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Gauge fields as constrained composite bosons

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ABSTRACT

We reconsider a scenario in which photons and other gauge fields appear as the composite vector bosons made of the fermion pairs that may happen with or without spontaneous violation of Lorentz invariance. The class of composite models for emergent gauge fields is proposed, where these fields are required to be restricted by the nonlinear covariant constraint of type $A_\mu A^\mu = M^2$. Such a constraint may only appear if the corresponding fermion currents in the prototype model, being invariant under some global internal symmetry G, are properly constrained as well. In contrast to the conventional approach, the composite bosons emerged in this way appear naturally massless, the global symmetry G in the model turns into the local symmetry G_{loc}, while the vector field constraint reveals itself as the gauge fixing condition. Finally, we consider the case when the constituent fermions generating emergent gauge bosons could be at the same time the preons composing the known quark-lepton species in the Standard Model and Grand Unified Theories.

1. Introduction

One can think that local gauge invariance, contrary to a global symmetry case, may look like a cumbersome geometrical input rather than a true physical principle, especially in the framework of an effective quantum field theory becoming, presumably, irrelevant at very high energies. In this connection, one could wonder whether there is any basic dynamical reason that necessitates gauge invariance and an associated masslessness of gauge fields as some emergent phenomenon arising from a more profound level of dynamics related to their truly elementary constituents. By analogy with a dynamical origin of massless scalar particle excitations, which is very well understood in terms of spontaneously broken global internal symmetries [1], one could think that the origin of composite massless gauge fields as the vector Nambu-Goldstone (NG) bosons are presumably related to spontaneous violation of Lorentz invariance which is in fact the minimal spacetime global symmetry underlying particle physics. This well-known approach which might in principle provide a viable alternative to quantum electrodynamics [2], gravity [3] and Yang-Mills theories [4,5] has a long history started over fifty years ago.

This scenario proposes that a given current-current interaction of basic constituent fermions induces some almost gauge invariant effective theory which, apart from the invariant kinetic terms, contain the nontrivial vector-field potential terms. These terms, no matter whether they exist at the tree level or appear through radiative corrections, may lead in principle to spontaneous Lorentz invariance violation (SLIV) according to which some of components of the emergent composite vector bosons can be viewed as the corresponding NG zero modes. It is worth pointing out that one can generally talk about SLIV regardless of whether it is observable or not. In both of cases, the corresponding zero modes associated with gauge fields are necessarily generated. Indeed, in the gauge invariance limit this violation may normally be hidden in gauge degrees of freedom of emergent vector fields. However, when these superfluous degrees are eliminated by gauge symmetry breaking, SLIV becomes observable. In this connection, we will use later on the terms “inactive SLIV” and “active SLIV”, respectively, in order to distinguish these two cases.

The important point, however, is that the potential-based vector field models which can only lead to the active or physical SLIV appear to be generically unstable and contradictory and, therefore, seems to be hardly acceptable in their present form, as we will argue below. On the other hand, why should one necessarily insist on physical Lorentz violation, if emergent gauge fields are anyway generated through the “safe” inactive SLIV models which recover a conventional Lorentz invariance?

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In the light of this reasoning, we propose here an alternative approach for composite gauge bosons replacing the potential-based vector field models by the constraint-based ones which we briefly sketch below in Sections 2 and 3, respectively. Later in Section 4, we show how such constrained gauge bosons are induced by the properly constrained currents of the constituent fermions involved. In contrast to the potential-based approach, the composite gauge bosons appear now to be naturally massless, though being restricted by some nonlinear constraint which is then treated as their nonlinear gauge. So, the starting global symmetry of constituent fermions G turns into the local internal symmetry G_{loc} of the gauge and matter fields involved. Further, in Section 5, we discuss a possible scenario when the constituent fermions generating emergent gauge bosons could be at the same time the preons composing the known quark-lepton species in the Standard Model and Grand Unified Theories. And, finally, our conclusion is provided in Section 6, where we discuss why basically the proposed constraint pattern for vector fields could make the whole physical field system involved to adjust itself in a gauge invariant way. Otherwise, as is argued, it could lose too many degrees of freedom, thus getting unphysical.

2. Models with vector field potential

For an emergence of the potential-based vector field models one can start with the generalized prototype Lagrangian with all possible multi-fermi current-current interactions [6] rather than with the original four-fermi one [2]

$$L(\psi) = \overline{\psi}_s(i\gamma\partial - m)\psi_s + N\sum_{p=1}^{\infty} \mathrm{G}_p \left(j_\mu j^\mu/N^2\right)^p \tag{1}$$

where the fermion set includes N constituent fermion species ψ_s (hereafter, summation over all repeated indices is implied). The Lagrangian $L(\psi)$ possesses the $U(N)$ global flavor symmetry under which the above all-fermion current j_μ

$$j_\mu(\psi) = \overline{\psi}_s\gamma_\mu\psi_s \quad (s = 1, 2, ..., N) \tag{2}$$

is invariant in itself. This model is evidently non-renormalizable and can only be considered as an effective theory valid at sufficiently low energies. The dimensionful couplings G_p are proportional to appropriate powers of the UV cutoff Λ being ultimately related to some energy scale up to which this effective theory is valid, $\mathrm{G}_p \sim \Lambda^{4-6p}$. Factors of N in (1) are chosen in such a way to provide a well defined large N limit.

The Lagrangian (1) can be re-written using the standard trick of introducing an auxiliary field A_μ

$$L(\psi, A_\mu) = \overline{\psi}_s(i\gamma\partial - \gamma A - m)\psi_s - N\mathrm{V}(A_\mu A^\mu) \tag{3}$$

The potential V is a power series in $A_\mu A^\mu$ that can generally be written in the form

$$\mathrm{V}(A_\mu A^\mu) = \sum_{p=1}^{\infty} \lambda_p \left(A_\mu A^\mu - M^2\right)^{2p} \tag{4}$$

with the coefficients λ_p and mass parameter M chosen such that by solving the algebraic equations of motion for A_μ and substituting back into (3) one recovers the starting Lagrangian (1). If instead one integrates out the fermions ψ_s, one gets an effective action in terms of the properly renormalized composite A_μ field which acquires its own dynamics

$$S_{eff} = N\int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathrm{V}(A_\mu A^\mu) + A_\mu J^\mu\right] \tag{5}$$

Since the fermions ψ_s are minimally coupled to the vector field A_μ in (3), its kinetic term generated in this way appears gauge invariant provided that a gauge invariant cutoff is chosen. Furthermore, since there are N species of fermions ψ_s the effective action (5) has an overall factor of N. And the last point is that one has to introduce in the basic Lagrangian (1), apart from the pure constituent fermions ψ_s, some actual matter fields Ψ_t interacting through their own conserved current J_μ

$$J_\mu(\Psi) = \overline{\Psi}_t\gamma_\mu\Psi_t \ \ (t = 1, 2, ..., N_\Psi) \tag{6}$$

This will generate in turn the standard gauge invariant matter coupling given in the action (5). Indeed, in contrast to the heavy fermions ψ_s whose masses are normally proposed to be of the cutoff order, the matter fermions Ψ_t can be taken to be light and even massless. This allows to keep them in the low energy effective action, whereas the heavy fermions ψ_s are integrated out. They are in some sense hidden ones which are solely needed to properly induce the composite vector bosons as appropriate gauge fields. Note that, while their number N is in fact somewhat technical one allowing to properly suppress the high-order dangerous terms, N_Ψ is a number of the actually observed similar matter fermion species, say, those being among known quarks and leptons.

At the first glance, everything looks good in the effective action (5). However, some generic problems related to SLIV and stability of the emerged theory may necessarily appear. Let us consider at the beginning only the first non-trivial term in the multi-fermi interactions (1) and, respectively, in the potential V (5) that just corresponds to the original Bjorken model [2]. As matter of fact, this model naturally leads to the massive QED theory rather than to the conventional massless QED one. Indeed, any conclusion that such a massive vector boson might be condensed due to its radiatively produced quartic term [2] or even be massless in itself through a strict cancellation of its tree-level and radiative masses [4], seems to be rather problematic since it is related to somewhat formal manipulations with divergent integrals. The same could be said about the non-Abelian symmetry case as well.[1]

It might seem that the above problem would be over when the higher-order terms beyond the four-fermi interaction are activated in the basic fermion Lagrangian (1) and, respectively, in the potential (4). As is readily seen from (4), the next term in it really gives the quartic A_μ field term in the effective action S_{eff} (5). This is enough to generate the familiar Mexican hat structure of the potential $\mathrm{V}(A_\mu A^\mu)$, thus coming to spontaneous Lorentz violation at a scale determined by the mass parameter $|M|$. Rewriting the action (5) in terms of the renormalized A_μ field and leaving in the potential only bilinear and quartic vector field terms one gets the effective theory being sometimes referred to as the "bumblebee" model [7]. This partially gauge invariant model means in fact that the vector field A_μ develops a constant background value and Lorentz symmetry $SO(1,3)$ breaks down to $SO(3)$ or $SO(1,2)$ depending on whether the M^2 is positive or negative, respectively. In both of cases, there are three zero massless modes and a heavy Higgs mode in the symmetry broken phase.

The point is, however, that not only this bumblebee-like model but all possible potential-based models appear generally unstable. Their Hamiltonians (as was argued specifically for the bumblebee

[1] An interesting non-Abelian model was presented in [5]. It starts with current-current interaction involving some large N sets of fermions assigned to the fundamental representation of some $SU(n)$ group. It was then shown that in the leading N order, an explicit computation of the infinite fermion chain allows to completely reproduce the massive $SU(n)$ Yang-Mills theory. The composite boson mass can not be made zero for any finite value of the binding current-current coupling constant in the starting fermion Lagrangian. Thus, as it happens, this emergent theory does not possess a true gauge symmetry as well.

case [8] but those arguments are applicable to a general V potential as well) are not bounded from below beyond the constrained phase space determined by the nonlinear condition put on the vector field,

$$A_\mu A^\mu = M^2 \tag{7}$$

as can be readily shown using Dirac constraint analysis [9]. With this condition imposed, the massive Higgs mode never appears, the Hamiltonian turns to be positive, and the model is physically equivalent to the nonlinear constraint-based QED, which we consider in the next section.

Note also that, apart from the instability, these models, in contrast to some viable effective field theories, do not posses a consistent ultraviolet completion [10]. This makes their application rather problematic in a sense that one can not draw relevant conclusions about the strength of any physical effects in such an effective theory.

And one more argument against the potential-based models may follow from their supersymmetric extension. As one can readily confirm, SUSY may only admit the bilinear mass term in the vector field potential energy (see [11] for more details). As a result, without the stabilizing quartic (and higher order) vector field terms, this type of spontaneous Lorentz violation can in no way be realized in the SUSY context. The same could be said about the prototype Lagrangian with the multi-fermi current-current interactions (3) which can not be constructed from any matter chiral superfields.

All that means that the above-mentioned composite models leading eventually to the vector field potential $V(A_\mu A^\mu)$, whether it contains only the mass term or the higher order terms as well, seems to be hardly acceptable in the present form. One might only expect that, since a mass scale $|M|$ of SLIV is normally presumed to be near the Planck scale, the quantum gravity theory would make the ultimate conclusion on physical viability of such models.

3. Models with vector field constraint

We give here some brief sketch of the constraint-based models for vector fields considering first the constraint-based QED model. This model starts directly through the vector field "length-fixing" constraint (7) implemented into the conventional QED. Such type of models were first studied by Dirac [12] and Nambu [13] a long ago, and in more detail in recent years [14–21].

The constraint (7) is in fact very similar to the constraint appearing in the nonlinear σ-model for pions, $\sigma^2 + \pi^2 = f_\pi^2$, where f_π is the pion decay constant [22]. As is well known, this constraint leads to spontaneous breaking of the underlying chiral symmetry $SU(2) \times SU(2)$ in the model. Analogously, as Nambu argued [13], the constraint (7) might lead to spontaneous Lorentz violation. Rather than impose by postulate, the constraint (7) may be implemented through the Lagrange multiplier term into the standard QED Lagrangian

$$L(\Psi, A, \lambda) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\Psi}_t (i\gamma\partial - e\gamma A - \mathrm{m}) \Psi_t - \frac{\lambda}{2}\left(A_\mu A^\mu - M^2\right) \tag{8}$$

for some charged matter fermion fields Ψ_t $(t = 1, 2, ..., N_\Psi)$. The variation under the multiplier field $\lambda(x)$ leads then to the vector field constraint (7).

Actually, this Lagrangian, when taken without matter, is the original Dirac theory [12] proposed for an alternative introduction of classical electric charge. In fact, the Lagrange multiplier term in (8) corresponds to interaction of vector field with the extra source current $J_\mu^{ext} = \lambda A_\mu$. However, when the charged matter fields are included there appears the vector field interaction with the conventional Noether current $J_\mu = \overline{\Psi}_t \gamma_\mu \Psi_t$ as well. As follows from the total Lagrangian (8), both of them are separately conserved

$$\partial^\mu J_\mu^{ext} = 0\,, \quad \partial^\mu J_\mu = 0 \tag{9}$$

This extended Dirac theory contains five equations for the five field quantities, A_μ and λ, that includes their equations of motion and the extra current conservation (9). The solutions of these equations are fixed when the appropriate initial conditions are prescribed. We propose, as in the original Dirac model [12], that the starting values for all fields (and their momenta) involved are chosen so as to restrict their phase space to values providing the infinitesimal multiplier function $\lambda(x)$. Remarkably, due to an automatic conservation of the extra source current, J_μ^{ext} (9),

$$\partial\lambda/\partial t = [\partial_i(\lambda A_i) - \lambda(\partial_0 A_0)]/A_0 \tag{10}$$

such type of λ field will then remain for all times. One can thus safely keep the multiplier term in the Lagrangian (8) to derive the vector field constraint (7) in an ordinary way. This Lagrangian, being the ground for our further consideration, goes to the standard QED in the vanishing λ limit being provided again by the choice of the proper initial conditions. The only difference is that the vector potentials must now satisfy not only their equations of motion but the constraint (7) as well.[2]

One way or another, the constraint (7) means in essence that the vector field A_μ develops presumably the VEV along the direction given by the unit Lorentz vector n_μ

$$\langle A_\mu \rangle = n_\mu M \quad (n^2 = n_\mu n^\mu = 1) \tag{11}$$

causing for certainty some time-like $(M^2 > 0)$ Lorentz violation at a scale M, while rotational invariance is still maintained. This type of SLIV produces an ordinary photon as a true Goldstone vector boson (a_μ) being orthogonal to the vacuum direction given by the vector n_μ

$$A_\mu = a_\mu + n_\mu \sqrt{M^2 - a^2}\,,\ n_\mu a_\mu = 0\ (a^2 \equiv a_\mu a^\mu) \tag{12}$$

The point is, however, that in sharp contrast to the nonlinear σ model for pions, the constrained QED theory (8) ensures that physical Lorentz invariance in it remains unbroken. Indeed, although the theory in the symmetry broken phase contains a plethora of Lorentz and CPT violating couplings when it is expressed in terms of emergent a_μ modes, the contributions of all these Lorentz violating couplings to physical processes completely cancel out among themselves. Actually, as was shown in the tree [13] and one-loop approximations [14], the nonlinear constraint (7) applied as a supplementary condition appears in essence as a possible gauge choice for the vector field A_μ, while the S-matrix remains unaltered under such a gauge convention. The similar result was also confirmed for spontaneously broken non-Abelian theories [16,17] and tensor field gravity [19].

Let us describe in some detail just the non-Abelian symmetry case since we are going to consider later the Yang-Mills theories with composite gauge fields. Let us assume there is such a theory possessing some internal symmetry G having D generators so that

[2] Note that an arbitrary finite λ field may generally cause instability of the theory making its Hamiltonian negative [8]. However, with the infinitesimal λ field chosen (that is quite enough to provide the vector field constraint (7)) the Hamiltonian is always positive once Gauss' law holds [12]. It is also worth pointing out that even the zero limit for the infinitesimal λ field is quite imaginable, since in this limit the above extra charges are successively disappeared, while the vector field constraint remains [12].

the nonlinear constraint for the vector field multiplet has now the form

$$A_\mu^i A^{\mu i} = M^2 \;\; (i = 1, 2, ..., D) \tag{13}$$

Remarkably, this time not only the pure Lorentz symmetry $SO(1,3)$, but the much larger accidental symmetry $SO(D,3D)$ of the SLIV constraint (13) also happens to be spontaneously broken. As a result, although the pure Lorentz violation still generates only one true Goldstone vector boson, the accompanying pseudo-Goldstone vector bosons related to the $SO(D,3D)$ breaking

$$\begin{aligned} &A_\mu^i = a_\mu^i + n_\mu^i \sqrt{M^2 - a^2}\,,\; n_\mu^i a_\mu^i = 0 \\ &(n^2 \equiv n_\mu^i n^{i\mu} = 1,\; a^2 \equiv a_\mu^i a^{i\mu}) \end{aligned} \tag{14}$$

also come into play properly completing the whole gauge multiplet of the internal symmetry group G taken. In contrast to the known scalar pseudo-Goldstone modes, they remain strictly massless, being protected by the simultaneously generated non-Abelian gauge invariance [16,17].

To conclude, the constraint-based theories are in fact emergent gauge theories appearing due to spontaneous Lorentz violation which we refer to as inactive SLIV. These theories, both Abelian and non-Abelian, when being expressed in terms of the pure Goldstone vector modes (a_μ and a_μ^i, respectively) look essentially nonlinear and contain in general a variety of Lorentz and CPT breaking couplings. However, due to total cancellations of their contributions among themselves, they appear to be physically indistinguishable from the conventional QED and Yang-Mills theories. Their emergent nature could only be seen when taking the covariant gauge condition (7) into account. Any other gauge, e.g. Coulomb gauge, is not in line with emergent picture, since it "breaks" Lorentz invariance in an explicit rather than spontaneous way. As to an observational evidence in favor of emergent theories, the only way for SLIV to become active, thus causing physical Lorentz violation, would only appear if gauge invariance in these theories were really broken [23] rather than merely constrained by the gauge-fixing term. In substance, the above SLIV ansatz, due to which the vector field $A_\mu(x)$ develops the VEV (11), may itself be treated as a pure gauge transformation with a gauge function linear in coordinates, $\omega(x) = n_\mu x^\mu M$. From this viewpoint, gauge invariance in QED or Yang-Mills theory leads to the conversion of SLIV into gauge degrees of freedom of the massless gauge fields emerged. This is what one could refer to as the generic non-observability of SLIV in gauge invariant theories. Moreover, as was shown some time ago [24], gauge theories, both Abelian and non-Abelian, can be obtained by themselves from the requirement of the physical non-observability of SLIV induced by condensation of vector fields rather than from the standard gauge principle.

4. Constrained composite bosons

We are coming now to the most interesting question: how can the constrained QED Lagrangian (8) be generated from some underlying dynamics of the elementary constituent fermions? One can readily confirm following the standard procedure that such basic Lagrangian for both hidden constituent and real matter fermions, ψ_s and Ψ_t, has to have the form

$$\begin{aligned} L(\psi, \Psi, \lambda) = &\bar{\psi}_s(i\gamma\partial - m)\psi_s + \overline{\Psi}_t(i\gamma\partial - \mathrm{m})\Psi_t \\ &+ \frac{1}{2\lambda}(j_\mu + J_\mu)^2 + \frac{\lambda}{2}M^2 \end{aligned} \tag{15}$$

where j_μ and J_μ stand for their Noether currents (2) and (6), respectively. Indeed, employing the standard trick of introducing an auxiliary field $A_\mu(x)$ one has instead

$$L(\psi, \Psi, \lambda, A_\mu) = L_0(\psi) + L_0(\Psi) - (j_\mu + J_\mu)A^\mu - \frac{\lambda}{2}(A_\mu A^\mu - M^2) \tag{16}$$

with $L_0(\psi)$ and $L_0(\Psi)$ being the free Lagrangians for fermions ψ_s and Ψ_t. Now, solving the algebraic equations of motion for A^μ,

$$A_\mu = -\frac{1}{\lambda}(j_\mu + J_\mu) \tag{17}$$

and substituting back into the Lagrangian (16) one recovers the starting fermion Lagrangian (15). Remarkably, as one can readily see, the equation (17) connects the Dirac extra source current (9) with the standard fermion currents in the theory

$$j_\mu + J_\mu = -\, J_\mu^{ext} \tag{18}$$

Now, variation of the prototype vector field Lagrangian (16) under the multiplier function λ leads to the vector field constraint (7), mentioned above, while a similar variation of the fermion Lagrangian (15) gives in turn the constraint for the total fermion current in itself

$$(j_\mu + J_\mu)^2 = \lambda^2 M^2 \tag{19}$$

And eventually, integrating all constituent fermions ψ_s out from the prototype Lagrangian (16) we come to the effective Lagrangian (8) expressed in terms of the properly renormalized composite A^μ field interacting with matter fermions Ψ_t in gauge invariant way. Indeed, since the constituent fermions ψ_s were minimally coupled to A_μ in the prototype Lagrangian, all the generated terms in the effective Lagrangian (including any high-order ones) appear gauge invariant provided a gauge invariant cutoff is chosen. The vector field kinetic term appears in the form

$$-Z_3 \times \frac{1}{4}F_{\mu\nu}F^{\mu\nu},\; Z_3 = \frac{N}{12\pi^2}\ln\frac{\Lambda^2}{m^2} \tag{20}$$

where the renormalization constant Z_3 is given by the usual vacuum polarization integral with some momentum-space cutoff Λ. The Z_3 is then absorbed into the wave-function renormalization of A^μ field which in turn renormalizes the gauge coupling, multiplier field and SLIV scale as well

$$A_\mu \to A_\mu/\sqrt{Z_3}\,,\; 1 \to e = 1/\sqrt{Z_3}\,,\; \lambda \to \lambda/Z_3\,,\; M^2 \to M^2 Z_3 \tag{21}$$

that leads eventually to the gauge invariant effective Lagrangian (8) (with all the former notations remained). The nonlinear vector field constraint term in it, though being noninvariant, appears in fact as gauge condition in an otherwise gauge invariant and Lorentz invariant theory. So, in sharp contrast to the potential-based models considered above, all radiative corrections which may appear in the effective Lagrangian (8) have to necessarily be both Lorentz invariant and gauge invariant. In fact, in one-loop approximation it was explicitly demonstrated some time ago [14].

Interestingly, while the nonlinear vector field constraint (7) turns into the gauge condition, the constraint (19) put on the currents could mean some relation between currents of the hidden constituents and matter fermions. Actually, this relation also includes an extra source of charge density given by the λ field which is taken to be infinitesimal in our model. This means that the hidden and the actual matter currents appear to be approximately opposite to each other, $j^\mu \approx -J^\mu$. In this connection, one could notice that our basic field-current identity (17) is in fact the ratio of two infinitesimal field quantities giving eventually the finite vector field A_μ. While in the prototype Lagrangian (16) it couples with the infinitesimal sum of the currents $j_\mu + J_\mu$, in the effective Lagrangian (8) emerging after integrating out of the hidden constituent fermions ψ_{sa} the vector field interaction is solely given by the finite coupling with the matter fermions

$$A_\mu J^\mu = A_\mu \overline{\Psi}_t \gamma^\mu \Psi_t \quad (t = 1, 2, ..., N_\Psi) \tag{22}$$

And last but not least, it is worth pointing out that the constraint-based models possess one more remarkable advantage as compared to the potential-based ones – they do not need in general to have the enormously large number N of the hidden constituent fermion species. Actually such a number N was mainly imposed to properly suppress the large physical Lorentz violation at low energies which could otherwise appear through the uncontrollably large radiative corrections to the effective Lagrangians. That is not expected at all in the constraint-based models and, therefore, the number of the constituent fermions ψ_{sa} $(s = 1, 2, ..., N)$ may be determined now by only their own dynamics.

Let us now briefly describe a possible extension of this approach to the non-Abelian G symmetry case employing the non-Abelian model with constrained vector field multiplet that we discussed in the previous section. Now, all the N and N_Ψ species of hidden constituent and matter fermions, ψ_s and Ψ_t belong to the fundamental representations of G, ψ_{sa} and Ψ_{ta} $(a = 1, 2, ..., n)$, while the emerging composite vector fields will complete its adjoint multiplet. This extension can readily be made just by the corresponding replacements in the above equations (15)-(19)

$$A_\mu \to A_\mu^i\,,\ j_\mu \to j_\mu^i\,,\ \ J_\mu \to J_\mu^i\ (i = 1, 2, ..., D) \tag{23}$$

(D stands for the G symmetry group dimension). One can then proceed in a conventional way [4] to generate the vector field kinetic energy term together with the three- and four-vector self-couplings from the relevant loop diagrams. We eventually arrive at the totally gauge invariant theory with an obvious replacement in the QED Lagrangian (8), $F_{\mu\nu} \to F_{\mu\nu}^i$, where $F_{\mu\nu}^i$ is the standard strength-tensor for the constrained vector field multiplet A_μ^i. The modified Lagrange multiplier term providing the constraint (13) appears again as the pure gauge-fixing condition.

5. Composite quarks, leptons and gauge bosons

We present here some scenario how one could in principle combine this type models of composite gauge bosons with models for quarks and leptons composed from preons (some significant references can be found in [25]). The preons are usually viewed as the truly elementary carriers of all known physical charges, such as weak isospin, color, family number, etc. (which we refer to as “metaflavors”). Apart from metaflavors, they normally possess some metacolor forces that bind them inside quarks and leptons. And last but not least, they should be massless (or having some tiny masses) that is required to have at large distances the observed quarks and leptons with masses which much less than their composition scale. For that, as is well known, the chiral symmetry of the preons should be remained so that the anomaly matching condition of preons at small distances and their composites at the large ones has to be satisfied [26].

In the light of this, the above massive constituent fermions ψ_s composing gauge bosons can not be used for composition of quarks and leptons. The most direct way would be to treat the matter fields as the massless preon candidates supplying them with both metacolor and metaflavor symmetries G_{MC} and G_{MF}

$$\Psi_{ak}\,,\ a = 1, 2, ..., n_{MC}\,,\ k = 1, 2, ..., n_{MF} \tag{24}$$

where indices a and k belong to their fundamental representations, respectively. They are still global symmetries which then will be converted into the local ones by the proper current-current interaction of the hidden constituent fermions, such as it was described above. We propose that they are assigned to the symmetry groups G_{MC} and G_{MF} in the same way

$$\psi_{ak}\,,\ a = 1, 2, ..., n_{MC}\,,\ \ k = 1, 2, ..., n_{MF} \tag{25}$$

Note that in the both cases (24) and (25), we identify the number of fermions, the hidden constituent and matter ones, with number of metaflavors in the symmetry group G_{MF}

$$N = N_\Psi = n_{MF} \tag{26}$$

One might, of course, introduce some independent large N number of the hidden constituent fermions (25). However, as was mentioned above, in the constraint-based models, one can have it as low as it is required by the composite dynamics itself.

So, technically, we can start with the pure fermion Lagrangian like that of (15) but being extended to the non-Abelian symmetry corresponding to the still global metacolor and metaflavor symmetries G_{MC} and G_{MF}, respectively. One can then readily use the above procedure (16), (17) of introducing the auxiliary vector fields

$$A_\mu^i = A_{b\mu}^a (T^i)_a^b\,,\ \mathcal{A}_\mu^r = \mathcal{A}_l^k (\mathcal{T}^r)_k^l$$
$$(i = 1, 2, ..., D_{MC}\,;\ \ r = 1, 2, ..., D_{MF}) \tag{27}$$

in the Lagrangian through the interrelated Noether currents of the above fermions (24) and (25)

$$A_\mu^i = -\frac{1}{2\lambda_{MC}}(j_\mu^i + J_\mu^i)\,,\ j_\mu^i = \overline{\psi}\gamma_\mu T^i \psi\,,\ J_\mu^i = \overline{\Psi}\gamma_\mu T^i \Psi$$
$$\mathcal{A}_\mu^r = -\frac{1}{2\lambda_{MF}}(\mathrm{j}_\mu^r + \mathcal{J}_\mu^r)\,,\ \mathrm{j}_\mu^r = \overline{\psi}\gamma_\mu \mathcal{T}^r \psi\,,\ \mathcal{J}_\mu^r = \overline{\Psi}\gamma_\mu \mathcal{T}^r \Psi \tag{28}$$

Here T^i and $\mathcal{T}^r$ stand for generators of the symmetry groups G_{MC} and G_{MF} having dimensions D_{MC} and D_{MF}, respectively, while $\lambda_{MC}(x)$ and $\lambda_{MF}(x)$ are the corresponding Lagrange multiplier fields. Note also that the metacolor currents j_μ^i and J_μ^i are invariant under the metaflavor symmetry G_{MF}, as well as the metaflavor currents j_μ^r and $\mathcal{J}_\mu^r$ are invariant under the metacolor symmetry G_{MC} (the corresponding fermion indices are properly summed up in them).

Further, a subsequent integration of the massive constituent fermion multiplets ψ_{ak} out will convert the global symmetries G_{MC} and G_{MF} into the local ones with the metacolor and metaflavor gauge fields A_μ^i and $\mathcal{A}_\mu^r$ interacting with the presumably massless preon multiplets Ψ_{ak}. After the appropriate renormalization of all the composite vector fields involved one arrives at the final preon Lagrangian

$$\begin{aligned} L(\Psi, A, \mathcal{A}) = L_0(\Psi) &- \frac{1}{4}F_{\mu\nu}^i F^{i\mu\nu} - \frac{1}{4}\mathcal{F}_{\mu\nu}^r \mathcal{F}^{r\mu\nu} \\ &- g J_\mu^i A^{i\mu} - g\mathcal{J}_\mu^r \mathcal{A}^{r\mu} - \lambda_{MC}(A_\mu^i A^{i\mu} - M_A^2) \\ &- \lambda_{MF}(\mathcal{A}_\mu^r \mathcal{A}^{r\mu} - \mathrm{M}_{\mathcal{A}}^2) \end{aligned} \tag{29}$$

with the vector field constraints included, which contain the corresponding Lagrange multiplier functions $\lambda_{MC}(x)$ and $\lambda_{MF}(x)$, as well as the SLIV scales M_A and $\mathrm{M}_{\mathcal{A}}$, respectively. The equations (28) show that both metacolor and metaflavor gauge field multiplets, A_μ^i and $\mathcal{A}_\mu^r$, apart from the hidden constituent fermions ψ_{ak} (25), are also consisted of the preons Ψ_{ak} (24) which at the same time compose the observed quarks and lepton through the metacolor forces. It goes without saying that their composition scale Λ_{MC} is always less than the cutoff Λ of the prototype fermion theory.

After all that, some whole scenario with composite quarks and leptons interacting with composite gauge bosons may be properly developed depending on the particular symmetry groups G_{MC} and G_{MF} taken. One possible scenario could be realized in the model of composite quarks and leptons that was recently presented in [27]. We give below some of its key elements:

(1) It is proposed that there are $2K$ elementary massless left-handed and right-handed preons at small distances, P_{kL} and Q_{kR} ($k = 1, \dots, K$), which possess a common local symmetry $SU(K)_{MF}$ unifying all known metaflavors, such as weak isospin, color, family number, etc. The preons, both P_{kL} and Q_{kR}, transform under fundamental representation of $SU(K)_{MF}$ and their metaflavor theory has presumably an exact L-R symmetry. Actually, the $SU(K)_{MF}$ appears at the outset as some vectorlike symmetry which then breaks down at large distances to some of its chiral subgroup.

(2) In contrast to their common metaflavors, the left-handed and right-handed preon multiplets are taken to be chiral under the local metacolor symmetry $G_{MC} = SO(3)_L \times SO(3)_R$. They appear with different metacolors, P^a_{kL} and $Q^{a'}_{kR}$, where a and a' are indices of the corresponding metacolor subgroups $SO(3)_L$ ($a = 1, 2, 3$) and $SO(3)_R$ ($a' = 1, 2, 3$), respectively. They are generically anomaly-free and provide the minimal three-preon configurations for composite quarks and leptons. Due to the chiral metacolor, there are two types of composites at large distances being composed individually from the left-right and left-handed preons, respectively.

(3) Obviously, the preon condensate $\langle \overline{P}_L Q_R \rangle$ which could cause the metacolor scale Λ_{MC} order masses for composites is principally impossible in the left-right metacolor model taken. This may be generally considered as a necessary but not yet a sufficient condition for masslessness of composites. The genuine massless fermion composites are presumably only those which preserve the chiral $SU(K)_L \times SU(K)_R$ symmetry of preons at large distances that is controlled by the 't Hooft's anomaly matching condition [26].

(4) The strengthening of this condition in a way that the massless fermion composites, both left-handed and right-handed, are required to complete a single representation of the $SU(K)_{MF}$ rather than some set of its representations, allows to fix the number of basic metaflavors K. Particularly, just eight left-handed and eight right-handed preons and their composites preserving the global chiral symmetry $SU(8)_L \times SU(8)_R$ are turned out to uniquely identify the local metaflavor symmetry $SU(8)_{MF}$ as the grand unified symmetry of preons at small distances.

(5) An appropriate violation or the starting L-R symmetry at large distances breaks then this vectorlike $SU(8)_{MF}$ symmetry for preons down to the chiral $SU(5) \times SU(3)_F$ symmetry for composites that contains the conventional $SU(5)$ GUT with an extra local family symmetry $SU(3)_F$ and three standard families of composite quarks and leptons.

We can see now that in order to adapt the constrained gauge bosons to this scenario one has to generate the local vectorlike metaflavor symmetry $SU(8)_{MF}$ together with the local chiral metacolor symmetry $SO(3)_L \times SO(3)_R$ binding separately the left-handed and right-handed preons. This means that, while the metaflavor part in the above preon Lagrangian (29) is left intact, its metacolor part has to be properly changed. Namely, it should now include separately both left-handed and right-handed preons, P^a_{kL} and $Q^{a'}_{kR}$, being assigned to the different metacolor groups $SO(3)_L$ and $SO(3)_R$ though to the same metaflavor symmetry $SU(8)_{MF}$. Also, there are the two metacolor currents, J^a_μ and $J^{a'}_\mu$ of $SO(3)_L$ and $SO(3)_R$ with the corresponding gauge bosons A^a_μ and $A^{a'}_\mu$, as well as the two constraining Lagrange multiplier terms with the λ_{MC} and λ'_{MC} fields in the modified Lagrangian. It is clear that this Lagrangian is emerged in turn after integration out of the corresponding massive constituent fermion fields ψ^a_k and $\psi^{a'}_k$ which, in contrast to chiral preons, are vectorlike ($\psi^a_k \equiv (\psi_{L,R})^a_k$, $\psi^{a'}_k \equiv (\psi_{L,R})^{a'}_k$) with respect to their own symmetry group, $SO(3)_L$ or $SO(3)_R$, respectively. It is worth noting that these constituents will produce themselves the composite states due metacolor forces they possess. However, they will definitely appear very heavy (of the order of the metacolor scale taken) and might only influence physics at high energies comparable with the grand unification scale or so.

We have thus briefly demonstrated one particular way of unification of the constrained composite gauge bosons with composite quarks and leptons that could appear due to the effective preon Lagrangian (29) and its extensions. There are presumably some other ways as well.

6. Conclusion

We have considered the class of composite models for gauge fields, which are emerged from the prototype fermion model taken in the current-current interaction form. In contrast to the conventional approach where such interactions lead generally to the unstable vector field potential, in our model with the properly constrained fermion currents the composite gauge bosons come out to be only restricted by the nonlinear covariant constraint of type $A^2_\mu = M^2$. They appear naturally massless as the NG modes of SLIV, so that the global internal symmetry G in the model turns into the local symmetry G_{loc}, while the vector field constraint reveals itself as the gauge fixing condition.[3]

Whereas the field-current identity for the massive vector fields, underlying the conventional argumentation, has a long story dating back to the paper of Kroll, Lee and Zumino [30], such an identity for massless vector fields seems hardly possible unless they are properly constrained. The crucial point, as we could see, happens to be the relation (18) between the standard matter currents and extra classical current introduced by Dirac [12]. Actually, one now has in a sense the current-current identity rather than field-current one.

We can go a bit further and wonder why basically the proposed constraint pattern for vector fields could make the whole physical field system involved to adjust itself in a gauge invariant way. The possible answer seems to be that the only theories compatible with the nonlinear vector field constraints taken are the gauge invariant ones, as has been generally argued in [17] (see also [28,29]). Indeed, once the SLIV constraint (7) or (13) is imposed, it is therefore not possible to satisfy another supplementary condition since this would superfluously restrict the number of degrees of freedom for the vector field. To avoid this, its equation of motion should be automatically divergenceless since otherwise one would have one more condition. However, such an equation of motion is only possible in the gauge invariant theory. Actually, gauge invariance in theories considered appears in essence as a response of an interacting field system to putting the covariant constraint (7), (13) on its dynamics, provided that we allow parameters in the corresponding Lagrangian density to be adjusted so as to ensure self-consistency without losing too many degrees of freedom. Otherwise, a given field system could get unphysical in a sense that a superfluous reduction in the number of degrees of freedom would make it impossible to set the required initial conditions in

[3] Generally, apart from the current-current interaction terms, there are many other fermion bilinears and their interactions that could be included in our prototype Lagrangian (15). The above "bosonization" procedure could then be generalized so as to introduce a new auxiliary field (scalar, vector or tensor one) for each bilinear that eventually would lead to an effective action for a set of interacting auxiliary fields [6]. In contrast to the above massless A_μ fields induced by the current bilinears, they are not constrained and, presumably, acquire large masses. On general grounds, these masses have to be of the cutoff order and, therefore, all such states can be neglected at low energies.

the appropriate Cauchy problem. Namely, it would be impossible to specify arbitrarily the initial values of the vector and other field components involved, as well as the initial values of the momenta conjugated to them. Furthermore, in quantum theory, to choose self-consistent equal-time commutation relations would also become impossible [31]. So, the nonlinear SLIV condition (7), (13), due to which true vacuum in the theory is chosen and massless gauge fields are generated, may provide a dynamical setting for all underlying internal symmetries involved through such an emergence conjecture.

One can see that the gauge theory framework, be it taken from the outset or emerged, makes in turn SLIV to be physically unobservable both in Abelian and non-Abelian symmetry case that is also favored by the supersymmetric SLIV version. We referred to it above as the inactive SLIV in contrast to the active SLIV case where physical Lorentz violation could effectively occur. From the present standpoint, the only way for an inactive SLIV to be activated would be if emergent gauge symmetries presented above were slightly broken at small distances being presumably controlled by quantum gravity. One might even think that quantum gravity could in principle hinder the setting of the required initial conditions in the appropriate Cauchy problem, thus admitting a superfluous restriction of vector fields in terms of some high-order operators which occur at the Planck scale order distances. This is just the range of distances where the composite gauge fields, as well as the composite quarks and leptons, should presumably emerge in order to be properly adapted to the grand unification landscape. So, some trace of the gauge symmetry breaking and therefore physical Lorentz violation might accompany their composition processes. We may return to this special issue elsewhere.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Y. Nambu, G. Jona-Lasinio, Phys. Rev. 122 (1961) 345;
J. Goldstone, Nuovo Cimento 19 (1961) 154.
[2] J.D. Bjorken, Ann. Phys. (N.Y.) 24 (1963) 174.
[3] P.R. Phillips, Phys. Rev. 146 (1966) 966.
[4] T. Eguchi, Phys. Rev. D 14 (1976) 2755;
H. Terazava, Y. Chikashige, K. Akama, Phys. Rev. D 15 (1977) 480.
[5] M. Suzuki, Phys. Rev. D 37 (1988) 210, Phys. Rev. D 82 (2010) 045026.
[6] Per Kraus, E.T. Tomboulis, Phys. Rev. D 66 (2002) 045015.
[7] V.A. Kostelecky, S. Samuel, Phys. Rev. D 39 (1989) 683.
[8] R. Bluhm, N.L. Gagne, R. Potting, A. Vrublevskis, Phys. Rev. D 77 (2008) 125007.
[9] P.A.M. Dirac, Lectures on Quantum Mechanics, Yeshiva University, New York, 1964.
[10] A. Hashimoto, J. High Energy Phys. 0808 (2008) 040.
[11] J.L. Chkareuli, Phys. Lett. B 721 (2013) 146, Phys. Rev. D 90 (2014) 065015.
[12] P.A.M. Dirac, Proc. R. Soc. A 209 (1951) 291.
[13] Y. Nambu, Progr. Theor. Phys. Suppl. Extra 190 (1968).
[14] A.T. Azatov, J.L. Chkareuli, Phys. Rev. D 73 (2006) 065026.
[15] R. Bluhm, S.-H. Fung, V.A. Kostelecky, Phys. Rev. D 77 (2008) 065020.
[16] J.L. Chkareuli, J.G. Jejelava, Phys. Lett. B 659 (2008) 754.
[17] J.L. Chkareuli, C.D. Froggatt, J.G. Jejelava, H.B. Nielsen, Nucl. Phys. B 796 (2008) 211.
[18] J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 821 (2009) 65.
[19] J.L. Chkareuli, J.G. Jejelava, G. Tatishvili, Phys. Lett. B 696 (2011) 124.
[20] O.J. Franca, R. Montemayor, L.F. Urrutia, Phys. Rev. D 85 (2012) 085008.
[21] C.A. Escobar, L.F. Urrutia, Phys. Rev. D 92 (2015) 025013, Phys. Rev. D 92 (2015) 025042.
[22] S. Weinberg, The Quantum Theory of Fields, vol. 2, Cambridge University Press, Cambridge, 2000.
[23] J.L. Chkareuli, Z. Kepuladze, G. Tatishvili, Eur. Phys. J. C 55 (2008) 309;
J.L. Chkareuli, Z. Kepuladze, Eur. Phys. J. C 72 (2012) 1954.
[24] J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, Phys. Rev. Lett. 87 (2001) 091601, Nucl. Phys. B 609 (2001) 46.
[25] R.N. Mohapatra, Unification and Supersymmetry, Springer-Verlag, New York, 2003.
[26] G. 't Hooft, in: G. 't Hooft, et al. (Eds.), Recent Developments in Gauge Theories, Plenum, New York, 1980.
[27] J.L. Chkareuli, Nucl. Phys. B 941 (2019) 425.
[28] J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 848 (2011) 498.
[29] J.L. Chkareuli, On emergent gauge and gravity theories, in: Low Dimensional Physics and Gauge Principles, World Scientific, Singapore, 2011, pp. 80–92, arXiv:1206.1368 [hep-th].
[30] N.M. Kroll, T.D. Lee, B. Zumino, Phys. Rev. 157 (1967) 1376.
[31] V.I. Ogievetsky, I.V. Polubarinov, Ann. Phys. (N.Y.) 25 (1963) 358, Nucl. Phys. 76 (1966) 677.