

# SEARCHING FOR NEW PHYSICS WITH $\bar{\mathcal{B}}(B_{s,d} \rightarrow \mu\bar{\mu})/\Delta M_{s,d}$ \*

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We reemphasize that the ratio  $R_{s\mu} \equiv \bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})/\Delta M_s$  is a measure of the tension of the Standard Model (SM) with the latest measurements of  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$  that does not suffer from the persistent puzzle on the  $|V_{cb}|$  determinations from inclusive *versus* exclusive  $b \rightarrow c\ell\bar{\nu}$  decays and which affects the value of the CKM element  $|V_{ts}|$  that is crucial for the SM predictions of both  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$  and  $\Delta M_s$ , but cancels out in the ratio  $R_{s\mu}$ . In our analysis, we include higher-order electroweak and QED corrections and adapt the latest hadronic input to find a tension of about  $2\sigma$  for  $R_{s\mu}$  measurements with the SM independently of  $|V_{ts}|$ . We also discuss the ratio  $R_{d\mu}$  which could turn out, in particular in correlation with  $R_{s\mu}$ , to be useful for the search for new physics, when data on both ratios improve.  $R_{d\mu}$  is also independent of  $|V_{cb}|$  or more precisely  $|V_{td}|$ .

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## 1. Introduction

Since the first observation of the decay  $B_s \rightarrow \mu\bar{\mu}$  in 2013, there have been steady improvements of the measurement of its branching ratio and also of the one for  $B_d \rightarrow \mu\bar{\mu}$  by CMS, LHCb and ATLAS collaborations [1–3]. In 2020, the three experimental collaborations combined their results to provide the world average of the two-dimensional likelihood in the space of  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$  and  $\mathcal{B}(B_d \rightarrow \mu\bar{\mu})$ , which give the one-dimensional results [4–6]

$$\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu}) = (2.69^{+0.37}_{-0.35}) \times 10^{-9}, \quad (1)$$

$$\mathcal{B}(B_d \rightarrow \mu\bar{\mu}) < 1.6(1.9) \times 10^{-10} \text{ at } 90\% (95\%) \text{ C.L.} \quad (2)$$

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Very recently the LHCb Collaboration presented their final results based on the full Run 2 data [7]

$$\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu}) = (3.09^{+0.46+0.15}_{-0.43-0.11}) \times 10^{-9}, \quad (3)$$

$$\mathcal{B}(B_d \rightarrow \mu\bar{\mu}) < 2.6 \times 10^{-10} \text{ at } 95\% \text{ C.L.}, \quad (4)$$

which imply new world averages. The world averages must be performed by the experimental collaborations themselves to account properly for all systematic uncertainties. Until then, only provisional averages with varying sophistication are available, as, for example, presented in [8, 9] and [10]. We will use here the exemplary value of [10]

$$\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu}) = (2.85^{+0.34}_{-0.31}) \times 10^{-9}, \quad (5)$$

$$\mathcal{B}(B_d \rightarrow \mu\bar{\mu}) < 2.05 \times 10^{-10} \text{ at } 95\% \text{ C.L.} \quad (6)$$

The other preliminary world averages of  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$  are given in [8] and [9] with very similar values of  $(2.84 \pm 0.33) \times 10^{-9}$  and  $(2.93 \pm 0.35) \times 10^{-9}$ , respectively. The upper bounds on  $\mathcal{B}(B_d \rightarrow \mu\bar{\mu})$  are read off from the  $2\sigma$  contours of the 2-dimensional likelihood plots in [8–10], where the first two find  $2.0 \times 10^{-10}$  and  $2.2 \times 10^{-10}$  as upper bounds.

On the other hand, the present SM values of  $\mathcal{B}(B_q \rightarrow \mu\bar{\mu})$ , based on the calculations over three decades by several groups [11–18], read

$$\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})_{\text{SM}} = (3.66 \pm 0.14) \times 10^{-9}, \quad (7)$$

$$\mathcal{B}(B_d \rightarrow \mu\bar{\mu})_{\text{SM}} = (1.03 \pm 0.05) \times 10^{-10}. \quad (8)$$

Comparing the results in (5) with (7) implies the tension between the SM and the data in the ballpark of  $2\sigma$  [8, 9].

We would like to point out that such a conclusion is premature because in obtaining the result in (7) the *inclusive* determination of  $|V_{cb}|$  has been used with the value of  $|V_{cb}|_{B \rightarrow X_c} = (42.00 \pm 0.64) \times 10^{-3}$  [19]. For the corresponding *exclusive* determination of  $|V_{cb}|$ , as for example  $|V_{cb}|_{B \rightarrow D} = (40.7 \pm 1.1) \times 10^{-3}$  from  $B \rightarrow D\ell\bar{\nu}$  [20], one finds the branching ratio in question in the ballpark of  $(3.44 \pm 0.20) \times 10^{-9}$  and the reduced tension of  $1.4\sigma$  deeming the hopes for seeing new physics in this decay at work. Full compatibility between theory and experiment can be found with the less reliable determination  $|V_{cb}|_{B \rightarrow D^*} = (38.8 \pm 1.4) \times 10^{-3}$  from  $B \rightarrow D^*\ell\bar{\nu}$  [20], which gives  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})_{\text{SM}} = (3.12 \pm 0.23) \times 10^{-9}$ . Therefore, taking all these results into account, in our view, the uncertainty of 4% in (7) does not represent properly the present uncertainty in the SM prediction for the branching ratio in question. It is significantly larger due to the  $V_{cb}$  puzzle. We stress that it is only the parametrical CKM uncertainty. The remaining theoretical ones are in the ballpark of a few percent.

In view of the fact that the tension between the inclusive and exclusive determinations of  $|V_{cb}|$  has not been satisfactorily resolved despite the efforts of world experts lasting already two decades (see [21] and references therein), it may still take a few years before we will be able to find out whether the tension between the data and the SM value for the branching ratio in question is  $2\sigma$  or significantly lower.

In this paper, we would like to demonstrate that a much better insight into what is going on can be obtained by using the strategy that one of us proposed already in 2003 [22]. In this strategy, one considers instead of the branching ratios the ratios

$$R_{q\mu} \equiv \frac{\bar{B}(B_q \rightarrow \mu\bar{\mu})}{\Delta M_q}, \quad q = d, s \quad (9)$$

that have the following advantages over the branching ratios themselves:

- The dependence on  $|V_{cb}|$  drops out. Even more, the dependences on  $|V_{ts}|$  and  $|V_{td}|$  that contain additional subleading uncertainties beyond  $|V_{cb}|$  cancel out.
- The dependence on the  $B_q$ -meson decay constant  $f_{B_q}$  drops out and present uncertainties in  $f_{B_q}$  from the lattice QCD (LQCD) are irrelevant in this strategy.
- The dependence on the top-quark mass is decreased lowering thereby the uncertainty due to  $m_t$ .
- Due to the negligible experimental errors on  $\Delta M_q$ , the experimental errors of  $R_{q\mu}$  are practically the same as in the branching ratios themselves.

This means that for the purpose of testing the SM now the decision of whether the inclusive or exclusive value of  $|V_{cb}|$  should be used is irrelevant and as a byproduct the parametric uncertainty related to  $f_{B_q}$  is absent as well and the one due to  $m_t$  is reduced. However, it should be emphasized that the main goal in the strategy of [22] is to test the SM when  $R_{s\mu}$  and  $R_{d\mu}$  are taken together to test the models with Constrained Minimal Flavour Violation (CMFV) [23, 24]. In this manner, the possible anomalies in the the ratios  $R_{q\mu}$  would signal NP not only beyond the SM but also beyond CMFV, that is non-SM operators and/or new flavour-violating parameters beyond the CKM ones, in particular, new CP-violating phases.

Yet nothing is for free. The use of  $\Delta M_q$  introduces the dependence on the non-perturbative parameters  $B_q$  or  $\hat{B}_q$ . However, these parameters are already known from LQCD and HQET sum rule calculations within a few percent accuracy and the prospects for obtaining even better determinations in coming years are good. We will be more explicit about it below.

At first sight, one would think that the same result could be obtained in global  $b \rightarrow s\ell\bar{\ell}$  fits, that use only  $\Delta B = 1$  transitions, by including now  $\Delta M_s$ . However, without the imposition of CMFV and without a careful inclusion of the correlation between  $\Delta M_s$  and  $B_s \rightarrow \mu\bar{\mu}$ , the cancellation of  $|V_{cb}|$  in question cannot be achieved.

The outline of our paper is as follows. In Section 2, we recall the SM expressions for the two quantities from which  $R_{q\mu}$  are constructed and introduce a qualitatively similar ratio  $\kappa_{q\mu}$ . In Section 3, we collect the numerical input and present the numerical analysis of  $R_{s\mu}$  and  $\kappa_{s\mu}$ . For completeness, we also present the result for  $R_{d\mu}$  and  $\kappa_{d\mu}$ . Further, we briefly discuss the double ratio  $R_{s\mu}/R_{d\mu}$ . In Section 4, a brief outlook is given.

## 2. Basic formulae

In this section, we recall the basic formulae for the branching ratios of the leptonic decays  $B_q \rightarrow \mu\bar{\mu}$  and the mass differences in neutral  $B$ -meson systems  $\Delta M_q$ . Besides the higher-order QCD corrections, we include the known next-to-leading (NLO) electroweak (EW) corrections as well as QED corrections.

The effective Lagrangian for  $|\Delta B| = 1$  decays ( $q = d, s$ )

$$\mathcal{L}_{\Delta B=1} = \mathcal{N}_q \sum_i C_i(\mu_b) O_i + \text{h.c.}, \quad \mathcal{N}_q = \frac{G_F^2 m_W^2}{\pi^2} V_{tb} V_{tq}^* \quad (10)$$

contains the normalization factor  $\mathcal{N}_q$ , which is chosen to facilitate the renormalization at NLO in EW interactions [15, 25]. The Wilson coefficients are evaluated at the scale  $\mu_b \sim m_b$  of the order of the  $b$ -quark mass and include NNLO QCD and NLO EW/QED corrections [15, 16, 26, 27]. At LO in EW/QED interactions the single operator

$$O_{10} = [\bar{q}\gamma^\mu P_L b] [\bar{\mu}\gamma_\mu \gamma_5 \mu], \quad P_L \equiv \frac{1 - \gamma_5}{2}, \quad (11)$$

is relevant only<sup>1</sup>. The time-integrated branching fraction [28], denoted by a bar, is given by

$$\bar{\mathcal{B}}(B_q \rightarrow \mu\bar{\mu}) = \frac{|\mathcal{N}_q|^2 M_{B_q}^3 f_{B_q}^2}{8\pi \Gamma_q^H} \beta_{q\mu} \left( \frac{m_\mu}{M_{B_q}} \right)^2 |C_{10}^{\text{eff}}|^2, \quad \beta_{q\mu} \equiv \sqrt{1 - \frac{4m_\mu^2}{M_{B_q}^2}}, \quad (12)$$

where  $C_{10}^{\text{eff}}$  includes

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<sup>1</sup> Here, we use the convention  $C_{10} = -2C_A$  compared to [14, 16] and  $C_{10} = \tilde{c}_{10}$  to [15]. It differs by a factor of sine-squared of the weak mixing angle to  $c_{10}$  of [26, 27]:  $C_{10} = s_W^2 c_{10}$  at LO in EW interactions.

1. the NLO EW corrections from matching the SM at the electroweak scale  $\mu_{\text{ew}} \sim 160$  GeV and resummed QED corrections to the scale  $\mu_b \sim m_b$  [15];
2. power-enhanced structure-dependent NLO QED corrections between the scales  $\mu_b$  and the scales  $\Lambda_{\text{QCD}}$  [17, 18].

It is the photon-inclusive branching fraction, recovered after including soft-photon final-state radiation [13, 18]. In the SM, the time-integration implies that the lifetime  $\Gamma_q^H$  of the heavy-mass eigenstate  $|B_q^H\rangle$  has to be used instead of the averaged one [28]. However, time-integration is at the current precision numerically only relevant for  $B_s$  decays. We follow [18] for the calculation of  $\bar{B}(B_q \rightarrow \mu\bar{\mu})$ .

The mass difference of the neutral meson system is governed in the SM by a single  $|\Delta B| = 2$  operator<sup>2</sup>

$$\mathcal{L}_{\Delta B=2} = -\frac{\mathcal{N}_q V_{tb} V_{tq}^*}{4} C_{\text{VLL}}(\mu_b) O_{\text{VLL}} + \text{h.c.}, \quad O_{\text{VLL}} = [\bar{q}\gamma^\mu P_L b] [\bar{q}\gamma_\mu P_L b] \quad (13)$$

with

$$C_{\text{VLL}}(\mu_{\text{ew}}) = S_0(x_t) + \dots, \quad S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}, \quad (14)$$

and  $x_t \equiv m_t^2/m_W^2$ . Here, we include besides the LO contribution  $S_0(x_t)$  also higher-order corrections indicated by the dots. These are  $S_{\text{QCD}}(x_t)$  at NLO in QCD [29] as well as  $S_{\text{ew}}(x_t)$  at NLO in EW corrections [30]. The RG evolution from  $\mu_{\text{ew}}$  to  $\mu_b$  is performed to NLO in QCD

$$C_{\text{VLL}}(\mu_b) = \eta^{6/23} \left[ S_0(x_t) + S_{\text{ew}}(x_t) + \frac{\alpha_s(\mu_b)}{4\pi} \left( \frac{5165}{3174} (1-\eta) S_0(x_t) + \eta S_{\text{QCD}}(x_t) \right) \right], \quad (15)$$

where  $\eta \equiv \alpha_s(\mu_{\text{ew}})/\alpha_s(\mu_b)$ . The hadronic matrix element of the  $|\Delta B| = 2$  operator in the  $\overline{\text{MS}}$  scheme at the scale  $\mu_b$  is defined by

$$\langle B_q | O_{\text{VLL}} | \bar{B}_q \rangle (\mu_b) = \frac{2}{3} M_{B_q}^2 f_{B_q}^2 B_q(\mu_b) \quad (16)$$

in terms of the  $B_q$ -meson decay constant  $f_{B_q}$  and the so-called bag factor  $B_q(\mu_b)$  in the  $\overline{\text{MS}}$  scheme. The latter is related to the renormalization group-

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<sup>2</sup> This is also the case of CMFV models, but the function  $S_0(x_t)$  receives additional flavour-universal contributions.

invariant bag factor  $\hat{B}_q$  at NLO in QCD as [29]

$$\hat{B}_q = \alpha_s(\mu)^{-6/23} \left( 1 + \frac{\alpha_s(\mu)}{4\pi} \frac{5165}{3174} \right) B_q(\mu) \stackrel{\mu=4.18 \text{ GeV}}{=} 1.520 B_q(\mu = 4.18 \text{ GeV}), \quad (17)$$

where  $\alpha_s(\mu = 4.18 \text{ GeV}) = 0.2241$  has been used in the second equation. Since the Wilson coefficient (15) is calculated in the  $\overline{\text{MS}}$  scheme, it is not advisable to convert the lattice results that were originally calculated in the  $\overline{\text{MS}}$  scheme to the RG-invariant bag factor<sup>3</sup>, as done for example by FLAG before averaging them. At least the numerical values for such a conversion should be always provided. An even more principal question concerns the factorization of the matrix element (16) into the decay constant and the bag factor, when lattice collaborations might actually calculate the l.h.s. of (16) directly. However, we stress that in our strategy only the bag factor is required and our predictions profit from cancellations of systematic uncertainties in lattice and sum rule predictions for this quantity.

The mass difference reads as

$$\Delta M_q = \frac{G_F^2 m_W^2}{6\pi^2} M_{B_q} f_{B_q}^2 B_q(\mu_b) |V_{tb} V_{tq}^*|^2 |C_{\text{VLL}}(\mu_b)|. \quad (18)$$

The phenomenologically most interesting case is for  $B_s$  mesons, since the leptonic decay has a larger branching ratio, enhanced by  $(V_{ts}/V_{td})^2 \sim 20$  compared to  $B_d$  mesons. In particular, due to the unitarity of the CKM matrix, the matrix element  $V_{ts}$  depends strongly on the input of  $V_{cb}$  that should be preferably determined in tree-level decays  $b \rightarrow c\ell\bar{\nu}$ . In fact, the ratio

$$\left| \frac{V_{tb} V_{ts}^*}{V_{cb}} \right|^2 = 1 - (1 - 2\rho)\lambda^2 + \mathcal{O}(\lambda^4), \quad \lambda \approx 0.22, \quad (19)$$

with  $\rho \approx 0.15$ , is rather independent of  $B$ -physics input, as can be seen in the Wolfenstein parametrization of the CKM matrix. This renders both,  $\tilde{B}(B_s \rightarrow \mu\bar{\mu})$  and  $\Delta M_s$ , very sensitive to the input value of  $V_{cb}$ . Unfortunately, the persisting discrepancy of the determination of  $V_{cb}$  from inclusive and exclusive  $b \rightarrow c\ell\bar{\nu}$  decays prevents stringent tests of the SM using charged-current tree-level decays *versus* FCNC decays  $\tilde{B}(B_q \rightarrow \mu\bar{\mu})$ . Previous predictions [14, 18] used the inclusive determination of  $|V_{cb}|_{B \rightarrow X_c}$ , because the theoretical predictions are more solid for  $B \rightarrow X_c \ell\bar{\nu}$ , thereby yielding larger values of  $\tilde{B}(B_s \rightarrow \mu\bar{\mu})$  compared to exclusive determinations. As pointed out in [22], the dependence on  $V_{cb}$  cancels out in the

<sup>3</sup> On the other hand, if the matching between the lattice UV regulator and the dimensional regulator is carried out non-perturbatively, one obtains directly  $\hat{B}_q$  and one should not convert it to the  $\overline{\text{MS}}$  scheme, because this would imply additional uncertainties. We thank Andreas Kronfeld for this insight.

ratio  $R_{s\mu}$ , see (9), thus removing the issue of  $V_{cb}$  in tests of the SM with  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$ , thereby introducing a correlation with  $\Delta M_s$ , which might involve further assumptions on NP, like CMFV, when extending the tests beyond the framework of the SM.

Analogous comments can be made about  $\mathcal{B}(B_d \rightarrow \mu\bar{\mu})$  and  $\Delta M_d$  in which the CKM element  $|V_{td}|$  is involved. It is also very sensitive to the value of  $|V_{cb}|$  but it cancels out in the ratio  $R_{d\mu}$ .

In summary, the SM expression for  $R_{q\mu}$  is given as follows:

$$R_{q\mu}|_{\text{SM}} = \frac{3(G_F m_W m_\mu)^2 \beta_{q\mu}}{4\pi^3 \Gamma_q^H} \frac{|C_{10}^{\text{eff}}|^2}{C_{\text{VLL}}(\mu_b) B_q(\mu_b)}. \quad (20)$$

In addition to  $R_{q\mu}$  that are dimensionful, slightly modified dimensionless ratios

$$\kappa_{q\mu} \equiv \frac{R_{q\mu} \Gamma_q^H}{(G_F m_W m_\mu)^2 \beta_{q\mu}} \stackrel{\text{SM}}{=} \frac{3}{4\pi^3} \frac{|C_{10}^{\text{eff}}|^2}{C_{\text{VLL}}(\mu_b) B_q(\mu_b)} \quad (21)$$

have been introduced in [14]. The theory prediction for  $\kappa_{ql}$  does not suffer from the uncertainty of  $\Gamma_q^H$ , in contrast to the theory prediction for  $R_{q\mu}$ . The uncertainty of  $\Gamma_q^H$  is now shifted to the experimental determination of  $\kappa_{q\mu}$ . This could be an advantage if the experimental determination of the ratio  $\Delta M_q/\Gamma_q^H$ , which enters  $\kappa_{q\mu}$ , allows for cancellation of systematic uncertainties that would otherwise be present in  $\Gamma_q^H$ . However currently, the uncertainty of the experimental determination of both,  $R_{q\mu}$  and  $\kappa_{q\mu}$ , is dominated by the one of  $\bar{\mathcal{B}}(B_q \rightarrow \mu\bar{\mu})$ . In the numerical analysis, we will mainly focus on  $R_{q\mu}$ . Further, the overall dependence on  $G_F$  in the SM prediction of  $R_{q\mu}$  is also removed in  $\kappa_{ql}$ . This would suggest that its prediction is even independent of new physics contributions in the  $\beta$  decay  $\mu \rightarrow e \nu_\mu \bar{\nu}_e$ , but  $G_F$  enters indirectly in the determination of  $m_W$  and the weak mixing angle when calculating  $C_{10}^{\text{eff}}$  and  $C_{\text{VLL}}(\mu_b)$ .

### 3. Numerical analysis

The numerical predictions of the Wilson coefficients depend on the values of the parameters of the SM from the electroweak sector, the strong coupling  $\alpha_s$  and the top-quark mass  $m_t$ , which enter the calculation of the Wilson coefficients. We collect their numerical values in Table I and proceed with the calculation of the Wilson coefficients as described in [15, 26, 27]. Note that we have chosen for the input value of the top-quark mass in the pole-scheme the one determined in cross-section measurements. For what concerns the electroweak renormalization, we use the on-shell scheme 2 (“OS-2”) introduced in [15], in which the mass of the  $W$ -boson is not an independent input, but calculated following [31]. Therefore, the value in Table I

slightly differs from the one in the PDG [32]. The central value of the matching scale is fixed to  $\mu_{\text{ew}} = 160$  GeV and the central value of the low-energy scale is set to  $\mu_b = 5.0$  GeV.

TABLE I

Numerical input values for parameters entering  $\bar{\mathcal{B}}(B_q \rightarrow \mu\bar{\mu})$  and  $\Delta M_q$ . The  $B_q$ -meson decay constants  $f_{B_q}$  are averages from the FLAG group for  $N_f = 2 + 1 + 1$  from [37–40]. They are almost identical to the single determination of FNAL/MILC  $f_{B_s} = 230.7(1.3)$  MeV and  $f_{B_d} = 190.5(1.3)$  MeV [37]. The bag factors have been converted from the  $\overline{\text{MS}}$  scheme to the RG-invariant form using the conversion factor 1.520 at  $\mu = 4.18$  GeV in (17).

Parameter	Value	Ref.	Parameter	Value	Ref.
$G_F$	$1.166379 \times 10^{-5} \text{ GeV}^{-2}$	[32]	$m_Z$	91.1876(21) GeV	[32]
$\alpha_s^{(5)}(m_Z)$	0.1179(10)	[32]	$m_W$	80.358(8) GeV	
$\alpha_{\text{em}}^{(5)}(m_Z)$	1/127.955	[33]	$m_t^{\text{OS}}$	172.4(7) GeV	[32]
$M_{B_s}$	5366.88(17) MeV	[32]	$M_{B_d}$	5279.65(12) MeV	[32]
$\Delta M_s$	17.749(20) $\text{ps}^{-1}$	[32]	$\Delta M_d$	0.5065(19) $\text{ps}^{-1}$	[32]
$1/\Gamma_s^H$	1.620(7) ps	[32]	$2/(\Gamma_d^H + \Gamma_d^L)$	1.519(4) ps	[32]
$f_{B_s}$	230.3(1.3) MeV	[34]	$f_{B_d}$	190.0(1.3) MeV	[34]
$B_s(4.18 \text{ GeV})$	0.849(23)	[35]	$B_d(4.18 \text{ GeV})$	0.835(28)	[35]
$\hat{B}_s$	1.291(35)		$\hat{B}_d$	1.269(43)	
$\lambda_{B_s}(1 \text{ GeV})$	400(150) MeV	[36]	$\lambda_{B_d}(1 \text{ GeV})$	350(150) MeV	[36]

The hadronic input would usually concern the  $B$ -meson decay constants  $f_{B_q}$  and the bag factors  $\hat{B}_q$  or  $B_q(\mu_b)$ , but in our procedure, the  $f_{B_q}$  do not enter. We provide their values in Table I for later purposes. The bag factors, on the other hand, are crucial in this strategy and we summarize their present status below.

The FLAG averages of several  $N_f = 2 + 1$  lattice calculations are [34]

$$B_s(4.18 \text{ GeV}) = 0.89(4), \quad B_d(4.18 \text{ GeV}) = 0.86(6). \quad (22)$$

They are based<sup>4</sup> on the calculations [41–43] from HPQCD, RBC-UKQCD and MILC/FNAL, respectively. The rather high values are driven by the calculation in [41]. The more recent  $N_f = 2 + 1 + 1$  lattice calculation from HPQCD [44]

$$B_s(4.16 \text{ GeV}) = 0.813(35), \quad B_d(4.16 \text{ GeV}) = 0.806(40) \quad (23)$$

<sup>4</sup> They have been converted to  $\overline{\text{MS}}$  using the conversion factor 1.5158 from [41] in (17).



finds lower values and has smaller uncertainties. In particular they provide an average with the  $N_f = 2 + 1$  results from MILC/FNAL [41]

$$B_s(4.16 \text{ GeV}) = 0.84(3), \quad B_d(4.16 \text{ GeV}) = 0.83(3). \quad (24)$$

Eventually, HQET sum rule calculations of the bag factors are also available [45], which have been averaged in [46]. These averages are based on [41, 44, 45] and listed in Table I. They will be used in the numerical evaluations.

### 3.1. $R_{q\mu}$ and $\kappa_{q\mu}$

The SM predictions for the ratios  $R_{q\mu}$  are then

$$\begin{aligned} R_{s\mu}|_{\text{SM}} &= 2.042 \left( 1 \begin{array}{c} +0.0274 \\ -0.0003 \end{array} \middle|_{\mu_{\text{ew}}} \begin{array}{c} +0.0028 \\ -0.0020 \end{array} \middle|_{\mu_b} \begin{array}{c} +0.0101 \\ -0.0100 \end{array} \middle|_{m_t} \begin{array}{c} +0.0278 \\ -0.0264 \end{array} \middle|_{B_s} \begin{array}{c} +0.0043 \\ -0.0043 \end{array} \middle|_{\Gamma_s^H} \right) \times 10^{-10} \text{ ps} \\ &= (2.042 \begin{array}{c} +0.083 \\ -0.058 \end{array}) \times 10^{-10} \text{ ps}, \end{aligned} \quad (25)$$

$$\begin{aligned} R_{d\mu}|_{\text{SM}} &= 1.947 \left( 1 \begin{array}{c} +0.0274 \\ -0.0003 \end{array} \middle|_{\mu_{\text{ew}}} \begin{array}{c} +0.0031 \\ -0.0022 \end{array} \middle|_{\mu_b} \begin{array}{c} +0.0101 \\ -0.0100 \end{array} \middle|_{m_t} \begin{array}{c} +0.0347 \\ -0.0324 \end{array} \middle|_{B_d} \begin{array}{c} +0.0026 \\ -0.0026 \end{array} \middle|_{\Gamma_d} \right) \times 10^{-10} \text{ ps} \\ &= (1.947 \begin{array}{c} +0.089 \\ -0.066 \end{array}) \times 10^{-10} \text{ ps}. \end{aligned} \quad (26)$$

They represent the most accurate predictions on these ratios to date. Note that the central values would be  $R_{s\mu}|_{\text{SM}} = 2.022 \times 10^{-10} \text{ ps}$  and  $R_{d\mu}|_{\text{SM}} = 1.928 \times 10^{-10} \text{ ps}$  when neglecting the NLO EW corrections [30] to  $C_{\text{VLL}}$ . The electroweak scale has been varied within  $\mu_{\text{ew}} \in [60, 300] \text{ GeV}$ , and exhibits a strong asymmetric effect, because the central value  $\mu_{\text{ew}} = 160 \text{ GeV}$  is close to the lowest predictions of  $R_{q\mu}$ . The variation with  $\mu_{\text{ew}}$  is rather large, up to +3%, mainly from  $\mu_{\text{ew}} \rightarrow 60 \text{ GeV}$ . This simple variation reproduces the more careful estimates of various higher-order EW and QCD scheme dependences discussed for  $\bar{\mathcal{B}}(B_q \rightarrow \mu\bar{\mu})$  in [14, 15]. The low-energy scale is varied within  $\mu_b \in [2.5, 10] \text{ GeV}$  and results in about 0.3% uncertainty. The top-quark mass dependence is about 1% and the one of the lifetime about (0.3–0.4)%. The largest uncertainty of about (3–4)% is due to the bag factors. The theoretical uncertainties of the observables have been obtained by varying consecutively each parameter within the error ranges given in Table I. Throughout these uncertainties are then combined by adding them in quadrature.

The experimental value of  $R_{s\mu}$  follows from the preliminary world average of  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$  in (5) and  $\Delta M_s$  given in Table I as

$$R_{s\mu}|_{\text{exp}} = (1.61 \begin{array}{c} +0.19 \\ -0.17 \end{array}) \times 10^{-10} \text{ ps}, \quad (27)$$

having a tension with the SM prediction (25) of about  $2.1\sigma$ . Similar values  $R_{s\mu}|_{\text{exp}} = (1.60 \pm 0.19) \times 10^{-10}$  ps and  $(1.65 \pm 0.20) \times 10^{-10}$  ps are found from the averages [8] and [9], respectively, with tensions of  $2.2\sigma$  and  $1.9\sigma$ . On the other hand, the preliminary world average of  $\mathcal{B}(B_d \rightarrow \mu\bar{\mu})$  in (6) provides only an upper limit for

$$R_{d\mu}|_{\text{exp}} < 4.05 \times 10^{-10} \text{ ps} \quad \text{at } 95\% \text{ C.L.} \quad (28)$$

The SM predictions of  $R_{q\mu}$  are compared in figure 1 with the experimental results in the plane of  $\mathcal{B}(B_q \rightarrow \mu\bar{\mu})$  versus  $\Delta M_q$ . In this plane, the  $R_{q\mu}$  are straight lines where the bands indicate the theoretical uncertainties. The case of  $R_{s\mu}$  shows a tension of about  $2\sigma$ , depending on the average used from (5). When interpreted in the context of physics beyond the SM, the tension between experiment and SM in  $R_{s\mu}$  could be caused by new physics in both,  $\Delta M_s$  and  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$ . However, given the very tiny slope  $R_{s\mu}|_{\text{SM}}$ , the new physics impact on  $\Delta M_s$  should be rather large, *i.e.* the experimental measurement should be around 14.5 ps to be fully compatible with  $R_{s\mu}|_{\text{SM}}$  and the measured value of  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$ . The experimental prospects from LHCb with 300/fb for the absolute uncertainty  $\delta\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu}) \approx 0.16 \times 10^{-9}$

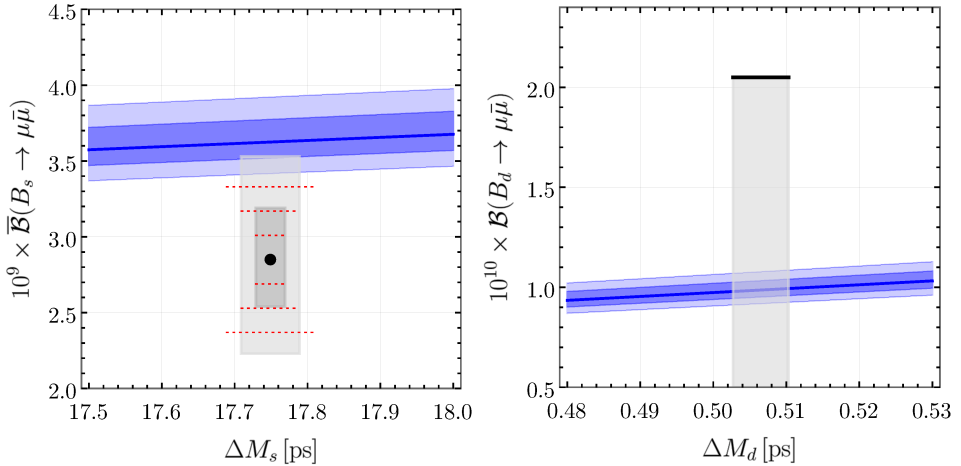


Fig. 1. (Colour on-line) The SM predictions of  $R_{s\mu}$  [left] and  $R_{d\mu}$  [right] are shown by the horizontal blue bands in the plane of  $\mathcal{B}(B_q \rightarrow \mu\bar{\mu})$  versus  $\Delta M_q$ . The lighter bands indicate twice the theory errors given in (25) and (26), respectively. The (lighter) darker gray areas show the experimental results at (95%) 68% C.L. from [10] with the dot at the central value and the solid line as upper bound, see (5) and (6), respectively. The dotted red lines show the 68%, 95% and 99% C.L. regions for the projections of LHCb with 300/fb, see the text for more details, assuming the central value from [9].

is about half the ones in (5), but it remains to be seen whether future measurements confirm the current central values. If so, the current tension would be increased to above  $3\sigma$ .

The SM predictions for the ratio  $\kappa_{q\mu}$  are

$$\kappa_{s\mu}|_{\text{SM}} = (1.286^{+0.052}_{-0.036}) \times 10^{-2}, \quad \kappa_{d\mu}|_{\text{SM}} = (1.308^{+0.059}_{-0.044}) \times 10^{-2}. \quad (29)$$

Note that the central values would be  $\kappa_{s\mu}|_{\text{SM}} = 1.274 \times 10^{-2}$  and  $\kappa_{d\mu}|_{\text{SM}} = 1.295 \times 10^{-2}$  when neglecting the NLO EW corrections [30] to  $C_{\text{VLL}}$ . The sources of uncertainty are the same as for  $R_{q\mu}$ , except that the one of  $\Gamma_q^H$  is removed. For the remaining, the relative uncertainties in  $\kappa_{q\mu}$  are the same as in the corresponding  $R_{q\mu}$ . The current experimental determinations are based on the world averages (5) and (6)

$$\kappa_{s\mu}|_{\text{exp}} = (1.011^{+0.121}_{-0.110}) \times 10^{-2}, \quad \kappa_{d\mu}|_{\text{exp}} < 2.7 \times 10^{-2} \text{ at } 95\% \text{ C.L.}, \quad (30)$$

where the experimental error from  $\Delta M_q$  and  $\Gamma_q^H$  are negligible at the current stage. Similar values  $\kappa_{s\mu}|_{\text{exp}} = (1.007 \pm 0.117) \times 10^{-2}$  and  $(1.039 \pm 0.124) \times 10^{-2}$  are obtained from [8] and [9], respectively. The corresponding upper bounds are  $\kappa_{d\mu}|_{\text{exp}} < 2.7 \times 10^{-2}$  and  $2.9 \times 10^{-2}$ , respectively. The tension between the SM prediction and the experimental measurement of  $\kappa_{s\mu}$  is in the range of  $(1.8\text{--}2.2)\sigma$ , depending on the averages presented in [8–10]. There is only an upper bound for  $\kappa_{d\mu}|_{\text{exp}}$  well compatible with the SM prediction.

As pointed out in [22], the following relation holds:

$$\frac{\Delta M_s}{\Delta M_d} \frac{\Gamma_d}{\Gamma_s^H} \frac{B_d(\mu_b)}{B_s(\mu_b)} \stackrel{\text{CMFV}}{=} \frac{\mathcal{B}(B_s \rightarrow \mu\bar{\mu})}{\mathcal{B}(B_d \rightarrow \mu\bar{\mu})}, \quad (31)$$

in the SM and also in any CMFV model, up to negligible effects. The l.h.s. of (31) involves only measurable quantities except for the ratio  $B_d/B_s$ . This ratio can be determined with higher precision than the individual bag factors. The most precise predictions are  $B_s/B_d = 0.9984(45)_{\text{stat}}(^{+80}_{-63})_{\text{syst}}$  [47],  $B_s/B_d = 1.008(25)$  [44] and  $B_s/B_d = 0.987(^{+7}_{-9})$  [45]. This leads to a relative uncertainty of about  $(1\text{--}2)\%$  when using the values of the  $\Delta M_q$  and the lifetimes from Table I together with the uncertainties of the ratio of bag factors. Translating this result into ratios  $R_{s\mu}$  and  $R_{d\mu}$ , we find

$$\frac{R_{s\mu}}{R_{d\mu}} \stackrel{\text{CMFV}}{=} \frac{\Gamma_d}{\Gamma_s^H} \frac{B_d(\mu_b)}{B_s(\mu_b)} = \begin{cases} 1.072 \pm 0.011 & [47] \\ 1.058 \pm 0.027 & \text{for } [44] \\ 1.081 \pm 0.011 & [45] \end{cases}, \quad (32)$$

a double ratio that is independent of CKM parameters and the Wilson coefficients. The prospects to measure the ratio  $\mathcal{B}(B_d \rightarrow \mu\bar{\mu})/\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$  at LHCb foresee a precision of 10% [48] with 300/fb.

It should be emphasized that although the ratio of the two ratios in question is common to all models with Constrained Minimal Flavour Violation (CMFV) [23], the ratios  $R_{q\mu}$  themselves are not. Indeed, CMFV models can only be distinguished from each other by the Wilson coefficients  $C_{10}$  and  $C_{VLL}$  entering (12) and (18), respectively, and varying them, one just moves on the straight lines shown in figure 1.

Despite the comments just made, our result for the size of the tension in the case of  $R_{s\mu}$ , that is independent of the value of  $|V_{cb}|$ , being in the ballpark of  $2\sigma$  is consistent with the results in [8, 9], where the inclusive value of  $|V_{cb}|$  was used. However, with such a value, the SM prediction for  $\Delta M_s$  is fully consistent with the data, although as analysed in [35], with the improved future theoretical calculations of  $B_s$  and  $f_{B_s}$  some amount of NP contributing to  $\Delta M_s$  could still be identified. Yet, these findings indicate that  $\Delta M_s$  is SM-like and it is some NP affecting  $B_s \rightarrow \mu\bar{\mu}$  that is dominantly responsible for the  $2\sigma$  tension in  $R_{s\mu}$  found by us.

### 3.2. $\Delta M_q$ and $V_{cb}$

Assuming then for the moment that  $\Delta M_q$  is SM-like, we would like to point out that the mass differences  $\Delta M_q$  provide currently in the framework of the SM one of the most precise probes of  $|V_{tb}V_{tq}^*|^2$ , and hence indirectly also on  $V_{cb}$ . This is thanks to the high experimental accuracy, but also to the high control over the hadronic uncertainties from  $f_{B_q}$  and  $B_q(\mu)$  in the theoretical predictions. In fact,  $\Delta M_q$  are presently the only loop-induced transitions in the SM in which both theoretical calculations and experimental data are very accurate, even better than  $B \rightarrow X_s\gamma$  decay and  $\varepsilon_K$ , both known at the NNLO level. In principle, also  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$ , being theoretically clean [49–51] and sensitive to the choice of  $|V_{cb}|$ , could be used for this purpose [52]<sup>5</sup>, but this would require dramatic improvements on the experimental side and from the present perspective, it is better to use them for the search of NP rather than the determination of the CKM parameters.

The various sources of uncertainties in  $\Delta M_q$  contribute as

$$\begin{aligned} \Delta M_s|_{\text{SM}} &= 10444.8 \times |V_{tb}V_{ts}^*|^2 \\ &\times \left( 1 \begin{array}{c} +0.0001 \\ -0.0269 \end{array} \middle|_{\mu_{\text{ew}}} \begin{array}{c} +0.0008 \\ -0.0009 \end{array} \middle|_{\mu_b} \begin{array}{c} +0.0098 \\ -0.0098 \end{array} \middle|_{m_t} \begin{array}{c} +0.0271 \\ -0.0271 \end{array} \middle|_{B_s} \begin{array}{c} +0.0113 \\ -0.0113 \end{array} \middle|_{f_{B_s}} \right) \text{ps}^{-1} \\ &= 10444.8 \times |V_{tb}V_{ts}^*|^2 \left( 1 \begin{array}{c} +0.031 \\ -0.041 \end{array} \right) \text{ps}^{-1}, \end{aligned} \quad (33)$$

---

<sup>5</sup> Note that the  $K \rightarrow \pi\nu\bar{\nu}$  branching ratios being proportional to  $|V_{td}V_{ts}^*|^2$  are even more sensitive to the choice of  $|V_{cb}|$  than the  $B$  observables considered here.

$$\begin{aligned}
 \Delta M_d|_{\text{SM}} &= 6878.3 \times |V_{tb}V_{td}^*|^2 \\
 &\times \left( 1 \begin{array}{cc} +0.0001 & +0.0008 \\ -0.0269 & -0.0009 \end{array} \middle|_{\mu_{\text{ew}}} \begin{array}{cc} +0.0098 & +0.0335 \\ -0.0098 & -0.0335 \end{array} \middle|_{m_t} \begin{array}{cc} +0.0137 & +0.0137 \\ -0.0137 & -0.0137 \end{array} \middle|_{f_{B_d}} \right) \text{ps}^{-1} \\
 &= 6878.3 \times |V_{tb}V_{td}^*|^2 \left( 1 \begin{array}{c} +0.038 \\ -0.046 \end{array} \right) \text{ps}^{-1}, \tag{34}
 \end{aligned}$$

where the CKM combinations are left unspecified. The SM predictions have about (4–5)% relative uncertainty, with the largest uncertainty from the bag factor. This allows to extract the CKM combinations with about 2% relative uncertainty, which is at the same level as the determination from the inclusive  $B \rightarrow X_c \ell \bar{\nu}$  with about 1.5% relative uncertainty:  $|V_{cb}|_{B \rightarrow X_c} = (42.00 \pm 0.64) \times 10^{-3}$  [19]. The experimental measurements of  $\Delta M_q$  yield the central values  $|V_{tb}V_{ts}^*| = 41.22 \times 10^{-3}$  and  $|V_{tb}V_{td}^*| = 8.58 \times 10^{-3}$  in the SM. On the basis of the inclusive determination of  $|V_{cb}|_{B \rightarrow X_c}$ , one finds in the framework of the SM for the ratio in (19) that  $|V_{tb}V_{ts}^*|/|V_{cb}| = 0.982$ . It would be interesting to verify whether CKM fits that do not include  $|\Delta B| = 2$  and  $b \rightarrow c \ell \bar{\nu}$  processes provide values that are compatible with this one. That the determinations of  $|V_{tb}V_{ts}^*|$  in the framework of the SM from  $\Delta M_s$  lead to branching ratios of  $\bar{B}(B_s \rightarrow \mu\bar{\mu})$  above the data (5) has been discussed previously, as for example in [35, 44].

### 3.3. The issue of $m_t$ in rare decays

The ratios  $\kappa_{q\mu} \sim (m_t^{\overline{\text{MS}}}/m_W)^2$  scale with the second power of the top-quark mass in the  $\overline{\text{MS}}$  scheme and, in principle, might be used to determine the top-quark mass in rare flavour processes under the assumption that  $\kappa_{q\mu}$  are not affected by NP contributions, which is still a possibility. A discussion of other examples in flavour physics that require the knowledge of CKM input and the corresponding prospects can be found in [53]. The  $\overline{\text{MS}}$  mass is actually the preferred scheme for rare decay calculations over the pole scheme (OS), which however is used in many collider physics applications and provided in the PDG [32]. In the numerical evaluation, we converted the top-quark mass from the pole to the  $\overline{\text{MS}}$  scheme (in QCD), see [14], using the perturbative expressions at 3-loops. For illustration, we show the dependence of  $\kappa_{s\mu}$  on  $m_t^{\text{OS}}$  in figure 2. The preferred values of  $m_t^{\text{OS}}$ , corresponding to the central value and 68% C.L. interval of  $\kappa_{s\mu}|_{\text{exp}}$  in (30), are  $m_t^{\text{OS}} \in [136, 158]$  GeV and are much lower than those from collider determinations. Such low values would correspond to absolute stability of the SM vacuum [54]. The current experimental uncertainty of  $\kappa_{s\mu}|_{\text{exp}}$  is dominated by the one of  $\bar{B}(B_s \rightarrow \mu\bar{\mu})$ . Assuming a future measurement with 4% relative uncertainty, as might be feasible at LHCb, a determination of  $m_t^{\text{OS}}$  with about 2% relative uncertainty can be expected if the theoretical uncertainties due to the bag factor will be negligible at this level in the

future. Clearly, this is not competitive to determinations based on collider observables. If the  $V_{cb}$  puzzle will be solved in the future, then  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu}) \sim (m_t^{\overline{\text{MS}}}/m_W)^4$  could offer a better opportunity, because it scales with the fourth power of  $m_t$  if sufficient control over  $V_{cb}$  and  $f_{B_s}$  are provided.

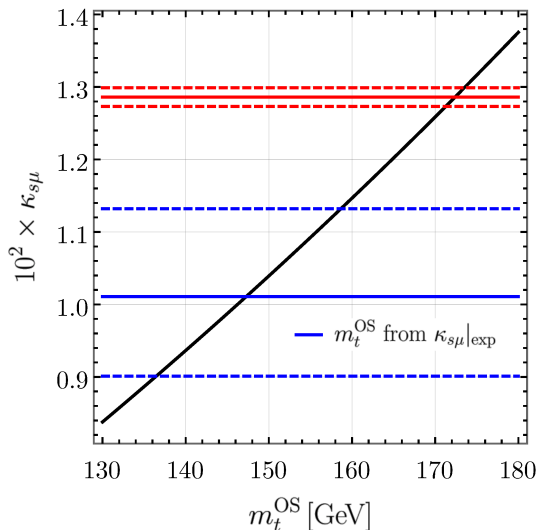


Fig. 2. (Colour on-line) The SM prediction of  $\kappa_{s\mu}$  depending on the value of  $m_t^{\text{OS}}$  [black]. The SM prediction in (29) [light grey/red] is for the value given in Table I from top-cross-section determinations, showing the uncertainty of about 1% on  $\kappa_{s\mu}$  [light grey/red dashed lines]. The experimental value  $\kappa_{s\mu}|_{\text{exp}}$  in (30) [grey/blue] requires much lower values of  $m_t^{\text{OS}}$ .

#### 4. Summary and outlook

It is evident that once both ratios  $R_{s\mu}$  and  $R_{d\mu}$  will be measured precisely, the strategy presented in [22] and executed here 18 years later will provide one of the theoretically cleanest tests of the SM and more generally of CMFV models.

Of course, a global analysis of a large set of processes in the framework of a specific model, as done in the standard analyses of the unitarity triangle or the recent global fits testing the violation of lepton flavour universality, will reveal any potential tension by lowering the goodness of the fit. However, a clear-cut insight on the origin of tensions is not so easily obtainable. We emphasize that in contrast, our proposed strategy involves only two observables on the experimental side and requires only well-accessible parametric input on the theoretical side, providing optimal control and a very transparent test of the SM (and CMFV models). Indeed, the global analyses involve usually

CKM uncertainties, in particular the one from  $|V_{cb}|$ , and also hadronic uncertainties present in other processes that are larger than the ones in  $\Delta M_s$  and  $B_s \rightarrow \mu\bar{\mu}$ . Moreover, NP could enter many observables used in such global fits and the transparent identification of the impact of NP on a given observable is a challenge. On the contrary, in the proposed ratios, all these uncertainties cancel out except for the bag factors, which are already precisely known from LQCD and importantly do not depend on NP. In this manner, concentrating just on  $\Delta M_s$  and  $B_s \rightarrow \mu\bar{\mu}$  allows us to test the SM (and CMFV models) independently of the values of CKM parameters and also independently of whether NP affects other observables or not. These ratios could turn out to be smoking guns of new physics.

However, one should emphasize that taking ratios of observables cancels not only parametric, theoretical and experimental uncertainties. It can, in principle, cancel also NP effects present in our case in the two branching ratios and in mass differences  $\Delta M_{s,d}$ . Therefore, the complete search for NP must also consider four observables separately that brings back CKM uncertainties. Yet, the analysis presented here allows to conclude, without any use of the CKM parameters and the decay constants  $f_{B_q}$ , that indeed new experimental results for  $B_s \rightarrow \mu\bar{\mu}$  exhibit some footprints of NP that affect the SM correlation between  $\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})$  and  $\Delta M_s$ . We are looking forward to improved results for  $B_s \rightarrow \mu\bar{\mu}$  and even more to improved results for  $B_d \rightarrow \mu\bar{\mu}$  which would allow to test the correlation between  $R_{s\mu}$  and  $R_{d\mu}$  that, as seen in (32), is already precisely known within CMFV models.

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