and its equivalent Lorentz scalars constructed from the geometric forms $\mathcal{H}_3$ and $\mathcal{F}_2$ can govern an emergent metric. Each of them possesses three local degrees in an emergent first-order formulation. The $\mathcal{F}_4$ is a Poincaré dual to a dynamical axionic scalar field ψ and possesses one local degree in an emergent second-order formulation. Together they describe four local degrees in an NP theory of an emergent $D = 5$ theory of gravity. In addition, the NP formulation is described by an appropriate topological coupling from

$$\left(B_2^{\rm (KR)} \wedge \mathcal{H}_3\ ,\ \ B_2^{\rm (NS)} \wedge H_3\ ,\ \ B_2^{\rm (NS)} \wedge \mathcal{F}_2 \wedge d\psi\right).$$

A geometric $\mathcal{F}_2$ in an emergent theory underlies the $U(1)$ gauge symmetry and is given by

$$\mathcal{F}_{\mu\nu} = \big(\mathcal{D}_\mu A_\nu - \mathcal{D}_\nu A_\mu\big) = \big(F_{\mu\nu} + \mathcal{H}_{\mu\nu}{}^{\lambda}A_\lambda\big), \tag{6}$$

where $F_{\mu\nu} = \big(\nabla_\mu A_\nu - \nabla_\nu A_\mu\big)$. The geometric forms $\mathcal{F}_2$ and $\mathcal{H}_3$ are worked out for their gauge-theoretic counterparts. The Lorentz scalar for a geometric two-form may be re-expressed with a mass (squared) matrix represented by a symmetric (emergent) curvature tensor of order two. It is given by

$$\mathcal{F}^2_{\mu\nu} = \left(F^2_{\mu\nu} - \mathcal{K}^{\mu\nu}A_\mu A_\nu + \frac{\epsilon^{\mu\nu\lambda\alpha\beta}}{\sqrt{-g}}A_\mu F_{\nu\lambda}\mathcal{F}_{\alpha\beta}\right), \tag{7}$$

where the symmetric curvature tensor of order two may be expressed in terms of a geometric three-form and its Poincaré dual. They are:

$$\mathcal{K}^{\mu\nu} = -\frac{1}{4}\mathcal{H}^{\mu\alpha\beta}\mathcal{H}_{\alpha\beta}{}^{\nu} = \big(g^{\mu\nu}\mathcal{F}_2^2 + 2\mathcal{F}^{\mu\lambda}\mathcal{F}_\lambda{}^{\nu}\big). \tag{8}$$

The geometric two-form in an emergent non-perturbation theory, Eq. (5), is replaced by the gauge-theoretic forms of Eq. (7). A priori, the effective non-perturbative dynamics is re-expressed as:

$$S_{\rm NP} = -\frac{1}{4}\int \sqrt{-g}\left[F_{\mu\nu}^2 - \mathcal{K}^{\mu\nu}A_\mu A_\nu + 2\big(\nabla_\mu\psi\big)^2\right] + \int \Big(A_1\wedge F_2\wedge F_2 - B_2^{(\rm NS)}\wedge H_3\Big). \tag{9}$$

At first sight the emergent curvature tensor $\mathcal{K}^{\mu\nu}$ appears to a mass (squared) matrix. A count of the local degrees enforces $\mathcal{F}_4 = 0$ in the effective gauge theory of Eq. (9). Thus a geometric torsion turns out to be a constant, which in turn defines a perturbative vacuum. However, $\mathcal{F}_4 \neq 0$ in an emergent gravity, Eq. (8), turns out to be non-trivial. Alternately, the perturbative gauge vacuum may be realized in a gauge choice for $\mathcal{F}_4 = 0$. A constant $\mathcal{H}_3$ leads to a constant $\mathcal{K}^{\mu\nu}$ which is diagonalized. Thus, $\mathcal{K}^{\mu\nu}$ is proportional to $g^{\mu\nu}$ in a perturbation theory:

$$\mathcal{K}^{\mu\nu} = m_1^2\, g^{\mu\nu} = \frac{1}{5}g^{\mu\nu}\mathcal{K}, \tag{10}$$

where m_1^2 is a proportionality constant. It assigns a mass to A_μ at the expense of a dynamical non-perturbative correction. Interestingly the non-perturbative tool to generate a mass for a gauge field is remarkable. In fact it helps to generate mass $m_p = \sqrt{\mathcal{K}/d}$ for a generic higher p-form field in a gauge theory in d dimensions. Furthermore, a mass m_1 can also be derived from a geometric two-form in an appropriate combination—Eq. (8). With a proportionality constant $\tilde{m}_1^2$, the symmetric tensor $\mathcal{K}^{\mu\nu} = \tilde{m}_1^2 g^{\mu\nu}$ and the curvature scalar $\mathcal{K} = 3\mathcal{F}_{\mu\nu}^2$. Then the mass $\tilde{m}_1$ for the A_μ field is re-expressed generically in d dimensions. It is given by

$$\tilde{m}_1^2 = \frac{d-2}{d}\,\mathcal{F}_{\mu\nu}^2. \tag{11}$$

It can be checked that $\tilde{m}_1 = m_1$ and hence the mass of a one-form is uniquely defined in a perturbative gauge theory using an NP technique. The Poincaré dual of a four-form ensures that the local degree of an axionic scalar signifying an NP dynamics is absorbed to generate a massive gauge field in a perturbation gauge theory which is equivalently described by a massless gauge field in an NP theory of emergent gravity.

The relation between an NP theory like Eq. (5) and massive gauge theory may provide a hint toward a strong–weak coupling duality symmetry [40] in an emergent gravity. Schematically, the wrong or unstable vacuum perturbation vacuum has been identified with a stable NP vacuum in Fig. 1. The dynamical axion is believed to describe the strong coupling regime, while a non-zero mass for the gauge field ensures a weak force in a perturbation theory. Nevertheless, detailed calculations for the strong and weak couplings need further attention. Interestingly, the axion in an NP theory may be identified with a Goldstone boson in a spontaneous local $U(1)$ symmetry-breaking phase of a peturbative vacuum. Then the NP theory effective action, Eq. (5), may formally be re-expressed in terms of a massive gauge-theoretic perturbative vacuum and is given by

$$S_{\rm PG} = -\frac{1}{4}\int d^5x\sqrt{-G}\ \left(F_2^2 - m_1^2 A^2\right) - \int \Big(A_1\wedge F_2\wedge F_2 + B_2^{(\rm KR)}\wedge \mathcal{H}_3\Big), \tag{12}$$

where $\mathcal{F}_2 \to F_2$ in a perturbation gauge theory. Importantly, a massless gauge field A_μ in an emergent $D = 5$ NP theory of gravity becomes massive at the expense of an NP dynamics. The non-perturbative

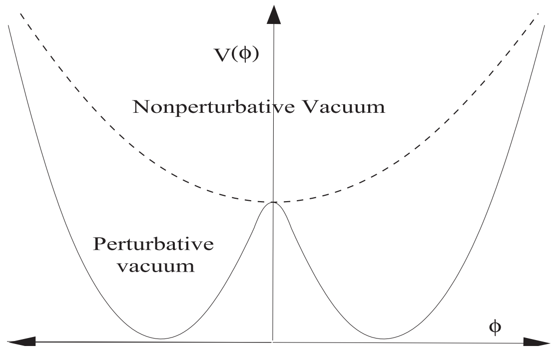

Fig. 1. Potential variation shows that a non-perturbative stable vacuum may be viewed as a perturbative unstable vacuum.

tool for mass generation of a gauge field in a perturbative vacuum is remarkable and appears to be a generic feature for higher forms. It is believed to be a viable NP tool to explore new physics underlying a strong–weak coupling duality. A massive gauge field dynamics for its Poincaré dual is worked out to assign a mass m_2 to the KR field in a perturbative gauge theory. Computation of a mass (squared) matrix for an NS field may be directly worked out from the curvature scalar:

$$\mathcal{K} \approx -\frac{1}{4}\left(H^{\lambda}{}_{\alpha\beta}H^{\alpha\beta}{}_{\rho}\right) B^{(\mathrm{NS})}_{\delta\lambda} B^{\delta\rho}_{(\mathrm{NS})}. \tag{13}$$

In a gauge choice for a non-propagating geometric torsion, the gauge-theoretic H_3 turns out to be a constant for a perturbative vacuum within a non-perturbative formulation. This is due to the fact that the NS field is covariantly constant on a D_4-brane where ∇_μ is an appropriate covariant derivative. Thus the mass (squared) matrix for the NS field in Eq. (13) can be diagonal and hence is proportional to g^{λ}_{ρ}. It implies that

$$\left(H^{\lambda}{}_{\alpha\beta}H^{\alpha\beta}{}_{\rho}\right) = m_2^2\, g^{\lambda}_{\rho}. \tag{14}$$

At this juncture we recall a transition from the KR gauge theory on a D_4-brane defined with a constant NS background with that of an NP formulation of an emergent gravity on a gravitational $(3\bar{3})$-brane pair [7,8]. Generically, it underlies a correspondence between a non-perturbation emergent gravity on a gravitational $(4\bar{4})$-brane pair and a perturbation CFT on a D_5-brane. The boundary/bulk correspondence $\mathrm{NP}_5/\mathrm{CFT}_6$ may be summarized with the relevant forms:

$$\left[\mathcal{H}_3\ ,\ \mathcal{F}_4\ ,\ B_2^{(\mathrm{KR})}\right]_{\mathrm{NP}} \longleftrightarrow \left[\ H_3\ ,\ B_2^{(\mathrm{NS})}\right]_{\mathrm{CFT}}.$$

The dynamical correspondence is primarily between a KR form in the world-volume gauge theory and an NS form in superstring theory. Equation (14) further ensures that a mass for the NS field is sourced by the KR field dynamics. Similarly, the analysis that follows from the derivation of the effective action in Eq. (12) confirms that a mass for a KR field is indeed sourced by an NS field dynamics. Both of them are two-forms and they are different due to their differences in backgrounds or connections. Intuitively the dynamical correspondence of Eq. (15) leading to two different formulations may be viewed with a single two-form with two different names for their masses in a perturbative vacuum.

The dynamical correspondence between a perturbative gauge theory and a non-perturbative emergent gravity is remarkable. It signifies a strong/weak coupling duality [40] between the two different formulations underlying a two-form gauge theory. The NP theory of emergent gravity is purely governed by $\mathcal{H}_3$ and hence generically describes a torsion geometry. However, in a gauge choice $\mathcal{F}_4 = 0$, the emergent gravity describes a torsion-free geometry purely sourced by a dynamical NS field.

A realization of the perturbative vacuum of Eq. (12) within an NP theory may be described for a KR field. It is given by

$$S_{\rm PG} = -\frac{1}{12}\int d^5x\sqrt{-G}\,\left(H_{\mu\nu\lambda}H^{\mu\nu\lambda} - m_2^2 B^2_{\rm (KR)}\right) - \int \left(\left(\mathcal{F}_2 - B_2^{\rm (NS)}\right)\wedge H_3\right). \tag{15}$$

The first term in the bulk topological action is a total divergence. However, it regains significance at the $4D$ boundary where the coupling $\left(B_2 \wedge \mathcal{F}_2\right)$ may be identified with the BF toplogical theory as discussed in Refs. [41,42]. A massive KR form in a perturbation gauge theory is generated by an NP correction sourced by a propagating geometric torsion which turns out to be an axion in $5D$. The NP tool to generate mass for a form field underlying a geometric torsion in a 1.5-order formulation is thought-provoking.

The emergent gravity scenarios [7–9,30–37] ensure that all the NP phenomena are sourced by a lower-dimensional D_p-brane whose fundamental unit is a D-instanton. Interestingly, in a recent article [10], the NP phenomenon has been shown to be sourced by the dynamics of the $\mathcal{H}_3$ potential in a 1.5-order formulation. The dynamical effect incorporates a quantum correction to the torsion-free vacua underlying an emergent metric. The correction breaks the Riemannian geometry and hence is hidden to the GTR underlying an emergent 3-brane universe within a gravitational $(3\bar{3})$-brane pair.

The NP idea leading to mass generation suggests that a dynamical axion (quintessence) or generically a higher essence is hidden to an emergent 3-brane universe and hence its significance to the GTR can only be revealed with a topological coupling.

5. Higgs mechanism in emergent gravity

We begin by recalling the perspectives of a CFT underlying a KR gauge theory on a D_5-brane. The gauge-theoretic vacuum may equivalently be described by an a priori massless NS form in an emergent $6D$ perturbation theory [36]. A pair-symmetric emergent curvature tensor of order four has been shown to be sourced by an NS field in an emergent (first-order) perturbation theory and possesses six local degrees. It has been shown to describe a torsion-free geometry and has been argued to describe a Riemann-type curvature in $6D$. A dynamical correction by $\tilde{\mathcal{F}}_4^2$ in an emergent gravity theory incorporates four non-perturbative local degrees. The effective dynamics is described by ten local degrees and is given by

$$S = \int d^6x\,\sqrt{-\tilde{g}}\,\left(\tilde{\mathcal{K}} - \frac{1}{48}\,\tilde{\mathcal{F}}_4^2\right). \tag{16}$$

Interestingly, the local degrees of an NS field in $6D$ precisely match with the local degrees of a metric tensor in $5D$ and a scalar field presumably underlying a quintessence. Generically, a two-form (NS field) theory in the bulk can be completely mapped to the boundary dynamics underlying a metric tensor and a scalar field ϕ. The bulk/boundary correspondence in an emergent gravity formulation on a gravitational $(4\bar{4})$-brane pair is remarkable. It is believed to attribute Riemannian geometry possibly at the expense of a local $U(1)$ gauge symmetry. We digress to mention that attempts have been made to use the gauge principle to realize Riemannian geometry in the recent past [45]. The effective action in the case is given by

$$S = \int_{4\bar{4}} d^5x\, \sqrt{-G}\, \Big(\mathcal{R} \; - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} \\ -\big(\mathcal{D}_\mu \Phi\big)^\star \big(\mathcal{D}^\mu \Phi\big) \; - V(\Phi, \Phi^\star)\Big). \tag{17}$$

The complex scalar field $\Phi = \frac{1}{\sqrt{2}}\,(\phi + i\psi)$ is defined with two real scalar fields, where ψ denotes an axionic scalar sourced by an NP correction. A geometric two-form in $5D$, though derived from NP dynamics in $6D$, describes a perturbative field strength. This is due to the fact that the $\mathcal{H}_3$ dynamics cannot be realized by $\mathcal{F}_2$ in $5D$. The canonical potential V is sourced by the gravitational interaction in $6D$. An explicit form may be assigned to V by taking account of the self-interaction of the complex scalar field with a wrong sign for the mass term underlying an unstable perturbative vacuum. It may suggest that the Higgs mechanism may find a natural place on a gravitational $(4\bar{4})$-brane pair. The potential may explicitly be given by

$$V(\Phi, \Phi^\star) \; = \Big(m^2 \big(\Phi^\star \Phi\big) - \lambda^2 \big(\Phi^\star \Phi\big)^2 \Big), \tag{18}$$

where m and λ are real constants. The emergent theory of Eq. (17) with the potential in Eq. (18) remains invariant under a global $U(1)$ symmetry: $\Phi \to e^{i\theta}\Phi$. The global $U(1)$ is replaced with a local $U(1)$ symmetry with a minimal gauge coupling in the action: $D_\mu \equiv \big(\mathcal{D}_\mu + ieA_\mu\big)$. Explicitly, the dynamics leading to an unstable vacuum is given by

$$S = -\int_{4\bar{4}} d^5x\, \sqrt{-G}\, \Big[\, \mathcal{R} \; - \frac{1}{4}\mathcal{F}^2_{\mu\nu} - \big(D_\mu \Phi\big)^\star \big(D^\mu \Phi\big) \\ - \; m^2 \big(\Phi^\star \Phi\big) + \lambda^2 \big(\Phi^\star \Phi\big)^2 \,\Big]. \tag{19}$$

The interaction energy function $V(\Phi^\star, \Phi)$ at its minima satisfies the equation of a circle:

$$\phi^2_{\text{min}} + \psi^2_{\text{min}} = \Big(\frac{m}{\lambda}\Big)^2 .$$

Thus a large number of stable ground/vacuum states, underlying the local $U(1)$ symmetry, are described by the circle equation. Any particular vacuum state, i.e. $\phi_{\text{min}} = \big(m/\lambda\big)$ and $\psi_{\text{min}} = 0$, spontaneously breaks the local $U(1)$ symmetry in an emergent geometric theory. In fact, the local symmetry-breaking phenomenon, i.e. the Higgs mechanism, takes place at the event horizon of an emergent black hole which is identified as a stable vacuum.

A shift from an unstable (a non-perturbation) vacuum, Eq. (21), to a stable (perturbative) vacuum may be realized with redefined real scalar fields: $\eta = \phi - \big(m/\lambda\big)$ and $\xi = 0$. The action is re-expressed in terms of η and ξ fields for a stable vacuum and is known to generate the mass term for the gauge field A_μ in addition to a few non-sensible interactions. For instance, see Ref. [13] for the detailed nature of interactions in the symmetry-breaking phase underlying the Higgs mechanism. The non-sensible interaction terms can be gauged away completely by the local $U(1)$ invariance under $\Phi \to \Phi'$ in the action of Eq. (21). In a gauge choice of $\Phi' = \big(\phi\cos\theta - \psi\sin\theta\big)$, i.e. restricting to the real parts, the complete perturbation theory on a gravitational pair is given by

$$S_{\text{PG}} = \int_{4\bar{4}} d^5x\sqrt{-G}\, \Big[\Big(\mathcal{R} - \frac{m^4}{4\lambda^2}\Big) + \frac{e^2}{2}\Big(\eta + \frac{m}{\lambda}\Big)^2 A^2 \\ - \frac{1}{4}\mathcal{F}^2_{\mu\nu} - \frac{1}{2}\big(\mathcal{D}\eta\big)^2 + \frac{1}{2}m^2\eta^2 + \big(m\lambda\big)\eta^3 + \frac{1}{4}\lambda^2\eta^4 \Big]. \tag{20}$$

This implies that a cosmological constant appears to possess its origin in the symmetry-breaking phase and is sourced by the Higgs mechanism. Keeping track of the four local degrees in $5D$ emergent gravity, the effective dynamics may explicitly be given on a 4-brane within a vacuum pair of gravitational brane/anti-brane. Thus some of the undesirable emergent curvatures are assigned to an anti 4-brane. This is equivalent to a consistent truncation of the effective action defined with a massive gauge field. It may suggest that analysis under a CFT leads to a study of the Higgs mechanism naturally in an emergent gravity in $5D$. Then the effective dynamics on an emergent gravitational 4-brane in the presence of a background $\bar{4}$-brane is re-expressed as

$$\begin{aligned} S_{\rm PG} = -\frac{1}{4}\int_4 d^5x\,\sqrt{-G}\left[\mathcal{F}^2_{\mu\nu}\;-\;\frac{e^2}{2}\left(\eta+\frac{m}{\lambda}\right)^2 A^2\right] \\ +\int_{\bar{4}} d^5x\,\sqrt{-G}\,\Bigg[\,\left(\mathcal{R}\;-\;\frac{m^4}{4\lambda^2}\right)\;-\frac{1}{2}(\mathcal{D}\eta)^2 \\ +\;\frac{1}{2}m^2\eta^2+\big(m\lambda\big)\eta^3+\frac{1}{4}\lambda^2\eta^4\,\Bigg]. \end{aligned} \tag{21}$$

The gauge field on an emergent 4-brane universe acquires a mass $M=\frac{e}{\sqrt{2}}\left(\eta_0+\frac{m}{\lambda}\right)$ via the Higgs mechanism, where the Higgs field takes a constant η_0 there. Apparently the local degree of the self-interacting Higgs scalar is described on an anti 4-brane and is hidden to the 4-brane universe. Thus the Higgs field underlying an NP formulation of emergent gravity may be identified with a missing scalar in a $5D$ metric theory. It is inspiring to interpret the Higgs scalar as a hidden essence to the gravitation theory in $5D$. The scalar field, being a generalized coordinate, determines the thickness of the brane/anti-brane configuration.

In an NP decoupling limit a gravitational 4-brane becomes independent from the anti 4-brane and hence $\eta\to\eta_0$. The effective dynamics of a 4-brane and anti 4-brane are approximated in the limit to yield

$$\begin{aligned} S_4 \to -\frac{1}{12}\int\; d^5x\,\sqrt{-G}\,\left(\mathcal{H}^2_{\mu\nu\lambda}\;-\tilde{M}^2 B^2_{\rm (NS)}\right) \\ \text{and} S_{\bar{4}} \to \int\; d^5x\,\sqrt{-G}\;(\mathcal{R}-\Lambda), \end{aligned} \tag{22}$$

where $\Lambda=(m^4/4\lambda^2)$ is a constant. A mass for an NS field ensures short-range interactions. Thus the effective dynamics on a 4-brane may be identified with a weak interacting phase for the NS boson in a decoupling limit. Remarkably, an anti 4-brane effective dynamics is purely governed by the Riemannian geometry in the limit. The idea of a Higgs mechanism with a two-form field has been explored in the past for both commutative and NC gauge theories [43,44]. Generically, the action in Eq. (21) signals a "gravito-weak" phase within an NP theory.

Furthermore, the emergent gravity on a 4-brane may further be viewed on a gravitational $(3\bar{3})$-brane pair. The effective action is given by

$$\begin{aligned} S_{\rm PG} = -\frac{1}{12}\int_3 d^4x\sqrt{-G}\,\left(\mathcal{H}^2_3\;-\tilde{M}^2 B^2_{\rm (NS)}\right) \\ -\;\frac{1}{4}\int_{\bar{3}}\sqrt{-G}\,\mathcal{F}^2_2-\int_{3\bar{3}}\,B^{\rm (NS)}_2\wedge\mathcal{F}_2. \end{aligned} \tag{23}$$

Four local degrees in $5D$ may rightfully be governed by two local degrees of a massive NS field on an emergent 3-brane and two for a massless gauge field A_μ on an anti 3-brane. This is due to the fact that GTR and their parallel are described by two local degrees each in an emergent scenario. Two local degrees of a massive NS field in $(3+1)$ dimensions is an NP phenomenon as the mass is generated by an NP-local degree in $5D$. The correspondence between the NS field in $5D$ and a metric field in $4D$ with a quintessence scalar further reconfirms two local degrees of a massive NS field in $(3+1)$ dimensions. It suggests that the massive NS field quanta in $(3+1)$ dimensions with a hidden NP axion may be a potential candidate to describe a graviton in $4D$.

A mass for an NS field in the action of Eq. (23) ensures that an emergent 3-brane may formally be identified with the weak interacting NS boson, whose role is analogous to the gauge bosons $(W^{\pm}, Z^0)$ in the standard model for particle physics. The dynamics on an anti 3-brane governs a $U(1)$ gauge theory and may be identified with an EM vacuum. Remarkably, the complete dynamics of Eq. (23) may a priori be viewed via "electro-weak" interactions.

On the other hand, the topological term in Eq. (23) precisely describes a coupling between an emergent gravitational 3-brane and an anti 3-brane within a vacuum pair. Generically, an emergent gravity on a 3-brane underlying a Riemann curvature may be derived from the anti 4-brane, Eq. (22). In a decoupling limit the quintessence freezes to describe the GTR. For constant values, $\eta_1 > \eta_0 > \eta_{-1}$, i.e. for $\eta_1 = (1.707)\eta_0$ and $\eta_{-1} = (0.293)\eta_0$, the non-perturbation correction decouples to yield

$$S_3 = \int d^4x \sqrt{-G}\ (\mathcal{R} - \Lambda_{\text{eff}}), \tag{24}$$

where

$$\Lambda_{\text{eff}} = \Lambda\ - \frac{\eta_o^2}{2}\Big[\big(m + \ \lambda\eta_1\big)\big(m + \ \lambda\eta_{-1}\big)\Big]. \tag{25}$$

Thus the Higgs scalar in an NP-decoupling limit ensures a small cosmological constant. Analysis suggests that the Einstein–Hilbert action in the presence of a small non-zero value for Λ_{eff} may alternately be realized by the Higgs phase of an NS field presumably underlying a "gravito-weak" interaction on a gravitational $(3\bar{3})$-brane pair. The Higgs scalar η and the scalar derived from $\mathcal{R}$ in Eq. (22) are identified as the quintessence(s) for two emergent pairs of $4D$ brane dynamics. Presumably this provides a hint toward four parallel brane-universes in $4D$ underlying an NP theory in $6D$. The approach of unifying the geometry of antisymmetric gauge fields and gravity [46] as well the unification idea [47] underlying a two-form CFT is thought-provoking and is believed to reveal new physics.

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