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Holographic computation of quantum corrections to the bulk cosmological constant

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We explore the construction of the dual bulk theory in the flow equation approach. We compute the vacuum expectation value of the Einstein operator at the next-to-leading order in the $1/n$ expansion using a free O(n) vector model. We interpret the next-to-leading correction as the quantum correction to the cosmological constant of the anti-de Sitter space. We finally comment on how to generalize this computation to matrix elements of the Einstein operator for excited states.

Subject Index B21

1. Introduction

Two decades have passed since the anti-de Sitter / conformal field theory (AdS/CFT) correspondence was discovered [1] (see Refs. [2–4] for reviews). This is a theoretical realization of holography [5,6] and provides an alternative framework to explore strongly coupled gauge theory or quantum gravity from its dual weakly coupled theory from a computable relation of observables [7,8]. Even though it is very difficult to give a complete proof for the AdS/CFT correspondence, as is usual with a strong–weak duality, there have been several attempts to deepen our understanding of the duality by extending a boundary CFT to a bulk gravitational one [9–11] (see also Refs. [12–14]).

One of the recent focuses in the study of the AdS/CFT correspondence is on how diffeomorphism invariance is encoded in a boundary CFT and the Einstein equation is reproduced from boundary data. Such a study was initiated at the linear order of perturbation around the AdS background by using the entanglement entropy [15–17]. A recent study in Ref. [18] elegantly reproduced the Einstein equation with a fixed gauge at the second order by incorporating a geometrical identity [19] (see also Ref. [20]). In the holographic renormalization group approach using the local renormalization group (RG) [21] (see Refs. [22–25], and also Ref. [26] and the references therein for a review), it was shown in an abstract way that the bulk diffeomorphism invariance is fully encoded in the form of the Poisson algebra of the RG Hamiltonian by its Wess–Zumino consistency condition [27,28].

In this setting, this paper aims to propose a new scheme to compute bulk dynamical observables from a boundary CFT by employing a new approach to AdS/CFT incorporating a flow equation [29–32], which was introduced to smear operators so as to resolve the UV divergence arising in the coincidence limit [33–35]. One of the virtues in the flow equation approach is to access a dual

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geometry directly even in a non-conformal case, which emerges after the metric operator condenses [29,30]. This enables one to elucidate that the boundary conformal transformation converts to the bulk AdS isometry precisely after taking into account the quantum effect at the boundary [31] (see also Refs. [36,37]), and to provide an AdS geometry whose boundary is a general conformally flat manifold [32]. In this paper we pursue this direction further in order to compute the quantum correction to the cosmological constant of the bulk AdS space through the vacuum expectation value of the Einstein tensor at the next-to-leading order of the $1/n$ expansion. This may provide a deeper understanding of the bulk dynamics in the proposal.

The rest of the paper is outlined as follows. In Sect. 2, we explain our approach to define bulk operators from a boundary CFT containing a scalar primary operator though the flow equation. We introduce various "quantum operators" here. In Sect. 3, we present our results. We first show that the vacuum expectation value of the metric operator describes the bulk AdS space at the leading order of the $1/n$ expansion for the free massless $O(n)$ vector model. We then calculate the next-to-leading order corrections and show that, while the metric operator receives no corrections, the Einstein tensor has $1/n$ correction proportional to the metric, which can be interpreted as a quantum correction to the cosmological constant of the AdS space. In Sect. 4, we explain why the quantum correction to the Einstein tensor is proportional to the metric, using the conformal symmetry of the original d-dimensional theory. In Sect. 5 we summarize our results and discuss some extensions for the future. Some technical details are given in appendices. In Appendix A, we set up a covariant formulation for the $1/n$ expansion around the background metric. In Appendix B, we calculate various two-point functions for the metric operator around its vacuum expectation value, which are necessary for the $1/n$ expansion.

2. Pre-geometric operators

In this section we propose a method to compute dual observables in the flow equation approach. In this approach we generally construct d-dimensional operators parametrized by a flow time which become seeds of geometric objects in the bulk theory. We refer to such operators as pre-geometric operators. Then, physical observables in the bulk are obtained by taking the vacuum expectation value of the pre-geometric operators.

2.1. *Metric operator*

In this subsection we illustrate how to construct a metric operator in the flow equation approach; see also Ref. [31] for more details. For this purpose, as well as for later convenience, we consider a d-dimensional quantum field theory whose elementary fields are n real scalar fields denoted by $\varphi^a(x)$ with $a = 1, 2, \ldots, n$. Then we smear the elementary fields so as to remove the short-distance singularity, which is described by a flow equation of a generic form given as

$$\frac{\partial \phi^a(x;t)}{\partial t} = - \left.\frac{\delta S_f(\varphi)}{\delta \varphi^a(x)}\right|_{\varphi(x)\to\phi(x;t)}, \quad \phi^a(x;0) = \varphi^a(x). \tag{2.1}$$

Here, S_f is a smearing functional describing how to smear operators, which is in principle independent of the action of the original theory controlling the dynamics of φ. This defines a procedure to smear the original field φ^a into a smeared field ϕ^a like the block spin transformation of φ^a in a non-local fashion, and ϕ^a is called the flow field corresponding to φ^a. The dynamics generated by the flowed operators $\phi^a(x;t)$ constitutes a holographic theory with the smearing scale $\sqrt{t}$ as the holographic direction.

To approach the dual AdS geometry from CFT, it is convenient to consider the free flow, which is realized by choosing S_f as the free action:

$$\frac{\partial \phi^a(x;t)}{\partial t} = (\partial^2 - m^2)\phi^a(x;t). \tag{2.2}$$

The solution is given by

$$\phi^a(x;t) = e^{t(\partial^2 - m^2)}\varphi^a(x) = \frac{e^{-tm^2}}{(4\pi t)^{d/2}} \int d^d y\, e^{-(x-y)^2/4t}\varphi(y). \tag{2.3}$$

The free flow equation is formally the same form as the heat equation, so that the smeared operator is given by superposing all the original operators inserted at each point of the space with the Gaussian weight whose standard deviation is the smearing scale $\sqrt{t}$.

Following the standard renormalization group transformation procedure, we (re)normalize the smeared field ϕ^a as

$$\sigma^a(x;t) := \frac{\phi^a(x;t)}{\sqrt{\langle \sum_{a=1}^n \phi^a(x;t)^2 \rangle}}, \tag{2.4}$$

where $\langle O(\varphi)\rangle$ denotes the quantum average with the original d-dimensional action S as

$$\langle O(\varphi)\rangle := \frac{1}{Z}\int \mathcal{D}\varphi\, O(\varphi)\, e^{-S(\varphi)},\ Z = \int \mathcal{D}\varphi\, e^{-S(\varphi)}. \tag{2.5}$$

Note that this operator is well defined due to the fact that the flowed operators are free from the contact singularity.

We can introduce the metric operator, which becomes the metric in the holographic space after taking the quantum average, as

$$\hat{g}_{MN}(x;t) := L^2 \sum_{a=1}^{n} \frac{\partial \sigma^a(x;t)}{\partial z^M}\frac{\partial \sigma^a(x;t)}{\partial z^N}, \tag{2.6}$$

where L is a constant with the dimension of length, and $z^M = (x^\mu, \tau)$ with $\tau = \sqrt{2dt}$, which will be regarded as the $(d+1)$-dimensional coordinates. The vacuum expectation value (VEV) of the metric is called the induced metric, which is denoted by $g_{MN}(z) := \langle \hat{g}_{MN}(x;t)\rangle$.

It was shown in Ref. [31] that the induced metric $g_{MN}(z)$ becomes a quantum information metric called the Bures or Helstrom metric for the Hilbert space generated by the flowed fields. This holds for a general (even non-conformal) quantum field theory.

2.2. *Other pre-geometric operators*

Once the metric operator is constructed, other pre-geometric operators are defined by replacing the metric that appears in the definition of the corresponding (classical) geometric object with the metric operator. For example, the Levi–Civita connection operator is defined by

$$\hat{\Gamma}^M_{LN}(x;t) = \frac{1}{2}\hat{g}^{MP}(x;t)(\hat{g}_{P\{N,L\}}(x;t) - \hat{g}_{NL,P}(x;t)), \tag{2.7}$$

where $X_{\{x,y\}} := X_{x,y} + X_{y,x}$. Curvature operators are defined by

$$\hat{R}_{LP}{}^M{}_N(x;t) = \partial_{[L}\hat{\Gamma}^M_{P]N}(x;t) + \hat{\Gamma}^M_{[LQ}(x;t)\hat{\Gamma}^Q_{P]N}(x;t), \tag{2.8}$$

$$\hat{R}_{PN}(x;t) = \hat{R}_{MP}{}^M{}_N(x;t), \tag{2.9}$$

$$\hat{R}(x;t) = \hat{g}^{PN}(x;t)\hat{R}_{PN}(x;t), \tag{2.10}$$

where $X_{[x,y]} := X_{x,y} - X_{y,x}$. Finally, the Einstein tensor operator is defined by

$$\hat{G}_{MN}(x;t) = \hat{R}_{MN}(x;t) - \frac{1}{2}\hat{g}_{MN}(x;t)\hat{R}(x;t). \tag{2.11}$$

3. Induced Einstein tensor and bulk interpretation

In this section we evaluate the VEV of the Einstein tensor operator for a free $O(n)$ vector model at the next-to-leading order (NLO) in the $1/n$ expansion. We then interpret the induced Einstein tensor as the bulk stress–energy tensor through the Einstein equation as

$$\langle \hat{G}_{AB} \rangle = T^{\text{bulk}}_{AB}. \tag{3.1}$$

Since the Einstein tensor is now evaluated on the vacuum, it is natural to think that the corresponding bulk stress–energy tensor consists only of the cosmological constant term:

$$T^{\text{bulk}}_{AB} = -\Lambda g_{AB}. \tag{3.2}$$

In what follows, we compute the cosmological constant Λ at the NLO in the $1/n$ expansion.

3.1. Leading order

Let us first compute induced geometric observables for a free $O(n)$ vector model at the leading order (LO) in the $1/n$ expansion. For this computation we can use the result in Ref. [31], where we computed the induced metric for an arbitrary CFT that contains a real scalar primary operator $\varphi(x)$ of a general conformal dimension Δ. The two-point function of the normalized field σ is

$$\langle \sigma^a(x;t)\sigma^b(y;s)\rangle_{\text{CFT}} = \frac{\delta^{ab}}{n}\left(\frac{2\sqrt{ts}}{t+s}\right)^{\Delta} F\left(\frac{(x-y)^2}{t+s}\right), \tag{3.3}$$

where $F(0) = 1$ and $2dF'(0) = -\Delta$. Explicitly, $F(x)$ is given by

$$F(x) = \frac{\Gamma(d/2)}{\Gamma(\Delta)\Gamma(d/2-\Delta)} \int_0^1 dv\, v^{\Delta-1}(1-v)^{d/2-\Delta-1} e^{-xv/4}. \tag{3.4}$$

In the current case, $\Delta = (d-2)/2$.

The VEV of the induced metric becomes

$$g_{AB} = \frac{L^2\Delta}{\tau^2}\delta_{AB}. \tag{3.5}$$

Using the factorization in the large-n limit as $\langle \hat{g}_{AB}\hat{g}_{CD}\rangle = g_{AB}g_{CD}$, etc., we then obtain the induced Levi–Civita connection at the LO as

$$\Gamma^A_{BC} = -\frac{1}{\tau}\rho^{A\tau}_{BC}, \qquad \rho^{A\tau}_{BC} := \delta_{B\tau}\delta_{AC} + \delta_{C\tau}\delta_{AB} - \delta_{A\tau}\delta_{BC},$$
$$\Gamma^A_{BC,D} = \frac{1}{\tau^2}\rho^{A\tau,D}_{BC}, \qquad \rho^{A\tau,D}_{BC} := \delta^{\tau}_{BD}\delta_{AC} + \delta^{\tau}_{CD}\delta_{AB} - \delta^{\tau}_{AD}\delta_{BC},\ \delta^{\tau}_{AB} := \delta_{A\tau}\delta_{B\tau}. \tag{3.6}$$

Induced curvatures at the LO are computed as

$$R_{AB}{}^{C}{}_{D} = -\frac{g^C_{[A}g_{B]D}}{L^2\Delta}, \quad R_{AB} = -\frac{d}{L^2\Delta}g_{AB}, \quad R = -\frac{d(d+1)}{L^2\Delta}, \tag{3.7}$$

which determine the VEV of the Einstein tensor at the LO as

$$G_{AB} = \frac{d(d-1)}{2L^2\Delta} g_{AB}. \tag{3.8}$$

Therefore, the cosmological constant of the dual geometry is evaluated through Eq. (3.2) at the LO as

$$\Lambda = -\frac{d(d-1)}{2L^2\Delta} + \mathcal{O}\left(\frac{1}{n}\right), \tag{3.9}$$

which implies that the AdS radius of the dual geometry is given at LO by $L^2_{\mathrm{AdS}} = L^2\Delta + \mathcal{O}(1/n)$.

3.2. *Next-to-leading order*

We proceed to the next-to-leading order computation of the induced Einstein tensor. For this purpose we employ a covariant perturbation expansion: that is, we expand an arbitrary operator $\hat{X}$ built from the metric operator $\hat{g}_{AB}$ around the vacuum by plugging in $\hat{g}_{AB} = g_{AB} + \hat{h}_{AB}$ and expanding in powers of the fluctuation field $\hat{h}_{AB}$. We indicate terms in this expansion with a certain power of $\hat{h}_{AB}$ with the corresponding number of dots, like

$$\hat{X} = X + \dot{X} + \ddot{X} + \cdots . \tag{3.10}$$

Terms with increasing numbers of $\hat{h}_{AB}$ fields are more and more suppressed in the $1/n$ expansion. Then, the VEV of the operator $\hat{X}$ reduces to that of correlation functions of the fluctuation field $\hat{h}$. In this notation the Einstein tensor at the NLO is given by

$$\langle \ddot{G}_{AB} \rangle = \langle \ddot{R}_{AB} \rangle - \frac{1}{2}\langle \hat{h}_{AB}\dot{R} \rangle - \frac{1}{2} g_{AB}\langle \ddot{R} \rangle. \tag{3.11}$$

We summarize the results of this expansion in Appendix A.

For a free $O(n)$ vector model, the result in Eq. (3.5) does not receive any correction. Using results for two-point functions of $\hat{h}_{AB}$ that are calculated in Appendix B, we obtain

$$\langle \ddot{R}_{AB} \rangle = \frac{1}{n(d+2)\tau^2}\left(\chi_{ABCC} - \chi_{ACBC}\right) = -\frac{d}{n\tau^2}\delta_{AB}, \tag{3.12}$$

$$-\frac{1}{2}\langle \hat{h}_{AB}\dot{R} \rangle = \frac{1}{n(d+2)\tau^2}\left(\chi_{ABCC} - \chi_{ACBC}\right) + \frac{1}{n}R_{AB} = -\frac{2d}{n\tau^2}\delta_{AB}, \tag{3.13}$$

and

$$g^{AB}\langle \ddot{R}_{AB} \rangle = \frac{1}{n(d+2)L^2\Delta}\left(\chi_{CCDD} - \chi_{CDCD}\right) = \langle \hat{h}^{AB}\dot{R}_{AB} \rangle, \tag{3.14}$$

$$\langle \hat{h}^{AC}\hat{h}_C{}^B \rangle R_{AB} = -\frac{d(d+1)(d+2)}{nL^2\Delta}, \tag{3.15}$$

so that we have

$$-\frac{1}{2} g_{AB}\langle \ddot{R} \rangle = \frac{d(d+1)(d+2)}{2n\tau^2}\delta_{AB}, \tag{3.16}$$

where we use

$$\chi_{ABCC} = (\Delta+1)\{(d+5)\delta_{AB} + 2(d+2)\delta^{\tau}_{AB}\} - (d+1)(d+2)\delta_{AB}, \tag{3.17}$$

$$\chi_{ACBC} = (\Delta+1)\{(d+5)\delta_{AB} + 2(d+2)\delta^{\tau}_{AB}\} - (d+2)\delta_{AB}, \tag{3.18}$$

$$\chi_{CDCD} = (\Delta+1)\{(d+5)(d+1) + 2(d+2)\} - (d+1)(d+2), \tag{3.19}$$

and the definition of χ_{ABCD} can be found in Appendix B. Finally, we obtain[1]

$$\langle \ddot{G}_{AB} \rangle = \frac{(d-1)d(d+4)}{2nL^2\Delta} g_{AB}. \tag{3.20}$$

This final result is manifestly covariant, even though the calculation in the intermediate step contains non-covariant terms. This is a non-trivial check of our result.

As a result, the induced Einstein tensor evaluated at the vacuum is given by

$$\langle \hat{G}_{AB} \rangle = \frac{d(d-1)}{2L^2\Delta} g_{AB} \left(1 + \frac{d+4}{n}\right) + \mathcal{O}\left(\frac{1}{n^2}\right). \tag{3.21}$$

As asserted at the beginning of this section, this quantity is related to the bulk stress energy tensor through the bulk Einstein equation, Eq. (3.1). Through Eq. (3.2), the cosmological constant and the AdS radius are determined at the NLO as

$$\Lambda = -\frac{d(d-1)}{2L^2\Delta}\left(1 + \frac{d+4}{n}\right) + \mathcal{O}\left(\frac{1}{n^2}\right), \quad L^2_{\mathrm{AdS}} = L^2\Delta\left(1 - \frac{d+4}{n}\right) + \mathcal{O}\left(\frac{1}{n^2}\right). \tag{3.22}$$

Since $\langle \hat{g}_{AB} \rangle$ has no NLO corrections and thus has the same classical relation to the Einstein tensor at the LO, the $1/n$ correction to the cosmological constant comes purely from the quantum effect to the Einstein tensor of the dual gravity theory to the $O(n)$ free vector model, which is conjectured as the free higher-spin theory on AdS_{d+1} [38].

4. Symmetry constraints

Finally, we discuss our results in terms of the conformal symmetry or AdS isometry. In the previous publications [31,32], we showed that the conformal transformation converts to the AdS isometry after taking the quantum average. In what follows, we show that pre-geometric operators computed at the $1/n$ level in the previous sections become covariant under the symmetry after taking the quantum average.

For this purpose, as in Refs. [31,32], we divide the infinitesimal conformal transformation for the $\sigma(x;t)$ as

$$\delta^{\mathrm{conf}}\sigma(x;t) = \delta^{\mathrm{diff}}\sigma(x;t) + \delta^{\mathrm{extra}}\sigma(x;t), \tag{4.1}$$

where δ^{diff} generates the isometry of the AdS space while δ^{extra} is the remaining contribution, which was shown to vanish after taking the vacuum expectation value for the metric operator. Then, in order to show that a pre-geometric operator $\hat{T}$ behaves in a covariant manner under the AdS isometry, we need to show that $\langle \delta^{\mathrm{diff}} \hat{T} \rangle = 0$, which is equivalent to $\langle \delta^{\mathrm{extra}} \hat{T} \rangle = 0$.

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[1] This result was checked in a slightly different computation by

$$\begin{aligned}\left\langle \ddot{G}_{MN} \right\rangle =& \left\langle \ddot{R}_{PQ} \right\rangle (\delta^P_M \delta^Q_N - \frac{1}{2} g_{MN} g^{PQ}) + \frac{1}{2}(-g^{PQ}\delta^R_M\delta^S_N + g_{MN}g^{RP}g^{SQ}) \left\langle \hat{h}_{RS}\dot{R}_{PQ} \right\rangle \\ &+ \frac{1}{2}\left\langle \hat{h}_{MN}\hat{h}^{PQ} \right\rangle R_{PQ} - \frac{1}{2} g_{MN} \left\langle \hat{h}^{QP}\hat{h}^R_P \right\rangle R_{QR},\end{aligned}$$

where $\left\langle \hat{h}_{RS}\dot{R}_{PQ} \right\rangle$ is evaluated as $\left\langle \hat{h}_{RS}\dot{R}_{PQ} \right\rangle = \dfrac{-g_{\{RQ}g_{S\}P} + 2g_{RS}g_{PQ}}{n\Delta}$.

To this end, explicit expressions for $\delta^{\rm diff}\sigma(x;t)$ and $\delta^{\rm extra}\sigma(x;t)$ are not necessary, but we only need the operation of $\delta^{\rm extra}$ on the two-point function, which is given by

$$\langle\delta^{\rm extra}\{\sigma(x_1;t_1)\sigma(x_2;t_2)\}\rangle = -16d\left(\frac{2\tau_1\tau_2}{\tau_1^2+\tau_2^2}\right)^{\Delta}\frac{(\tau_1^2-\tau_2^2)}{(\tau_1^2+\tau_2^2)^2}b\cdot(x_1-x_2)(x_1-x_2)^2 \times F''\left(\frac{2d(x_1-x_2)^2}{\tau_1^2+\tau_2^2}\right). \tag{4.2}$$

Here, b_μ is the parameter of an infinitesimal conformal transformation. Using this, we have

$$\langle\delta^{\rm extra}\{\hat h_{AB;E}\hat h_{CD;F}\}\rangle = -C(\tau)b_\mu\Big[\delta_{BD}\Big\{\delta_{E\tau}\rho^d_{ACF\mu}+\delta_{F\tau}\rho^d_{ACE\mu}+\delta_{A\tau}\rho^d_{CEF\mu}+\delta_{C\tau}\rho^d_{AEF\mu}\Big\} + (C\leftrightarrow D)+(A\leftrightarrow B)+\text{both}\Big], \tag{4.3}$$

where

$$C(\tau) := \frac{16dL^4\Delta F''(0)}{n\tau^5},\quad \rho^d_{ABC\mu} := \delta^\tau_{AB}\delta_{C\mu}+\delta^\tau_{AC}\delta_{B\mu}+\delta^\tau_{BC}\delta_{A\mu}. \tag{4.4}$$

We need to consider

$$\langle\ddot R_{AB}\rangle = \frac{\tau^4}{4L^4\Delta^2}\Big[\langle\hat h_{CC;D}(\hat h_{DA;B}+\hat h_{DB;A}-\hat h_{BA;D})\rangle + 2\langle\hat h_{AC;D}\hat h_{BD;C}-\hat h_{AD;C}\hat h_{BD;C})\rangle + \langle\hat h_{CD;A}\hat h_{CD;B}\rangle\Big], \tag{4.5}$$

$$\langle\hat h_{AB}\dot R\rangle = \frac{\tau^4}{L^4\Delta^2}\langle\hat h_{AB;C}(\hat h_{DD;C}-\hat h_{CD;D})\rangle + \frac{d\tau^2}{L^4\Delta^2}\langle\hat h_{AB}\hat h_{CC}\rangle. \tag{4.6}$$

Equation (4.3) leads to

$$\langle\delta^{\rm extra}\{\hat h_{CC;D}\hat h_{DA;B}\}\rangle = -4(d+2)C(\tau)\left[\delta_{A\tau}b_B+b_A\delta_{B\tau}\right] = \langle\delta^{\rm extra}\{\hat h_{CC;D}\hat h_{AB;D}\}\rangle = \langle\delta^{\rm extra}\{\hat h_{CD;D}\hat h_{AB;C}\}\rangle, \tag{4.7}$$

$$\langle\delta^{\rm extra}\{\hat h_{AC;D}\hat h_{BD;C}\}\rangle = -3(d+2)C(\tau)\left[\delta_{A\tau}b_B+b_A\delta_{B\tau}\right], \tag{4.8}$$

$$\langle\delta^{\rm extra}\{\hat h_{AD;C}\hat h_{BD;C}\}\rangle = -(d+3)(d+2)C(\tau)\left[\delta_{A\tau}b_B+b_A\delta_{B\tau}\right], \tag{4.9}$$

$$\langle\delta^{\rm extra}\{\hat h_{CD;A}\hat h_{CD;B}\}\rangle = -2(d+2)(d+2)C(\tau)\left[\delta_{A\tau}b_B+b_A\delta_{B\tau}\right], \tag{4.10}$$

where $b_\tau = 0$. Then we obtain

$$\langle\delta^{\rm extra}\ddot R_{AB}\rangle = \frac{\tau^4C(\tau)}{4L^4\Delta^2}(d+2)\left[\delta_{A\tau}b_B+b_A\delta_{B\tau}\right] \times\{-4-4+4+2(-3+d+3)-2(d+2)\}=0, \tag{4.11}$$

$$\langle\delta^{\rm extra}\{\hat h_{AB}\dot R\}\rangle = \frac{\tau^4C(\tau)}{L^4\Delta^2}(d+2)\left[\delta_{A\tau}b_B+b_A\delta_{B\tau}\right](4-4)=0. \tag{4.12}$$

These guarantee that each term is covariant under isometry, and thus proportional to g_{AB}, as seen in the previous section.

5. Discussion

In this paper, we have constructed the holographic space from the primary scalar field in a free massless $O(n)$ vector model by a flow equation at the next-to-leading order in the $1/n$ expansion. We investigated some properties of the bulk theory by calculating the induced Einstein tensor in the $1/n$ expansion. After defining pre-geometric operators, we have calculated the VEV of the Einstein tensor operator at the NLO in the $1/n$ expansion. As a result, the NLO correction appeared in the VEV of the Einstein operator but not in the induced metric g_{AB}. We therefore regarded the NLO correction of the induced Einstein operator as the quantum correction to the cosmological constant from the dual gravity theory on AdS space, which is proposed as the free higher-spin theory [38]. We have also shown that the stress–energy tensor for the vacuum state is covariant and proportional to g_{AB}, thanks to the conformal symmetry of the boundary theory, which turns into the bulk isometry of the AdS space.

In our approach, the bulk AdS radial direction emerges as the smearing scale for the boundary CFT. It is important to clarify how to determine the whole structure of the bulk theory. The bulk stress–energy tensor corresponding to the vacuum state calculated in this paper may give a hint to constructing the bulk theory.

In this paper we computed the one-loop correction to the cosmological constant in the bulk theory, which is supposed to be the free higher-spin theory [38], from the dual free $O(n)$ vector model in the proposed framework. On the other hand, one-loop tests of higher-spin/vector model duality have already appeared in Refs. [39,40] (see also Refs. [41–45]). Their intriguing result is that the logarithmic divergence in the one-loop correction to the free energy cancels for higher-spin gauge theories employing a standard zeta function regularization in accordance with the analysis of sphere partition functions of their dual vector models. In particular, it was confirmed that the finite part vanishes for a free higher-spin theory, with its scalar field obeying the standard boundary condition. In the flow equation approach, the one-loop correction to the cosmological constant, which presumably corresponds to that of the vacuum energy, is free from UV divergence without specifying any regularization scheme. This is because the computation of the one-loop correction reduces to the two-point function of the flowed field, which has no UV divergence by construction. Our result for the finite part correction to the cosmological constant for a free higher-spin theory does not vanish in any dimension. It is highly important to fill the gap between these results, which could be done by identifying a bulk local operator in the flow equation approach.

The next important step is to evaluate the bulk stress–energy tensor corresponding to excited states. Indeed, we can easily generalize the computation of the VEV for the Einstein operator presented in this paper to that of arbitrary states as follows. We consider a set of states $\{|O\rangle\}$ in CFT with the inner product $\langle O|O'\rangle = \delta_{O,O'}$, where the meaning of $\delta_{O,O'}$ depends on the type of states. Then we evaluate the matrix element of the Einstein operator in the $1/n$ expansion by using the covariant perturbation given in Appendix A as

$$\begin{aligned}\langle O|\hat{G}_{AB}|O'\rangle &= \langle O|G_{AB}|O'\rangle + \langle O|\dot{G}_{AB}|O'\rangle + \langle O|\ddot{G}_{AB}|O'\rangle + \cdots \\ &= \left\{G_{AB} + \langle 0|\ddot{G}_{AB}|0\rangle\right\}\delta_{O,O'} + \langle O|\dot{G}_{AB}|O'\rangle + \langle O|\ddot{G}_{AB}|O'\rangle_c + O\left(\frac{1}{n^2}\right),\end{aligned} \tag{5.1}$$

where $\langle O|\hat{X}|O'\rangle_c := \langle O|\hat{X}|O'\rangle - \langle 0|\hat{X}|0\rangle\delta_{O,O'}$ for an arbitrary operator $\hat{X}$. As asserted in Sect. 3, we interpret the matrix element of the Einstein operator as the bulk stress–energy tensor through Eq. (3.1), which we may call the quantum Einstein equation. It is natural to interpret in this way that the corresponding bulk stress–energy tensor consists of the cosmological constant and the contribution from the matter field in the bulk:

$$T_{AB}^{\rm bulk} = -\Lambda g_{AB}^{\rm mat} + T_{AB}^{\rm mat}, \qquad g_{AB}^{\rm mat} := \langle O|\hat{g}_{AB}|O'\rangle. \tag{5.2}$$

Notice that we have already calculated the first term in Eq. (5.1) as

$$G_{AB} + \langle 0|\ddot{G}_{AB}|0\rangle = -\Lambda g_{AB}, \qquad \Lambda = -\frac{d(d-1)}{2L^2\Delta}\left(1+\frac{d+4}{n}\right)+\cdots, \tag{5.3}$$

which represents the vacuum contribution. Therefore, the contribution of the matter field to the bulk stress–energy tensor is given by

$$T_{AB}^{\rm mat} = \langle O|\dot{G}_{AB}|O'\rangle + \langle O|\ddot{G}_{AB}|O'\rangle_c + \Lambda\langle O|\hat{g}_{AB}|O'\rangle_c. \tag{5.4}$$

It is very important to compute this bulk stress–energy tensor in the construction of the dual bulk theory beyond the vacuum or geometry level. We are currently calculating $T_{AB}^{\rm mat}$, and will report the result elsewhere.

This program can be extended to the case of the $\lambda\varphi^4$ theory in three dimensions. In the previous investigation [46], while the induced metric describes the AdS_4 space at the leading order, the next-to-leading-order corrections make the space asymptotically AdS only in the UV and IR limits with different radii. These two limits correspond to the asymptotically free UV fixed point and the Wilson–Fischer IR fixed point of the boundary theory, respectively. It would be interesting to investigate how the stress–energy tensor in the bulk behaves from UV to IR at the NLO. We expect that this behaves in a similar manner to the one computed from the corresponding solution for the dual bulk (higher-spin) theory.

We hope to report on the progress with these issues in the near future.

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Appendix A. Covariant perturbation

We introduce the fluctuation of the metric operator around its VEV as

$$\hat{g}_{AB} = g_{AB} + \hat{h}_{AB}, \tag{A.1}$$

where $g_{AB} = \langle \hat{g}_{AB}\rangle$. Note that $\hat{h}_{AB} = \hat{h}_{BA}$. The inverse is thus expanded as

$$\hat{g}^{AB} = g^{AB} - g^{AC}\hat{h}_{CD}g^{DB} + \cdots = g^{AB} - \hat{h}^{AB} + \hat{h}^{AC}\hat{h}_C^{\ B} + \cdots, \tag{A.2}$$

where the space-time indices are raised or lowered by the VEV of the metric so that $\hat{h}^{AB} = g^{AC}\hat{h}_{CD}g^{DB}$ and $\hat{h}_C^{\ B} = \hat{h}_{CD}g^{DB}$.

We then expand the Levi–Civita connection as

$$\hat{\Gamma}^A_{BC} = \Gamma^A_{BC} + \dot{\Gamma}^A_{BC} + \ddot{\Gamma}^A_{BC} + \cdots , \tag{A.3}$$

where

$$\dot{\Gamma}^A_{BC} = \frac{1}{2} g^{AD} \left(\hat{h}_{D\{B;C\}} - \hat{h}_{CB;D} \right), \quad \hat{h}_{AB;C} := \hat{h}_{AB,C} - \Gamma^D_{CA} \hat{h}_{DB} - \Gamma^D_{CB} \hat{h}_{AD}. \tag{A.4}$$

Similarly, we have

$$\ddot{\Gamma}^A_{BC} = -\frac{1}{2} \hat{h}^{AD} \left(\hat{h}_{D\{B;C\}} - \hat{h}_{CB;D} \right). \tag{A.5}$$

Riemann curvatures are expanded as

$$\hat{R}^A{}_{BCD} = R^A{}_{BCD} + \dot{R}^A{}_{BCD} + \ddot{R}^A{}_{BCD} + \cdots , \tag{A.6}$$

$$\hat{R}_{AB} = R_{AB} + \dot{R}_{AB} + \ddot{R}_{AB} + \cdots , \tag{A.7}$$

$$\hat{R} = R + \dot{R} + \ddot{R} + \cdots , \tag{A.8}$$

where

$$R^A{}_{BCD} = \Gamma^A_{B[D,C]} + \Gamma^A_{E[C} \Gamma^E_{BD]}, \tag{A.9}$$

$$\dot{R}^A{}_{BCD} = \dot{\Gamma}^A_{B[D,C]} + \dot{\Gamma}^A_{E[C} \Gamma^E_{BD]} + \Gamma^A_{E[C} \dot{\Gamma}^E_{BD]} = \dot{\Gamma}^A_{B[D;C]}, \tag{A.10}$$

$$\ddot{R}^A{}_{BCD} = \ddot{\Gamma}^A_{B[D;C]} + \dot{\Gamma}^A_{E[C} \dot{\Gamma}^E_{BD]}, \tag{A.11}$$

$$R_{AB} = \Gamma^C_{A[B,C]} + \Gamma^C_{E[C} \Gamma^E_{AB]}, \tag{A.12}$$

$$\dot{R}_{AB} = \dot{\Gamma}^C_{A[B;C]}, \tag{A.13}$$

$$\ddot{R}_{AB} = \ddot{\Gamma}^C_{A[B;C]} + \dot{\Gamma}^C_{E[C} \dot{\Gamma}^E_{AB]}, \tag{A.14}$$

$$R = g^{AB} R_{AB}, \tag{A.15}$$

$$\dot{R} = g^{AB} \dot{R}_{AB} - \hat{h}^{AB} R_{AB}, \tag{A.16}$$

$$\ddot{R} = g^{AB} \ddot{R}_{AB} - \hat{h}^{AB} \dot{R}_{AB} + \hat{h}^{AC} \hat{h}_C^{\ B} R_{AB}. \tag{A.17}$$

Using Eqs. (A.4) and (A.5), we have

$$\dot{R}_{AB} = \frac{1}{2} \left(\hat{h}^C_{\ \{A;B\}C} - \hat{h}_{BA}{}^{;C}{}_C - \hat{h}^C{}_{C;AB} \right), \tag{A.18}$$

$$\begin{aligned} \ddot{R}_{AB} = \frac{1}{4} g^{CE} g^{FD} \Big\{ & -2 \left[\hat{h}_{EF} \left(\hat{h}_{D\{A;B\}} - \hat{h}_{BA;D} \right) \right]_{;C} + 2 \left[\hat{h}_{EF} \left(\hat{h}_{D\{A;C\}} - \hat{h}_{CA;D} \right) \right]_{;B} \\ & + \hat{h}_{EC;D} \left(\hat{h}_{F\{A;B\}} - \hat{h}_{BA;F} \right) - (\hat{h}_{E\{D;B\}} - \hat{h}_{BD;E})(\hat{h}_{F\{A;C\}} - \hat{h}_{CA;F}) \Big\}. \end{aligned} \tag{A.19}$$

At the next-to-leading order, the Einstein tensor is evaluated as

$$\langle \ddot{G}_{AB} \rangle = \langle \ddot{R}_{AB} \rangle - \frac{1}{2} \langle \hat{h}_{AB} \dot{R} \rangle - \frac{1}{2} g_{AB} \langle \ddot{R} \rangle, \tag{A.20}$$

where

$$\langle \hat{h}_{AB}\dot{R}\rangle = \langle \hat{h}_{AB}\{g^{GH}g^{CF}(\hat{h}_{FG;HC} - \hat{h}_{HG;FC}) - \hat{h}^{CD}R_{CD}\}\rangle,$$
$$\langle \ddot{R}\rangle = g^{AB}\langle \ddot{R}_{AB}\rangle - \frac{1}{2}g^{AE}g^{BF}g^{CD} \times \left\{\left\langle \hat{h}_{EF}\left(\hat{h}_{D\{A;B\}C} - \hat{h}_{BA;DC} - \hat{h}_{DC;AB}\right)\right\rangle - 2R_{AB}\langle \hat{h}_{ED}\hat{h}_{CF}\rangle\right\}.$$

Appendix B. Two-point functions of the metric fluctuation

In this appendix we calculate various two-point functions of $\hat{h}_{AB}$.

Appendix B.1. Preparation

We define

$$G(z_i, z_j) := L^2 f(\tau_j, \tau_j) F\left(\frac{2d(x_i - x_j)^2}{\tau_i + \tau_j}\right), \tag{B.1}$$

$$f(x,y) = g(x,y)^{\Delta},\ g(x,y) := \frac{2xy}{x^2+y^2}, \tag{B.2}$$

where

$$F(0) = 1, \quad F'(0) = -\frac{\Delta}{2d}, \quad F''(0) = \frac{\Delta(\Delta+1)}{4d(d+2)}. \tag{B.3}$$

Derivatives of f at $x = y$ are given by

$$f_x(x,x) = f_y(x,x) = 0, \quad f_{xx}(x,x) = -\frac{\Delta}{x^2}, \quad f_{xy}(x,x) = \frac{\Delta}{x^2}, \tag{B.4}$$

$$f_{xxx}(x,x) = \frac{3\Delta}{x^3}, \quad f_{xxy}(x,x) = -\frac{\Delta}{x^3}, \tag{B.5}$$

$$f_{xxyy}(x,x) = \frac{3\Delta(\Delta+1)}{x^4}, \quad f_{xxxy}(x,x) = -\frac{3\rho^2}{x^4}, \tag{B.6}$$

where the subscripts x and y denote derivatives with respect to these variables.

Appendix B.2. The connected part of propagators

The simplest one can be calculated from G as follows:

$$\langle \hat{h}_{AB}\hat{h}_{CD}\rangle = \langle \hat{g}_{AB}\hat{g}_{CD}\rangle_c = \frac{1}{n}\partial^1_A\partial^2_B\partial^3_C\partial^4_D G(z_1,z_3)G(z_2,z_4)\Big|_{z_i=z} + (C \leftrightarrow D)$$
$$= \frac{1}{n}\frac{L^4\Delta^2}{\tau^4}(\delta_{AC}\delta_{BD} + \delta_{AD}\delta_{BC}), \tag{B.7}$$

where

$$\partial^1_A\partial^3_C G(z_1,z_3)\big|_{z_1=z_3} = \frac{L^2\Delta}{\tau^2}\delta_{AC}. \tag{B.8}$$

Similarly,

$$\langle \hat{h}_{AB,E}\hat{h}_{CD}\rangle = \langle \hat{g}_{AB,E}\hat{g}_{CD}\rangle_c = \frac{1}{n}\partial^1_E\partial^1_A\partial^2_C G_\Delta(z_1,z_2)\partial^1_B\partial^2_D G_\Delta(z_1,z_2)\Big|_{z_1=z_2=z}$$
$$+ (C \leftrightarrow D) + (A \leftrightarrow B) + (\{A,C\} \leftrightarrow \{B,D\}), \tag{B.9}$$

$$\langle \hat{h}_{AB,E}\hat{h}_{CD,F}\rangle = \frac{1}{n}\partial^1_E\partial^1_A\partial^2_C G(z_1,z_2)\partial^1_F\partial^1_D\partial^2_B G(z_1,z_2)\Big|_{z_1=z_2=z} + \frac{1}{n}\partial^1_E\partial^2_F\partial^1_A\partial^2_C G(z_1,z_2)\partial^1_D\partial^2_B G(z_1,z_2)\Big|_{z_1=z_2=z} + (C\leftrightarrow D)+(A\leftrightarrow B)+(\{A,C\}\leftrightarrow\{B,D\}), \tag{B.10}$$

$$\langle \hat{h}_{AB,EF}\hat{h}_{CD}\rangle = \frac{1}{n}\partial^1_E\partial^1_A\partial^2_C G(z_1,z_2)\partial^1_F\partial^1_B\partial^2_D G(z_1,z_2)\Big|_{z_1=z_2=z} + \frac{1}{n}\partial^1_E\partial^1_F\partial^1_A\partial^2_C G(z_1,z_2)\partial^1_B\partial^2_D G(z_1,z_2)\Big|_{z_1=z_2=z} + (C\leftrightarrow D)+(A\leftrightarrow B)+(\{A,C\}\leftrightarrow\{B,D\}). \tag{B.11}$$

We now evaluate derivatives of G as

$$\partial^1_E\partial^1_A\partial^2_C G(z_1,z_2)\big|_{z_1=z_2=z} = -\frac{L^2\Delta}{\tau^3}\rho^{C\tau}_{AE}, \tag{B.12}$$

$$\begin{aligned}\partial^1_C\partial^2_D\partial^1_A\partial^2_B G(z_1,z_2)\big|_{z_{1,2}=z} &= \frac{L^2\Delta}{\tau^4}\Big[3(\Delta+1)\delta^\tau_{ABCD} + (\Delta-1)\left(\delta^\tau_{AC}\delta^d_{BD}+\delta^\tau_{BD}\delta^d_{AC}\right)\\ &\quad + (\Delta+2)\left(\delta^\tau_{AB}\delta^d_{CD}+\delta^\tau_{AD}\delta^d_{BC}+\delta^\tau_{CD}\delta^d_{AB}+\delta^\tau_{BC}\delta^d_{AD}\right)\\ &\quad + \frac{d}{d+2}(\Delta+1)\left(\delta^d_{AB}\delta^d_{CD}+\delta^d_{AD}\delta^d_{BC}+\delta^d_{AC}\delta^d_{BD}\right)\Big] \qquad \text{(B.13)}\\ &= \frac{L^2\Delta}{(d+2)\tau^4}\big[(\Delta+1)(d\,\rho_{ABCD}-6\delta^\tau_{ABCD})\\ &\quad + (2\Delta+d+4)\left(\delta^\tau_{AB}\delta_{CD}+\delta^\tau_{AD}\delta_{BC}+\delta^\tau_{CD}\delta_{AB}+\delta^\tau_{BC}\delta_{AD}\right)\\ &\quad + (2\Delta-2d-2)\left(\delta^\tau_{AC}\delta_{BD}+\delta^\tau_{BD}\delta_{AC}\right)\big], \qquad \text{(B.14)}\end{aligned}$$

with $\delta^\tau_{AB} := \delta_{A\tau}\delta_{B\tau}$, $\delta^d_{AB} := \delta_{AB}-\delta^\tau_{AB}$, $\rho_{ABCD} := \delta_{AB}\delta_{CD}+\delta_{AC}\delta_{BD}+\delta_{AD}\delta_{BC}$, $\delta^\tau_{ABCD} := \delta^\tau_{AB}\delta^\tau_{CD}$, and

$$\begin{aligned}\partial^1_C\partial^1_D\partial^1_A\partial^2_B G(z_1,z_2)\big|_{z_{1,2}=z} &= -\frac{L^2\Delta}{\tau^4}\Big[3\Delta\delta^\tau_{ABCD} + (\Delta-1)\left(\delta^\tau_{AC}\delta^d_{BD}+\delta^\tau_{AD}\delta^d_{BC}+\delta^\tau_{CD}\delta^d_{AB}\right)\\ &\quad + (\Delta+2)\left(\delta^\tau_{AB}\delta^d_{CD}+\delta^\tau_{BC}\delta^d_{AD}+\delta^\tau_{BD}\delta^d_{AC}\right)\\ &\quad + \frac{d}{d+2}(\Delta+1)\left(\delta^d_{AB}\delta^d_{CD}+\delta^d_{AD}\delta^d_{BC}+\delta^d_{AC}\delta^d_{BD}\right)\Big] \qquad \text{(B.15)}\\ &= -\frac{L^2\Delta}{(d+2)\tau^4}\big[(\Delta+1)(d\,\delta_{ABCD}-6\delta^\tau_{ABCD})\\ &\quad + (2\Delta+d+4)\left(\delta^\tau_{AB}\delta_{CD}+\delta^\tau_{BC}\delta_{AD}+\delta^\tau_{BD}\delta_{AC}\right)\\ &\quad + (2\Delta-2d-2)\left(\delta^\tau_{CD}\delta_{AB}+\delta^\tau_{AD}\delta_{BC}+\delta^\tau_{AC}\delta_{BD}\right)\big]. \qquad \text{(B.16)}\end{aligned}$$

Combining the above results, we finally obtain

$$\langle \hat{h}_{AB}\hat{h}_{CD}\rangle = \frac{1}{n}\frac{L^4\Delta^2}{\tau^4}\left(\delta_{AC}\delta_{BD}+\delta_{AD}\delta_{BC}\right), \tag{B.17}$$

$$\langle \hat{h}_{AB;E}\hat{h}_{CD}\rangle = 0, \tag{B.18}$$

$$\langle \hat{h}_{AB;E}\hat{h}_{CD;F}\rangle = -\langle \hat{h}_{AB;EF}\hat{h}_{CD}\rangle = \frac{1}{n}\frac{L^4\Delta^2}{(d+2)\tau^6}[\chi_{AECF}\delta_{BD}+\chi_{AEDF}\delta_{BC} + \chi_{BECF}\delta_{AD}+\chi_{BEDF}\delta_{AC}], \tag{B.19}$$

where

$$\chi_{AECF} := 2(\Delta+1)\left\{\frac{d}{2}\rho_{AECF} - 3\delta^{\tau}_{AECF} + \rho^{\tau}_{AECF}\right\} - (d+2)\delta_{AE}\delta_{CF}, \tag{B.20}$$

$$\rho_{AECF} := \delta_{AE}\delta_{CF}+\delta_{AC}\delta_{EF}+\delta_{AF}\delta_{CE}, \tag{B.21}$$

$$\rho^{\tau}_{AECF} := \delta^{\tau}_{AE}\delta_{CF}+\delta_{AE}\delta^{\tau}_{CF}+\delta^{\tau}_{AC}\delta_{EF}+\delta_{AC}\delta^{\tau}_{EF}+\delta^{\tau}_{AF}\delta_{CE}+\delta_{AF}\delta^{\tau}_{CE}. \tag{B.22}$$

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999) [arXiv:hep-th/9711200] [Search INSPIRE].
[2] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111] [Search INSPIRE].
[3] I. R. Klebanov, in Proc. TASI 1999: Strings, Branes, and Gravity, p. 615 (2001) [arXiv:hep-th/0009139] [Search INSPIRE].
[4] E. D'Hoker and D. Z. Freedman, in Proc. TASI 2001: Strings, Branes and Extra Dimensions, p. 3 (2002) [arXiv:hep-th/0201253] [Search INSPIRE].
[5] G. 't Hooft, in Proc. Salamfest 1993, p. 284 (1993) [arXiv:gr-qc/9310026] [Search INSPIRE].
[6] L. Susskind, J. Math. Phys. **36**, 6377 (1995) [arXiv:hep-th/9409089] [Search INSPIRE].
[7] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109] [Search INSPIRE].
[8] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150] [Search INSPIRE].
[9] T. Banks, M. R. Douglas, G. T. Horowitz, and E. Martinec, arXiv:hep-th/9808016 [Search INSPIRE].
[10] V. Balasubramanian, P. Kraus, and A. Lawrence, Phys. Rev. D **59**, 046003 (1999) [arXiv:hep-th/9805171] [Search INSPIRE].
[11] V. Balasubramanian, P. Kraus, A. Lawrence, and S. P. Trivedi, Phys. Rev. D **59**, 104021 (1999) [arXiv:hep-th/9808017] [Search INSPIRE].
[12] V. Balasubramanian, S. B. Giddings, and A. Lawrence, J. High Energy Phys. **9903**, 001 (1999) [arXiv:hep-th/9902052] [Search INSPIRE].
[13] I. Bena, Phys. Rev. D **62**, 066007 (2000) [arXiv:hep-th/9905186] [Search INSPIRE].
[14] G. Lifschytz and V. Periwal, J. High Energy Phys. **0004**, 026 (2000) [arXiv:hep-th/0003179] [Search INSPIRE].
[15] M. Nozaki, T. Numasawa, A. Prudenziati, and T. Takayanagi, Phys. Rev. D **88**, 026012 (2013) [arXiv:1304.7100 [hep-th]] [Search INSPIRE].
[16] J. Bhattacharya and T. Takayanagi, J. High Energy Phys. **1310**, 219 (2013) [arXiv:1308.3792 [hep-th]] [Search INSPIRE].
[17] T. Faulkner, M. Guica, T. Hartman, R. C. Myers, and M. Van Raamsdonk, J. High Energy Phys. **03**, 051 (2014) [arXiv:1312.7856 [hep-th]] [Search INSPIRE].
[18] T. Faulkner, F. M. Haehl, E. Hijano, O. Parrikar, C. Rabideau, and M. Van Raamsdonk, J. High Energy Phys. **1708**, 057 (2017) [arXiv:1705.03026 [hep-th]] [Search INSPIRE].
[19] S. Hollands and R. M. Wald, Commun. Math. Phys. **321**, 629 (2013) [arXiv:1201.0463 [gr-qc]] [Search INSPIRE].
[20] G. Sárosi and T. Ugajin, J. High Energy Phys. **1801**, 012 (2018) [arXiv:1705.01486 [hep-th]] [Search INSPIRE].

[21] H. Osborn, Nucl. Phys. B **363**, 486 (1991).
[22] C. Imbimbo, A. Schwimmer, S. Theisen, and S. Yankielowicz, Class. Quantum Grav. **17**, 1129 (2000) [arXiv:hep-th/9910267] [Search INSPIRE].
[23] I. Papadimitriou and K. Skenderis, IRMA Lect. Math. Theor. Phys. **8**, 73 (2005) [arXiv:hep-th/0404176] [Search INSPIRE].
[24] S.-S. Lee, J. High Energy Phys. **1210**, 160 (2012) [arXiv:1204.1780 [hep-th]] [Search INSPIRE].
[25] S.-S. Lee, J. High Energy Phys. **1401**, 076 (2014) [arXiv:1305.3908 [hep-th]] [Search INSPIRE].
[26] K. Skenderis, Class. Quantum Grav. **19**, 5849 (2002) [arXiv:hep-th/0209067] [Search INSPIRE].
[27] V. Shyam, Phys. Rev. D **95**, 066003 (2017) [arXiv:1611.05315 [gr-qc]] [Search INSPIRE].
[28] V. Shyam, J. High Energy Phys. **1803**, 171 (2018) [arXiv:1712.07955 [hep-th]] [Search INSPIRE].
[29] S. Aoki, K. Kikuchi, and T. Onogi, Prog. Theor. Exp. Phys. **2015**, 101B01 (2015) [arXiv:1505.00131 [hep-th]] [Search INSPIRE].
[30] S. Aoki, J. Balog, T. Onogi, and P. Weisz, Prog. Theor. Exp. Phys. **2016**, 083B04 (2016) [arXiv:1605.02413 [hep-th]] [Search INSPIRE].
[31] S. Aoki and S. Yokoyama, Prog. Theor. Exp. Phys. **2018**, 031B01 (2018) [arXiv:1707.03982 [hep-th]] [Search INSPIRE].
[32] S. Aoki and S. Yokoyama, Nucl. Phys. B **933**, 262 (2018) [arXiv:1709.07281 [hep-th]] [Search INSPIRE].
[33] M. Albanese et al. [APE Collaboration], Phys. Lett. B **192**, 163 (1987).
[34] R. Narayanan and H. Neuberger, J. High Energy Phys. **0603**, 064 (2006) [arXiv:hep-th/0601210] [Search INSPIRE].
[35] M. Lüscher, J. High Energy Phys. **1008**, 071 (2010); **1403**, 092 (2014) [erratum].
[36] A. Jevicki, Y. Kazama, and T. Yoneya, Phys. Rev. Lett. **81**, 5072 (1998) [arXiv:hep-th/9808039] [Search INSPIRE].
[37] S. R. Das and A. Jevicki, Phys. Rev. D **68**, 044011 (2003) [arXiv:hep-th/0304093] [Search INSPIRE].
[38] I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **550**, 213 (2002) [arXiv:hep-th/0210114] [Search INSPIRE].
[39] S. Giombi and I. R. Klebanov, J. High Energy Phys. **1312**, 068 (2013) [arXiv:1308.2337 [hep-th]] [Search INSPIRE].
[40] S. Giombi, I. R. Klebanov, and B. R. Safdi, Phys. Rev. D **89**, 084004 (2014) [arXiv:1401.0825 [hep-th]] [Search INSPIRE].
[41] R. Camporesi and A. Higuchi, Phys. Rev. D **47**, 3339 (1993).
[42] S. Giombi, I. R. Klebanov, and A. A. Tseytlin, Phys. Rev. D **90**, 024048 (2014) [arXiv:1402.5396 [hep-th]] [Search INSPIRE].
[43] A. Jevicki, K. Jin, and J. Yoon, Phys. Rev. D **89**, 085039 (2014) [arXiv:1401.3318 [hep-th]] [Search INSPIRE].
[44] M. Beccaria and A. A. Tseytlin, J. High Energy Phys. **1411**, 114 (2014) [arXiv:1410.3273 [hep-th]] [Search INSPIRE].
[45] J.-B. Bae, E. Joung, and S. Lal, J. High Energy Phys. **1604**, 061 (2016) [arXiv:1603.05387 [hep-th]] [Search INSPIRE].
[46] S. Aoki, J. Balog, T. Onogi, and P. Weisz, Prog. Theor. Exp. Phys. **2017**, 043B01 (2017) [arXiv:1701.00046 [hep-th]] [Search INSPIRE].