



On some implications of the BaBar data on the $\gamma^* \eta'$ transition form factor

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ABSTRACT

At leading-twist accuracy the form factors for the transitions from a virtual photon to the η or η' can be expanded into a power series of the variable ω , being related to the difference of the two photon virtualities. The series possess the remarkable feature that only the Gegenbauer coefficients of the meson distribution amplitudes of order $l \leq m$ contribute to the term $\sim \omega^m$. Thus, for $\omega \rightarrow 0$ only the asymptotic meson distribution amplitude contributes, allowing for a test of the mixing of the η and η' decay constants. Employing the Gegenbauer coefficients determined in an analysis of the form factors in the real photon limit, we present predictions for the $\gamma^* \eta$ and $\gamma^* \eta'$ form factors and compare them to the BaBar data.

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1. Introduction

The photon-meson transition form factors have always found much attention; there is a rich literature about these simple observables. These form factors have been measured in a rather large range of photon virtualities and the data are analyzed within the framework of collinear factorization. An interesting generalization of these observables are the form factors for the transitions from a virtual photon to a meson. Also these form factors have repeatedly been studied theoretically. Recently, the BaBar collaboration has measured such a form factor for the first time [1], namely the $\gamma^* \eta'$ one. Although the data are not very accurate this measurement is important since it demonstrates the feasibility of measuring such form factors at large photon virtualities. The prospects of getting better and more data from future experiments, as for instance BELLE 2, are high. Ji and Vladimirov [2] already analyzed the BaBar data within the collinear factorization approach. The authors showed agreement of the data with perturbative QCD and put the emphasize on special features like power corrections and effects of the binning of the data. They also elaborated on the

kinematic regions that are especially sensitive to the underlying dynamics. A particular aspect of the theoretical description of the transition form factors is not investigated in [2] although the authors are aware of it: the separation of Gegenbauer coefficients in dependence of the difference between the two photon virtualities, $Q_1^2 = -q_1^2$ and $Q_2^2 = -q_2^2$ (with the q_i being the momenta of the photons), or rather in dependence on the variable

$$\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2}. \quad (1)$$

This property of the collinear factorization approach has first been pointed out in [3]. The purpose of the present paper is to study this property in some detail and to generalize it to next-to-leading order (NLO) of perturbative QCD for the case of the $\gamma^* \eta$ and $\gamma^* \eta'$ form factors. A comparison with the BaBar data will also be made.

2. The general idea

Consider the process $\gamma^* \gamma^* \rightarrow M$ where M is an unflavored, charge-parity even meson. The behavior of the transition form factors appearing in that process is, at large photon virtualities, determined by the expansion of a product of two electromagnetic currents about light-like distances [4]. The form factors then factorize into a hard scattering amplitude, T_H , and a soft meson matrix element, parametrized as a process-independent meson distribution

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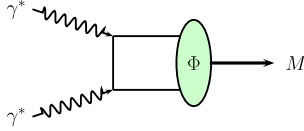


Fig. 1. A LO Feynman graph for $\gamma^* \gamma^* \rightarrow M$.

amplitude, $\Phi_M(\xi)$, where $\xi = 2x - 1$ and x is the usual momentum fraction carried by the quark inside the meson. We assume that the distribution amplitude possesses a Gegenbauer expansion

$$\Phi_M(\xi, \mu_F) = (1 - \xi^2)^{\lambda-1/2} \sum a_{Mn}(\mu_F) C_n^{(\lambda)}(\xi) \quad (2)$$

where $C_n^{(\lambda)}$ is the n -th Gegenbauer polynomial of order λ . In general several Fock components of a meson may contribute to a particular form factor; the Gegenbauer expansions of the corresponding distribution amplitudes may have different values of λ . The Gegenbauer coefficients, a_{Mn} , depend on the factorization scale, μ_F , at which the distribution amplitude is probed. The hard scattering amplitude, T_H , has a very simple structure with the quark propagator $\sim 1/(1 \pm \xi\omega)$, see the LO Feynman graph shown in Fig. 1. At leading-twist accuracy and for $\omega < 1$ the hard scattering amplitude can be expanded into a double power series $\sum_m \omega^m p(\xi)$. Here, p is a polynomial of ξ of order $m' = m + l'$ where l' is a small integer, typically $|l'| \leq 1$. Consider a term $\omega^m \xi^k$ (m, k positive integers) in that expansion. Its convolution with a distribution amplitude reads

$$I_{mk} = \int_{-1}^1 d\xi (1 - \xi^2)^{\lambda-1/2} \sum_n a_n C_n^{(\lambda)}(\xi) \omega^m \xi^k. \quad (3)$$

Any power of ξ can be expressed in terms of the Gegenbauer polynomials [5]

$$\xi^k = \sum_{l, k-l \equiv 0 \pmod{2}}^k d_{kl}^\lambda C_l^{(\lambda)}(\xi) \quad (4)$$

where

$$d_{kl}^\lambda = \frac{(l + \lambda)k! \Gamma(\lambda)}{2^k \left(\frac{k-l}{2}\right)! \Gamma\left(\frac{k+l+2\lambda}{2}\right)}. \quad (5)$$

Using this property and applying the orthogonality of the Gegenbauer polynomials, one arrives at

$$I_{mk} = \omega^m \sum_n a_n \sum_{l, k-l \equiv 0 \pmod{2}}^k d_{kl}^\lambda \frac{\pi 2^{1-2\lambda} \Gamma(l + 2\lambda)}{l!(l + \lambda)[\Gamma(\lambda)]^2} \delta_{nl}. \quad (6)$$

Obviously, Gegenbauer coefficients a_n with $n > k$ do not contribute to the integral I_{mk} . This is the observation made in [3] for the $\gamma^* \pi^0$ form factor to NLO (the latter corrections were taken from [6]) and, to LO, for the $\gamma^* \eta$ and $\gamma^* \eta'$ ones. Below we are going to generalize the latter case to NLO. In [7] it has been shown that, for the $\gamma^* \pi^0$ form factor, this property of the ω -expansion even holds to NNLO.¹ The correlation between the power of ω and the Gegenbauer coefficients also holds for the $\gamma^* f_0(980)$ [8] to LO and, exploiting the NLO corrections given in [6], also to that order. For the axial-vector, e.g. $\gamma^* a_1(1260)$ [9], and tensor, e.g. $\gamma^* f_2(1270)$

[9,10], form factors the correlation holds to LO; the NLO corrections are unknown as yet. Meson-mass corrections, taken into account for instance in [9,10], do not spoil this property of the ω expansion provided \bar{Q}^2 is much larger than the meson mass. Under the same premise the η_c form factor is another example. As shown in [11] such an expansion holds for the vertex function of the annihilation of two virtual gluons into a pseudoscalar meson too. Results presented in [12] are in agreement with the findings in [11]. We anticipate similar properties for other mesons. It should be stressed that for $\omega \rightarrow 1$, i.e. in the real photon limit, the hard scattering amplitude cannot be expanded this way and the sum of all Gegenbauer coefficients controls the transition form factors. However, if power corrections to the leading-twist result, accumulated in the soft end-point regions $\xi \rightarrow -1, 1$, are taken into account the higher Gegenbauer coefficients are gradually suppressed [13–15].

3. The $\gamma^* \eta$ and $\gamma^* \eta'$ form factors

Because of $\eta - \eta'$ mixing the $\gamma^* \eta$ and $\gamma^* \eta'$ form factors are much more complicated than the $\gamma^* \pi^0$ one. Even more so, there is an additional complication at NLO due to contributions from the gluon-gluon Fock component of the η and η' mesons. Thus, we have to take into account three distribution amplitudes for each of the mesons:

$$\begin{aligned} \Phi_{P_i}(\xi, \mu_F) &= \frac{3}{2}(1 - \xi^2) \left[1 + \sum_{n=2,4,\dots} a_{Pn}^i(\mu_F) C_n^{(3/2)}(\xi) \right], \\ \Phi_{Pg}(\xi, \mu_F) &= \frac{15}{8}(1 - \xi^2)^2 \sum_{n=2,4,\dots} a_{Pn}^g(\mu_F) C_{n-1}^{(5/2)}(\xi) \end{aligned} \quad (7)$$

where $P = \eta, \eta'$ and $i = 1, 8$ refers to the flavor singlet and octet contributions.² The full $\gamma^* P$ transition form factor is the sum of the flavor octet and singlet contributions

$$F_{P\gamma^*}(\bar{Q}^2, \omega) = F_{P\gamma^*}^8(\bar{Q}^2, \omega) + F_{P\gamma^*}^1(\bar{Q}^2, \omega) \quad (8)$$

where

$$\bar{Q}^2 = \frac{1}{2}(Q_1^2 + Q_2^2). \quad (9)$$

The two parts of the form factor read

$$\begin{aligned} F_{P\gamma^*}^8 &= \frac{1}{3\sqrt{6}} \frac{f_P^8}{\bar{Q}^2} \int_{-1}^1 d\xi \Phi_{P8}(\xi, \mu_F) \frac{1}{1 - \xi^2 \omega^2} \\ &\quad \times \left[1 + \frac{\alpha_S(\mu_R)}{\pi} \mathcal{K}(\omega, \xi, \bar{Q}/\mu_F) \right], \\ F_{P\gamma^*}^1 &= \frac{2}{3\sqrt{3}} \frac{f_P^1}{\bar{Q}^2} \left\{ \int_{-1}^1 d\xi \Phi_{P1}(\xi, \mu_F) \frac{1}{1 - \xi^2 \omega^2} \right. \\ &\quad \times \left[1 + \frac{\alpha_S(\mu_R)}{\pi} \mathcal{K}(\omega, \xi, \bar{Q}/\mu_F) \right] \\ &\quad \left. + \frac{\alpha_S(\mu_R)}{\pi} \int_{-1}^1 d\xi \Phi_{Pg}(\xi, \mu_F) \mathcal{K}_{gg}(\omega, \xi, \bar{Q}/\mu_F) \right\} \end{aligned} \quad (10)$$

where f_P^i is the constant of the decay of the meson P through the action of either a singlet or octet axial-vector current. One

¹ The result generalizes to higher order of perturbative QCD provided no terms $\sim \ln \xi$ or $\sim \ln(1 - \xi)$ occur.

² As compared to previous work [11,16] we changed the definition of the gluon distribution amplitude by a factor of 30 in order to facilitate comparison with other work.

notices that the LO term in (10) has the simple expansion $\sim \sum_{n=0,2,\dots} (\xi\omega)^n$. The quark parts of the NLO leading-twist contributions, \mathcal{K} , in (10) (evaluated in the $\overline{\text{MS}}$ -scheme) are the same as for the case of $\gamma^*\pi^0$ form factor and can be taken from [3]. The hard scattering amplitude for the gluon contribution which contributes to NLO, can be adapted from double DVCS [17–19]. The result is ³

$$\begin{aligned} \mathcal{K}_{gg} &= \frac{1}{3(1-\xi^2)\omega^2} \left\{ -\frac{4(1-\omega)}{1-\xi} \ln(1-\omega) + \frac{1-\omega}{1-\xi} \ln^2(1-\omega) \right. \\ &\quad + \frac{4-\omega(1+\xi)^2}{1-\xi} \ln^2(1-\xi\omega) \\ &\quad - \frac{2-\omega(1+\xi)}{2(1-\xi)} \ln^2(1-\xi\omega) \\ &\quad + \frac{1-\omega}{1-\xi} \left(2\ln(1-\omega) - (1+\xi)\ln(1-\xi\omega) \right) \ln \frac{\bar{Q}^2}{\mu_F^2} \left. \right\} \\ &\quad + (\omega \rightarrow -\omega) - (\xi \rightarrow -\xi) - (\xi \rightarrow -\xi, \omega \rightarrow -\omega) \\ &= -\frac{5}{9}\xi\omega^2 \left(1 - \frac{2}{5} \ln \frac{\bar{Q}^2}{\mu_F^2} \right) - \frac{37}{135}\xi(1+2\xi^2) \\ &\quad \times \omega^4 \left(1 - \frac{12}{37} \ln \frac{\bar{Q}^2}{\mu_F^2} \right) + \mathcal{O}(\omega^6). \end{aligned} \quad (11)$$

As one sees this is an expansion of the type discussed in Sect. 2. Power corrections to the above leading-twist result are mainly accumulated in the soft end-point regions where $\xi \rightarrow \pm 1$. They are expected to be small for small ω and large \bar{Q}^2 . This is obvious from the parton propagator $1/(1-\xi^2\omega^2)$ in (10). For $\omega \rightarrow 0$ the form factor becomes less sensitive to the end-point regions. Estimates of power corrections arising from quark transverse momentum [3] or from meson-mass corrections [15] support this expectation. Thus, for the case of interest, it seems to be reasonable to work at leading-twist accuracy.

4. Expansion of the NLO leading-twist $\gamma^*\eta$ and $\gamma^*\eta'$ form factors

Expanding the form factor (8) upon ω leads to

$$F_{P\gamma^*}(\bar{Q}^2, \omega) = \frac{\sqrt{2}}{3\bar{Q}^2} \sum_{n=0,2,\dots} c_{Pn}(\bar{Q}^2) \omega^n \quad (12)$$

where the first coefficients of the series read

$$\begin{aligned} c_{P0} &= f_P \left(1 - \frac{\alpha_S}{\pi} \right), \\ c_{P2} &= \frac{f_P}{5} \left(1 - \frac{5\alpha_S}{3\pi} \right) + \frac{12}{35} a_{P2}^{\text{eff}} \left(1 + \frac{5\alpha_S}{12\pi} \left(1 - \frac{10}{3} \ln \frac{\bar{Q}^2}{\mu_F^2} \right) \right) \\ &\quad - \sqrt{\frac{2}{3}} \frac{50}{63} f_P^1 a_{P2}^g \frac{\alpha_S}{\pi} \left(1 - \frac{2}{5} \ln \frac{\bar{Q}^2}{\mu_F^2} \right), \\ c_{P4} &= \frac{3f_P}{35} \left(1 - \frac{59\alpha_S}{27\pi} \right) \\ &\quad + \frac{8}{35} a_{P2}^{\text{eff}} \left(1 + \frac{173}{216} \frac{\alpha_S}{\pi} \left(1 - \frac{300}{173} \ln \frac{\bar{Q}^2}{\mu_F^2} \right) \right) \\ &\quad + \frac{8}{77} a_{P4}^{\text{eff}} \left(1 + \frac{523}{270} \frac{\alpha_S}{\pi} \left(1 - \frac{546}{523} \ln \frac{\bar{Q}^2}{\mu_F^2} \right) \right) \end{aligned}$$

³ In [15] a part of the gluonic hard scattering amplitude is given. A part corresponding to $\xi \rightarrow -\xi$ is lacking and when it is added agreement with our results is to be seen.

$$- \sqrt{\frac{2}{3}} \frac{370}{567} \frac{\alpha_S}{\pi} \left(1 - \frac{12}{37} \ln \frac{\bar{Q}^2}{\mu_F^2} \right) f_P^1 \left(a_{P2}^g + \frac{28}{55} a_{P4}^g \right). \quad (13)$$

The effective decay constant is defined by

$$f_P = \frac{1}{\sqrt{3}} \left[f_P^8 + 2\sqrt{2} f_P^1 \right], \quad (14)$$

and the effective quark Gegenbauer coefficients by

$$a_{Pn}^{\text{eff}}(\mu_F) = \frac{1}{\sqrt{3}} \left[f_P^8 a_{Pn}^8(\mu_F) + 2\sqrt{2} f_P^1 a_{Pn}^1(\mu_F) \right]. \quad (15)$$

The various Gegenbauer coefficients depend on the factorization scale, μ_F . Thus,

$$\begin{aligned} a_{Pn}^8(\mu_F) &= a_{Pn}^8(\mu_0) L^{\gamma_n^{qq}/\beta_0}, \\ a_{Pn}^1(\mu_F) &= \frac{1}{1 - \rho_n^{(+)} \rho_n^{(-)}} \left[(L^{\gamma_n^{(+)}/\beta_0} - \rho_n^{(+)} \rho_n^{(-)} L^{\gamma_n^{(-)}/\beta_0}) a_{Pn}^1(\mu_0^2) \right. \\ &\quad \left. + (L^{\gamma_n^{(-)}/\beta_0} - L^{\gamma_n^{(+)}/\beta_0}) \rho_n^{(-)} a_{Pn}^g(\mu_0^2) \right], \\ a_{Pn}^g(\mu_F) &= \frac{1}{1 - \rho_n^{(+)} \rho_n^{(-)}} \left[(L^{\gamma_n^{(-)}/\beta_0} - \rho_n^{(+)} \rho_n^{(-)} L^{\gamma_n^{(+)}/\beta_0}) a_{Pn}^g(\mu_0^2) \right. \\ &\quad \left. + (L^{\gamma_n^{(+)}/\beta_0} - L^{\gamma_n^{(-)}/\beta_0}) \rho_n^{(+)} a_{Pn}^1(\mu_0^2) \right]. \end{aligned} \quad (16)$$

The parameters $\rho_n^{(\pm)}$ read

$$\rho_n^{(+)} = \frac{1}{5} \frac{\gamma_n^{gq}}{\gamma_n^{(+)} - \gamma_n^{gg}}, \quad \rho_n^{(-)} = 5 \frac{\gamma_n^{qg}}{\gamma_n^{(-)} - \gamma_n^{qq}}, \quad (17)$$

and

$$L = \frac{\alpha_S(\mu_0)}{\alpha_S(\mu_F)} \quad (18)$$

The anomalous dimensions, γ_n^i , can, for our conventions, be found in [16] ($\beta_0 = 25/3$ for four flavors).⁴ As we see from (13) the Gegenbauer coefficients $a_{Pn}^{i(g)}$ are suppressed in the transition form factors by a power ω^n . Thus, accurate data on the transition form factors, $F_{P\gamma^*}$, offer the possibility to measure at least the lowest Gegenbauer coefficients of the meson distribution amplitudes. This is to be contrasted with other hard exclusive processes where frequently the $1/\xi$ -moment of the distribution amplitudes controls the observables. To this moment all Gegenbauer coefficients contribute equally. The expansion coefficients c_{Pn} depend on \bar{Q}^2 only logarithmically through α_S and the evolution. The transition form factors $F_{P\gamma^*}$ scale as $1/\bar{Q}^2$.

5. Comparison with the BaBar data

The most interesting result is that, for $\omega \rightarrow 0$, the leading term of the transition form factor only depends on the asymptotic meson distribution amplitudes:

$$\bar{Q}^2 F_{P\gamma^*} = \frac{\sqrt{2}}{3} f_P \left(1 - \frac{\alpha_S}{\pi} \right) + \mathcal{O}(\omega^2, \alpha_S^2). \quad (19)$$

This result has been already given in [3]. In contrast to the case of the pion where the decay constant f_π is known, f_P depends on the $\eta - \eta'$ mixing parameters. In the two-angle mixing scheme [20] the decay constants, f_P^i , are parametrized as

⁴ Because of the definition of the gluon distribution amplitude in Eq. (7) the quantities $\rho_n^{(\pm)}$ differ from those quoted in [16] by the factor of 30.

Table 1

Predictions for the scaled $\gamma^*\eta$ and $\gamma^*\eta'$ transition factors. The data are taken from [1] and the mixing parameters from [20] (phenomenological values). For $\omega \neq 0$ the Gegenbauer coefficients (23) are used. Parametric errors of the theoretical results are also quoted. The χ^2 values are evaluated with regard to the experimental errors.

\bar{Q}^2 [GeV ²]	ω	$\bar{Q}^2 F_{\eta\gamma^*}^{\text{exp}}$ [MeV]	$\bar{Q}^2 F_{\eta\gamma^*}$ [MeV]	χ^2	$\bar{Q}^2 F_{\eta\gamma^*}$ [MeV]
6.48	0.000	92.8 ± 13.8	92.7 ± 3.9	0.00	56.2 ± 3.3
16.85	0.000	90.1 ± 37.3	93.8 ± 3.9	0.01	56.8 ± 3.3
9.55	0.553	78.7 ± 13.5	98.7 ± 4.1	2.19	59.9 ± 3.5
26.53	0.436	161.0 ± 44.2	97.7 ± 4.1	2.05	59.2 ± 3.5
45.63	0.000	397.4 ± 400.9	94.6 ± 4.0	0.57	57.4 ± 3.4

$$\begin{aligned} f_{\eta'}^8 &= f_8 \sin \theta_8, & f_{\eta'}^1 &= f_1 \cos \theta_1, \\ f_{\eta}^8 &= f_8 \cos \theta_8, & f_{\eta}^1 &= -f_1 \sin \theta_1. \end{aligned} \quad (20)$$

The various mixing parameters are taken from the phenomenological set presented in [20] ($f_{\pi} = 131$ MeV)

$$\begin{aligned} f_8 &= (1.26 \pm 0.06) f_{\pi}, & \theta_8 &= -21.2 \pm 1.4, \\ f_1 &= (1.17 \pm 0.04) f_{\pi}, & \theta_1 &= -9.2 \pm 1.4. \end{aligned} \quad (21)$$

In contrast to f_8 and the mixing angles the singlet decay constant, f_1 , is renormalization scale dependent [21]. The anomalous dimension controlling this scale dependence is of order α_s^2 and therefore small. In the determination of the mixing parameters this scale dependence is usually ignored. It is therefore not clear at which scale (21) holds. Since the data on the $\gamma\eta$ and $\gamma\eta'$ transition form factors [26] as well as a number of charmonium decays play an important role in the analysis of $\eta - \eta'$ mixing [20] a possible initial scale of f_1 presumably lies in the range of 2 – 4 GeV². If so, the scale dependence of f_1 is weak; its effect on the form factors is of the order of the theoretical errors quoted in Table 1. We therefore ignore the scale dependence of f_1 in the following. This procedure is consistent with the determination of the mixing parameters (21).

For the evaluation of the form factors the QCD coupling, α_s , is evaluated from the two-loop expression with $\Lambda_{QCD} = 319$ MeV for four flavors in the \overline{MS} -scheme [22]. The renormalization scale is chosen as $2\bar{Q}^2$. This is all we need for an evaluation of the transition form factors at $\omega = 0$. The results of the computation are presented in Table 1 and compared to the three BaBar data points at this value of ω . Excellent agreement is to be observed, all predicted values agree with the experimental ones within the admittedly large experimental errors. The mixing parameters have been determined repeatedly, using more recent but often less data, e.g. [23,24]. Also these sets of mixing parameters agree with the BaBar data within the experimental errors. Even the theoretical mixing parameters quoted in [25] which, with the help of the divergences of the axial-vector currents, are expressed in terms of particle masses, agree with experiment. The predictions for the $\gamma^*\eta'$ form factor obtained from the various sets of mixing parameters, even agree within the parametric phenomenological errors quoted in Table 1, except for the mixing parameters of [23] which lead to values of the $\gamma^*\eta'$ form factor about 2σ (with respect to the phenomenological errors) larger than the predictions listed in the table. A somewhat larger spread of the predictions for the $\gamma^*\eta$ form factor is obtained.

The BaBar collaboration has also measured the $\gamma^*\eta'$ form factor for two non-zero but adjacent values of ω . The two face values of the form factor data differ by about a factor of two. It seems difficult to accommodate this difference within the NLO leading-twist theory since, as we mentioned above, the expansion coefficients, c_{Pn} , only depend on \bar{Q}^2 logarithmically. Anyway the present experimental information on the ω -dependence of the transition

form factors is too limited for an attempt of fitting even the lowest Gegenbauer coefficients of the meson distribution amplitudes to the BaBar data. Nevertheless, we can carry out the following check: We make use of the second Gegenbauer coefficients which we extracted in [16] from the data on the meson-photon transition form factors [26,27]. In this analysis we have had to assume that all higher Gegenbauer coefficients do not contribute in the real-photon limit. Thus, although the extracted Gegenbauer coefficients are to be regarded as effective ones, perhaps contaminated by higher-order Gegenbauer coefficients, we here identify them with the real second order coefficients. It should also be mentioned that in [16] particle-independence of the meson distribution amplitudes is assumed, i.e.

$$\Phi_P^{i(g)} = \Phi^{i(g)}. \quad (22)$$

This plausible assumption has also been made in [15,20]. The values of the Gegenbauer coefficients read [16]

$$\begin{aligned} a_2^8 &= -0.05 \pm 0.02, & a_2^1 &= -0.12 \pm 0.01, \\ a_2^g &= 0.63 \pm 0.17, \end{aligned} \quad (23)$$

valid at the scale $\mu_0 = 1$ GeV. In order to match the choice of the factorization scale made in [16] in the real photon case we choose $\mu_F^2 = 2\bar{Q}^2$. From the Gegenbauer coefficients (23) we obtain the values for the $\gamma^*\eta$ and $\gamma^*\eta'$ transition form factors quoted in Table 1. Terms $\propto \omega^4$ are included in that evaluation. Both the results at the non-zero values of ω deviate by about 2σ from experiment. One of the theoretical values is too low as compared to the BaBar data, the other one too high. We emphasize – a change of the Gegenbauer coefficients (23) either increase or decreases the values of the form factors for both the ω values. Thus, it seems that we cannot improve the predictions. However, we do not think that serious conclusions should be drawn from this result; more accurate data are required for this. We remark that the ω^2 -term affects the results by about 5%, the ω^4 term (with zero fourth-order Gegenbauer coefficients) by less than 1%. The contribution from the gluon-gluon Fock component of the mesons, $\propto a_n^g$, is tiny but, implicitly, it substantially affects the results through the evolution of a_n^1 , see (16).

In Fig. 2 we display predictions for the scaled form factors versus ω^2 at $\bar{Q}^2 = 5$ GeV². In this computation we have naturally used the expressions (8) and (10) for the form factors instead of the expansion (12). We stress again – the scaled form factors depend only logarithmically on \bar{Q}^2 . For comparison we have made an alternative evaluation of the transition form factors for which we have assumed $a_2^8 = a_2^1 = 0.25$, positive values for these Gegenbauer coefficients are favored by QCD sum rules [15], and, in order to have the same effective a_2^1 values in the real photon limit, we have chosen the following (effective) fourth order Gegenbauer coefficient: $a_4^8 = -0.31$ and $a_4^1 = -0.38$. The gluon distribution amplitude is left unchanged. As one sees from Fig. 2 the two sets of

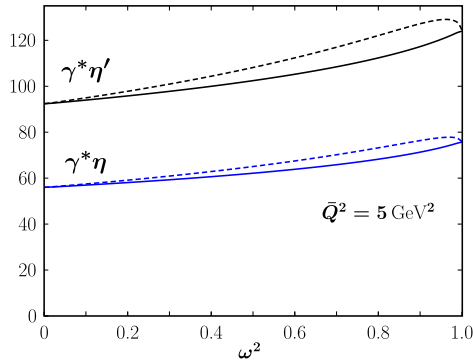


Fig. 2. Predictions for the scaled $\gamma^*\eta$ and $\gamma^*\eta'$ form factors versus ω^2 at $\bar{Q}^2 = 5 \text{ GeV}^2$. Solid (dashed) lines: fit (23) (alternative fit – see text).

Gegenbauer coefficients lead to the same form factors for $\omega \rightarrow 0$ and, indeed, in the real photon limit. However, for large, but < 1 , values of ω the predictions differ and for sufficiently accurate data on the γ^*P form factors one may distinguish between the two sets of Gegenbauer coefficients. Thus, we conclude data on the γ^*P transition form factors may provide more detailed information on the meson distribution amplitudes than one obtains in the real photon limit.

6. Summary

We have discussed the ω -expansion of the form factors for the annihilation of two virtual photons into a meson to leading-twist accuracy and have, in particular, investigated the correlation between the power of ω and the Gegenbauer coefficients of the corresponding meson distribution amplitudes in some detail. We have applied this property to the $\gamma^*\eta$ and $\gamma^*\eta'$ form factors and have shown that to the ω^m -term only the Gegenbauer coefficients of the quark octet and singlet distribution amplitudes as well as of the gluon distribution amplitude to order $n \leq m$ contribute. While for the quark distribution amplitudes this property has already been discussed in [3], it is a new result for the gluon contribution.

The correlation between the power of ω and the Gegenbauer coefficients is a possibility to learn more about the meson distribution amplitudes as it is possible from the transition form factors in the real photon limit. In the latter case the sum of all Gegenbauer coefficients makes up the form factors. While for the $\gamma^*\pi^0$ form factor the application of the ω expansion is very simple and straightforward it is more involved for the η and η' because of the $\eta - \eta'$ mixing and the contributions from the gluon-gluon Fock components of these mesons. But it is feasible with the plausible assumption of particle-independence of the corresponding distribution amplitudes. Still there are three Gegenbauer coefficients at any order but only two independent form factor measurements. The gluon distribution amplitude is only separated from the two quarks ones through the α_S -corrections which merely provide a small lever arm in practice. In any case, accurate form factor data

for large ω , close to 1, will certainly allow to check whether the effective value of a_2 extracted in the real photon limit, results from the cancellation of rather large individual terms or from the smallness of the a_n for $n > 2$ as stated in [3]. The much higher accuracy of the data on $F_{P\gamma}$ than those for $F_{P\gamma^*}$ could thus be overcompensated. From the high luminosity available at BELLE2 one may expect that an accurate measurement of γ^*P form factors seems to be possible there although this is a challenging task.

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