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A uniqueness theorem for warped $N > 16$ Minkowski backgrounds with fluxes

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ABSTRACT

We demonstrate that warped Minkowski space backgrounds, $\mathbb{R}^{n-1,1}\times_w M^{d-n}$, $n\geq 3$, that preserve strictly more than 16 supersymmetries in $d=11$ and type II $d=10$ supergravities and with fields which may not be smooth everywhere are locally isometric to the $\mathbb{R}^{d-1,1}$ Minkowski vacuum. In particular, all such flux compactification vacua of these theories have the same local geometry as the maximally supersymmetric vacuum $\mathbb{R}^{n-1,1}\times T^{d-n}$.

Recently, all warped anti-de-Sitter (AdS) backgrounds with fluxes that preserve $N>16$ supersymmetries in $d=11$ and $d=10$ supergravities have been classified up to a local isometry in [1–3]. In this note, we extend this result to include all warped $\mathbb{R}^{n-1,1}\times_w M^{d-n}$ backgrounds of these theories. In particular, we demonstrate that all warped $\mathbb{R}^{n-1,1}\times_w M^{d-n}$, $n\geq 3$, solutions with fluxes of $d=11$, IIA $d=10$ and IIB $d=10$ supergravities that preserve $N>16$ supersymmetries are locally isometric to the $\mathbb{R}^{d-1,1}$ maximally supersymmetric vacuum of these theories. Massive IIA supergravity does not admit such solutions. A consequence of this is that all $N>16$ flux compactification vacua, $\mathbb{R}^{n-1,1}\times_w M^{d-n}$, of these theories are locally isometric to the maximally supersymmetric toroidal vacuum $\mathbb{R}^{n-1,1}\times T^{d-n}$. To prove these results we have made an assumption that the translation isometries along the $\mathbb{R}^{n-1,1}$ subspace of these backgrounds commute with all the odd generators of their Killing superalgebra. The necessity and justification of this assumption will be made clear below.

To begin, we shall first describe the steps of the proof of our result which are common to all $d=11$ and $d=10$ theories and then specialize at the end to present the special features of the proof for each individual theory. Schematically, the fields of $\mathbb{R}^{n-1,1}\times_w M^{d-n}$ backgrounds are

$$\begin{aligned} ds^2 &= A^2 ds^2(\mathbb{R}^{n-1,1}) + ds^2(M^{d-n})\,, \\ F &= W\wedge \mathrm{dvol}_A(\mathbb{R}^{n-1,1}) + Z\,, \end{aligned} \tag{1}$$

where A is the warp factor that depends only on the coordinates of the internal space M^{d-n}, $\mathrm{dvol}_A(\mathbb{R}^{n-1,1})$ denotes the volume form of $\mathbb{R}^{n-1,1}$ evaluated in the warped metric and F denotes collectively all the k-form fluxes of the supergravity theories. We take $ds^2(\mathbb{R}^{n-1,1}) = 2dudv + dz^2 + \delta_{ab}dx^a dx^b$, where we have singled out a spatial coordinate z which will be useful later. W and Z are $(k-n)-$ and $k-$ forms on M^{d-n} which depend only on the coordinates of M^{d-n}. Clearly if $n>k$, $W=0$. Therefore these backgrounds are invariant under the Poincaré isometries of the $\mathbb{R}^{n-1,1}$ subspace. It is known that there are no smooth compactifications with non-trivial fluxes of $d=10$ and $d=11$ supergravities [4,5], i.e. solutions for which all fields are smooth including the warp factor and M^{d-n} is compact without boundary. However here we do not make these assumptions. M^{d-k} is allowed to be non-compact and the fields may not be smooth.

To continue following the description of $\mathbb{R}^{n-1,1}\times_w M^{d-n}$ backgrounds in [6–8], where one can also find more details about our notation, we introduce a light-cone orthonormal frame

$$\mathbf{e}^+ = du\,,\quad \mathbf{e}^- = (dr - 2rA^{-1}dA)\,,\quad \mathbf{e}^m = A\,dx^m\,,\quad \mathbf{e}^i = e^i_I dy^I\,, \tag{2}$$

on the spacetime with $ds^2(M^{d-n}) = \delta_{ij}\mathbf{e}^i\mathbf{e}^j$. Then the Killing spinors of the $\mathbb{R}^{n-1,1}\times_w M^{d-n}$ backgrounds can be written as

$$\begin{aligned} \epsilon &= \sigma_+ + u\Gamma_+\Gamma_z\Xi^{(-)}\sigma_- + A\sum_m x^m\Gamma_m\Gamma_z\Xi^{(+)}\sigma_+ \\ &\quad + \sigma_- + r\Gamma_-\Gamma_z\Xi^{(+)}\sigma_+ + A\sum_m x^m\Gamma_m\Gamma_z\Xi^{(-)}\sigma_-\,, \end{aligned} \tag{3}$$

where $x^m = (z, x^a)$, all the gamma matrices are in the frame basis (2) and the spinors $\sigma_\pm$, $\Gamma_\pm\sigma_\pm = 0$, depend only on the coordinates

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of M^{d-n}. The remaining independent KSEs are a restriction of the gravitino and algebraic KSEs of the supergravity theories on $\sigma_\pm$ which schematically can be written as

$$D_i^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{A}^{(\pm)}\sigma_\pm = 0\,, \tag{4}$$

respectively, and in addition an integrability condition

$$(\Xi^{(\pm)})^2\sigma_\pm = 0\,, \tag{5}$$

where $\Xi^{(\pm)}$ is a Clifford algebra element that depends on the fields. $\Xi^{(\pm)}$ will not be given here and can be found in the references above. The latter arises as a consequence of integrating the gravitino KSE of the theories along the $\mathbb{R}^{n-1,1}$ subspace.

Notice that the (spacetime) Killing spinors (3) may depend on the coordinates of the $\mathbb{R}^{n-1,1}$ subspace. Such a dependence arises whenever $\sigma_\pm$ is not in the kernel of $\Xi^{(\pm)}$. Of course $\sigma_\pm$ is required to lie in the kernel of $(\Xi^{(\pm)})^2$. To see why this dependence can arise for $\mathbb{R}^{n-1,1}\times_w M^{d-n}$ backgrounds, notice that AdS_{n+1} in Poincaré coordinates can be written as a warped product of $\mathbb{R}^{n-1,1}\times_w \mathbb{R}$. Therefore all AdS backgrounds, warped or otherwise, can be interpreted as warped Minkowski space backgrounds. It is also known that the former admit Killing spinors that depend on all AdS coordinates including those of the Minkowski subspace. Therefore $\mathbb{R}^{n-1,1}\times_w M^{d-n}$ may also admit Killing spinors that depend on the coordinates of $\mathbb{R}^{n-1,1}$, see also [9] for a more detailed explanation.

The assumption we have made that the commutator of the translations P along $\mathbb{R}^{n-1,1}$ and the odd generators Q of the Killing superalgebra [10,11] must vanish, $[P,Q]=0$, is required for Killing spinors ϵ not to exhibit a dependence on the coordinates of $\mathbb{R}^{n-1,1}$. Indeed, if the Killing spinors have a dependence on the Minkowski subspace coordinates, then the commutator $[P,Q]$ of the Killing superalgebra will not vanish. This can be verified with an explicit computation of the spinorial Lie derivative of ϵ in (3) along the translations of $\mathbb{R}^{n-1,1}$. Although this may seem as a technical assumption, it also has a physical significance in the context of flux compactifications. Typically the reduced theory is invariant under the Killing superalgebra of the compactification vacuum. So for the reduced theory to exhibit at most super-Poincaré invariance, one must set $[P,Q]=0$ for all P and Q generators. This physical justification applies only to compactification vacua but we shall take it to be valid for all backgrounds that we shall investigate below. Of course such an assumption excludes all AdS solutions of supergravity theories re-interpreted as warped Minkowski backgrounds. Therefore from now on we shall take $\Xi^{(\pm)}\sigma_\pm = 0$ and so *all* Killing spinors ϵ will not depend on the coordinates of the $\mathbb{R}^{n-1,1}$ subspace.

Before we proceed further, let us describe the Killing spinors of $\mathbb{R}^{n-1,1}\times_w M^{d-n}$ backgrounds in more detail. It turns out that if σ_+ is a Killing spinor, then $\sigma_- := A\Gamma_{-z}\sigma_+$ is also a Killing spinor. Similarly if σ_- is a Killing spinor, then $\sigma_+ := A^{-1}\Gamma_{+z}\sigma_-$ is also a Killing spinor. Furthermore if σ_+ is a Killing spinor, then $\sigma'_+ := \Gamma_{mn}\sigma_+$ are also Killing spinors for every m,n. Therefore the Killing spinors form multiplets under these Clifford algebra operations. The counting of Killing spinors of a background proceeds with identifying the linearly independent Killing spinors in each multiplet and then counting the number of different multiplets that can occur [6–8]. As all Killing spinors are generated from σ_+ Killing spinors, we shall express all key formulae in terms of the latter.

The 1-form bilinears that are associated with spacetime Killing vectors ϵ^r, $r=1,\dots,N$, that also leave all other fields invariant are

$$X(\epsilon^r,\epsilon^s) := \langle(\Gamma_+ - \Gamma_-)\epsilon^r, \Gamma_A\epsilon^s\rangle_s\,\mathbf{e}^A\,, \tag{6}$$

where in $d=11$ and IIA supergravity theories $\langle(\Gamma_+-\Gamma_-)\cdot,\cdot\rangle_s$ is the Dirac inner product restricted on the Majorana representation of $Spin(10,1)$ and $Spin(9,1)$, respectively, while in IIB it is the real part of the Dirac inner product. Note that $X(\epsilon^r,\epsilon^s)=X(\epsilon^s,\epsilon^r)$. In particular, one finds that

$$\begin{aligned}
X(\sigma_-^r,\sigma_-^s) &= 2A^2\langle\sigma_+^r, \Gamma_z\Gamma_A\Gamma_-\Gamma_z\sigma_+^s\rangle_s\mathbf{e}^A\,,\\
X(\sigma_-^r,\sigma_+^s) &= 2A\langle\sigma_+^r, \Gamma_z\Gamma_A\sigma_+^s\rangle_s\mathbf{e}^A\,,\\
X(\sigma_+^r,\sigma_+^s) &= -\langle\sigma_+^r, \Gamma_+\Gamma_A\sigma_+^s\rangle_s\mathbf{e}^A\,.
\end{aligned}\tag{7}$$

Clearly the last 1-form bilinear is $X(\sigma_+^r,\sigma_+^s) = -2\langle\sigma_+^r,\sigma_+^s\rangle_s\mathbf{e}^-$. The requirement that $X(\sigma_+^r,\sigma_+^s)$ is Killing implies that $\langle\sigma_+^r,\sigma_+^s\rangle_s$ are constants. In particular, one can choose without loss of generality that $\langle\sigma_+^r,\sigma_+^s\rangle_s = (1/2)\delta^{rs}$. Then first 1-form bilinear in (7) is $X(\sigma_-^r,\sigma_-^s) = 2A^2\delta^{rs}\mathbf{e}^+$.

Next consider the middle 1-form bilinear in (7). If σ_+^r is in the same multiplet as σ_+^s, i.e. $\sigma_+^s = \Gamma_{za}\sigma_+^r$, then $X(\sigma_-^r,\sigma_+^s) = -\delta^{rs}A\,\mathbf{e}^a$. On the other hand if $\sigma_+^s = \sigma_+^r$, then $X(\sigma_-^r,\sigma_+^r) = A\,\mathbf{e}^z$. Thus all these bilinears generate the translations in $\mathbb{R}^{n-1,1}$. However, if σ_+^r and σ_+^s are not in the same multiplet, then the bilinear

$$\tilde{X}_{rs} := X(\sigma_-^r,\sigma_+^s) = 2A\langle\sigma_+^r, \Gamma_z\Gamma_i\sigma_+^s\rangle_s\,\mathbf{e}^i\,, \tag{8}$$

will generate the isometries of the internal space. The Killing condition of $\tilde{X}$ implies that

$$\tilde{X}^i_{rs}\partial_i A = 0\,. \tag{9}$$

As the $\tilde{X}$ isometries commute with the translations, the even part, $\mathfrak{g}_0$, of the Killing superalgebra decomposes as $\mathfrak{g}_0 = \mathfrak{p}_0\oplus\mathfrak{t}_0$, where $\mathfrak{p}_0$ is the Lie algebra of translations in $\mathbb{R}^{n-1,1}$ and $\mathfrak{t}_0$ is the Lie algebra of isometries in the internal space M^{d-n}.

So far we have not used the assumption that the backgrounds preserve $N>16$ supersymmetries. If this is the case, the Killing vectors generated by $\mathfrak{g}_0$ span the tangent space of the spacetime at each point. This is a consequence of the homogeneity theorem proven for $d=11$ and $d=10$ supergravity backgrounds in [12,13]. This states that all solutions of these theories that preserve more than 16 supersymmetries must be locally homogeneous. In this particular case because of the decomposition of $\mathfrak{g}_0$, the Killing vector fields generated by $\mathfrak{t}_0$ span the tangent space of M^{d-n} at every point. As a result the condition (9) implies that A is constant. The main result of this note then follows as a consequence of the field equation of the warp factor and those of the rest of the scalar fields of these theories. So to complete the proof we shall state the relevant equations on a case by case basis.

In $d=11$ supergravity, the 4-form field strength of the theory for $\mathbb{R}^{n-1,1}\times_w M^{11-n}$, $n\geq 3$, backgrounds can be expressed as

$$F = d\mathrm{vol}_A(\mathbb{R}^{n-1,1})\wedge W^{4-n} + Z\,, \tag{10}$$

with $W^{4-n}=0$ for $n>4$, and the warp factor field equation is

$$\tilde{\nabla}^2\log A = -n(\partial\log A)^2 + \frac{1}{3\cdot(4-n)!}(W^{4-n})^2 + \frac{1}{144}Z^2\,, \tag{11}$$

where $\tilde{\nabla}$ is the Levi-Civita connection on M^{11-n}. The superscripts on the forms denote their degree whenever it is required for clarity. Clearly if A is constant, as it has been demonstrated above for $N>16$ backgrounds, then $W^{n-4}=Z=0$. So $F=0$ and thus all the fluxes vanish. In fact, this is also the case for $n=2$ provided that A is taken to be constant. As $F=0$ and A is constant, the gravitino KSE in (4) implies that all the Killing spinors $\sigma_\pm$ are parallel with respect to the Levi-Civita connection, $\tilde{\nabla}$, on M^{11-n}. In turn this gives that all the Killing vector fields $\tilde{X}$ in (8), which span the tangent space of M^{11-n}, are also parallel with respect to

$\tilde{\nabla}$. Thus M^{11-n} is locally isometric to $\mathbb{R}^{11-n}$. Therefore the backgrounds $\mathbb{R}^{n-1,1}\times_w M^{11-n}$ are locally isometric to the maximally supersymmetric vacuum $\mathbb{R}^{10,1}$. Notice that the last step of the proof requires the use of the homogeneity theorem.

In (massive) IIA supergravity, the 4-form F, 3-form H and 2-form G field strengths of the theory for $\mathbb{R}^{n-1,1}\times_w M^{10-n}$, $n\geq 3$, backgrounds can be written as

$$\begin{aligned} F &= \mathrm{dvol}_A(\mathbb{R}^{n-1,1})\wedge W^{4-n}+Z\,,\\ H &= \mathrm{dvol}_A(\mathbb{R}^{n-1,1})\wedge P^{3-n}+Q\,,\\ G &= L\,, \end{aligned} \tag{12}$$

where W^{4-n} vanishes for $n>4$ and similarly P^{3-n} vanishes for $n>3$. The field equations for the warp factor A and dilaton field Φ, $n>2$, are

$$\begin{aligned} \tilde{\nabla}^2\log A &= -n(\partial\log A)^2+2\partial_i\log A\,\partial^i\Phi+\frac{1}{2}(P^{3-n})^2+\frac{1}{4}S^2\\ &\quad+\frac{1}{8}L^2+\frac{1}{96}Z^2+\frac{1}{4}(W^{4-n})^2\,,\\ \tilde{\nabla}^2\Phi &= -n\partial_i\log A\,\partial^i\Phi+2(d\Phi)^2-\frac{1}{12}Q^2+\frac{1}{2}(P^{3-n})^2\\ &\quad+\frac{5}{4}S^2+\frac{3}{8}L^2+\frac{1}{96}Z^2\\ &\quad-\frac{1}{4}(W^{4-n})^2\,, \end{aligned} \tag{13}$$

where $S=e^{\Phi}m$ and m is the cosmological constant of (massive) IIA supergravity. Clearly if both A and Φ are constant, which is the case for all $N>16$ backgrounds, then the above two field equations imply that all the form fluxes will vanish. Significantly, the cosmological constant must vanish as well. There are no $\mathbb{R}^{n-1,1}\times_w M^{10-n}$ solutions in massive IIA supergravity that preserve $N>16$ supersymmetries. In IIA supergravity, an argument similar to the one presented above in $d=11$ supergravity reveals that M^{10-n} is locally isometric to $\mathbb{R}^{10-n}$ and so all $N>16$ $\mathbb{R}^{n-1,1}\times_w M^{10-n}$ backgrounds are locally isometric to the maximally supersymmetric vacuum $\mathbb{R}^{10,1}$. It is not apparent that the theorem holds for $n=2$ even if A and Φ are taken to be constant.

In IIB supergravity the self-dual real 5-form F and complex 3-form H field strengths of the theory for $\mathbb{R}^{n-1,1}\times_w M^{10-n}$, $n\geq 3$, backgrounds can be expressed as

$$\begin{aligned} F &= \mathrm{dvol}_A(\mathbb{R}^{n-1,1})\wedge W^{5-n}+\star W^{5-n}\,,\\ H &= \mathrm{dvol}_A(\mathbb{R}^{n-1,1})\wedge P^{3-n}+Q\,, \end{aligned} \tag{14}$$

where W^{5-n} vanishes for $n>5$ and P^{3-n} vanishes for $n>3$. The field equation of the warp factor is

$$\begin{aligned} \tilde{\nabla}^2\log A &= -n(\partial\log A)^2+\frac{3}{8}|P^{3-n}|^2+\frac{1}{48}|Q|^2\\ &\quad+\frac{4}{(5-n)!}(W^{5-n})^2\,. \end{aligned} \tag{15}$$

Clearly if A is constant, which we have demonstrated that it holds for $N>16$ backgrounds, then $F=H=0$. Moreover the homogeneity theorem implies that the two scalar fields of IIB supergravity, the axion and the dilaton, are also constant. A similar argument to that used in $d=11$ supergravity implies that M^{10-n} is locally isometric to $\mathbb{R}^{10-n}$. Thus $\mathbb{R}^{n-1,1}\times_w M^{10-n}$ are locally isometric to the $\mathbb{R}^{9,1}$ maximally supersymmetric vacuum of the theory. The same conclusion holds for $n=2$ as well provided that A is taken to be constant.

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References

[1] S.W. Beck, J.B. Gutowski, G. Papadopoulos, AdS_5 backgrounds with 24 supersymmetries, J. High Energy Phys. 1606 (2016) 126, https://doi.org/10.1007/JHEP06(2016)126, arXiv:1601.06645 [hep-th].

[2] A.S. Haupt, S. Lautz, G. Papadopoulos, AdS_4 backgrounds with $N>16$ supersymmetries in 10 and 11 dimensions, J. High Energy Phys. 1801 (2018) 087, https://doi.org/10.1007/JHEP01(2018)087, arXiv:1711.08280 [hep-th].

[3] A.S. Haupt, S. Lautz, G. Papadopoulos, A non-existence theorem for $N>16$ supersymmetric AdS_3 backgrounds, arXiv:1803.08428 [hep-th].

[4] G.W. Gibbons, Aspects of supergravity theories, in: F. del Aguila, J.A. de Azcarraga, L.E. Ibanez (Eds.), GIFT Seminar on Supersymmetry, Supergravity and Related Topics, World Scientific, 1984.

[5] J.M. Maldacena, C. Nunez, Supergravity description of field theories on curved manifolds and a no go theorem, Int. J. Mod. Phys. A 16 (2001) 822, https://doi.org/10.1142/S0217751X01003935, arXiv:hep-th/0007018.

[6] J.B. Gutowski, G. Papadopoulos, Supersymmetry of AdS and flat backgrounds in M-theory, J. High Energy Phys. 1502 (2015) 145, https://doi.org/10.1007/JHEP02(2015)145, arXiv:1407.5652 [hep-th].

[7] S.W. Beck, J.B. Gutowski, G. Papadopoulos, Supersymmetry of AdS and flat IIB backgrounds, J. High Energy Phys. 1502 (2015) 020, https://doi.org/10.1007/JHEP02(2015)020, arXiv:1410.3431 [hep-th].

[8] S. Beck, J.B. Gutowski, G. Papadopoulos, Supersymmetry of IIA warped flux AdS and flat backgrounds, J. High Energy Phys. 1509 (2015) 135, https://doi.org/10.1007/JHEP09(2015)135, arXiv:1501.07620 [hep-th].

[9] U. Gran, J.B. Gutowski, G. Papadopoulos, On supersymmetric Anti-de-Sitter, de-Sitter and Minkowski flux backgrounds, Class. Quantum Gravity 35 (6) (2018) 065016, https://doi.org/10.1088/1361-6382/aaac8c, arXiv:1607.00191 [hep-th].

[10] J.P. Gauntlett, R.C. Myers, P.K. Townsend, Supersymmetry of rotating branes, Phys. Rev. D 59 (1998) 025001, https://doi.org/10.1103/PhysRevD.59.025001, arXiv:hep-th/9809065.

[11] J.M. Figueroa-O'Farrill, On the supersymmetries of Anti-de Sitter vacua, Class. Quantum Gravity 16 (1999) 2043, https://doi.org/10.1088/0264-9381/16/6/330, arXiv:hep-th/9902066.

[12] J. Figueroa-O'Farrill, N. Hustler, The homogeneity theorem for supergravity backgrounds, J. High Energy Phys. 1210 (2012) 014, https://doi.org/10.1007/JHEP10(2012)014, arXiv:1208.0553 [hep-th].

[13] J. Figueroa-O'Farrill, N. Hustler, Symmetric backgrounds of type IIB supergravity, Class. Quantum Gravity 30 (2013) 045008, https://doi.org/10.1088/0264-9381/30/4/045008, arXiv:1209.4884 [hep-th].