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On the one loop $\gamma^{(*)} \to q\bar{q}$ impact factor and the exclusive diffractive cross sections for the production of two or three jets

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Abstract: We present the calculation of the impact factor for the $\gamma^{(*)} \to q\bar{q}$ transition with one loop accuracy in arbitrary kinematics. The calculation was done within Balitsky's high energy operator expansion. Together with our previous result for the $\gamma^{(*)} \to q\bar{q}g$ Born impact factor it allows one to derive cross sections for 2- (one loop) and 3-jet (Born) difractive electroproduction. We write such cross sections for the 2 and 3 jet exclusive diffractive electroproduction off a proton in terms of hadronic matrix elements of Wilson lines. For the 2-jet cross section we demonstrate the cancellation of infrared, collinear and rapidity singularities. Our result can be directly exploited to describe the recently analyzed data on exclusive dijet production at HERA and used for the study of jet photoproduction in ultraperipheral proton or nuclear scattering.

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1 Introduction

For several decades diffraction has been one of the most intriguing phenomena of strong interaction. The HERA research program has shown for the first time that diffractive processes in the semi-hard regime can be measured and studied based on QCD, giving one of the main tools to access the internal dynamics of the nucleon in a regime of very high gluon densities.[1] One of the most important legacies of HERA is the fact that almost 10 % of the $\gamma^* p \to X$ deep inelastic scattering (DIS) events contain a rapidity gap between the proton remnants and the hadrons coming from the fragmentation region of the initial virtual photon. This subset of events, called diffractive deep inelastic scattering (DDIS), thus looks like $\gamma^* p \to X\,Y$ [3–10], where Y is the outgoing proton or one of its low-mass excited states, and X is the diffractive final state. Apart from the inclusive DDIS data, one can further focus on more specific interesting observables, like diffractive jet(s) production, or exclusive meson production.

Due to the existence of a rapidity gap between X and Y, it is natural to describe diffraction through a Pomeron exchange in the $t-$channel between these X and Y states. This is a common concept underlying the approaches to describe diffraction within perturbative QCD.

In the collinear framework, justified by the existence of a hard scale (the photon virtuality Q^2 of DIS), a QCD factorization theorem was derived [11]. Similarly to DIS on a proton, here one introduces diffractive structure functions, which are convolutions of coefficient functions with diffractive parton distributions. In this resolved Pomeron model, those distributions describe the partonic content of the Pomeron, similarly to the usual parton distribution functions for proton in DIS.

At high energies, it is natural to model the diffractive events by a *direct* Pomeron contribution involving the coupling of a Pomeron with the diffractive state X of invariant mass M. For low values of M^2, X can be modeled by a $q\bar{q}$ pair, while for larger values of M^2, the cross section with an additional produced gluon, i.e. $X = q\bar{q}g$, is enhanced. Based on such a model, with a simplified two-gluon exchange picture for the Pomeron, a good description of HERA data for diffraction was achieved [12]. Interestingly, the $q\bar{q}$ component with a longitudinally polarized photon plays a crucial role in the region of small diffractive mass M, although it is a twist-4 contribution. It is a typical signature of such models.

In this direct Pomeron contribution, the $q\bar{q}g$ diffractive state has been studied in two particular limits. First, at large Q^2, a collinear approximation can be used, based on the fact that the transverse momentum of the gluon is much smaller than the transverse momentum of the emitter [13–15]. Second, for very large M^2, contributions with a strong ordering of longitudinal momenta are enhanced [16, 17]. These two limiting results were combined in a single model, and applied to HERA data for DDIS in ref. [18].

In the present article, we extend our work started in [19] towards a complete next-to-leading order (NLO) description of the direct coupling of the Pomeron to the diffractive X state. To be more specific, the "Pomeron" should be here understood as a color singlet QCD shockwave, in the spirit of Balitsky's high energy operator expansion [20–23] or in its color glass condensate formulation [24–32].

[1] For reviews, see refs. [1, 2].

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In our previous study [19], we computed the $\gamma^{(*)} \to q\bar{q}g$ impact factor and rederived the $\gamma^{(*)} \to q\bar{q}$ impact factor, both at tree level.[2] In the present article,[3] we complete the results of ref. [19] by a study of the virtual contributions, and compute the one-loop $\gamma^{(*)} \to q\bar{q}$ impact factor. We emphasize that in these results, the impact factors are computed without any soft or collinear approximation for the emitted gluon, in contrast with the results reported in the literature. This thus presents an important step towards a consistent description of inclusive DDIS, or exclusive two-jet diffractive production, in the fragmentation region of the scattered photon, i.e. in the forward rapidity region, with NLO precision. Since the results we derive are obtained in the QCD shock-wave approach, and depend on the total available center-of-mass energy, the present framework is rather general and can have many applications. Indeed, below the saturation regime, one might describe the $t-$channel exchanged state in the linear Balitsky-Fadin-Kuraev-Lipatov (BFKL) regime [35–38], here with NLO precision [39–42]. At higher energies, beyond the saturation limit, the Wilson-line operators, whose matrix element describes the $t-$channel exchanged state, evolve with respect to rapidity according to the Balitsky hierarchy. In the case of a dipole operator, it reduces to the Balitsky-Kovchegov (BK) equation [20–23, 43, 44] in the large N_c limit.

In the present paper we calculate the matrix element for the $\gamma^{(*)} \to q\bar{q}$ transition in the shockwave background of the target. It depends on the target via the matrix elements of two Wilson line operators $\mathrm{tr}(U_1U_2^\dagger)$ and $\mathrm{tr}(U_1U_3^\dagger)\,\mathrm{tr}(U_3U_2^\dagger) - N_c\mathrm{tr}(U_1U_2^\dagger)$ between the *in* and *out* target states. The Wilson lines are functions of the rapidity which separates the gluons belonging to the impact factor and the gluons from the Wilson lines. For hadron targets these matrix elements are to be described by some models. For example for the former one there are several saturation models, inspired by the Golec-Biernat and Wüsthoff model [45, 46], while for the latter, to the best of our knowledge, we are not aware about any such model. These Wilson line operators can also be calculated as solutions of the NLO BK and the LO double dipole evolution equations with the initial conditions at the rapidity of the target. In the linear limit (BFKL) for forward scattering these solutions are known analytically with a running coupling [47, 48]. In addition, in the low density regime one can always linearize the second Wilson line operator and write the cross section in terms of matrix elements of color dipoles only.

Here we will focus on the details of the coupling of these Wilson-line operators to the diffractive state. The various possible regimes and the related appropriate projections which are required for phenomenological applications will be the subject of a future study. This choice is motivated by the fact that one of the technical difficulties in this framework is to prove explicitly that the various infrared (IR) and ultraviolet (UV) singularities cancel properly.

Next, motivated by the phenomenological importance of our results, we study in detail the cross section for exclusive dijet production in diffraction, as was recently reported by ZEUS [49], to show how these cancellations occur in a detailed way. For that, we derived the differential cross section for the $\gamma^* P \to q\bar{q}P'$ transition. Taking the corresponding

[2] Here the photon can be either on-shell or off-shell, hence the notation $\gamma^{(*)}$.

[3] Partial results of the present study were already presented in refs. [33, 34].

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matrix element from ref. [19] we also calculated the $\gamma^* P \to q\bar{q}gP'$ cross section. Combining them we wrote the $\gamma^* P \to 2jets\, P'$ exclusive cross section canceling the soft and collinear singularities in the small cone approximation. Besides, outside the jet cones one can use the $\gamma^* P \to q\bar{q}gP'$ cross section to study the electroproduction of 3 jets as well.

The paper is organized as follows. The next section contains the definitions and necessary results. Section 3 briefly introduces the basic notations and reproduces the LO $\gamma^{(*)} \to q\bar{q}$ impact factor and gives the general expression for the $\gamma^{(*)} \to q\bar{q}$ impact factor at one-loop accuracy. Section 4 gives the $\gamma^{(*)} \to q\bar{q}g$ impact factor at Born order in arbitrary dimensions. Section 5 gives the $\gamma^{(*)} P \to q\bar{q}P'$ cross section at leading and next-to-leading order. Section 6 gives the $\gamma^{(*)} P \to q\bar{q}gP'$ cross section at leading order. Section 7 gives the final result for exclusive $\gamma^* P \to dijetP'$ transition, showing explicitly the cancellation of divergencies, based on the two previous sections. Section 8 concludes the paper. The appendices comprise the necessary technical details.

2 Definitions and building blocks

We introduce the light cone vectors n_1 and n_2 as follows:

$$n_1 \equiv (1, 0_\perp, 1)\,, \quad n_2 \equiv \frac{1}{2}\,(1, 0_\perp, -1)\,, \quad n_1^+ = n_2^- = (n_1 \cdot n_2) = 1. \tag{2.1}$$

For any vector p we note

$$p^+ = p_- \equiv (p \cdot n_2) = \frac{1}{2}\left(p^0 + p^3\right), \qquad p_+ = p^- \equiv (p \cdot n_1) = p^0 - p^3, \tag{2.2}$$

$$p = p^+ n_1 + p^- n_2 + p_\perp, \tag{2.3}$$

so that

$$(p \cdot k) = p^\mu k_\mu = p^+ k^- + p^- k^+ + (p_\perp \cdot k_\perp) = p_+ k_- + p_- k_+ - (\vec{p} \cdot \vec{k}). \tag{2.4}$$

For a moment, we will consider the open production of partons, the conversion into jets will be discussed later in this paper. We denote the initial photon vector as p_γ, and the outgoing quark and antiquark vectors as p_q, and $p_{\bar{q}}$. In the real correction, an additional external gluon is emitted. Its momentum will be denoted as p_g. We will focus on diffraction off a proton P which remains intact after the interaction. We denote the initial and final proton momenta as p_0 and p_0'. Our calculation can be used for other processes later on with minor modifications. We consider semihard kinematics with the hard scale

$$s = (p_\gamma + p_0)^2 \gg |p_\gamma^2|,\ M_P^2,\ |p_{00'}^2|. \tag{2.5}$$

Here and throughout this paper we use the notation $p_{ij} = p_i - p_j$ for two given vectors p_i and p_j. M_P is the proton mass. The semihard scale comes from either the photon virtuality $|p_\gamma^2|$, the momentum transfer $|p_{00'}^2|$, or the invariant mass of the produced jets. In this kinematics one can write

$$s \simeq 2p_\gamma^+ p_0^-\,, \tag{2.6}$$

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and choose the reference frame where

$$p_\gamma^+, p_0^- \sim \sqrt{s}. \tag{2.7}$$

In the case of our process, one can write

$$p_\gamma^+ \sim p_q^+ \sim p_{\bar{q}}^+ \gg p_0^+, p_0'^+, \quad p_0^- \gg p_\gamma^-, p_{\bar{q}}^-, p_q^-. \tag{2.8}$$

The longitudinal momentum fractions of the $q\bar{q}$ pair are defined by

$$\frac{p_q^+}{p_\gamma^+} \equiv x_q, \quad \frac{p_{\bar{q}}^+}{p_\gamma^+} \equiv x_{\bar{q}}. \tag{2.9}$$

For simplicity we consider a forward photon with virtuality Q and no transverse momentum:

$$\vec{p}_\gamma = 0, \quad p_\gamma^\mu = p_\gamma^+ n_1^\mu + \frac{p_\gamma^2}{2p_\gamma^+} n_2^\mu, \quad -p_\gamma^2 \equiv Q^2 > 0. \tag{2.10}$$

We will denote its transverse polarization ε_T. Its longitudinal polarization vector reads

$$\varepsilon_L^\alpha = \frac{1}{\sqrt{-p_\gamma^2}} \left(p_\gamma^+ n_1^\alpha - \frac{p_\gamma^2}{2p_\gamma^+} n_2^\alpha \right), \quad \varepsilon_L^+ = \frac{p_\gamma^+}{Q}, \quad \varepsilon_L^- = \frac{Q}{2p_\gamma^+}.$$

We work in the light-cone gauge $\mathcal{A} \cdot n_2 = 0$. In this gauge, the bare gluon propagator is given by

$$\hat{G}_0^{\mu\nu}(p) = \frac{-id^{\mu\nu}(p)}{p^2 + i0}, \tag{2.11}$$

where

$$d^{\mu\nu}(p) = d_0^{\mu\nu}(p) - \frac{n_2^\mu n_2^\nu p^2}{(p^+)^2}, \quad d_0^{\mu\nu}(p) = g_\perp^{\mu\nu} - \frac{p_\perp^\mu n_2^\nu + p_\perp^\nu n_2^\mu}{p^+} - \frac{n_2^\mu n_2^\nu \vec{p}^{\,2}}{(p^+)^2}. \tag{2.12}$$

The bare fermion propagator reads (we give expressions in the massive case for completeness)

$$G_0(p) = \frac{i(\hat{p} + m)}{p^2 - m^2 + i0}. \tag{2.13}$$

We will need the external lines and the propagators in the shockwave background [19, 20]

$$\begin{aligned} \overline{u}(p,y)|_{0>y^+} &= \langle q_p | T(\overline{\psi}(y)\, e^{i\int \mathcal{L}_i(z)dz}) |0\rangle_{sw}, \\ v(p,y)|_{0>y^+} &= \langle \bar{q}_p | T(\psi(y)\, e^{i\int \mathcal{L}_i(z)dz}) |0\rangle_{sw}, \end{aligned} \tag{2.14}$$

$$\begin{aligned} G_{\mu\nu}^{ab}(x,y) &= \langle 0 | T(A_\mu^a(x)\, A_\nu^b(y)\, e^{i\int \mathcal{L}(z)dz}) |0\rangle_{sw}, \\ \hat{G}(x,y) &= \langle 0 | T(\psi(x)\, \bar{\psi}(y)\, e^{i\int \mathcal{L}_i(z)d^D z}) |0\rangle_{sw}. \end{aligned} \tag{2.15}$$

The external line for a particle with momentum p which steams from the coordinate y with lightcone time $y^+ < 0$ can be written as

$$\begin{aligned} \overline{u}(p,y)|_{0>y^+} = \frac{\theta(p^+)}{\sqrt{2p^+}} e^{ip^+y^-} \int \frac{dp_{2\perp}^{D-2}}{(2\pi)^{D-2}} e^{i(p_{2\perp}\cdot y_\perp) - i\frac{y^+}{2p^+}\left(p_{2\perp}^2 - m^2 + i0\right)} \\ \times \overline{u}_p \gamma^+ U(p_\perp - p_{2\perp}) \frac{[\gamma^- p^+ + \hat{p}_{2\perp} + m]}{2p^+} \end{aligned} \tag{2.16}$$

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in the case of a quark (where U is an operator in the fundamental representation of SU(N)),

$$v\left(p,y\right)|_{0>y^+} = \frac{\theta\left(p^+\right)}{\sqrt{2p^+}} e^{ip^+y^-} \int \frac{dp_{2\perp}^{D-2}}{(2\pi)^{D-2}} e^{i(p_{2\perp}\cdot y_\perp) - i\frac{y^+}{2p^+}\left(p_{2\perp}^2 - m^2 + i0\right)} \times \frac{[\gamma^- p^+ + \hat{p}_{2\perp} - m]}{2p^+} U^\dagger\left(p_{2\perp} - p_\perp\right)\gamma^+ v_p \tag{2.17}$$

in the case of an antiquark (again with $U^\dagger$ being an operator in the fundamental representation of SU(N)), and

$$\epsilon_\nu^*\left(p,\, y\right)|_{0>y^+} = \frac{\theta\left(p^+\right)}{\sqrt{2p^+}} e^{ip^+y^-} \int \frac{d^{D-2}p_{2\perp}}{(2\pi)^{D-2}} e^{i(p_{2\perp}\cdot y_\perp) - i\frac{y^+}{2p^+}\left(p_{2\perp}^2 + i0\right)} \times \epsilon_{p\perp\sigma}^* \left[g_{\perp\nu}^\sigma - \frac{p_{2\perp}^\sigma}{p^+} n_{2\nu}\right] U\left(p_\perp - p_{2\perp}\right) \tag{2.18}$$

in the case of a gluon (now with U in the adjoint representation of SU(N)). The propagators read

$$\begin{aligned} &\hat{G}(y,x)|_{y^+>0>x^+} \\ &= \int \frac{dp_1^+ d^{D-2}p_{1\perp}}{(2\pi)^{D-1}} \int \frac{dp_2^+ d^{D-2}p_{2\perp}}{(2\pi)^{D-1}} e^{-iy^- p_2^+ - i(p_{2\perp}\cdot y_\perp)} e^{ix^- p_1^+ + i(p_{1\perp}\cdot x_\perp)} 2\pi\delta(p_{12}^+)\theta(p_2^+) \\ &\quad \times \theta(-x^+)\theta(y^+) e^{iy^+\frac{p_{2\perp}^2 - m^2 + i0}{2p_2^+}} e^{-ix^+\frac{p_{1\perp}^2 - m^2 + i0}{2p_1^+}} \frac{\gamma^- p_2^+ + \hat{p}_{2\perp} + m}{2p_2^+} \gamma^+ U(p_{21\perp}) \frac{\gamma^- p_1^+ + \hat{p}_{1\perp} + m}{2p_1^+} \end{aligned} \tag{2.19}$$

for a quark,

$$\begin{aligned} &\hat{G}(x,y)|_{y^+>0>x^+} \\ &= -\int \frac{dp_1^+ d^{D-2}p_{1\perp}}{(2\pi)^{D-1}} \int \frac{dp_2^+ d^{D-2}p_{2\perp}}{(2\pi)^{D-1}} e^{-iy^- p_2^+ - i(p_{2\perp}\cdot y_\perp)} e^{ix^- p_1^+ + i(p_{1\perp}\cdot x_\perp)} 2\pi\delta(p_{12}^+)\theta(p_2^+) \\ &\quad \times \theta(-x^+)\theta(y^+) e^{iy^+\frac{p_{2\perp}^2 - m^2 + i0}{2p_2^+}} e^{-ix^+\frac{p_{1\perp}^2 - m^2 + i0}{2p_1^+}} \frac{\gamma^- p_1^+ + \hat{p}_{1\perp} - m}{2p_1^+} \gamma^+ U^\dagger(p_{12\perp}) \frac{\gamma^- p_2^+ + \hat{p}_{2\perp} - m}{2p_2^+} \end{aligned} \tag{2.20}$$

for an antiquark, and

$$\begin{aligned} G_{\mu\nu}(x,y)|_{x^+>0>y^+} = &-\int \frac{dp_1^+ d^{D-2}p_{1\perp}}{(2\pi)^{D-1}} \int \frac{dp_2^+ d^{D-2}p_{2\perp}}{(2\pi)^{D-1}} e^{-ip_2^+ x^- + ip_1^+ y^-} e^{-i(p_{2\perp}\cdot x_\perp) + i(p_{1\perp}\cdot y_\perp)} \\ &\times \pi \frac{\delta(p_{12}^+)\theta(p_2^+)}{p_1^+} e^{i\frac{p_{2\perp}^2 + i0}{2p_2^+}x^+ - i\frac{p_{1\perp}^2 + i0}{2p_1^+}y^+} d_{0\mu\alpha}(p_2^+, p_{2\perp}) U(p_{21\perp}) g_\perp^{\alpha\delta} d_{0\delta\nu}(p_1^+, p_{1\perp}) \end{aligned} \tag{2.21}$$

for a gluon. In these formulas $D \equiv 2 + d \equiv 4 + 2\epsilon$ is the space-time dimension and

$$U(p_\perp) = \int d^{D-2}r_\perp e^{i(p_\perp\cdot r_\perp)} U_{r_\perp} \text{ and } U^\dagger(p_\perp) = \int d^{D-2}r_\perp e^{-i(p_\perp\cdot r_\perp)} U_{r_\perp}^\dagger \tag{2.22}$$

are the Fourier transforms of the Wilson lines

$$U_r = U\left(\vec{r},\eta\right) = Pe^{ig\mu^\epsilon \int_{-\infty}^{+\infty} b_\eta^-(r^+,\vec{r})dr^+}\,. \tag{2.23}$$

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For convenience we will write $U_i \equiv U_{r_i}$ for a point r_i. The external shock-wave field b_η^- is built from the gluons that are slow in the asymmetric boosted frame where the non-perturbative dynamics only occur in the proton. Its form in coordinate space can be written in a simple way, as

$$b_\eta^- = \int \frac{d^4p}{(2\pi)^4} e^{-i(p\cdot z)} b^-(p)\,\theta\left(e^\eta - |\frac{p^+}{p_\gamma^+}|\right), \quad b^\mu(z) = b^-(z^+,\vec{z}) n_2^\mu = \delta(z^+) B(\vec{z})\, n_2^\mu. \tag{2.24}$$

Here η is the rapidity divide, which separates the slow gluons in the Wilson lines and the fast ones in the impact factors.

To construct the cross section after calculating the impact factor one has to integrate w.r.t. the field b generated by the proton. Technically it means that one has to treat the field b as an operator and use the matrix element of the total Wilson operator between the proton states

$$U_i \cdots \to \langle P'_{p'_0} | T(U_i \ldots) | P_{p_0} \rangle. \tag{2.25}$$

For simplicity of the notations we will still use the operator U instead of its matrix element during the calculation of the impact factor, and return to the matrix element later on.

We introduced the regularization scale μ in (2.23) because in dimensional regularization the coupling constant is a dimensional quantity

$$g_0 = g\,\mu^{-\epsilon}, \quad \alpha_{s0} = \alpha_s\,\mu^{-2\epsilon}. \tag{2.26}$$

In our computation we do not need the $\beta-$function correction since renormalization effects start at the next-to-next-to leading order in the impact factor. We also introduce a regularization cutoff α for the spurious light cone singularity $p_g^+ \to 0$. Evolving the operators $U(r,\rho)$ from $\rho = \alpha$ to $\rho = e^\eta$ with the help of the BK equation will allow us to cancel rapidity singularities, as shown in section 3.2.2.

In the following we will need the BK equation in d dimensions. It reads [50]

$$\begin{aligned}\frac{\partial tr(U_1 U_2^\dagger)}{\partial \eta} = \frac{\alpha_s \mu^{2-d}}{2\pi^d}\Gamma^2\left(\frac{d}{2}\right)\int d^d r_{3\perp}\left[tr(U_1U_3^\dagger)tr(U_3U_2^\dagger) - N_c tr(U_1U_2^\dagger)\right] \\ \times\left[\frac{2(r_{13\perp}\cdot r_{23\perp})}{\left(-r_{13\perp}^2+i0\right)^{\frac{d}{2}}\left(-r_{23\perp}^2+i0\right)^{\frac{d}{2}}} + \frac{1}{\left(-r_{13\perp}^2+i0\right)^{d-1}} + \frac{1}{\left(-r_{23\perp}^2+i0\right)^{d-1}}\right]\end{aligned} \tag{2.27}$$

in coordinate space, and

$$\begin{aligned}\frac{\partial tr(U(p_{1\perp})U^\dagger(-p_{2\perp}))}{\partial\eta} &= \delta(k_{1\perp}+k_{2\perp}+k_{3\perp}-p_{1\perp}-p_{2\perp})2\alpha_s\mu^{2-d} \\ &\times\int\frac{d^dk_{1\perp}d^dk_{2\perp}d^dk_{3\perp}}{(2\pi)^{2d}}\left[-\frac{2(k_{1\perp}-p_{1\perp})\cdot(k_{2\perp}-p_{2\perp})}{(k_1-p_1)_\perp^2(k_2-p_2)_\perp^2}\right. \\ &\left.+\frac{\pi^{\frac{d}{2}}\Gamma(1-\frac{d}{2})\Gamma(\frac{d}{2})^2}{\Gamma(d-1)}\left(\frac{\delta(k_{2\perp}-p_{2\perp})}{(-(k_1-p_1)_\perp^2)^{1-\frac{d}{2}}}+\frac{\delta(k_{1\perp}-p_{1\perp})}{(-(k_2-p_2)_\perp^2)^{1-\frac{d}{2}}}\right)\right] \\ &\times\left[tr(U_1U_3^\dagger)tr(U_3U_2^\dagger)-N_c tr(U_1U_2^\dagger)\right](k_{1\perp},k_{2\perp},k_{3\perp})\end{aligned} \tag{2.28}$$

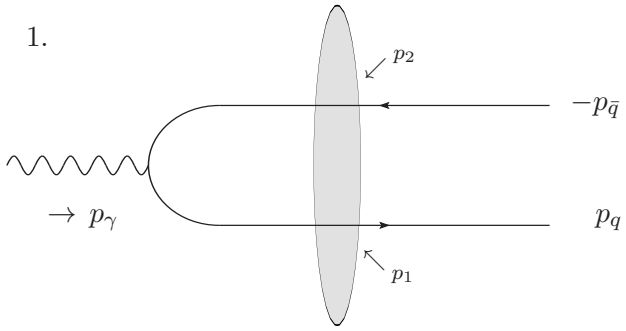

Figure 1. LO impact factor. The momenta p_1 and p_2 go from the shockwave to the quark and antiquark.

in momentum space. Here we introduced the Fourier transform of the operator

$$\left[tr(U_1U_3^\dagger)tr(U_3U_2^\dagger) - N_c tr(U_1U_2^\dagger)\right](k_{1\perp},k_{2\perp},k_{3\perp}) \tag{2.29}$$
$$= \int d^dr_{1\perp}d^dr_{2\perp}d^dr_{3\perp}e^{i[(r_{1\perp}\cdot k_{1\perp})+(r_{2\perp}\cdot k_{2\perp})+(r_{3\perp}\cdot k_{3\perp})]}\left[tr(U_1U_3^\dagger)tr(U_3U_2^\dagger) - N_c tr(U_1U_2^\dagger)\right].$$

3 Impact factor for the $\gamma^* \to q\bar{q}$ transition

The matrix element for the EM current in the shockwave background reads

$$\tilde{M}^\alpha = -ie_q\int d^Dy_0\frac{e^{-i(p_\gamma\cdot y_0)}}{\sqrt{2p_\gamma^+}}\frac{\delta_l^n}{\sqrt{N_c}}\langle 0|T(b^l_{p_{\bar{q}}}(a_{p_q})_n\overline{\psi}(y_0)\gamma^\alpha\psi(y_0)e^{i\int\mathcal{L}_i(z)dz})|0\rangle_{sw}. \tag{3.1}$$

Here a and b are the quark and antiquark annihilation operators, e_q is the quark electric charge, and $\frac{\delta_l^n}{\sqrt{N_c}}$ is the projector on the color singlet. To shorten the notation we will work with the reduced matrix element $\tilde{T}^\alpha$

$$\tilde{M}^\alpha \equiv \frac{-ie_q}{\sqrt{2p_\gamma^+}}\frac{-i\delta(p_q^+ + p_{\bar{q}}^+ - p_\gamma^+)}{\sqrt{N_c}\,(2\pi)^{D-3}\sqrt{2p_{\bar{q}}^+}\sqrt{2p_q^+}}\tilde{T}^\alpha. \tag{3.2}$$

3.1 LO impact factor

Its expression at LO is obtained with the help of (2.16) and (2.17), by computing the diagram depicted in figure 1. The result can be written as

$$\tilde{T}_0^\alpha = \int d^dp_{1\perp}d^dp_{2\perp}\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{\gamma\perp})\;tr\left[U(p_{1\perp})U^\dagger(-p_{2\perp})\right]\Phi_0^\alpha. \tag{3.3}$$

After subtraction of the noninteracting part one gets

$$T_0^\alpha = \int d^dp_{1\perp}d^dp_{2\perp}\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{\gamma\perp})\,\Phi_0^\alpha\left[tr\left(U_1U_2^\dagger\right)-N_c\right](p_{1\perp},p_{2\perp}). \tag{3.4}$$

Here

$$\left[tr\left(U_1U_2^\dagger\right)-N_c\right](p_{1\perp},p_{2\perp}) = \int d^dr_{1\perp}d^dr_{2\perp}e^{i(p_{1\perp}\cdot r_{1\perp})+i(p_{2\perp}\cdot r_{2\perp})}\left[tr\left(U_1U_2^\dagger\right)-N_c\right] \tag{3.5}$$

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is the dipole operator. The function

$$\Phi_0^\alpha \equiv \Phi_0^\alpha\left(p_{1\perp}, p_{2\perp}\right) \tag{3.6}$$

is the LO impact factor and we will often suppress its dependence on variables for brevity. Its components have the form (3.7)–(3.8), in which $x \equiv x_q$ and $\bar{x} = 1 - x = x_{\bar{q}}$:

$$\Phi_0^+ = -\frac{p_\gamma^+}{p_\gamma^-}\Phi_0^- = \frac{2x\bar{x}p_\gamma^+}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\left(\overline{u}_{p_q}\gamma^+ v_{p_{\bar{q}}}\right), \tag{3.7}$$

$$\Phi_0^i = \frac{\overline{u}_{p_q}\left((1-2x)\,p_{q1\perp}^i + \frac{1}{2}\left[\hat{p}_{q1\perp}, \gamma^i\right]\right)\gamma^+ v_{p_{\bar{q}}}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}. \tag{3.8}$$

The first equality in (3.7) holds thanks to the electromagnetic gauge invariance, which allows us to calculate only the $^+$ component of the impact factor.

3.2 NLO impact factor

At NLO the subtracted matrix element generalizing expression (3.4) can be split into two terms, depending on the type of Wilson line operators involved. Expressing all the Wilson operators in the fundamental representation, we can show that one term involves a single dipole operator $\mathcal{U}_{12} = tr(U_1U_2^\dagger) - N_c$, while the other one involves the double-dipole operator $tr(U_1U_3^\dagger)tr(U_3U_2^\dagger) - N_c tr(U_1U_2^\dagger) = \mathcal{U}_{13}\,\mathcal{U}_{32} + N_c\left(\mathcal{U}_{13} + \mathcal{U}_{32} - \mathcal{U}_{12}\right)$:

$$\begin{aligned} T_1^\alpha = \alpha_s\frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}\int d^dp_{1\perp}d^dp_{2\perp}\Bigg\{&\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{\gamma\perp})\Phi_1^\alpha\frac{N_c^2-1}{N_c}[tr(U_1U_2^\dagger)-N_c](p_{1\perp},p_{2\perp}) \\ &+\int\frac{d^dp_{3\perp}}{(2\pi)^d}\delta(p_{q1}+p_{\bar{q}2}-p_{\gamma\perp}-p_{3\perp})\Phi_2^\alpha[tr(U_1U_3^\dagger)tr(U_3U_2^\dagger) \\ &-N_c tr(U_1U_2^\dagger)](p_{1\perp},p_{2\perp},p_{3\perp})\frac{1}{2}\Bigg\}. \end{aligned} \tag{3.9}$$

The Wilson operators here are defined in (2.29), (3.5) and the dependence of the coupling constant on the regularization scale (2.26) is included in the definition of Φ_1^α and Φ_2^α.

There are 8 one-loop diagrams contributing to the matrix element T_1. Five of them are presented in figure 2. The remaining ones can be obtained from diagrams 3, 5 and 6 via the substitution $p_q \leftrightarrow p_{\bar{q}}$, $\bar{u}_q \leftrightarrow v_{\bar{q}}$, $p_1 \leftrightarrow p_2$ and the reversal of the order of the gamma matrices, which we will denote $(q \leftrightarrow \bar{q})$. Diagrams 2, 3 and 4 contribute only to the dipole term Φ_1 while diagrams 5 and 6 contribute to both Φ_1 and the double-dipole term Φ_2. Indeed, after projecting on the color singlet state and subtracting the non-interacting part, it is straightforward to see that diagrams 2, 3 and 4 involve $tr(U_1U_2^\dagger) - N_c$, while diagrams 5 and 6 involve the Fourier transform of an operator which can be decomposed as follows:

$$tr(t^aU_1t^bU_2^\dagger)U_3^{ab} - \frac{N_c^2-1}{2} = \frac{1}{2}[tr(U_1U_3^\dagger)tr(U_3U_2^\dagger) - N_c tr(U_1U_2^\dagger)] + \frac{N_c^2-1}{2N_c}[tr(U_1U_2^\dagger) - N_c]. \tag{3.10}$$

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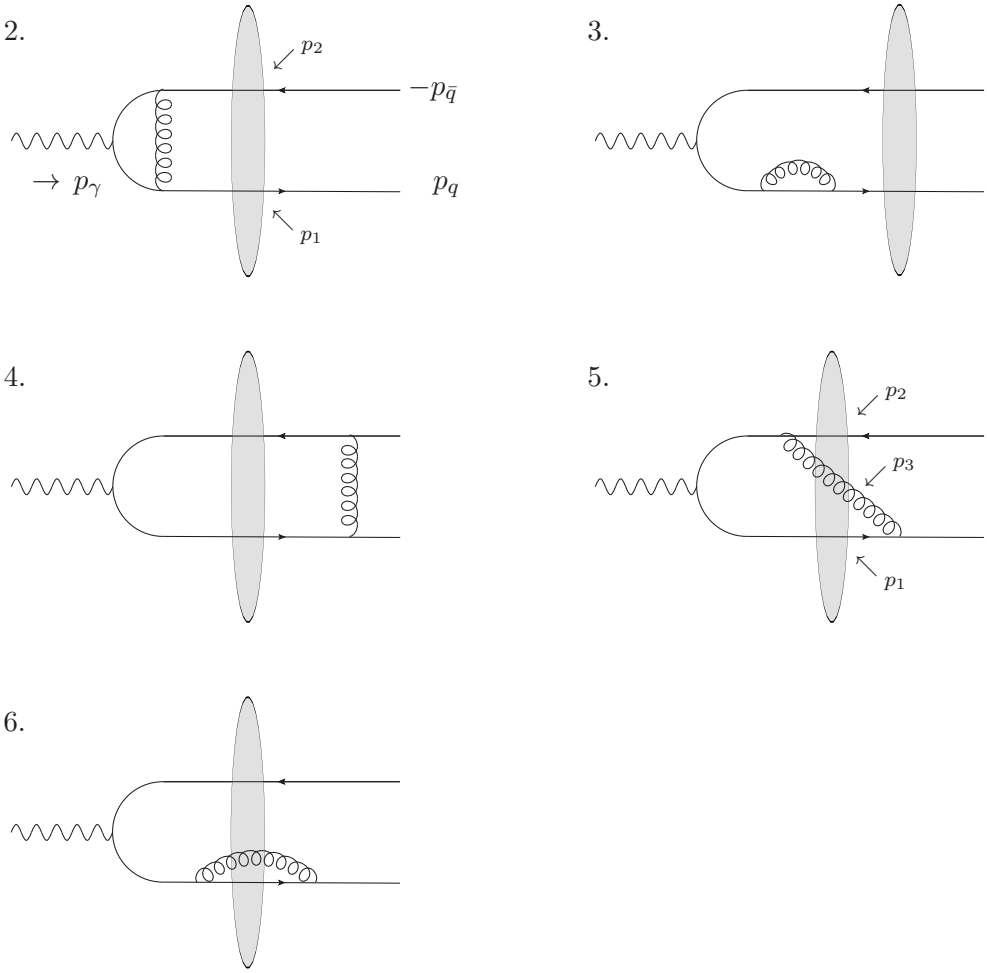

Figure 2. One-loop diagrams for the $\gamma^* \to q\bar{q}$ transition. The momenta p_1, p_2, and p_3 go from the shockwave to the quark, antiquark and gluon.

3.2.1 Method of calculation of the NLO corrections

Due to the presence of the rapidity singularity $p^+ \to 0$ in lightcone gauge, we cannot integrate directly in D dimensions with the usual Feynman integration methods. Dimensional regularization can be used for the transverse components with dimension $d = D - 2$, while the longitudinal divergences will be regularized by the cutoff α.

We will now present the essential steps required to compute the diagrams in figure 2. For this purpose, we will consider the simple case of diagram 3, the contribution of the other diagrams being obtained in a similar way. The initial expression for this diagram is the projection on the color singlet state of the following expression:

$$\tilde{M}^\alpha|_3 = \frac{ig^2 e_q}{\sqrt{N_c}} \int d^D y_2 d^D y_1 d^D y_0 \bar{u}\left(p_q,\, y_2\right) \gamma^\mu \hat{G}_0\left(y_{21}\right) \gamma^\nu \hat{G}_0\left(y_{10}\right) \gamma^\alpha \frac{1}{\sqrt{2p_\gamma^+}}$$
$$\times\, \theta\left(p_\gamma^+\right) \theta\left(-y_2^+\right) \theta\left(-y_1^+\right) \theta\left(-y_0^+\right) v\left(p_{\bar{q}},\, y_0\right) G_{\mu\nu}\left(y_{21}\right) e^{-i(p_\gamma \cdot y_0)}. \qquad (3.11)$$

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Using the building blocks defined by eqs. (2.16) to (2.21) and projecting on the singlet, we get:

$$\begin{aligned}\tilde{M}^{\alpha}|_3 = &\frac{ig^2 e_q}{\sqrt{N_c}}\int d^D y_2 d^D y_1 d^D y_0 \int \frac{d^D q_2}{(2\pi)^D}\frac{d^D q_1}{(2\pi)^D}\frac{d^D l}{(2\pi)^D}\frac{N_c^2-1}{2N_c}\\ &\times tr\left[U(p_{1\perp})U^{\dagger}(-p_{2\perp})\right]\theta\left(-y_2^+\right)\theta\left(-y_1^+\right)\theta\left(-y_0^+\right)\\ &\times\frac{\theta\left(p_q^+\right)}{(2\pi)^{D-2}\sqrt{2p_q^+}}e^{ip_q^+ y_2^-}\,\bar{u}_{p_q}\gamma^+\frac{p_q^+\gamma^- + \hat{p}_{q1\perp}}{2p_q^+}e^{\,i\left(p_{q1\perp}\cdot y_{2\perp}\right)-i\frac{y_2^+}{2p_q^+}\left(p_{q1\perp}^2+i0\right)}\\ &\times e^{-i(q_2+l)\cdot y_{21}-i(q_1\cdot y_{10})}\gamma^{\mu}\hat{G}_0\left(q_2\right)\gamma^{\nu}\hat{G}_0\left(q_1\right)\gamma^{\alpha}\frac{1}{\sqrt{2p_\gamma^+}}e^{-i(p_\gamma\cdot y_0)}\theta\left(p_\gamma^+\right)G_{\mu\nu}\left(l\right)\\ &\times\frac{\theta\left(p_{\bar{q}}^+\right)}{(2\pi)^{D-2}\sqrt{2p_{\bar{q}}^+}}e^{\,ip_{\bar{q}}^+ y_0^- + i\left(p_{2\bar{q}\perp}\cdot y_{0\perp}\right)-i\frac{y_0^+}{2p_{\bar{q}}^+}\left(p_{2\bar{q}\perp}^2+i0\right)}\frac{p_{\bar{q}}^+\gamma^- + \hat{p}_{\bar{q}2\perp}}{2p_{\bar{q}}^+}\gamma^+ v_{p_{\bar{q}}}. \end{aligned}\tag{3.12}$$

We now apply the following procedure:

- Integrating w.r.t. the $^-$ and transverse components of the coordinates y_2, y_1 and y_0 gives relations for the conservation of $^+$ and transverse momentum components, such as
$$\delta\left(p_q^+ - q_2^+ - l^+\right)\delta\left(p_{q\perp} - q_{2\perp} - l_\perp\right).$$
- The $^+$ and transverse momentum integrations are now taken, as trivial δ integrations.
- We integrate the $^-$ momenta by pole integration.

This way, one obtains

$$\begin{aligned}\tilde{M}^{\alpha}|_3 = &-\frac{ig^2 e_q}{\sqrt{N_c}}\frac{tr\left[U(p_{1\perp})U^{\dagger}(-p_{2\perp})\right]}{(2\pi)^{2D-4}}\left(\frac{N_c^2-1}{2N_c}\right)\frac{\theta\left(p_q^+\right)\theta\left(p_{\bar{q}}^+\right)\theta\left(p_\gamma^+\right)}{\sqrt{2p_\gamma^+ 2p_q^+ 2p_{\bar{q}}^+}}\\ &\times\int dy_2^+ dy_1^+ dy_0^+\,\theta\left(-y_2^+\right)\theta\left(y_{21}^+\right)\theta\left(-y_0^+\right)\theta\left(p_q^+ - l^+\right)\theta\left(l^+\right)\\ &\times\int dl^+ d^{D-2}l_\perp\,\delta\left(p_q^+ + p_{\bar{q}}^+ - p_\gamma^+\right)\delta\left(p_{q1\perp}+p_{2\bar{q}\perp}-p_{\gamma\perp}\right)\bar{u}_{p_q}\gamma^+\left[\frac{p_q^+\gamma^- + \hat{p}_{q1\perp}}{2p_q^+ 2\left(p_q^+ - l^+\right)}\right]\gamma^{\mu}\\ &\times\left[\left(p_q^+ - l^+\right)\gamma^- - \frac{\left(p_{q1\perp}-l_\perp\right)^2}{2\left(p_q^+ - l^+\right)}\gamma^+ + \left(\hat{p}_{q1\perp} - \hat{l}_\perp\right)\right]\\ &\times\gamma^{\nu}\left(\left[p_q^+\gamma^- - \frac{p_{q1\perp}^2}{2p_q^+}\gamma^+ + \hat{p}_{q1\perp}\right]\theta\left(y_{10}^+\right) + i\delta\left(y_{10}^+\right)\gamma^+\right)\gamma^{\alpha}\left[\frac{p_{\bar{q}}^+\gamma^- + \hat{p}_{\bar{q}2\perp}}{2p_q^+ 2p_{\bar{q}}^+}\right]\gamma^+ v_{p_{\bar{q}}}\\ &\times\frac{1}{2l^+}d_{0\mu\nu}(l)\exp\left[-i\left(p_\gamma^- + \frac{p_{2\bar{q}\perp}^2+i0}{2p_{\bar{q}}^+} + \frac{p_{q1\perp}^2+i0}{2p_q^+}\right)y_0^+\right]\\ &\times\exp\left[i\left(\frac{l_\perp^2+i0}{2l^+} + \frac{\left(p_{q1\perp}-l_\perp\right)^2+i0}{2\left(p_q^+ - l^+\right)} - \frac{p_{q1\perp}^2+i0}{2p_q^+}\right)y_{21}^+\right]. \end{aligned}\tag{3.13}$$

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In this example, one can easily see that the contributions from the terms with $\delta(y_{21}^+)$ or $\theta(y_{21}^+)$ give zero, due to either the gamma structure or $^+$-momentum ordering.

The integration w.r.t. the $^+$ components of the coordinates finally gives two contributions:

$$\begin{aligned}\tilde{M}^\alpha|_3^{(1)} &= i\frac{ie_q g^2}{\sqrt{N_c}}\frac{tr\left[U(p_{1\perp})U^\dagger(-p_{2\perp})\right]}{(2\pi)^{2D-4}}\left(\frac{N_c^2-1}{2N_c}\right) \\ &\times \frac{\theta\left(p_q^+\right)\theta\left(p_{\bar{q}}^+\right)\theta\left(p_\gamma^+\right)}{\sqrt{2p_\gamma^+ 2p_q^+ 2p_{\bar{q}}^+}}\frac{\delta\left(p_q^+ + p_{\bar{q}}^+ - p_\gamma^+\right)\delta\left(p_{q1\perp}+p_{2\bar{q}\perp}\right)}{\left(p_\gamma^- + \frac{p_\perp^2+i0}{2p_q^+} + \frac{p_\perp^2+i0}{2p_{\bar{q}}^+}\right)^2} \\ &\times \bar{u}_{p_q}\gamma^+\left[\frac{p_q^+\gamma^- + \hat{p}_\perp}{2p_q^+}\right]\int_{\alpha p_\gamma^+}^{p_q^+}\frac{dl^+}{\left(2p_q^+\right)^2}\int\frac{d^{D-2}l_\perp}{\left[\left(l_\perp - \frac{l^+}{p_q^+}p_\perp\right)^2 + \frac{2l^+\left(p_q^+ - l^+\right)}{p_q^+}\left(p_\gamma^- + \frac{p_\gamma^+}{2p_q^+ p_{\bar{q}}^+}p_\perp^2\right)\right]} \\ &\times \gamma^\mu\left[\left(p_q^+ - l^+\right)\gamma^- - \frac{\left(p_\perp - l_\perp\right)^2}{2\left(p_q^+ - l^+\right)}\gamma^+ + \left(\hat{p}_\perp - \hat{l}_\perp\right)\right]\gamma^\nu \\ &\times \left[g_{\perp\mu\nu} - \frac{l_{\perp\mu}n_{2\nu} + l_{\perp\nu}n_{2\mu}}{l^+} + \frac{n_{2\mu}n_{2\nu}}{\left(l^+\right)^2}l_\perp^2\right]\left[p_q^+\gamma^- - \frac{p_\perp^2}{2p_q^+}\gamma^+ + \hat{p}_\perp\right]\gamma^\alpha\left[\frac{p_{\bar{q}}^+\gamma^- - p_\perp}{2p_{\bar{q}}^+}\right]\gamma^+ v_{p_{\bar{q}}}\,,\end{aligned} \tag{3.14}$$

and

$$\begin{aligned}\tilde{M}^\alpha|_3^{(2)} &= -\frac{ig^2 e_q}{\sqrt{N_c}}\frac{tr\left[U(p_{1\perp})U^\dagger(-p_{2\perp})\right]}{(2\pi)^{2D-4}}\left(\frac{N_c^2-1}{2N_c}\right)\frac{\theta\left(p_q^+\right)\theta\left(p_{\bar{q}}^+\right)\theta\left(p_\gamma^+\right)}{\sqrt{2p_\gamma^+ 2p_q^+ 2p_{\bar{q}}^+}} \\ &\times i\delta\left(p_q^+ + p_{\bar{q}}^+ - p_\gamma^+\right)\delta\left(p_{q1\perp} + p_{2\bar{q}\perp} - p_{\gamma\perp}\right) \\ &\times \int dl^+ d^d l_\perp \theta\left(p_q^+ - l^+\right)\theta\left(l^+\right)\frac{1}{2l^+}\bar{u}_{p_q}\gamma^+\left[\frac{p_q^+\gamma^- + \hat{p}_{q1\perp}}{2p_q^+ 2\left(p_q^+ - l^+\right)}\right]\gamma^\mu \\ &\times \frac{\left[\left(p_q^+ - l^+\right)\gamma^- + \left(\hat{p}_{q1\perp} - \hat{l}_\perp\right)\right]\left[\gamma_{\perp\mu} - \frac{\hat{l}_\perp n_{2\mu}}{l^+}\right]\gamma^+\gamma^\alpha\left[\frac{p_{\bar{q}}^+\gamma^- + p_{\bar{q}2\perp}}{2p_q^+ 2p_{\bar{q}}^+}\right]\gamma^+ v_{p_{\bar{q}}}}{\left(p_\gamma^- + \frac{p_{2\bar{q}\perp}^2+i0}{2p_{\bar{q}}^+} + \frac{l_\perp^2+i0}{2l^+} + \frac{\left(p_{q1\perp}-l_\perp\right)^2+i0}{2\left(p_q^+ - l^+\right)}\right)\left(p_\gamma^- + \frac{p_{q1\perp}^2+i0}{2p_q^+} + \frac{p_{2\bar{q}\perp}^2+i0}{2p_{\bar{q}}^+}\right)}\,.\end{aligned} \tag{3.15}$$

We introduced the regularization cutoff α for the divergences which will emerge from the longitudinal integration.

The transverse momentum integration can be performed straightforwardly with the Feynman parameter or Schwinger representation method. We then decompose the result as a divergent term in $\epsilon = \frac{D-4}{2}$ and a constant term. Thus:

$$\begin{aligned}\tilde{M}^\alpha|_3 &\simeq -\frac{e_q g^2}{\sqrt{N_c}}\frac{\Gamma(1-\epsilon)}{(16\pi^3)^{1+\epsilon}}tr\left[U(p_{1\perp})U^\dagger(-p_{2\perp})\right]\left(\frac{N_c^2-1}{2N_c}\right)\frac{\theta\left(p_q^+\right)\theta\left(p_{\bar{q}}^+\right)\theta\left(p_\gamma^+\right)}{\sqrt{2p_\gamma^+ 2p_q^+ 2p_{\bar{q}}^+}} \\ &\times \frac{\delta\left(p_q^+ + p_{\bar{q}}^+ - p_\gamma^+\right)\delta\left(p_{q1\perp} + p_{2\bar{q}\perp}\right)}{\left(4p_\gamma^+\right)\left(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2\right)} \\ &\times \int_\alpha^x \frac{dz}{x^2}\left[\frac{1}{\epsilon} + \ln\left(\frac{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}{x\bar{x}\mu^2}\right) + \ln\left(\frac{z(x-z)}{x}\right)\right]\left(zd + 4\frac{x(x-z)}{z}\right) \\ &\times \bar{u}_{p_q}\left[\left(\gamma^+\hat{p}_{q1\perp}\right) + p_q^+\left(\gamma^+\gamma^-\right)\right]\gamma^\alpha\left(p_{\bar{q}}^+\gamma^- - p_{q1\perp}\right)\gamma^+ v_{p_{\bar{q}}}\,.\end{aligned} \tag{3.16}$$

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To regularize the integration over parameter z, we will always write

$$\int_\alpha^{z_0} dz\,\phi(z) \simeq \int_\alpha^{z_0} dz\,\phi_0(z) + \int_0^{z_0} dz\,[\phi(z)-\phi_0(z)]$$
$$\equiv \int_\alpha^{z_0} dz\,\phi_0(z) + \int_0^{z_0} dz\,[\phi(z)]_+\,, \tag{3.17}$$

where we explicitly extract the non-integrable part ϕ_0 of ϕ, writing $\phi(z)=\phi_0(z)+O(\ln z)$ for $z\to 0$.

In the case of diagram 3, both these terms can be computed analytically to the end. However it is not the case for some other diagrams, although the first term of the second line of eq. (3.17) is always straightforward to obtain.

The method for the computation of the other NLO virtual diagrams is similar although more elaborate. We will not give the details for them in the body text and relegate the details of the results to the appendices. We will only give the divergent part, by which we mean the first part of the second line of eq. (3.17) and the $\frac{1}{\epsilon}$ term in the second part of this line.

3.2.2 Double dipole contribution Φ_2

For reader's convenience, we will only write here explicitly the divergent parts of diagrams in figure 2. The expressions for the constant parts can be found in appendix A.

The contribution of diagram 5 to Φ_2, including the $(q\leftrightarrow\bar q)$ terms, reads:

$$\Phi_2^+|_5 = 2p_\gamma^+(\bar u_{p_q}\gamma^+ v_{p_{\bar q}})\frac{x\bar x(\vec p_3^{\,2}-\vec p_{\bar q2}^{\,2}-\vec p_{q1}^{\,2}-2x\bar xQ^2)}{(\vec p_{\bar q2}^{\,2}+x\bar xQ^2)\left(\vec p_{q1}^{\,2}+x\bar xQ^2\right)-x\bar xQ^2\vec p_3^{\,2}}$$
$$\times\ln\left(\frac{x\bar x}{\alpha^2}\right)\ln\left(\frac{(\vec p_{\bar q2}^{\,2}+x\bar xQ^2)\left(\vec p_{q1}^{\,2}+x\bar xQ^2\right)}{x\bar xQ^2\vec p_3^{\,2}}\right)+(\bar u_{p_q}C_{2\|}^5 v_{p_{\bar q}}) \tag{3.18}$$

for a longitudinal photon, and

$$\Phi_2^i|_5 = \ln\left(\frac{x\bar x}{\alpha^2}\right)\left\{\overline{u}_{p_q}(p_{q1\perp}^i(1-2x)+\frac{1}{2}[\hat p_{q1\perp},\gamma_\perp^i])\gamma^+ v_{p_{\bar q}}\right.$$
$$\times\left[\frac{1}{\vec p_{q1}^{\,2}}\ln\left(\frac{\vec p_{q1}^{\,2}+x\bar xQ^2}{x\bar xQ^2}\right)-\frac{\vec p_{\bar q2}^{\,2}+x\bar xQ^2}{\left(\vec p_{q1}^{\,2}+x\bar xQ^2\right)(\vec p_{\bar q2}^{\,2}+x\bar xQ^2)-x\bar xQ^2\vec p_3^{\,2}}\right.$$
$$\left.\left.\times\ln\left(\frac{(\vec p_{q1}^{\,2}+x\bar xQ^2)\left(\vec p_{\bar q2}^{\,2}+x\bar xQ^2\right)}{x\bar xQ^2\vec p_3^{\,2}}\right)\right]+(q\leftrightarrow\bar q)\right\}+\bar u_{p_q}C_{2\perp}^{5i}v_{p_{\bar q}} \tag{3.19}$$

for a transverse photon. The contribution of diagram 6 reads:

$$\Phi_2^+|_6 = -\left(\frac{x\bar xp_\gamma^+(\bar u_{p_q}\gamma^+v_{p_{\bar q}})}{\vec p_{q1}^{\,2}+x\bar xQ^2}\left[4\ln\left(\frac{\bar x}{\alpha}\right)\left(\frac{1}{\epsilon}+\ln\left(\frac{\vec p_3^{\,2}}{\mu^2}\right)\right)-\frac{3}{\epsilon}\right]+(q\leftrightarrow\bar q)\right)+\bar u_{p_q}C_{2\|}^6 v_{p_{\bar q}} \tag{3.20}$$

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for a longitudinal photon, and

$$\Phi_2^i|_6 = -\left(\frac{\overline{u}_{p_q}(p^i_{q1\perp}(1-2x)+\frac{1}{2}[\hat{p}_{q1\perp},\,\gamma^i_\perp])\gamma^+ v_{p_{\bar{q}}}}{\vec{p}^{\,2}_{q1}+x\bar{x}Q^2}\left[2\ln\left(\frac{\bar{x}}{\alpha}\right)\left(\frac{1}{\epsilon}+\ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right)-\frac{3}{2\epsilon}\right]\right.$$
$$\left.+\,(q\leftrightarrow\bar{q})\right)+\bar{u}_{p_q}C^{6i}_{2\perp}v_{p_{\bar{q}}} \tag{3.21}$$

for a transverse photon.

In these expressions the functions C do not contain singularities. Their exact form will be given in appendix A. The remaining (divergent) part contains a rapidity divergence of the form $\ln\alpha$. Such terms have to be absorbed into the renormalized Wilson operators with the help of the BK equation. Indeed the LO contribution as defined in (3.4) involves the Wilson line operators at rapidity $\ln\alpha$. We thus have to use the BK evolution from α to e^η, by writing

$$U(x,e^\eta)=U(x,\alpha)+\int_\alpha^{e^\eta}d\rho\left(\frac{\partial U(x,\rho)}{\partial\rho}\right). \tag{3.22}$$

Let us note that the BK equation is of order α_s so $U(x,\alpha)$ can be directly replaced by $U(x,e^\eta)$ in NLO corrections to impact factors without concern.

Plugging this eq. (3.22) into eq. (3.4) and using the explicit BK equation (2.28) allows one to evolve the LO dipole contribution into an NLO double-dipole contribution. This contribution reads:

$$\begin{aligned}\langle T_0^\alpha\rangle^\eta=&\int d^dp_{1\perp}d^dp_{2\perp}\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{\gamma\perp})\Phi_0^\alpha(p_{1\perp},p_{2\perp})\\&\times\ln\left(\frac{e^\eta}{\alpha}\right)\delta\left(k_{1\perp}+k_{2\perp}+k_{3\perp}-p_{1\perp}-p_{2\perp}\right)2\alpha_s\mu^{2-d}\int\frac{d^dk_{1\perp}d^dk_{2\perp}d^dk_{3\perp}}{(2\pi)^{2d}}\\&\times\left[-\frac{2(k_{1\perp}-p_{1\perp})\cdot(k_{2\perp}-p_{2\perp})}{(k_1-p_1)^2_\perp(k_2-p_2)^2_\perp}\right.\\&\left.\quad+\frac{\pi^{\frac{d}{2}}\Gamma(1-\frac{d}{2})\Gamma(\frac{d}{2})^2}{\Gamma(d-1)}\left(\frac{\delta(k_{2\perp}-p_{2\perp})}{(-(k_1-p_1)^2_\perp)^{1-\frac{d}{2}}}+\frac{\delta(k_{1\perp}-p_{1\perp})}{(-(k_2-p_2)^2_\perp)^{1-\frac{d}{2}}}\right)\right]\\&\times\left[tr\left(U_1U_3^\dagger\right)tr\left(U_3U_2^\dagger\right)-N_ctr\left(U_1U_2^\dagger\right)\right](k_{1\perp},k_{2\perp},k_{3\perp}).\end{aligned} \tag{3.23}$$

After integrating w.r.t. $p_{2\perp}$ and renaming the variables, we get

$$\begin{aligned}\langle T_0^\alpha\rangle^\eta=&\int\frac{d^dp_{1\perp}d^dp_{2\perp}d^dp_{3\perp}}{(2\pi)^d}\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{3\perp}-p_{\gamma\perp})\\&\times\ln\left(\frac{e^\eta}{\alpha}\right)2\alpha_s\mu^{2-d}\int\frac{d^dp_\perp}{(2\pi)^d}\Phi_0^\alpha(p_{1\perp}+p_\perp,p_{2\perp}+p_{3\perp}-p_\perp)\\&\times\left[\frac{2(p_\perp\cdot(p_\perp-p_{3\perp}))}{p_\perp^2(p-p_3)^2_\perp}+\frac{\pi^{\frac{d}{2}}\Gamma(1-\frac{d}{2})\Gamma(\frac{d}{2})^2}{\Gamma(d-1)}\left(\frac{\delta(p_\perp-p_{3\perp})}{(-p_\perp^2)^{1-\frac{d}{2}}}+\frac{\delta(p_\perp)}{(-(p-p_3)^2_\perp)^{1-\frac{d}{2}}}\right)\right]\\&\times[tr(U_1U_3^\dagger)tr(U_3U_2^\dagger)-N_ctr(U_1U_2^\dagger)](p_{1\perp},p_{2\perp},p_{3\perp}).\end{aligned} \tag{3.24}$$

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Integrating w.r.t. $p_\perp$, one can get the contribution from this convolution:

$$\Phi^+_{BK}(p_{1\perp},p_{2\perp},p_{3\perp}) = -4x\bar{x}p^+_\gamma(\bar{u}_{p_q}\gamma^+ v_{p_{\bar{q}}})\ln\left(\frac{e^\eta}{\alpha}\right) \tag{3.25}$$
$$\times\left[\left(\ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)+\frac{1}{\epsilon}\right)\left(\frac{-1}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}+\frac{-1}{\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2}\right)\right.$$
$$\left.+\frac{\vec{p}_3^{\,2}-\vec{p}_{q1}^{\,2}-\vec{p}_{\bar{q}2}^{\,2}-2x\bar{x}Q^2}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)-x\bar{x}\vec{p}_3^{\,2}Q^2}\ln\left(\frac{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)}{x\bar{x}\vec{p}_3^{\,2}Q^2}\right)\right]$$

in the longitudinal case, and

$$\Phi^i_{BK} = -2\ln\left(\frac{e^\eta}{\alpha}\right)\left\{\bar{u}_{p_q}\left((1-2x)p^i_{q1\perp}+\frac{1}{2}[\hat{p}_{q1\perp},\gamma^i_\perp]\right)\gamma^+ v_{p_{\bar{q}}}\right.$$
$$\times\left[\frac{-1}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\left(\ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)+\frac{1}{\epsilon}\right)+\frac{1}{\vec{p}_{q1}^{\,2}}\ln\left(\frac{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}{x\bar{x}Q^2}\right)\right.$$
$$-\frac{\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)-x\bar{x}\vec{p}_3^{\,2}Q^2}$$
$$\left.\left.\times\ln\left(\frac{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)}{x\bar{x}\vec{p}_3^{\,2}Q^2}\right)\right]+(q\leftrightarrow\bar{q})\right\} \tag{3.26}$$

in the transverse case.

Combining this subtraction term with the results from diagrams 5 and 6 above, we can cancel the rapidity divergence in $\ln\alpha$ and obtain the actual double-dipole part $\Phi'_2 = \Phi_2+\Phi_{BK}$ of the impact factor:

$$\Phi'^+_2 = 2p^+_\gamma(\bar{u}_{p_q}\gamma^+ v_{p_{\bar{q}}})\left\{\frac{x\bar{x}(\vec{p}_3^{\,2}-\vec{p}_{\bar{q}2}^{\,2}-\vec{p}_{q1}^{\,2}-2x\bar{x}Q^2)}{(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)\left(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2\right)-x\bar{x}Q^2\vec{p}_3^{\,2}}\right.$$
$$\times\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\ln\left(\frac{(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)\left(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2\right)}{x\bar{x}Q^2\vec{p}_3^{\,2}}\right)$$
$$\left.-\left(\frac{x\bar{x}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\left[2\ln\left(\frac{\bar{x}}{e^\eta}\right)\left(\frac{1}{\epsilon}+\ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right)-\frac{3}{2\epsilon}\right]+(q\leftrightarrow\bar{q})\right)\right\}$$
$$+\bar{u}_{p_q}(C^5_{2\parallel}+C^6_{2\parallel})v_{p_{\bar{q}}} \tag{3.27}$$

in the longitudinal case, or

$$\Phi'^i_2 = \left\{\overline{u}_{p_q}\left(p^i_{q1\perp}(1-2x)+\frac{1}{2}[\hat{p}_{q1\perp},\,\gamma^i_\perp]\right)\gamma^+ v_{p_{\bar{q}}}\left(\frac{-1}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\right.\right. \tag{3.28}$$
$$\times\left[2\ln\left(\frac{\bar{x}}{e^\eta}\right)\left(\frac{1}{\epsilon}+\ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right)-\frac{3}{2\epsilon}\right]+\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)$$
$$\times\left[\frac{1}{\vec{p}_{q1}^{\,2}}\ln\left(\frac{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}{x\bar{x}Q^2}\right)-\frac{\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)-x\bar{x}Q^2\vec{p}_3^{\,2}}\right.$$
$$\left.\left.\left.\times\ln\left(\frac{\left(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2\right)\left(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2\right)}{x\bar{x}Q^2\vec{p}_3^{\,2}}\right)\right]\right)+(q\leftrightarrow\bar{q})\right\}+\bar{u}_{p_q}(C^{5i}_{2\perp}+C^{6i}_{2\perp})v_{p_{\bar{q}}}$$

in the transverse case.

These impact factors still contain $\frac{1}{\epsilon}$ terms, although by construction they should not have any IR, UV or collinear singularity. These poles are artificial UV poles and already appear in the momentum representation of the BK equation (2.28). They originate from the fact that when we transform the Wilson line operator (2.29) into its momentum space representation straightforwardly, we do not take into account its property of vanishing when $r_3 = r_2$ or $r_3 = r_1$. This property reveals in the convolution of the impact factor and the operator (2.29) killing all the artificial singularities. Indeed, the divergent terms depend only on $\vec{p}_1$ and are independent of $\vec{p}_3$ and $\vec{p}_2$ (up to a (1↔2) permutation). Writing those terms as $F(p_{1\perp})$ and covoluting them as in (3.9) gives

$$\begin{aligned}
&\int d^d p_{1\perp} d^d p_{2\perp} d^d p_{3\perp} \delta(p_{q1} + p_{\bar{q}2} - p_{\gamma\perp} - p_{3\perp}) F\left(p_{1\perp}\right) \\
&\times [tr(U_1 U_3^\dagger) tr(U_3 U_2^\dagger) - N_c tr(U_1 U_2^\dagger)](p_{1\perp}, p_{2\perp}, p_{3\perp}) \\
=& \int d^d p_{1\perp} d^d p_{3\perp} d^d r_{1\perp} d^d r_{2\perp} d^d r_{3\perp} \\
&\times F\left(p_{1\perp}\right) e^{i(r_{1\perp}\cdot p_{1\perp}) + i r_{2\perp}\cdot(p_{q1\perp} + p_{\bar{q}\gamma\perp}) + i(p_{3\perp}\cdot r_{32\perp})} [tr(U_1 U_3^\dagger) tr(U_3 U_2^\dagger) - N_c tr(U_1 U_2^\dagger)] \\
\sim& \int d^d p_{1\perp} d^d r_{1\perp} d^d r_{2\perp} F\left(p_{1\perp}\right) e^{i(r_{1\perp}\cdot p_{1\perp}) + i r_{2\perp}\cdot(p_{q1\perp} + p_{\bar{q}\gamma\perp})} \\
&\times \int d^d r_{3\perp} \delta(r_{32\perp}) [tr(U_1 U_3^\dagger) tr(U_3 U_2^\dagger) - N_c tr(U_1 U_2^\dagger)] = 0.
\end{aligned} \tag{3.29}$$

Thus the artificially divergent part

$$F(p_{1\perp}) = \frac{x\bar{x}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} \left[2 \ln\left(\frac{\bar{x}}{e^\eta}\right) \frac{1}{\epsilon} - \frac{3}{2\epsilon} \right] \tag{3.30}$$

will cancel once convoluted, so it can be omitted. For a more involved discussion about such terms, see ref. [51].

The same computation can allow one to omit the $\ln(\mu^2)$ contribution. However, we will keep it so that no dimensional log appears, keeping in mind that there is no actual μ dependence. Therefore hereafter we will use

$$\begin{aligned}
\Phi_2'^{+} = 2p_\gamma^+ (\bar{u}_{p_q} \gamma^+ v_{p_{\bar{q}}}) \Bigg\{ & \frac{x\bar{x}(\vec{p}_3^{\,2} - \vec{p}_{\bar{q}2}^{\,2} - \vec{p}_{q1}^{\,2} - 2x\bar{x}Q^2)}{(\vec{p}_{\bar{q}2}^{\,2} + x\bar{x}Q^2)\left(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2\right) - x\bar{x}Q^2\vec{p}_3^{\,2}} \\
& \times \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \ln\left(\frac{(\vec{p}_{\bar{q}2}^{\,2} + x\bar{x}Q^2)\left(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2\right)}{x\bar{x}Q^2\vec{p}_3^{\,2}} \right) \\
& + \left(\frac{-2x\bar{x}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} \ln\left(\frac{\bar{x}}{e^\eta}\right) \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right) + (q \leftrightarrow \bar{q}) \right) \Bigg\} + \bar{u}_{p_q}(C_{2\|}^5 + C_{2\|}^6) v_{p_{\bar{q}}}
\end{aligned} \tag{3.31}$$

and

$$\begin{aligned}
\Phi_2'^{i} = \Bigg\{ & \overline{u}_{p_q}(p_{q1\perp}^i(1-2x) + \frac{1}{2}[\hat{p}_{q1\perp}, \gamma_\perp^i])\gamma^+ v_{p_{\bar{q}}} \left(\frac{-2}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} \ln\left(\frac{\bar{x}}{e^\eta}\right) \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right) \right. \\
& + \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\vec{p}_{q1}^{\,2}} \ln\left(\frac{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}{x\bar{x}Q^2} \right) - \frac{\vec{p}_{\bar{q}2}^{\,2} + x\bar{x}Q^2}{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2)(\vec{p}_{\bar{q}2}^{\,2} + x\bar{x}Q^2) - x\bar{x}Q^2\vec{p}_3^{\,2}} \right. \\
& \left. \left. \times \ln\left(\frac{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2)\left(\vec{p}_{\bar{q}2}^{\,2} + x\bar{x}Q^2\right)}{x\bar{x}Q^2\vec{p}_3^{\,2}} \right) \right] \right) + (q \leftrightarrow \bar{q}) \Bigg\} + \bar{u}_{p_q}(C_{2\perp}^{5i} + C_{2\perp}^{6i}) v_{p_{\bar{q}}}.
\end{aligned} \tag{3.32}$$

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3.2.3 Dipole contribution Φ_1

The combined contributions of diagrams 2, 3 and the diagram obtained from 3 via $(q \leftrightarrow \bar{q})$ reads

$$\Phi_1^+|_{23} = -\frac{p_\gamma^+}{p_\gamma^-}\Phi_1^-|_{23} = \frac{x\bar{x}p_\gamma^+(\overline{u}_{p_q}\gamma^+ v_{p_{\bar{q}}})}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} \times \left[\left(2\ln\left(\frac{x\bar{x}}{\alpha^2}\right) - 3\right)\left(\ln\left(\frac{\left(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2\right)^2}{x\bar{x}\mu^2 Q^2}\right) + \frac{1}{\epsilon}\right) + \ln^2\left(\frac{\bar{x}}{x}\right) - \frac{\pi^2}{3} + 6\right] \tag{3.33}$$

for a longitudinal photon and

$$\Phi_1^i|_{23} = \frac{\overline{u}_{p_q}((1-2x)p_{q1\perp}^i + \frac{1}{2}[\hat{p}_{q1\perp}, \gamma_\perp^i])\gamma^+ v_{p_{\bar{q}}}}{2(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2)}\left\{\left(2\ln\left(\frac{x\bar{x}}{\alpha^2}\right) - 3\right) \times \left(\ln\left(\frac{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}{\mu^2}\right) + \frac{x\bar{x}Q^2}{\vec{p}_{q1}^{\,2}}\ln\left(\frac{x\bar{x}Q^2}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\right) + \frac{1}{\epsilon}\right) + \ln^2\left(\frac{\bar{x}}{x}\right) - \frac{\pi^2}{3} + 6\right\} \tag{3.34}$$

for a transverse photon.

This set of diagrams is invariant under the QED gauge transformations. They are directly related to the NLO photon wave-function.

The contribution of diagram 4 reads

$$\Phi_1^+|_4 = \frac{x\bar{x}p_\gamma^+(\overline{u}_{p_q}\gamma^+ v_{p_{\bar{q}}})}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\left\{\ln^2\left(\frac{x\bar{x}}{\alpha^2}\right) - \ln^2\left(\frac{\bar{x}}{x}\right) + 2\ln\left(\frac{x\bar{x}}{\alpha^2}\right)\left(\ln\left(\frac{\left(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2\right)^2}{Q^2(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)^2}\right) + i\pi\right)\right\} + \overline{u}_{p_q}C_\parallel^4 v_{p_{\bar{q}}} \tag{3.35}$$

for a longitudinal photon and

$$\Phi_1^i|_4 = \frac{\overline{u}_{p_q}((1-2x)p_{q1\perp}^i + \frac{1}{2}[\hat{p}_{q1\perp}, \gamma_\perp^i])\gamma^+ v_{p_{\bar{q}}}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\left\{\frac{1}{2}\ln^2\left(\frac{x\bar{x}}{\alpha^2}\right) - \frac{1}{2}\ln^2\left(\frac{\bar{x}}{x}\right) + \ln\left(\frac{x\bar{x}}{\alpha^2}\right)\left(\frac{Q^2 x\bar{x}}{\vec{p}_{q1}^{\,2}}\ln\left(\frac{x\bar{x}Q^2}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\right) + \ln\left(\frac{x\bar{x}\left(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2\right)}{(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)^2}\right) + i\pi\right)\right\} + \overline{u}_{p_q}C_\perp^{4i} v_{p_{\bar{q}}} \tag{3.36}$$

for a transverse photon.

The contribution of diagram 5 reads

$$\Phi_1^+|_5 = \frac{x\bar{x}p_\gamma^+(\bar{u}_{p_q}\gamma^+ v_{p_{\bar{q}}})}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\left(2\ln\left(\frac{x\bar{x}}{\alpha^2}\right)\ln\left(\frac{x\bar{x}Q^4}{\left(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2\right)^2}\right) - \ln^2\left(\frac{x\bar{x}}{\alpha^2}\right) + \ln^2\left(\frac{\bar{x}}{x}\right)\right) + \bar{u}_{p_q}C_{1\parallel}^5 v_{p_{\bar{q}}}\,, \tag{3.37}$$

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for a longitudinal photon, and

$$\Phi_1^i|_5 = -\frac{\overline{u}_{p_q}((1-2x)p^i_{q1\perp} + \frac{1}{2}[\hat{p}_{q1\perp}, \gamma^i_\perp])\gamma^+ v_{p_{\bar{q}}}}{\vec{p}^{\,2}_{q1} + x\bar{x}Q^2}\left(\frac{1}{2}\ln^2\left(\frac{x\bar{x}}{\alpha^2}\right) - \frac{1}{2}\ln^2\left(\frac{\bar{x}}{x}\right)\right.$$
$$\left. + \ln\left(\frac{x\bar{x}}{\alpha^2}\right)\left[\frac{2x\bar{x}Q^2}{\vec{p}^{\,2}_{q1}}\ln\left(\frac{x\bar{x}Q^2}{\vec{p}^{\,2}_{q1} + x\bar{x}Q^2}\right) + \ln(x\bar{x})\right]\right) + \bar{u}_{p_q} C^{5i}_{1\perp} v_{p_{\bar{q}}}\,, \tag{3.38}$$

for a transverse photon.

The contribution of diagram 6 reads

$$\Phi_1^+|_6 = -\frac{x\bar{x}p^+_\gamma(\bar{u}_{p_q}\gamma^+ v_{p_{\bar{q}}})}{\vec{p}^{\,2}_{q1} + x\bar{x}Q^2}\left(\ln\left(\frac{x\bar{x}}{\alpha^2}\right)\left[\frac{4}{\epsilon} - 2\ln\left(\frac{x\bar{x}\mu^4}{(\vec{p}^{\,2}_{q1} + x\bar{x}Q^2)^2}\right)\right]\right.$$
$$\left. - \ln^2\left(\frac{x\bar{x}}{\alpha^2}\right) + \ln^2\left(\frac{\bar{x}}{x}\right) - \frac{6}{\epsilon}\right) + \bar{u}_{p_q} C^6_{1\parallel} v_{p_{\bar{q}}}\,, \tag{3.39}$$

for a longitudinal photon and

$$\Phi_1^i|_6 = -\frac{\overline{u}_{p_q}\left((1-2x)\,p^i_{q1\perp} + \frac{1}{2}\left[\hat{p}_{q1\perp}, \gamma^i_\perp\right]\right)\gamma^+ v_{p_{\bar{q}}}}{x\bar{x}Q^2 + \vec{p}^{\,2}_{q1}}\left(\frac{1}{2}\ln^2\left(\frac{\bar{x}}{x}\right) - \frac{1}{2}\ln^2\left(\frac{x\bar{x}}{\alpha^2}\right)\right.$$
$$\left. + \ln\left(\frac{x\bar{x}}{\alpha^2}\right)\left[\ln\left(\frac{\left(\vec{p}^{\,2}_{q1} + x\bar{x}Q^2\right)^2}{x\bar{x}\mu^4}\right) + \frac{2}{\epsilon}\right] - \frac{3}{\epsilon}\right) + \bar{u}_{p_q} C^{6i}_{1\perp} v_{p_{\bar{q}}}\,, \tag{3.40}$$

for a transverse photon.

As in the previous section, the C functions do not contain singularities. They are presented in appendix A.

Summing the contributions from all diagrams finally gives:

$$\Phi_1^\alpha = \frac{S_V}{2}\Phi_0^\alpha + \Phi_{1R}^\alpha\,, \tag{3.41}$$

where the singular term reads

$$\frac{S_V}{2} = \left[\ln\left(\frac{x\bar{x}}{\alpha^2}\right) - \frac{3}{2}\right]\left[\ln\left(\frac{x\bar{x}\mu^2}{(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)^2}\right) - \frac{1}{\epsilon}\right] + i\pi\ln\left(\frac{x\bar{x}}{\alpha^2}\right) + \frac{1}{2}\ln^2\left(\frac{x\bar{x}}{\alpha^2}\right) - \frac{\pi^2}{6} + 3\,, \tag{3.42}$$

and the regular terms read

$$\Phi^+_{1R} = \frac{3}{2}\Phi_0^+\ln\left(\frac{x^2\bar{x}^2\mu^4Q^2}{(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)^2\left(\vec{p}^{\,2}_{q1} + x\bar{x}Q^2\right)^2}\right) + \bar{u}_{p_q}(C^4_\parallel + C^5_{1\parallel} + C^6_{1\parallel})v_{p_{\bar{q}}}\,, \tag{3.43}$$

and

$$\Phi^i_{1R} = \frac{3}{2}\Phi_0^i\left[\ln\left(\frac{x\bar{x}\mu^4}{(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)^2(\vec{p}^{\,2}_{q1} + x\bar{x}Q^2)}\right) - \frac{x\bar{x}Q^2}{\vec{p}^{\,2}_{q1}}\ln\left(\frac{x\bar{x}Q^2}{\vec{p}^{\,2}_{q1} + x\bar{x}Q^2}\right)\right]$$
$$+ \bar{u}_{p_q}\left(C^{4i}_\perp + C^{5i}_{1\perp} + C^{6i}_{1\perp}\right)v_{p_{\bar{q}}}\,. \tag{3.44}$$

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Note that the $i\pi \ln\left(\frac{x\bar{x}}{\alpha^2}\right)$ term will never contribute, since in the cross sections $\frac{S_V}{2}$ will actually always appear as $\frac{1}{2}(S_V + S_V^*)$.

One can actually easily check that for diagrams 5 and 6 and for any photon polarization,

$$C_1 = C_2|_{\vec{p}_3=\vec{0}}\,. \tag{3.45}$$

This way in appendix A we will only write the expressions for the C_2 coefficients, the C_1 coefficients then being trivial to obtain.

4 The $\gamma^* \to q\bar{q}g$ impact factor

We will now derive the $\gamma^* \to q\bar{q}g$ impact factor. In the body of this paper, it will be used to construct a well defined cross section for dijet production, free of the soft and the collinear singularities. The IR finiteness of the cross-section is discussed in details in section 7. The complete expression for the $\gamma^* \to q\bar{q}g$ cross section is included in appendix B.

The computation of this impact factor in dimension 4 was already presented in [19]. For the purpose of the present study we need its divergent part in dimension D, therefore we will rewrite our results for an arbitrary value of D. The corresponding matrix element for the EM current in the shockwave background reads

$$\tilde{M}'^{\alpha} = -ie_q \int d^D y_0 \frac{e^{-i(p_\gamma \cdot y_0)}}{\sqrt{2p_\gamma^+}} \sqrt{\frac{2}{N_c^2-1}} (t^r)_l^n \langle 0|T(b^l_{p_{\bar{q}}}(a_{p_q})_n c^r_{p_g} \overline{\psi}\,(y_0)\,\gamma^\alpha \psi\,(y_0)\, e^{i\int \mathcal{L}_i(z)dz})|0\rangle_{sw}, \tag{4.1}$$

where c is the gluon annihilation operator and $\sqrt{\frac{2}{N_c^2-1}}(t^r)_l^n$ is the projector on the color singlet. We label the emitted gluon momentum as

$$p_g^\mu = z p_\gamma^+ n_1^\mu + \frac{-p_{g\perp}^2}{2z p_\gamma^+} n_2^\mu + p_{g\perp}^\mu. \tag{4.2}$$

Again, we will work with the reduced matrix element $\tilde{T}'^{\alpha}$

$$\tilde{M}'^{\alpha} = \frac{-ie_q}{\sqrt{2p_\gamma^+}} \sqrt{\frac{2}{N_c^2-1}} \frac{-i\delta(p_q^+ + p_{\bar{q}}^+ + p_g^+ - p_\gamma^+)}{(2\pi)^{D-3}\sqrt{2p_{\bar{q}}^+}\sqrt{2p_q^+}\sqrt{2p_g^+}} \tilde{T}'^{\alpha}, \tag{4.3}$$

which after subtraction of the noninteracting part can be parametrized as

$$\begin{aligned} T'^{\alpha} = g\mu^{-\epsilon} \int d^d p_{1\perp} d^d p_{2\perp} \Bigg\{ & \delta(p_{q1\perp} + p_{\bar{q}2\perp} + p_{g\gamma\perp}) \Phi_3^\alpha \frac{N_c^2-1}{N_c} [tr(U_1 U_2^\dagger) - N_c](p_{1\perp}, p_{2\perp}) \\ & + \int \frac{d^d p_{3\perp}}{(2\pi)^d} \delta(p_{q1\perp} + p_{\bar{q}2\perp} + p_{g\gamma\perp} - p_{3\perp}) \Phi_4^\alpha \\ & \times [tr(U_1 U_3^\dagger) tr(U_3 U_2^\dagger) - N_c tr(U_1 U_2^\dagger)](p_{1\perp}, p_{2\perp}, p_{3\perp}) \Bigg\}. \end{aligned} \tag{4.4}$$

There are four diagrams contribution to the matrix element T', two of them are shown in figure 3, the remaining one are obtained from them by the substitution $p_q \leftrightarrow p_{\bar{q}}$, $\bar{u}_q \leftrightarrow v_{\bar{q}}$, $p_1 \leftrightarrow p_2$ and the reversal of the order of the gamma matrices, which we will denote $(q \leftrightarrow \bar{q})$.

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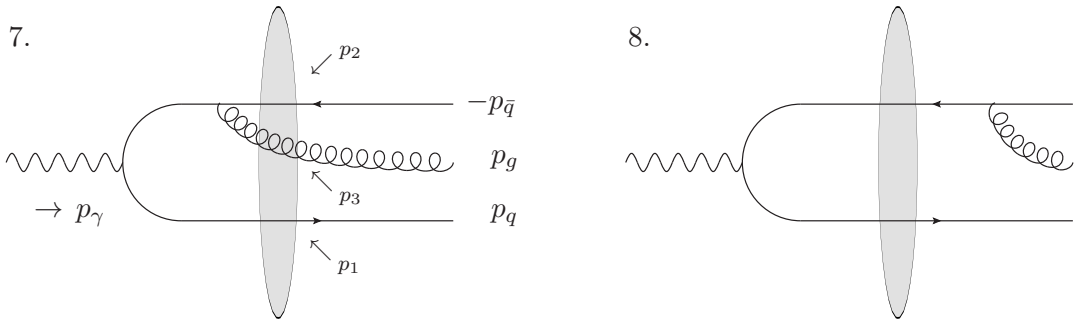

Figure 3. LO diagrams for the $\gamma^* \to q\bar{q}g$ impact factor. The momenta p_1, p_2 and p_3 go from the shockwave to the quark, antiquark and gluon.

The expressions for the impact factors in D-dimensional space with a longitudinal photon read

$$\Phi_4^+ = \frac{p_\gamma^+ \overline{u}_{p_q}[2x_q g_\perp^{\mu\nu} + z(\gamma_\perp^\nu \gamma_\perp^\mu)]\gamma^+ v_{p_{\bar{q}}} \varepsilon^*_{g\perp\mu}(z p_{q1\nu\perp} - x_q p_{g3\nu\perp})}{x_q z(x_q+z)\left(Q^2 + \frac{\vec{p}^{\,2}_{\bar{q}2}}{x_{\bar{q}}(1-\bar{x})}\right)\left(Q^2 + \frac{\vec{p}^{\,2}_{q1}}{x_q} + \frac{\vec{p}^{\,2}_{\bar{q}2}}{x_{\bar{q}}} + \frac{\vec{p}^{\,2}_{g3}}{z}\right)} - (q \leftrightarrow \bar{q})\,, \tag{4.5}$$

and

$$\Phi_3^+ = \Phi_4^+|_{p_3=0} + \left(-\frac{z p_\gamma^+ \overline{u}_{p_q} \hat{\varepsilon}^*_g (\hat{p}_q + \hat{p}_g)\gamma^+ v_{p_{\bar{q}}}}{x_q(\vec{p}_g - \frac{z}{x_q}\vec{p}_q)^2\left(Q^2 + \frac{\vec{p}^{\,2}_{\bar{q}2}}{x_{\bar{q}}(x_q+z)}\right)} - (q \leftrightarrow \bar{q}) \right). \tag{4.6}$$

For a transverse photon, they read

$$\begin{aligned}\Phi_{4\perp}^i = {} & \frac{\epsilon^*_{g\perp\mu}\bar{u}_{p_q}\gamma^+}{2x_q\bar{x}_q\,(x_q+z)\left(Q^2 + \frac{\vec{p}^{\,2}_{\bar{q}2}}{x_{\bar{q}}(1-x_{\bar{q}})}\right)\left(Q^2 + \frac{\vec{p}^{\,2}_{q1}}{x_q} + \frac{\vec{p}^{\,2}_{\bar{q}2}}{x_{\bar{q}}} + \frac{\vec{p}^{\,2}_{g3}}{z}\right)} \\ & \times \Big[x_q x_{\bar{q}} Q^2\left(\gamma_\perp^\mu \gamma_\perp^i\right) + \left(\hat{p}_{q1\perp}\gamma_\perp^\mu \gamma_\perp^i \hat{p}_{\bar{q}2\perp}\right) + 4\frac{x_q}{z}p^i_{\bar{q}2\perp}\left(x_{\bar{q}}p^\mu_{g3\perp} - z p^\mu_{\bar{q}2\perp}\right) \\ & -2\frac{x_q}{z}p^\mu_{g3\perp}\left(\gamma_\perp^i \hat{p}_{\bar{q}2\perp}\right) + 2x_q p^i_{\bar{q}2\perp}\left(\hat{p}_{\bar{q}2\perp}\gamma_\perp^\mu\right) - 2x_{\bar{q}}p^i_{\bar{q}2\perp}\left(\hat{p}_{q1\perp}\gamma_\perp^\mu\right)\Big] v_{p_{\bar{q}}} - (q \leftrightarrow \bar{q})\end{aligned} \tag{4.7}$$

and

$$\Phi_3^i = \Phi_4^i|_{p_3=0} + \left(-\frac{z\overline{u}_{p_q}\hat{\varepsilon}^*_g\left(\hat{p}_q + \hat{p}_g\right)\gamma^+\left(\gamma_\perp^i \hat{p}_{\bar{q}2\perp} - 2x_{\bar{q}}p^i_{\bar{q}2\perp}\right)v_{p_{\bar{q}}}}{2x_q x_{\bar{q}}\,(1-x_{\bar{q}})\left(\vec{p}_g - \frac{z}{x_q}\vec{p}_q\right)^2\left(Q^2 + \frac{\vec{p}^{\,2}_{\bar{q}2}}{x_{\bar{q}}(1-x_{\bar{q}})}\right)} - (q \leftrightarrow \bar{q}) \right). \tag{4.8}$$

5 Construction of the $\gamma^* P \to q\bar{q}P'$ cross section

Let us define the reduced matrix element A_3 such that the $\gamma^* P \to q\bar{q}P'$ cross section reads

$$d\sigma = \frac{1}{4s}\,(2\pi)^D\,\delta^{(D)}\left(p_\gamma + p_0 - p_q - p_{\bar{q}} - p_0'\right)|A_3|^2 d\rho_3. \tag{5.1}$$

We will need the parametrization of the proton matrix elements in the shockwave background

$$\langle P'\left(p_0'\right)|T(tr(U_{\frac{z_\perp}{2}}U^\dagger_{-\frac{z_\perp}{2}})-N_c)|P(p_0)\rangle \equiv 2\pi\delta(p^-_{00'})F_{p_{0\perp}p'_{0\perp}}(z_\perp) \equiv 2\pi\delta(p^-_{00'})F(z_\perp), \tag{5.2}$$

$$\begin{aligned}\langle P'(p_0')|T(tr(U_{\frac{z}{2}}U^\dagger_x)tr(U_xU^\dagger_{-\frac{z}{2}})-N_ctr(U_{\frac{z}{2}}U^\dagger_{-\frac{z}{2}}))|P(p_0)\rangle &\equiv 2\pi\delta(p^-_{00'})\tilde{F}_{p_{0\perp}p'_{0\perp}}(z_\perp,x_\perp)\\ &\equiv 2\pi\delta(p^-_{00'})\tilde{F}(z_\perp,x_\perp).\end{aligned} \tag{5.3}$$

We dropped the dependence on the proton transverse momenta $p_{0\perp}$ and $p'_{0\perp}$ for convenience, and we assumed the following proton state normalization:

$$\langle P'(p_0')|P(p_0)\rangle = (2\pi)^{D-1}\delta(p^-_{00'})\delta^{D-2}_\perp(p_{00'\perp})\delta_{s_Ps_{P'}} \tag{5.4}$$

The corresponding Fourier transforms read

$$\int d^dz_\perp e^{i(z_\perp\cdot p_\perp)}F(z_\perp) \equiv \mathbf{F}(p_\perp), \tag{5.5}$$

$$\int d^dz_\perp d^dx_\perp e^{i(p_\perp\cdot x_\perp)+i(z_\perp\cdot q_\perp)}\tilde{F}(z_\perp,x_\perp) \equiv \tilde{\mathbf{F}}(q_\perp,p_\perp). \tag{5.6}$$

These hadronic matrix elements naturally appear when we insert the Wilson line operators between the proton states and we extract the overall momentum conservation delta functions. The matrix element for the dipole operator reads

$$\begin{aligned}&\langle P'(p_0')|T(tr(U_1U_2^\dagger)-N_c)[p_{1\perp},p_{2\perp}]|P(p_0)\rangle \\ &=(2\pi)^d\,\delta\left(p_{1\perp}+p_{2\perp}+p_{0'0\perp}\right)\int d^dz_\perp e^{i\frac{(z_\perp\cdot p_{12\perp})}{2}}\langle P'\left(p_0'\right)|T\left(tr\left(U_{\frac{z_\perp}{2}}U^\dagger_{-\frac{z_\perp}{2}}\right)-N_c\right)|P\left(p_0\right)\rangle.\end{aligned} \tag{5.7}$$

For the double dipole operator the analogous formula has the form:

$$\begin{aligned}&\langle P'(p_0')|T(tr(U_1U_3^\dagger)tr(U_3U_2^\dagger)-N_ctr(U_1U_2^\dagger))[p_{1\perp},p_{2\perp},p_{3\perp}]|P(p_0)\rangle \\ &=(2\pi)^d\delta(p_{1\perp}+p_{2\perp}+p_{3\perp}+p_{0'0\perp}) \\ &\times\int d^dz_\perp d^dx_\perp e^{i\frac{(z_\perp\cdot p_{12\perp})}{2}+i(p_{3\perp}\cdot x_\perp)}\langle P'(p_0')|T(tr(U_{\frac{z}{2}}U^\dagger_x)tr(U_xU^\dagger_{-\frac{z}{2}})-N_ctr(U_{\frac{z}{2}}U^\dagger_{-\frac{z}{2}}))|P(p_0)\rangle.\end{aligned} \tag{5.8}$$

In our kinematics, momentum conservation reads

$$\delta^{(D)}(p_\gamma+p_0-p_q-p_{\bar{q}}-p_0') = \delta(p^-_{00'})\delta(p_q^++p_{\bar{q}}^+-p_\gamma^+)\delta^{(d)}(p_{q\perp}+p_{\bar{q}\perp}-p_{\gamma\perp}+p_{0'0\perp}), \tag{5.9}$$

with the phase space measure

$$d\rho_3 = \frac{dp_q^+d^dp_{q\perp}}{2p_q^+(2\pi)^{d+1}}\frac{dp_{\bar{q}}^+d^dp_{\bar{q}\perp}}{2p_{\bar{q}}^+(2\pi)^{d+1}}\frac{dp_0'^-d^dp'_{0\perp}}{2p_0'^-(2\pi)^{d+1}}\,. \tag{5.10}$$

The reduced matrix element A_3 includes the LO and NLO dipole contributions and the NLO double dipole contribution, as defined in section 3. It reads:

$$\begin{aligned}A_3 = &\frac{-2p_0^-e_q\varepsilon_\alpha}{\sqrt{N_c}\,(2\pi)^{D-4}}\int d^dp_{1\perp}d^dp_{2\perp} \\ &\times\left[\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{\gamma\perp})\left\{\Phi_0^\alpha+\alpha_s\frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}\frac{N_c^2-1}{N_c}\Phi_1^\alpha\right\}\mathbf{F}\left(\frac{p_{12\perp}}{2}\right)\right. \\ &\left.+\,\alpha_s\frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}\int\frac{d^dp_{3\perp}}{(2\pi)^d}\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{\gamma\perp}-p_{3\perp})\Phi_2^\alpha\tilde{\mathbf{F}}\left(\frac{p_{12\perp}}{2},p_{3\perp}\right)\right].\end{aligned} \tag{5.11}$$

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Since the photon in the initial state can appear with different polarizations, we construct the density matrix from the cross sections

$$d\sigma_{JI} = \begin{pmatrix} d\sigma_{LL} & d\sigma_{LT} \\ d\sigma_{TL} & d\sigma_{TT} \end{pmatrix}, \qquad d\sigma_{TL} = d\sigma^*_{LT}. \tag{5.12}$$

Each element of this matrix has a LO contribution $d\sigma_0$, an NLO contribution $d\sigma_1$ involving two dipole operators and an NLO contribution $d\sigma_2$ involving a dipole operator and a double-dipole operator.

$$d\sigma_{JI} = d\sigma_{0JI} + d\sigma_{1JI} + d\sigma_{2JI}. \tag{5.13}$$

The leading order cross section can be written as

$$\begin{aligned} d\sigma_{0JI} = {} & \frac{\alpha_{\mathrm{em}} Q_q^2}{(2\pi)^{4(d-1)} N_c} \frac{\left(p_0^-\right)^2}{2x\bar{x}s^2} dx d\bar{x} d^d p_{q\perp} d^d p_{\bar{q}\perp} \delta\left(1-x-\bar{x}\right) \left(\varepsilon_{I\beta}\varepsilon^*_{J\gamma}\right) \\ & \times \int d^d p_{1\perp} d^d p_{2\perp} d^d p_{1'\perp} d^d p_{2'\perp} \delta\left(p_{q1\perp} + p_{\bar{q}2\perp}\right) \delta\left(p_{11'\perp} + p_{22'\perp}\right) \\ & \times \Phi_0^{\beta}\left(p_{1\perp}, p_{2\perp}\right) \Phi_0^{\gamma *}\left(p_{1'\perp}, p_{2'\perp}\right) \mathbf{F}\left(\frac{p_{12\perp}}{2}\right) \mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right). \end{aligned} \tag{5.14}$$

The dipole $\times$ dipole NLO cross section is given by

$$\begin{aligned} d\sigma_{1JI} = {} & \alpha_s \frac{\Gamma\left(1-\epsilon\right)}{(4\pi)^{1+\epsilon}} \left(\frac{N_c^2-1}{N_c}\right) \frac{\alpha_{\mathrm{em}} Q_q^2}{(2\pi)^{4(d-1)} N_c} \frac{\left(p_0^-\right)^2}{2x\bar{x}s^2} dx d\bar{x} d^d p_{q\perp} d^d p_{\bar{q}\perp} \delta\left(1-x-\bar{x}\right) \left(\varepsilon_{I\beta}\varepsilon^*_{J\gamma}\right) \\ & \times \int d^d p_{1\perp} d^d p_{2\perp} d^d p_{1'\perp} d^d p_{2'\perp} \delta\left(p_{q1\perp} + p_{\bar{q}2\perp}\right) \delta\left(p_{11'\perp} + p_{22'\perp}\right) \mathbf{F}\left(\frac{p_{12\perp}}{2}\right) \mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right) \\ & \times \left[\Phi_1^{\beta}\left(p_{1\perp}, p_{2\perp}\right) \Phi_0^{\gamma *}\left(p_{1'\perp}, p_{2'\perp}\right) + \Phi_0^{\beta}\left(p_{1\perp}, p_{2\perp}\right) \Phi_1^{\gamma *}\left(p_{1'\perp}, p_{2'\perp}\right)\right] \end{aligned} \tag{5.15}$$

We can separate this cross section into its divergent part and its convergent part. To get the convergent part, one only has to replace Φ_1 in eq. (5.15) by Φ_{1R} from eq. (3.41) and to set ϵ to 0. The remaining divergent part reads

$$\left(d\sigma_{1JI}\right)_{div} = \alpha_s \frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}} \left(\frac{N_c^2-1}{2N_c}\right) \left(S_V + S_V^*\right) d\sigma_{0JI}. \tag{5.16}$$

Replacing Φ_2 by the contribution Φ_2' from eqs. (3.31), (3.32) which includes the BK evolution (see the discussion in section 3.2.2), one gets a non-divergent dipole $\times$ double dipole NLO contribution, which reads

$$\begin{aligned} d\sigma_{2JI} = {} & \alpha_s \frac{\Gamma\left(1-\epsilon\right)}{(4\pi)^{1+\epsilon}} \frac{\alpha_{\mathrm{em}} Q_q^2}{(2\pi)^{4(d-1)} N_c} \frac{\left(p_0^-\right)^2}{2x\bar{x}s^2} dx d\bar{x} d^d p_{q\perp} d^d p_{\bar{q}\perp} \delta\left(1-x-\bar{x}\right) \left(\varepsilon_{I\beta}\varepsilon^*_{J\gamma}\right) \\ & \times \int d^d p_{1\perp} d^d p_{2\perp} d^d p_{1'\perp} d^d p_{2'\perp} \frac{d^d p_{3\perp} d^d p_{3'\perp}}{(2\pi)^d} \delta\left(p_{q1\perp} + p_{\bar{q}2\perp} - p_{3\perp}\right) \delta\left(p_{11'\perp} + p_{22'\perp} + p_{33'\perp}\right) \\ & \times \left[\Phi_2'^{\beta}\left(p_{1\perp}, p_{2\perp}, p_{3\perp}\right) \Phi_0^{\gamma *}\left(p_{1'\perp}, p_{2'\perp}\right) \mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right) \tilde{\mathbf{F}}\left(\frac{p_{12\perp}}{2}, p_{3\perp}\right) \delta\left(p_{3'\perp}\right) \right. \\ & \left. + \Phi_2'^{\gamma *}\left(p_{1'\perp}, p_{2'\perp}, p_{3'\perp}\right) \Phi_0^{\beta}\left(p_{1\perp}, p_{2\perp}\right) \mathbf{F}\left(\frac{p_{12\perp}}{2}\right) \tilde{\mathbf{F}}^*\left(\frac{p_{1'2'\perp}}{2}, p_{3'\perp}\right) \delta\left(p_{3\perp}\right)\right]. \end{aligned} \tag{5.17}$$

This expression is now finite so one can set $\epsilon = 0$.

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5.1 Results for the Born cross section

Using (3.7) and (3.8) and summing over helicities of the quark and the antiquark, one gets

$$\sum_{\text{helicities}} \Phi_0^+(p_{1\perp},p_{2\perp})\Phi_0^+(p'_{1\perp},p'_{2\perp}) = \frac{32(p_\gamma^+)^4x^3\bar{x}^3}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{q1'}^{\,2}+x\bar{x}Q^2)}, \tag{5.18}$$

$$\sum_{\text{helicities}} \Phi_0^+(p_{1\perp},p_{2\perp})\Phi_0^i(p'_{1\perp},p_{2'\perp})^* = \frac{16(p_\gamma^+)^3x^2\bar{x}^2p^i_{q1'\perp}(1-2x)}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{q1'}^{\,2}+x\bar{x}Q^2)}, \tag{5.19}$$

and

$$\sum_{\text{helicities}} \Phi_0^i(p_{1\perp},p_{2\perp})\Phi_0^k(p'_{1\perp},p_{2'\perp})^* = \frac{8(p_\gamma^+)^2x\bar{x}[(1-2x)^2g_\perp^{ri}g_\perp^{lk}-g_\perp^{rk}g_\perp^{li}+g_\perp^{rl}g_\perp^{ik}]p_{q1\perp r}p_{q1'\perp l}}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)(\vec{p}_{q1'}^{\,2}+x\bar{x}Q^2)}. \tag{5.20}$$

As a result, the LO density matrix elements read

$$d\sigma_{0LL} = \frac{4\alpha_{\rm em}Q_q^2}{(2\pi)^4N_c}dxd\bar{x}d^2p_{q\perp}d^2p_{\bar{q}\perp}\delta\left(1-x-\bar{x}\right)x^2\bar{x}^2Q^2\times\left|\int\frac{d^2p_{1\perp}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\mathbf{F}\left(p_{1q\perp}+\frac{p_{q\bar{q}\perp}}{2}\right)\right|^2, \tag{5.21}$$

$$\begin{aligned} d\sigma_{0TL} = {}& \frac{2\alpha_{\rm em}Q_q^2}{(2\pi)^4N_c}dxd\bar{x}d^2p_{q\perp}d^2p_{\bar{q}\perp}\delta(1-x-\bar{x})x\bar{x}(1-2x)Q \\ & \times\left[\int\frac{d^2p_{1\perp}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\mathbf{F}\left(p_{1q\perp}+\frac{p_{q\bar{q}\perp}}{2}\right)\right]\left[\int\frac{d^2p'_{1\perp}(\varepsilon_\perp\cdot p_{q1'\perp})}{\vec{p}_{q1'}^{\,2}+x\bar{x}Q^2}\mathbf{F}\left(p_{1'q\perp}+\frac{p_{q\bar{q}\perp}}{2}\right)\right]^*, \end{aligned} \tag{5.22}$$

and

$$\begin{aligned} d\sigma_{0TT} = {}& \frac{\alpha_{\rm em}Q_q^2}{(2\pi)^4N_c}dxd\bar{x}d^2p_{q\perp}d^2p_{\bar{q}\perp}\delta(1-x-\bar{x})[(1-2x)^2g_\perp^{ki}g_\perp^{lj}-g_\perp^{kj}g_\perp^{li}+g_\perp^{kl}g_\perp^{ij}] \\ & \times\left[\int\frac{d^2p_{1\perp}(\varepsilon_{\perp i}p_{q1\perp k})}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\mathbf{F}\left(p_{1q\perp}+\frac{p_{q\bar{q}\perp}}{2}\right)\right]\left[\int\frac{d^2p'_{1\perp}(\varepsilon_{\perp j}p_{q1'\perp l})}{\vec{p}_{q1'}^{\,2}+x\bar{x}Q^2}\mathbf{F}\left(p_{1'q\perp}+\frac{p_{q\bar{q}\perp}}{2}\right)\right]^*. \end{aligned} \tag{5.23}$$

5.2 Dipole - dipole NLO cross section $d\sigma_1$

5.2.1 LL photon transition

Combining (5.15), (3.41) and (3.7), and summing over the polarization components $\varepsilon^+\Phi_0^- + \varepsilon^-\Phi_0^+$ with the help of the gauge invariance relation $\Phi_0^- = \frac{Q^2}{2(p_\gamma^+)^2}\Phi_0^+$, we get

$$\begin{aligned} d\sigma_{1LL} = {}& \alpha_s\frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}\left(\frac{N_c^2-1}{2N_c}\right)(S_V+S_V^*)d\sigma_{0LL} \\ & +\frac{\alpha_sQ^2}{4\pi}\left(\frac{N_c^2-1}{N_c}\right)\frac{\alpha_{\rm em}Q_q^2}{(2\pi)^4N_c}dxd\bar{x}d^2p_{q\perp}d^2p_{\bar{q}\perp}\delta(1-x-\bar{x}) \\ & \times\int d^2p_{1\perp}d^2p_{2\perp}d^2p'_{1\perp}d^2p'_{2\perp}\delta(p_{q1\perp}+p_{\bar{q}2\perp})\frac{\delta(p_{11'\perp}+p_{22'\perp})}{\vec{p}_{q1'}^{\,2}+x\bar{x}Q^2}\mathbf{F}\left(\frac{p_{12\perp}}{2}\right)\mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right) \\ & \times\left[\frac{6x^2\bar{x}^2}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\ln\left(\frac{x^2\bar{x}^2\mu^4Q^2}{(x\vec{p}_{\bar{q}}-\bar{x}\vec{p}_q)^2(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)^2}\right)\right. \\ & \left.+\frac{(p_0^-)^2}{s^2p_\gamma^+}tr((C_\parallel^4+C_{1\parallel}^5+C_{1\parallel}^6)\hat{p}_{\bar{q}}\gamma^+\hat{p}_q)\right]+h.c. \end{aligned} \tag{5.24}$$

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We will parametrize the finite contribution of the C functions as:

$$\frac{(p_0^-)^2}{s^2 p_\gamma^+} tr(C^4_{||}\hat{p}_{\bar{q}}\gamma^+\hat{p}_q) = \int_0^x dz\,[(\phi_4)_{LL}]_+ + (q \leftrightarrow \bar{q})\,, \tag{5.25}$$

and

$$\frac{(p_0^-)^2}{s^2 p_\gamma^+} tr(C^n_{1||}\hat{p}_{\bar{q}}\gamma^+\hat{p}_q) = \int_0^x dz\,[(\phi_n)_{LL}]_+ |_{\vec{p}_3=\vec{0}} + (q \leftrightarrow \bar{q})\,, \tag{5.26}$$

where $n = 5$ or 6. The expressions for $(\phi_n)_{LL}$ are given in appendix A.

However for $n = 6$ the integral can be performed analytically to the end, so that:

$$\begin{aligned}\frac{(p_0^-)^2}{s^2 p_\gamma^+} tr(C^6_{1||}\hat{p}_{\bar{q}}\gamma^+\hat{p}_q) = {} & \frac{-2x^2\bar{x}^3\vec{p}_1^{\,2}}{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2 - \bar{x}\vec{p}_1^{\,2})^2}\ln\left(\frac{x\bar{x}Q^2 + \vec{p}_{q1}^{\,2}}{\bar{x}\vec{p}_1^{\,2}}\right) \\ & + \frac{2x^2\bar{x}^2}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\left[4\,\mathrm{Li}_2\left(\frac{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2 - \bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\right) + 3\ln\left(\frac{\vec{p}_1^{\,2}}{\mu^2}\right) - 8\right] \\ & + \frac{2x^2\bar{x}^2}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2 - \bar{x}\vec{p}_1^{\,2}}\left[3\ln\left(\frac{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}{\bar{x}\vec{p}_1^{\,2}}\right) + 1\right] + (q \leftrightarrow \bar{q}).\end{aligned} \tag{5.27}$$

5.2.2 LT photon transition

Using the same method as for the LL component, we get

$$\begin{aligned} d\sigma_{1TL} = {} & \alpha_s\frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}\left(\frac{N_c^2-1}{2N_c}\right)(S_V + S_V^*)\,d\sigma_{0TL} \\ & + \frac{\alpha_s Q}{(4\pi)}\left(\frac{N_c^2-1}{N_c}\right)\frac{\alpha_{\rm em}Q_q^2}{(2\pi)^4 N_c}dxd\bar{x}d^2p_{q\perp}d^2p_{\bar{q}\perp}\delta(1-x-\bar{x})\varepsilon^*_{Ti} \\ & \times\int d^2p_{1\perp}d^2p_{2\perp}d^2p'_{1\perp}d^2p_{2'\perp}\delta(p_{q1\perp}+p_{\bar{q}2\perp})\delta(p_{11'\perp}+p_{22'\perp})\mathbf{F}\left(\frac{p_{12\perp}}{2}\right)\mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right) \\ & \times\left[\frac{(p_0^-)^2}{s^2}\frac{tr((C'^{4i}_\perp + C'^{5i}_{1\perp} + C'^{6i}_{1\perp})\hat{p}_{\bar{q}}\gamma^+\hat{p}_q)^\dagger}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} + \frac{3x\bar{x}(1-2x)p^i_{q1'\perp}}{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2)(\vec{p}_{q1'}^{\,2} + x\bar{x}Q^2)}\right. \\ & \quad\times\left(\ln\left(\frac{x^3\bar{x}^3\mu^8Q^2(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)^{-4}}{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2)^2(\vec{p}_{q1'}^{\,2} + x\bar{x}Q^2)}\right) - \frac{x\bar{x}Q^2}{\vec{p}_{q1'}^{\,2}}\ln\left(\frac{x\bar{x}Q^2}{\vec{p}_{q1'}^{\,2} + x\bar{x}Q^2}\right)\right) \\ & \quad\left. + \frac{(p_0^-)^2}{s^2}\frac{tr((C^4_\parallel + C^5_{1\parallel} + C^6_{1\parallel})\hat{p}_{\bar{q}}(\gamma^i\hat{p}_{q1'\perp} - 2xp^i_{q1'\perp})\gamma^+\hat{p}_q)}{2p_\gamma^+ x\bar{x}\left(\vec{p}_{q1'}^{\,2} + x\bar{x}Q^2\right)}\right]. \end{aligned} \tag{5.28}$$

Once more we will parametrize the contributions from the C functions, as

$$\frac{(p_0^-)^2 tr(C^4_{||}\hat{p}_{\bar{q}}((1-2x)p^i_{q1'\perp} - \frac{1}{2}[\hat{p}_{q1'\perp},\gamma^i_\perp])\gamma^+\hat{p}_q)}{s^2 p_\gamma^+} = \int_0^x dz[(\phi_4)^i_{LT}]_+ + (q \leftrightarrow \bar{q})\,, \tag{5.29}$$

$$\frac{(p_0^-)^2 tr(C^n_{1||}\hat{p}_{\bar{q}}((1-2x)p^i_{q1'\perp} - \frac{1}{2}[\hat{p}_{q1'\perp},\gamma^i_\perp])\gamma^+\hat{p}_q)}{s^2 p_\gamma^+} = \int_0^x dz[(\phi_n)^i_{LT}]_+|_{\vec{p}_3=\vec{0}} + (q \leftrightarrow \bar{q})\,, \tag{5.30}$$

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and

$$\frac{(p_0^-)^2}{s^2} tr(C_\perp^{4i} \hat{p}_{\bar{q}} \gamma^+ \hat{p}_q) = \int_0^x dz [(\phi_4)^i_{TL}]_+ + (q \leftrightarrow \bar{q}) \,, \tag{5.31}$$

$$\frac{(p_0^-)^2}{s^2} tr(C_{1\perp}^{ni} \hat{p}_{\bar{q}} \gamma^+ \hat{p}_q) = \int_0^x dz [(\phi_n)^i_{TL}]_+|_{\vec{p}_3=\vec{0}} + (q \leftrightarrow \bar{q}) \,, \tag{5.32}$$

with $n = 5, 6$. The values for (ϕ_n) are given in appendix A.

However, the contribution from diagram 6 can be computed to the end:

$$\frac{(p_0^-)^2 tr(C_{1||}^6 \hat{p}_{\bar{q}}((1-2x)p^i_{q1'\perp} - \frac{1}{2}[\hat{p}_{q1'\perp}, \gamma^i_\perp])\gamma^+ \hat{p}_q)}{s^2 p_\gamma^+} = (1-2x) p^i_{q1'\perp} \frac{(p_0^-)^2}{s^2 p_\gamma^+} tr(C_{1||}^6 \hat{p}_{\bar{q}} \gamma^+ \hat{p}_q), \tag{5.33}$$

and

$$\begin{aligned}
\frac{(p_0^-)^2}{s^2} tr(C_{1\perp}^{6i} \hat{p}_{\bar{q}} \gamma^+ \hat{p}_q) = {} & \frac{-x\bar{x}(1-2x) p^i_{q1\perp}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2 - \bar{x}\vec{p}_1^{\,2}} \ln\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\right) \\
& + \frac{x\bar{x}(1-2x) p^i_{q1\perp}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} \left[4\mathrm{Li}_2\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} + 1\right) + 3\ln\left(\frac{\vec{p}_1^{\,2}}{\mu^2}\right) - 8\right] \\
& + \frac{-x\bar{x} p^i_{1\perp}}{3\vec{p}_1^{\,2}} \left[\pi^2 - 6\mathrm{Li}_2\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2} + 1\right)\right] + x\bar{x}\left(x p^i_{q1\perp} - \bar{x} p^i_{q\perp}\right) \\
& \times \left(\frac{-\bar{x}\vec{p}_1^{\,2}}{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2 - \bar{x}\vec{p}_1^{\,2})^2} \ln\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\right)\right. \\
& \left. + \frac{1}{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2 - \bar{x}\vec{p}_1^{\,2})} \left[2\ln\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\right) - 1\right]\right) + (q \leftrightarrow \bar{q}).
\end{aligned} \tag{5.34}$$

5.2.3 TT photon transition

The cross section for the TT transition reads

$$\begin{aligned}
d\sigma_{1TT} = {} & \alpha_s \frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}} \left(\frac{N_c^2 - 1}{2N_c}\right) (S_V + S_V^*)\, d\sigma_{0TT} \\
& + \frac{\alpha_s}{4\pi} \left(\frac{N_c^2 - 1}{N_c}\right) \frac{\alpha_{\mathrm{em}} Q_q^2}{(2\pi)^4 N_c} dx d\bar{x} d^2 p_{q\perp} d^2 p_{\bar{q}\perp} \delta(1 - x - \bar{x}) (\varepsilon_{Ti} \varepsilon^*_{Tk}) \\
& \times \int d^2 p_{1\perp} d^2 p_{2\perp} d^2 p'_{1\perp} d^2 p'_{2\perp} \delta(p_{q1\perp} + p_{\bar{q}2\perp}) \delta(p_{11'\perp} + p_{22'\perp}) \mathbf{F}\left(\frac{p_{12\perp}}{2}\right) \mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right) \\
& \times \left\{ \frac{3}{2} \frac{p_{q1\perp r} p_{q1'\perp l}}{(\vec{p}_{q1}^{\,2} + x\bar{x}Q^2)(\vec{p}_{q1'}^{\,2} + x\bar{x}Q^2)} [(1-2x)^2 g_\perp^{ri} g_\perp^{lk} - g_\perp^{rk} g_\perp^{li} + g_\perp^{rl} g_\perp^{ik}] \right. \\
& \times \left[\ln\left(\frac{x\bar{x}\mu^4}{(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)^2 (\vec{p}_{q1}^{\,2} + x\bar{x}Q^2)}\right) - \frac{x\bar{x}Q^2}{\vec{p}_{q1}^{\,2}} \ln\left(\frac{x\bar{x}Q^2}{\vec{p}_{q1}^{\,2} + x\bar{x}Q^2}\right)\right] \\
& \left. + \frac{(p_0^-)^2}{2s^2 x\bar{x}} \frac{tr[(C_\perp^{4i} + C_{1\perp}^{5i} + C_{1\perp}^{6i}) \hat{p}_{\bar{q}} (p^k_{q1'\perp}(1-2x) - \frac{1}{2}[\hat{p}_{q1'\perp}, \gamma^k]\gamma^+ \hat{p}_q]}{\vec{p}_{q1'}^{\,2} + x\bar{x}Q^2} + h.c.|_{\substack{p_1 \leftrightarrow p'_1 \\ i \leftrightarrow k}} \right\}.
\end{aligned} \tag{5.35}$$

The C functions are given by:

$$\frac{(p_0^-)^2}{2s^2} tr\left(C_\perp^{4i} \hat{p}_{\bar{q}} \left(p^j_{q1'\perp}(1-2x) - \frac{1}{2}\left[\hat{p}_{q1'\perp}, \gamma^j_\perp\right]\right) \gamma^+ \hat{p}_q\right) = \int_0^x dz \left[(\phi_4^{ij})_{TT}\right]_+ + (q \leftrightarrow \bar{q}) \,, \tag{5.36}$$

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and

$$\frac{\left(p_0^-\right)^2}{2s^2} tr\left(C_{1\perp}^{ni}\hat{p}_{\bar{q}}\left(p_{q1'\perp}^j\left(1-2x\right)-\frac{1}{2}\left[\hat{p}_{q1'\perp},\gamma_\perp^j\right]\right)\gamma^+\hat{p}_q\right)=\int_0^x dz\left[(\phi_n^{ij})_{TT}\right]_+|_{\vec{p}_3=\vec{0}}+(q\leftrightarrow\bar{q}),\tag{5.37}$$

with $n=5\,,\,6$. The values for ϕ_n are given in appendix A. The contribution from diagram 6 finally reads:

$$\begin{aligned}
&\frac{\left(p_0^-\right)^2}{s^2} tr\left(C_{1\perp}^{6i}\hat{p}_{\bar{q}}\left(p_{q1'\perp}^k\left(1-2x\right)-\frac{1}{2}\left[\hat{p}_{q1'\perp},\gamma_\perp^k\right]\right)\gamma^+\hat{p}_q\right)\\
&\quad= x\bar{x}\left[g_\perp^{ik}(\vec{p}_1\cdot\vec{p}_{q1'})+p_{1\perp}^k p_{q1'\perp}^i+(2x-1)p_{1\perp}^i p_{q1'\perp}^k\right]\\
&\qquad\times\left[\frac{-\bar{x}^2\vec{p}_1^{\,2}}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2-\bar{x}\vec{p}_1^{\,2})^2}\ln\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\right)\right.\\
&\qquad\quad+\frac{\bar{x}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2-\bar{x}\vec{p}_1^{\,2}}\left[2\ln\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\right)-1\right]\\
&\qquad\quad\left.+\frac{2}{\vec{p}_1^{\,2}}\left(\frac{\pi^2}{6}-\mathrm{Li}_2\left(\frac{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2-\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\right)\right)\right]\\
&\qquad-x\bar{x}\left[p_{q1\perp}^i p_{q1'\perp}^k(1-2x)^2-g_\perp^{ik}(\vec{p}_{q1}\cdot\vec{p}_{q1'})-p_{q1\perp}^k p_{q1'\perp}^i\right]\\
&\qquad\times\left[\frac{-1}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2-\bar{x}\vec{p}_1^{\,2}}\left(1-3\ln\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\right)\right)\right.\\
&\qquad\quad-\frac{1}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\left(3\ln\left(\frac{\vec{p}_1^{\,2}}{\mu^2}\right)+4\mathrm{Li}_2\left(\frac{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2-\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\right)-8\right)\\
&\qquad\quad\left.+\frac{-\bar{x}\vec{p}_1^{\,2}}{(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2-\bar{x}\vec{p}_1^{\,2})^2}\ln\left(\frac{\bar{x}\vec{p}_1^{\,2}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\right)\right]+(q\leftrightarrow\bar{q})\,.
\end{aligned}\tag{5.38}$$

5.3 Dipole-double dipole cross section $d\sigma_2$

5.3.1 LL photon transition

$$\begin{aligned}
d\sigma_{2LL}=&\frac{\alpha_s Q^2}{4\pi}\frac{\alpha_{\mathrm{em}}Q_q^2}{(2\pi)^4 N_c}dxd\bar{x}d^2p_{q\perp}d^2p_{\bar{q}\perp}\delta(1-x-\bar{x})\\
&\times\int d^2p_{1\perp}d^2p_{2\perp}d^2p'_{1\perp}d^2p'_{2\perp}\int\frac{d^2p_{3\perp}}{(2\pi)^2}\delta(p_{q1\perp}+p_{\bar{q}2\perp}-p_{3\perp})\delta(p_{11'\perp}+p_{22'\perp}+p_{3\perp})\\
&\times\frac{1}{\vec{p}_{q1'}^{\,2}+x\bar{x}Q^2}\tilde{\mathbf{F}}\left(\frac{p_{12\perp}}{2},p_{3\perp}\right)\int\mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right)\\
&\times\left(4x\bar{x}\left\{\frac{x\bar{x}(\vec{p}_3^{\,2}-\vec{p}_{\bar{q}2}^{\,2}-\vec{p}_{q1}^{\,2}-2x\bar{x}Q^2)}{(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)-x\bar{x}Q^2\vec{p}_3^{\,2}}\right.\right.\\
&\qquad\times\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\ln\left(\frac{(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2)\,(\vec{p}_{q1}^{\,2}+x\bar{x}Q^2)}{x\bar{x}Q^2\vec{p}_3^{\,2}}\right)\\
&\qquad\left.\left.-\left(\frac{2x\bar{x}}{\vec{p}_{q1}^{\,2}+x\bar{x}Q^2}\ln\left(\frac{x}{e^{\eta}}\right)\ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)+(q\leftrightarrow\bar{q})\right)\right\}\right.\\
&\quad\left.+\frac{Q^2(p_0^-)^2}{p_\gamma^+ s^2}tr((C_{2\parallel}^5+C_{2\parallel}^6)\hat{p}_{\bar{q}}\gamma^+\hat{p}_q)\right)+h.c.
\end{aligned}\tag{5.39}$$

We will write

$$\frac{(p_0^-)^2}{s^2 p_\gamma^+} tr(C^n_{2\|}\hat{p}_{\bar{q}}\gamma^+\hat{p}_q) = \int_0^x dz\,[(\phi_n)_{LL}]_+ + (q \leftrightarrow \bar{q})\ , \tag{5.40}$$

with $n = 5, 6$. The values for ϕ_n are given in appendix A.

5.3.2 LT photon transition

$$\begin{aligned}
d\sigma_{2TL} = {} & \frac{\alpha_s Q}{4\pi}\frac{\alpha_{\rm em}Q_q^2}{(2\pi)^4 N_c} dx d\bar{x} d^2p_{q\perp} d^2p_{\bar{q}\perp}\delta(1-x-\bar{x})\int d^2p_{1\perp}d^2p_{2\perp}d^2p'_{1\perp}d^2p'_{2\perp} \\
& \times \int \frac{d^2p_{3\perp}d^2p'_{3\perp}}{(2\pi)^2}\delta(p_{q1\perp}+p_{\bar{q}2\perp}+p_{g3\perp})\delta(p_{11'\perp}+p_{22'\perp}+p_{33'\perp}) \\
& \times \varepsilon^*_{Ti}\left[\frac{\delta(p'_{3\perp})}{\vec{p}^{\,2}_{q1'}+x\bar{x}Q^2}\tilde{\mathbf{F}}\left(\frac{p_{12\perp}}{2},p_{3\perp}\right)\mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right)\right. \\
& \times \left(2(1-2x)p^i_{q1'\perp}\left\{\frac{x\bar{x}(\vec{p}^{\,2}_3-\vec{p}^{\,2}_{\bar{q}2}-\vec{p}^{\,2}_{q1}-2x\bar{x}Q^2)}{(\vec{p}^{\,2}_{\bar{q}2}+x\bar{x}Q^2)(\vec{p}^{\,2}_{q1}+x\bar{x}Q^2)-x\bar{x}Q^2\vec{p}^{\,2}_3}\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\right.\right. \\
& \times \ln\left(\frac{(\vec{p}^{\,2}_{\bar{q}2}+x\bar{x}Q^2)\,(\vec{p}^{\,2}_{q1}+x\bar{x}Q^2)}{x\bar{x}Q^2\vec{p}^{\,2}_3}\right) \\
& \left.\left. - \left(\frac{2x\bar{x}}{\vec{p}^{\,2}_{q1}+x\bar{x}Q^2}\ln\left(\frac{x}{e^\eta}\right)\ln\left(\frac{\vec{p}^{\,2}_3}{\mu^2}\right)+(q\leftrightarrow\bar{q})\right)\right\} \\
& + \frac{(p_0^-)^2}{2s^2x\bar{x}p_\gamma^+}tr[(C^5_{2\|}+C^6_{2\|})\hat{p}_{\bar{q}}(p^i_{q1'\perp}(1-2x)-\frac{1}{2}[\hat{p}_{q1'\perp}\,,\gamma^i])\gamma^+\hat{p}_q]\Big) \\
& + \frac{\delta(p_{3\perp})}{\vec{p}^{\,2}_{q1}+x\bar{x}Q^2}\mathbf{F}\left(\frac{p_{12\perp}}{2}\right)\tilde{\mathbf{F}}^*\left(\frac{p_{1'2'\perp}}{2},p_{3'\perp}\right) \\
& \times \left(\left\{2x\bar{x}(1-2x)p^i_{q1'\perp}\left(\frac{-2}{\vec{p}^{\,2}_{q1'}+x\bar{x}Q^2}\ln\left(\frac{x}{e^\eta}\right)\ln\left(\frac{\vec{p}^{\,\prime 2}_3}{\mu^2}\right)\right.\right.\right. \\
& + \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\left[-\frac{\vec{p}^{\,2}_{\bar{q}2'}+x\bar{x}Q^2}{(\vec{p}^{\,2}_{q1'}+x\bar{x}Q^2)(\vec{p}^{\,2}_{\bar{q}2'}+x\bar{x}Q^2)-x\bar{x}Q^2\vec{p}^{\,\prime 2}_3}\right. \\
& \times \ln\left(\frac{(\vec{p}^{\,2}_{q1'}+x\bar{x}Q^2)(\vec{p}^{\,2}_{\bar{q}2'}+x\bar{x}Q^2)}{x\bar{x}Q^2\vec{p}^{\,\prime 2}_3}\right) \\
& \left.\left.\left. + \frac{1}{\vec{p}^{\,2}_{q1'}}\ln\left(\frac{\vec{p}^{\,2}_{q1'}+x\bar{x}Q^2}{x\bar{x}Q^2}\right)\right]\right)+(q\leftrightarrow\bar{q})\right\} + \frac{(p_0^-)^2}{s^2}tr((C'^{5i}_{2\perp}+C'^{6i}_{2\perp})\hat{p}_{\bar{q}}\gamma^+\hat{p}_q)^*\Big)\Big]\,.
\end{aligned} \tag{5.41}$$

Again we will write

$$\frac{(p_0^-)^2}{s^2}tr(C^{ni}_{2\perp}\hat{p}_{\bar{q}}\gamma^+\hat{p}_q) = \int_0^x dz\left[(\phi^i_n)_{LT}\right]_+ dz + (q\leftrightarrow\bar{q})\ , \tag{5.42}$$

and

$$\frac{\left(p_0^-\right)^2}{s^2p_\gamma^+}tr\left(C^n_{2\|}\hat{p}_{\bar{q}}\left(p^i_{q1'\perp}\,(1-2x)-\frac{1}{2}\left[\hat{p}_{q1'\perp},\gamma^i_\perp\right]\right)\gamma^+\hat{p}_q\right) = \int_0^x dz\left[\left(\phi^i_n\right)_{TL}\right]_+ + (q\leftrightarrow\bar{q})\,. \tag{5.43}$$

The values for $(\phi_{5,6})_{LT}$ and $(\phi_{5,6})_{TL}$ are given in appendix A.

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5.3.3 TT photon transition

$$
\begin{aligned}
d\sigma_{2TT} = {} & \frac{\alpha_s}{4\pi}\frac{\alpha_{\rm em}Q_q^2}{(2\pi)^4 N_c} dx d\bar{x} d^2p_{q\perp} d^2p_{\bar{q}\perp} \delta(1-x-\bar{x}) \int d^2p_{1\perp} d^2p_{2\perp} d^2p'_{1\perp} d^2p'_{2\perp} \int \frac{d^2p_{3\perp}}{(2\pi)^d} \\
& \times \frac{(\varepsilon_{Ti}\varepsilon^*_{Tj})}{\vec{p}^{\,2}_{q1'} + x\bar{x}Q^2} \left[\tilde{\mathbf{F}}\left(\frac{p_{12\perp}}{2}, p_{3\perp}\right) \mathbf{F}^*\left(\frac{p_{1'2'\perp}}{2}\right) \delta(p_{q1\perp} + p_{\bar{q}2\perp} - p_{3\perp}) \delta(p_{11'\perp} + p_{22'\perp} + p_{3\perp}) \right. \\
& \times \left(\left\{ p_{q1'\perp l} p_{q1\perp k} [(1-2x)^2 g_\perp^{ki} g_\perp^{lj} - g_\perp^{kj} g_\perp^{li} + g_\perp^{kl} g_\perp^{ij}] \left(\frac{-2}{\vec{p}^{\,2}_{q1} + x\bar{x}Q^2} \ln\left(\frac{x}{e^\eta}\right) \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right) \right.\right.\right. \\
& + \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\vec{p}^{\,2}_{q1}} \ln\left(\frac{\vec{p}^{\,2}_{q1} + x\bar{x}Q^2}{x\bar{x}Q^2} \right) - \frac{\vec{p}^{\,2}_{\bar{q}2} + x\bar{x}Q^2}{(\vec{p}^{\,2}_{q1} + x\bar{x}Q^2)(\vec{p}^{\,2}_{\bar{q}2} + x\bar{x}Q^2) - x\bar{x}Q^2\vec{p}_3^{\,2}} \right. \\
& \left.\left.\left.\left. \times \ln\left(\frac{(\vec{p}^{\,2}_{q1} + x\bar{x}Q^2)(\vec{p}^{\,2}_{\bar{q}2} + x\bar{x}Q^2)}{x\bar{x}Q^2\vec{p}_3^{\,2}} \right) \right] \right) + (q \leftrightarrow \bar{q}) \right\} \\
& \left. + \frac{(p_0^-)^2}{2s^2 x\bar{x}} tr\left(\left(C_{2\perp}^{5i} + C_{2\perp}^{6i}\right) \hat{p}_{\bar{q}} \left[p^j_{q1'\perp}(1-2x) - \frac{1}{2}\left[\hat{p}_{q1'\perp}, \gamma^j\right] \right) \gamma^+ \hat{p}_q \right] \right) + h.c.|_{\substack{p_1 p_3 \leftrightarrow p'_1 p'_3 \\ i \leftrightarrow j}} \Bigg] .
\end{aligned}
\tag{5.44}
$$

As for the other contributions, we will write

$$
\frac{(p_0^-)^2}{2s^2} tr\left(C_{2\perp}^{ni} \hat{p}_{\bar{q}} \left(p^j_{q1'\perp}(1-2x) - \frac{1}{2}\left[\hat{p}_{q1'\perp}, \gamma^j_\perp\right] \right) \gamma^+ \hat{p}_q \right) = \int_0^x dz \left[(\phi_n^{ij})_{TT} \right]_+ dz + (q \leftrightarrow \bar{q}) \,. \tag{5.45}
$$

The expressions of $(\phi_5^{ij})_{TT}$ and $(\phi_6^{ij})_{TT}$ can be found in appendix A.

6 Cross section for the $\gamma^* P \to q\bar{q}gP'$ transition

As in section 5 we define a reduced matrix element A_4 such that the $\gamma^* P \to q\bar{q}gP'$ cross section reads

$$
d\sigma_{(q\bar{q}g)} = \frac{1}{4s} (2\pi)^D \delta^{(D)}\left(p_\gamma + p_0 - p_q - p_{\bar{q}} - p_g - p'_0\right) |A_4|^2 d\rho_4 \,, \tag{6.1}
$$

where

$$
\begin{aligned}
\delta^{(D)}\left(p_\gamma + p_0 - p_q - p_{\bar{q}} - p_g - p'_0\right) = {} & \delta\left(p^-_{00'}\right) \delta\left(p_q^+ + p_{\bar{q}}^+ + p_g^+ - p_\gamma^+\right) \\
& \times \delta^{(d)}\left(p_{q\perp} + p_{\bar{q}\perp} + p_{g\perp} - p_{\gamma\perp} + p_{0'0\perp}\right) ,
\end{aligned}
\tag{6.2}
$$

with the 4-body phase space measure

$$
d\rho_4 = \frac{dp_q^+ d^d p_{q\perp}}{2p_q^+ (2\pi)^{d+1}} \frac{dp_{\bar{q}}^+ d^d p_{\bar{q}\perp}}{2p_{\bar{q}}^+ (2\pi)^{d+1}} \frac{dp_g^+ d^d p_{g\perp}}{2p_g^+ (2\pi)^{d+1}} \frac{dp_0'^- d^d p'_{0\perp}}{2p_0'^- (2\pi)^{d+1}} \,. \tag{6.3}
$$

The reduced matrix element can be derived from section 4 and reads

$$
\begin{aligned}
A_4 = {} & \frac{-e_q 2p_0^- \varepsilon_\alpha}{(2\pi)^{D-4}} \sqrt{\frac{2}{N_c^2 - 1}} g\mu^{-\epsilon} \int d^d p_{1\perp} d^d p_{2\perp} \left\{ \delta\left(p_{q1\perp} + p_{\bar{q}2\perp} + p_{g\gamma\perp}\right) \Phi_3^\alpha \frac{N_c^2 - 1}{N_c} \mathbf{F}\left(\frac{p_{12\perp}}{2}\right) \right. \\
& \left. + \int \frac{d^d p_{3\perp}}{(2\pi)^d} \delta(p_{q1\perp} + p_{\bar{q}2\perp} + p_{g\gamma\perp} - p_{3\perp}) \Phi_4^\alpha \tilde{\mathbf{F}}\left(\frac{p_{12\perp}}{2}, p_{3\perp}\right) \right\} .
\end{aligned}
\tag{6.4}
$$

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This cross section has a contribution $d\sigma_3$ with 2 dipole operators, a contribution $d\sigma_4$ with a dipole operator and a double dipole operator, and a contribution $d\sigma_5$ with 2 double dipole operators,

$$d\sigma_{(q\bar{q}g)} = d\sigma_3 + d\sigma_4 + d\sigma_5. \tag{6.5}$$

The dipole $\times$ dipole contribution reads

$$\begin{aligned} d\sigma_{3JI} = {} & \frac{\alpha_s}{\mu^{2\epsilon}} \left(\frac{N_c^2 - 1}{N_c} \right) \frac{\alpha_{\rm em} Q_q^2}{(2\pi)^{4(d-1)} N_c} \frac{(p_0^-)^2}{s^2 x_q x_{\bar{q}}} (\varepsilon_{I\alpha} \varepsilon^*_{J\beta}) \\ & \times dx_q \, dx_{\bar{q}} \, d^d p_{q\perp} \, d^d p_{\bar{q}\perp} \frac{dz d^d p_{g\perp}}{z(2\pi)^d} \delta(1 - x_q - x_{\bar{q}} - z) \\ & \times \int d^d p_{1\perp} d^d p_{2\perp} d^d p'_{1\perp} d^d p'_{2\perp} \delta(p_{q1\perp} + p_{\bar{q}2\perp} + p_{g\perp}) \delta(p_{11'\perp} + p_{22'\perp}) \\ & \times \Phi_3^{\alpha}(p_{1\perp}, p_{2\perp}) \Phi_3^{\beta *}(p'_{1\perp}, p'_{2\perp}) \mathbf{F} \left(\frac{p_{12\perp}}{2} \right) \mathbf{F}^* \left(\frac{p_{1'2'\perp}}{2} \right) . \end{aligned} \tag{6.6}$$

The dipole $\times$ double dipole contribution reads

$$\begin{aligned} d\sigma_{4JI} = {} & \frac{\alpha_s}{\mu^{2\epsilon}} \frac{\alpha_{\rm em} Q_q^2}{(2\pi)^{4(d-1)} N_c} \frac{(p_0^-)^2}{s^2 x_q x_{\bar{q}}} (\varepsilon_{I\alpha} \varepsilon^*_{J\beta}) dx_q dx_{\bar{q}} d^d p_{q\perp} d^d p_{\bar{q}\perp} \frac{dz d^d p_{g\perp}}{z(2\pi)^d} \delta(1 - x_q - x_{\bar{q}} - z) \\ & \times \int d^d p_{1\perp} d^d p_{2\perp} d^d p'_{1\perp} d^d p'_{2\perp} \frac{d^d p_{3\perp} d^d p'_{3\perp}}{(2\pi)^d} \delta(p_{q1\perp} + p_{\bar{q}2\perp} + p_{g3\perp}) \delta(p_{11'\perp} + p_{22'\perp} + p_{33'\perp}) \\ & \times \Big[\Phi_3^{\alpha}(p_{1\perp}, p_{2\perp}) \Phi_4^{\beta *}(p'_{1\perp}, p'_{2\perp}, p'_{3\perp}) \mathbf{F} \left(\frac{p_{12\perp}}{2} \right) \tilde{\mathbf{F}}^* \left(\frac{p_{1'2'\perp}}{2}, p'_{3\perp} \right) \delta(p_{3\perp}) \\ & \quad + \Phi_4^{\alpha}(p_{1\perp}, p_{2\perp}, p_{3\perp}) \Phi_3^{\beta *} \left(\frac{p_{1'2'\perp}}{2} \right) \tilde{\mathbf{F}} \left(\frac{p_{12\perp}}{2}, p_{3\perp} \right) \mathbf{F}^* \left(\frac{p_{1'2'\perp}}{2} \right) \delta(p'_{3\perp}) \Big] , \end{aligned} \tag{6.7}$$

and the double dipole $\times$ double dipole contribution is given by

$$\begin{aligned} d\sigma_{5JI} = {} & \frac{\alpha_s}{\mu^{2\epsilon}} \frac{\alpha_{\rm em} Q_q^2}{(2\pi)^{4(d-1)}} \frac{(p_0^-)^2}{s^2 x_q x_{\bar{q}}} \frac{(\varepsilon_{I\alpha} \varepsilon'_{J\beta})}{N_c^2 - 1} dx_q dx_{\bar{q}} d^d p_{q\perp} d^d p_{\bar{q}\perp} \frac{dz d^d p_{g\perp}}{z(2\pi)^d} \delta(1 - x_q - x_{\bar{q}} - z) \\ & \times \int d^d p_{1\perp} d^d p_{2\perp} d^d p'_{1\perp} d^d p'_{2\perp} \frac{d^d p_{3\perp} d^d p'_{3\perp}}{(2\pi)^{2d}} \delta(p_{q1\perp} + p_{\bar{q}2\perp} + p_{g3\perp}) \delta(p_{11'\perp} + p_{22'\perp} + p_{33'\perp}) \\ & \times \Phi_4^{\alpha}(p_{1\perp}, p_{2\perp}, p_{3\perp}) \Phi_4^{\beta *}(p'_{1\perp}, p'_{2\perp}, p'_{3\perp}) \tilde{\mathbf{F}} \left(\frac{p_{12\perp}}{2}, p_{3\perp} \right) \tilde{\mathbf{F}}^* \left(\frac{p_{1'2'\perp}}{2}, p'_{3\perp} \right) . \end{aligned} \tag{6.8}$$

We present the results for the products $\Phi_a \Phi_b^*$ in appendix B in D-dimensional space. They can be used directly in dimension 4 to describe the exclusive production of 3 jets.

The cross sections here seem to have a singularity at $z = 0$. The dipole dipole part $d\sigma_3$ describes gluon emission after the shockwave. Therefore it is natural for this term to have soft and collinear divergences, which will be discussed in the next section. However, each $d\sigma_{3,4,5}$ also get logarithmically large terms from the gluons with fixed transverse momenta integrated over a large area $\frac{\vec{p}^{\,2}}{s} \ll z \ll 1$.

To apply these formulas for the exclusive production of 3 jets one has to restrict the z integration with the rapidity cutoff $\theta(z - e^{\eta})$ (2.24) from the definition of the impact factor and use it only for the fast gluons (jets). As a result, the cross section gets explicit dependence on the rapidity cutoff of the 3 observed jets.

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The situation is different in the inclusive case when one has to integrate over the produced gluon. Since the z integration gives $\ln s$, one has to resum all such contributions. One can do it via the evolution equation for the double dipole operator with a color singlet projector following the logic of ref. [52].

However, since the main motivation of the present paper is to study the production of a dijet with NLO accuracy, in the next section we will only extract the soft and collinear divergences in these real terms to construct a well defined cross section for our process.

7 Cross section for the $\gamma^* P \to 2jets\, P'$ exclusive transition

The expressions for $\gamma^* \to q\bar{q}$ and $\gamma^* \to q\bar{q}g$ impact factors can be used to construct IR stable cross sections for dijet production. Whatever the experimental conditions are, one has to combine the $q\bar{q}$ and $q\bar{q}g$ production cross sections obtained above to cancel the soft and collinear singularities in the virtual part. They cancel with the singular contribution of $q\bar{q}g$ production arising from the emitted gluon phase space area where the gluon is soft or collinear to the quark or the antiquark.

We will explicitly show this cancellation on the example of the $\gamma^* P \to 2jets P'$ exclusive production cross section experimentally studied in ref. [49]. By exclusive production we understand that only two jets and the scattered proton are seen in the detector and there is nothing else. Since we want our result for the cross section to be differential only in the jets' momenta, we integrate over the transverse momentum of the outgoing proton as before. We define jets using the small cone algorithm, as in ref. [53].

Let us define a jet cone radius R^2. For convenience, we will assume that $R^2 \ll 1$. Two given particles will form a jet with a momentum equal to the sum of their momenta if they both satisfy the following condition:

$$\Delta\phi^2 + \Delta Y^2 < R^2, \tag{7.1}$$

where $\Delta\phi$ is the azimuthal angle difference between the particle and the jet, and ΔY is the rapidity difference between the particle and the jet. Let us consider for example a jet built from the quark and the gluon. Its momentum will be given by

$$p_j = x_j p_\gamma^+ n_1^\mu + \frac{\vec{p}_j^{\,2}}{2p_\gamma^+ x_j} n_2^\mu + p_{j\perp}^\mu, \qquad x_j = x_q + z, \quad \vec{p}_j = \vec{p}_q + \vec{p}_g. \tag{7.2}$$

In the small cone limit, $p_q^- + p_g^- \sim \frac{\vec{p}_j^{\,2}}{2p_\gamma^+ x_j}$ up to a $O(R)$ correction, so the jet is on-shell in this approximation. The azimuthal angle and rapidity differences read:

$$\Delta\phi = \arccos \frac{\vec{p}_j \cdot \vec{p}_g}{|\vec{p}_j|\,|\vec{p}_g|}, \quad \Delta Y = \frac{1}{2} \ln \frac{x_j^2 \vec{p}_g^{\,2}}{z^2 \vec{p}_j^{\,2}}. \tag{7.3}$$

Introducing the variable

$$\vec{\Delta}_q = \frac{x_q}{x_j}\vec{p}_g - \frac{z}{x_j}\vec{p}_q \tag{7.4}$$

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which approaches 0 when the quark and the gluon are collinear, we get the condition for the gluon to be inside the cone:

$$\vec{\Delta}_q^2 < R^2 \frac{\vec{p}_j^{\,2} z^2}{x_j^2}. \tag{7.5}$$

The corresponding condition for the quark reads

$$\vec{\Delta}_q^2 < R^2 \frac{\vec{p}_j^{\,2} x_q^2}{x_j^2}. \tag{7.6}$$

To obtain the 2-jet exclusive cross section in the small cone limit, we only need the contributions for the (LO + NLO) $q\bar{q}$ production, and the part of the contribution of the $q\bar{q}g$ production where the gluon is either soft or collinear to the quark or to the antiquark, so that they both form a single jet.

We denote the jet variables as x_j, $x_{\bar{j}}$, $p_{j\perp}$ and $p_{\bar{j}\perp}$. In the case of the virtual contribution one gets immediately the jet cross section by performing the following change of variables in eqs. (5.14)–(5.17):

$$(x, p_{q\perp}) \to (x_j, p_{j\perp}), \quad (\bar{x}, p_{\bar{q}\perp}) \to (x_{\bar{j}}, p_{\bar{j}\perp}), \tag{7.7}$$

and by symmetrizing $j \leftrightarrow \bar{j}$.

For a given contribution $d\sigma_n$ for partons in (6.5) or (5.13), we will denote the corresponding contribution to the cross section for jets as $d\sigma'_n$.

One can find the contribution of the collinear real gluons from the quasi-real electron approximation [54]. Indeed, the real contribution with a jet formed by the quark and the gluon and the other jet formed by the antiquark reads

$$d\sigma'_{3JI}(x_q, \vec{p}_q)|_{\rm col} = d\sigma'_{0JI}(x_j, \vec{p}_j)\, \alpha_s \frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}} \frac{N_c^2 - 1}{2N_c} n_j, \tag{7.8}$$

where n_j is proportional to the "number of jets in the quark"

$$\begin{aligned} n_j &= \frac{(4\pi)^{1+\epsilon}}{\Gamma(1-\epsilon)} \int_\alpha^{x_j} \frac{dz}{2z} \int_{\vec{\Delta}_q^2 < \frac{R^2 \vec{p}_j^{\,2}}{x_j^2} \min(z^2, (x_j - z)^2)} \frac{d^d \vec{\Delta}_q}{(2\pi)^d} \frac{\mu^{-2\epsilon}}{2p_j^+ 2p_q^+} \frac{tr(\hat{p}_q \gamma^\mu \hat{p}_j \gamma^\nu) d_{\mu\nu}(p_g)}{(p_q^- + p_g^- - p_j^-)^2} \\ &= 4 \int_\alpha^{x_j} \frac{x_j dz}{z(x_j - z)} \frac{\mu^{-2\epsilon}}{\Gamma(1-\epsilon)\pi^{\frac{d}{2}}} \int_{\vec{\Delta}_q^2 < \frac{R^2 \vec{p}_j^{\,2}}{x_j^2} \min(z^2, (x_j - z)^2)} d^d \vec{\Delta}_q \\ &\quad \times \frac{1}{4} \frac{(x_j - z)\left(dz^2 + 4x_j (x_j - z)\right)}{x_j^3 \vec{\Delta}_q^2}. \end{aligned} \tag{7.9}$$

This result can also be obtained by taking the collinear limit in the squares of the real impact factors, as shown in appendix B.

Here we introduced the jet j by performing the change of variables (7.2)

$$(\vec{p}_q, \vec{p}_g) \to \left(\vec{p}_j \equiv \vec{p}_q + \vec{p}_g, \vec{\Delta}_q\right), \quad (x, z) \to (x_j \equiv x + z, z) \tag{7.10}$$

and the jet $\bar{j}$ by

$$\vec{p}_{\bar{q}} \to \vec{p}_{\bar{j}}, \quad x_{\bar{q}} \to x_{\bar{j}} = 1 - x_j, \tag{7.11}$$

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and integrated inside the jet cone (7.5)–(7.6). The contribution of the jet built from the antiquark and the gluon is recovered via the $j \leftrightarrow \bar{j}$ symmetry. In appendix B we explicitly show that in the collinear limit the convolutions of the impact factors for real gluon production reproduce the last line in eq. (7.9), while all the other factors come from $d\sigma_{3JI}$ (6.6). The total collinear contribution reads

$$n_j + n_{\bar{j}} = 4\left[\frac{1}{2}\left(\ln\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right) - \frac{3}{2}\right)\ln\left(\frac{R^4\vec{p}_j^{\,2}\vec{p}_{\bar{j}}^{\,2}}{\mu^4}\right) + \frac{1}{\epsilon}\left(\ln\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right) - \frac{3}{2}\right)\right. \tag{7.12}$$
$$\left. - \frac{1}{2}\ln^2\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right) + \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{\vec{p}_j^{\,2}}{\vec{p}_{\bar{j}}^{\,2}}\right) - \frac{1}{2}\ln^2\left(\frac{x_j}{x_{\bar{j}}}\right) - \frac{\pi^2}{3} + \frac{7}{2} + \ln(8)\right].$$

In the soft gluon limit, the real cross section has the form

$$d\sigma'_{3JI}|_{soft} = d\sigma'_{0JI}\alpha_s\frac{N_c^2-1}{2N_c}\frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}S, \quad S \equiv \frac{(4\pi)^{1+\epsilon}}{\Gamma(1-\epsilon)}\int\left|\frac{p_q^\mu}{(p_q\cdot p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}}\cdot p_g)}\right|^2\frac{dz}{z}\frac{d^dp_g}{(2\pi)^d}, \tag{7.13}$$

as shown in appendix B. We have to integrate this formula over

$$\omega_g = \frac{1}{2}\left(zp_\gamma^+ + \frac{\vec{p}_g^{\,2}}{zp_\gamma^+}\right) < E \ll p_\gamma^+, \tag{7.14}$$

where ω_g is the emitted gluon energy and E is the energy resolution. The small energy limit for the gluon occurs when all the components of the gluon momentum approach 0 simultaneously. We achieve this by rescaling the gluon transverse momentum as

$$\vec{p}_g = z\vec{u} \tag{7.15}$$

and going to the limit $z \to 0$. In this limit the integration area reads

$$z\left(1 + \frac{\vec{u}^{\,2}}{(p_\gamma^+)^2}\right) < \frac{2E}{p_\gamma^+} \ll 1, \tag{7.16}$$

and we have

$$S = \int_\alpha^{\frac{2E}{p_\gamma^+}} z^{d-3}dz\int_{\vec{u}^{\,2}<(p_\gamma^+)^2(\frac{2E}{zp_\gamma^+}-1)}\frac{\mu^{-2\epsilon}d^du_\perp}{\Gamma(1-\epsilon)\pi^{\frac{d}{2}}}\frac{4\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2\left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2}, \tag{7.17}$$

as shown explicitly in appendix B. We have restored the rapidity cutoff α which of course will play a role to regularize the soft divergence.

However, in the sum $n_j + n_{\bar{j}} + S$ the region with a gluon both soft and collinear to the quark or to the antiquark is calculated twice. To avoid double counting we restrict the integration in S so that the gluons sit outside the cones (7.5). The new integration region reads

$$\Omega = \left\{\vec{u}^{\,2} < (p_\gamma^+)^2\left(\frac{2E}{zp_\gamma^+} - 1\right)\right\} \cap \Omega_{nc}\,, \tag{7.18}$$

$$\Omega_{nc} \equiv \left\{\left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2 > \frac{R^2\vec{p}_j^{\,2}}{x_j^2}\right\} \cap \left\{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 > \frac{R^2\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}^2}\right\}. \tag{7.19}$$

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Let us denote S' the new definition of S with this integration area:

$$\begin{aligned} S' &\equiv 4\int_\alpha^{\frac{2E}{p_\gamma^+}} \frac{dz}{z}\int_\Omega \frac{d\vec{u}}{\pi}\frac{4\left(\frac{\vec{p}_j}{x_j}-\frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u}-\frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2\left(\vec{u}-\frac{\vec{p}_j}{x_j}\right)^2} \\ &= 4\int_\alpha^{\frac{2E}{p_\gamma^+}} \frac{dz}{z}\int_{\Omega_{nc}} \frac{d\vec{u}}{\pi}\frac{\left(\frac{\vec{p}_j}{x_j}-\frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u}-\frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2\left(\vec{u}-\frac{\vec{p}_j}{x_j}\right)^2} + 4\,I(R,E) \\ &= 4\ln\left(\frac{2E}{\alpha p_\gamma^+}\right)\ln\left(\frac{(\vec{p}_j x_{\bar{j}}-x_j\vec{p}_{\bar{j}})^4}{(R^2\vec{p}_j^{\,2}x_j^2)(R^2\vec{p}_{\bar{j}}^{\,2}x_{\bar{j}}^2)}\right) + 4\,I(R,E)\,, \end{aligned} \tag{7.20}$$

where we defined

$$I(R,E) \equiv -\int_0^{\frac{2E}{p_\gamma^+}} \frac{dz}{z}\int_{\{\vec{u}^{\,2}>(p_\gamma^+)^2(\frac{2E}{zp_\gamma^+}-1)\}\cap\Omega_{nc}} \frac{d\vec{u}}{\pi}\frac{\left(\frac{\vec{p}_j}{x_j}-\frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u}-\frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2\left(\vec{u}-\frac{\vec{p}_j}{x_j}\right)^2}. \tag{7.21}$$

The integral $I(R,E)$ is convergent and depends neither on α nor on ϵ. In the appendix C, we show that this integral gives a contribution which is $1/s$ suppressed.

Finally,

$$S' = 4\ln\left(\frac{2E}{\alpha p_\gamma^+}\right)\ln\left(\frac{(\vec{p}_j x_{\bar{j}}-x_j\vec{p}_{\bar{j}})^4}{(R^2\vec{p}_j^{\,2}x_j^2)(R^2\vec{p}_{\bar{j}}^{\,2}x_{\bar{j}}^2)}\right). \tag{7.22}$$

Combining (7.12) and (7.22) we have

$$\begin{aligned} S'+n_j+n_{\bar{j}} = 2\Bigg[&\ln\left(\frac{(x_{\bar{j}}\vec{p}_j-x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2x_j^2R^4\vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right) \\ &+2\ln\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon}-\ln\left(\frac{x_{\bar{j}}x_j\mu^2}{(x_{\bar{j}}\vec{p}_j-x_j\vec{p}_{\bar{j}})^2}\right)\right)-\ln^2\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right) \\ &+\frac{3}{2}\ln\left(\frac{16\mu^4}{R^4\vec{p}_j^{\,2}\vec{p}_{\bar{j}}^{\,2}}\right)-\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right)-\frac{3}{\epsilon}-\frac{2\pi^2}{3}+7\Bigg]. \end{aligned} \tag{7.23}$$

Adding the singular part of the virtual correction (3.42), one finally cancels the $\ln\alpha$ and $\frac{1}{\epsilon}$ divergences and gets:

$$\begin{aligned} S_R &= S'+n_j+n_{\bar{j}}+S_V+S_V^* \\ &= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j-x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2x_j^2R^4\vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)+\frac{3}{2}\right)\right. \\ &\quad\left.+\ln(8)-\frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right)+\frac{13-\pi^2}{2}\right]. \end{aligned} \tag{7.24}$$

This demonstrates the IR finiteness of our cross section.

To get the IR-safe exclusive diffractive dijet production cross section in the small cone approximation, one has to take the $q\bar{q}$ production cross section from section 6, rename the quark momenta via (7.7), and substitute $S_V+S_V^*\to S_R$ in eq. (5.24) for the LL transition, in eq. (5.28) for the LT transition, and in eq. (5.35) for the TT transition.

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8 Summary and prospects for further studies

Using the QCD shock-wave approach [20, 55, 56], we have obtained for the first time the $\gamma^* \to q\bar{q}$ impact factor with one loop accuracy. Combined with our previous study of the $\gamma^* \to q\bar{q}g$ impact factor [19], we calculated the cross section for exclusive diffractive dijet electroproduction off the proton. For this specific example, we have shown in a detailed way the cancellation of UV and IR soft, collinear and rapidity divergencies. All presented results were obtained without any collinear or soft approximations, in an arbitrary kinematics: i.e. for nonzero incoming photon virtuality, arbitrary $t-$channel momentum transfer and invariant mass of the produced state.

There are several theoretical developments to be pursued based on our study.

First, after applying a suitable Fierz projection, one can obtain the NLO impact factor for the $\gamma^{(*)} \to \rho-$meson transition in arbitrary kinematics, therefore extending the forward result of ref. [57]. At leading twist, this process is dominated by the $\gamma_L^* \to \rho_L$ transition, while transitions with other polarizations start at twist 3. The impact factor for the transition $\gamma_T^* \to \rho_T$ in the forward limit was obtained at LO in ref. [58, 59], including both the kinematical twist 3 (the so-called Wandzura Wilczek (WW) [60] contribution, where the produced meson Fock state is only made of a $q\bar{q}$) and the genuine twist 3 contributions (i.e. including a $q\bar{q}g$ Fock state). The present result opens the way to a computation of LO $\gamma^* \to \rho$ transitions for arbitrary polarizations and kinematics (using our $\gamma^{(*)} \to q\bar{q}g$ Born order result), as well as of the NLO $\gamma^{(*)} \to \rho$ impact factor in the WW approximation, using our one-loop $\gamma^{(*)} \to q\bar{q}$ result.

Second, one could extend the results of our studies to massive quarks. This would allow for a study of diffractive open charm production, measured at HERA [61], and studied in the large M limit based on the direct coupling between a Pomeron and a $q\bar{q}$ or a $q\bar{q}g$ state, with massive quarks [17]. In the context of the Color Evaporation Model or studying a process where in the NRQCD formalism the color octet contribution is expected to dominate, the required NLO $\gamma^{(*)} \to J/\psi$ impact factor could be obtained.

Third, there are two ways to apply our result to phenomenology. A linearization procedure of the U operators allows one to make connection with the linear BFKL regime. On the other hand, one can also construct a phenomenological model for the matrix elements of the Wilson operators acting on the proton states to approach the saturated regime of the proton or nucleus target.

Fourth, although we here restricted ourselves to a color-singlet exchange in the $t-$channel, and thus to diffractive processes, an extension to the octet case can be performed, i.e. to the inclusive case. At LO, such a study was done in ref. [62]. In this case, one encounters a quadrupole Wilson line operator whose NLO evolution equation was derived in ref. [63].

Fifth integrating our results for the $q\bar{q}$ and $q\bar{q}g$ cross sections w.r.t. the external momenta, one can directly obtain the results for NLO $\gamma^* \to \gamma^*$ which were presented in refs. [55, 56]. A detailed comparison is left for further studies.

On the phenomenological side, the applications of our results are multiple, and are expected to improve essentially the precision of models based on the k_T-factorization picture, since several observables could now be made accessible theoretically with NLO

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accuracy. Indeed, it is known that passing from LO to NLO impact factors can have major effects in BFKL predictions. The only available process for which such a complete NLO description was obtained [64–69] is Mueller-Navelet dijet production [70]. In particular, the azimuthal decorrelation was recently extracted by CMS [71] and confronted with its very good theoretical description of refs. [72, 73]. Furthermore, the fact that the $t-$channel exchanged state in our present computation is very general allows one to study not only the linear BFKL regime, but also saturation effects in a proton or a nucleus, here with NLO accuracy.

First, the NLO impact factor of the present study could be used to describe exclusive dijet diffractive electroproduction [49], as well as non-exclusive dijet diffractive electroproduction, available at HERA [74]. In the limit $Q^2 \to 0$, our general result could be also applied to photo-production of diffractive jets [75, 76], with a hard scale given by the invariant mass of the produced state, and a precise comparison, now at NLO in the BFKL framework, could be performed with the NLO collinear factorization approach [77, 78]. More generally, at future ep and eA colliders, like EIC [79] and LHeC [80], a large set of observables will give a possibility to enter the saturation regime in a controllable way, since the saturation scale becomes perturbative for large center of mass energy and/or large values of A. This includes photoproduction of heavy quarkonia, exclusive diffractive production of light mesons, e.g. $\rho-$meson, either in electroproduction or in large t photoproduction. In particular, our result allows one to use diffractive dijet production, considered as a very promising observable to probe the color glass condensate and more generally to perform proton and nucleus tomography at low x, in connection to Wigner distributions, now beyond some recent LO analyses [81, 82].

Second, before the advent of future high energy and high luminosity ep and eA colliders, ultraperipheral collisions (UPCs) at high energy which provide a source of photons from a projectile proton or nucleus, are perfect playgrounds in order to probe the high-energy partonic content of the target proton or nucleus. These are already accessible at the LHC. In particular, during the Run I of the LHC, the LHCb collaboration have measured exclusive photoproduction of J/ψ and $\psi(2S)$ mesons [83, 84] in pp collision (later extended to Υ in ref. [85]), while the ALICE collaboration measured this process in pPb [86] and $PbPb$ [87–89] collisions. CMS very recently released a similar analysis for $PbPb$ [90]. The physics potential of UPCs will improve very significantly thanks to several very forward detectors which are installed, under test, or planned in each of the four LHC experiments, in particular the CMS-TOTEM Proton Spectrometer, AD-ALICE, HERSCHEL at LHCb and AFP at ATLAS [91]. For example, the protoproduction of large invariant mass diffractive dijet could be studied in UPC during Run II at LHC.[4]

Note. We are aware of a similar computation based on old-fashioned perturbation theory [93] for the NLO $\gamma^* \to q\bar{q}$ wavefunction. The results of this study should match our results for Φ_{23}^{α}, see eqs. (3.33), (3.34), when setting the shockwave momenta $p_{1\perp}$ and $p_{2\perp}$ to 0. Previous results by the same author [94] were confirmed by our previous study [19].

[4] In the usual collinear picture, a recent study of this process has been performed in ref. [92].

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A Finite part of the virtual correction

A.1 Building-block integrals

Throughout this section, we will need the following integrals

$$I_1^k(\vec{q}_1,\,\vec{q}_2,\,\Delta_1,\,\Delta_2) \equiv \frac{1}{\pi}\int \frac{d^d\vec{l}\,(l_\perp^k)}{\left[(\vec{l}-\vec{q}_1)^2+\Delta_1\right]\left[(\vec{l}-\vec{q}_2)^2+\Delta_2\right]\vec{l}^{\,2}}, \tag{A.1}$$

$$I_2(\vec{q}_1,\,\vec{q}_2,\,\Delta_1,\,\Delta_2) \equiv \frac{1}{\pi}\int \frac{d^d\vec{l}}{\left[(\vec{l}-\vec{q}_1)^2+\Delta_1\right]\left[(\vec{l}-\vec{q}_2)^2+\Delta_2\right]}, \tag{A.2}$$

$$I_3^k(\vec{q}_1,\,\vec{q}_2,\,\Delta_1,\,\Delta_2) \equiv \frac{1}{\pi}\int \frac{d^d\vec{l}\,(l_\perp^k)}{\left[(\vec{l}-\vec{q}_1)^2+\Delta_1\right]\left[(\vec{l}-\vec{q}_2)^2+\Delta_2\right]}, \tag{A.3}$$

$$I^{jk}(\vec{q}_1,\,\vec{q}_2,\,\Delta_1,\,\Delta_2) \equiv \frac{1}{\pi}\int \frac{d^d\vec{l}\left(l_\perp^j\,l_\perp^k\right)}{\left[(\vec{l}-\vec{q}_1)^2+\Delta_1\right]\left[(\vec{l}-\vec{q}_2)^2+\Delta_2\right]\vec{l}^{\,2}}\,. \tag{A.4}$$

The arguments of these integrals will be different for each diagram so we will write them explicitly before giving the expression of each diagram, but we will omit them in the equations for reader's convenience.

Explicit results for the first 3 integrals in (A.1)–(A.4) are obtained by a straightforward Feynman parameter integration. We will express them using the following variables:

$$\rho_1 \equiv \frac{\left(\vec{q}_{12}^{\,2}+\Delta_{12}\right)-\sqrt{\left(\vec{q}_{12}^{\,2}+\Delta_{12}\right)^2+4\vec{q}_{12}^{\,2}\Delta_2}}{2\vec{q}_{12}^{\,2}}, \tag{A.5}$$

$$\rho_2 \equiv \frac{\left(\vec{q}_{12}^{\,2}+\Delta_{12}\right)+\sqrt{\left(\vec{q}_{12}^{\,2}+\Delta_{12}\right)^2+4\vec{q}_{12}^{\,2}\Delta_2}}{2\vec{q}_{12}^{\,2}}\,, \tag{A.6}$$

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where $\Delta_{ij} = \Delta_i - \Delta_j$.

One gets:

$$I_1^k = \frac{q_{1\perp}^k}{2\left[\vec{q}_{12}^{\,2}\left(\vec{q}_1^{\,2}+\Delta_1\right)\left(\vec{q}_2^{\,2}+\Delta_2\right)-\left(\vec{q}_1^{\,2}-\vec{q}_2^{\,2}+\Delta_{12}\right)\left(\vec{q}_1^{\,2}\Delta_2-\vec{q}_2^{\,2}\Delta_1\right)\right]}$$
$$\times\left\{\frac{\left(\vec{q}_2^{\,2}+\Delta_2\right)\vec{q}_{12}^{\,2}+\vec{q}_2^{\,2}\left(\Delta_1+\Delta_2\right)+\Delta_2\left(\Delta_{21}-2\vec{q}_1^{\,2}\right)}{\left(\rho_1-\rho_2\right)\vec{q}_{12}^{\,2}}\ln\left[\left(\frac{-\rho_1}{1-\rho_1}\right)\left(\frac{1-\rho_2}{-\rho_2}\right)\right]\right.$$
$$\left.+\left(\vec{q}_2^{\,2}+\Delta_2\right)\ln\left[\frac{\Delta_2\left(\vec{q}_1^{\,2}+\Delta_1\right)^2}{\Delta_1\left(\vec{q}_2^{\,2}+\Delta_2\right)^2}\right]+(1\leftrightarrow 2)\right\}, \tag{A.7}$$

$$I_2 = \frac{1}{\vec{q}_{12}^{\,2}\left(\rho_1-\rho_2\right)}\ln\left[\left(\frac{-\rho_1}{1-\rho_1}\right)\left(\frac{1-\rho_2}{-\rho_2}\right)\right], \tag{A.8}$$

and

$$I_3^k = \frac{\left(\vec{q}_{12}^{\,2}+\Delta_{12}\right)q_1^k+\left(\vec{q}_{21}^{\,2}+\Delta_{21}\right)q_2^k}{2\left(\rho_1-\rho_2\right)\left(\vec{q}_{12}^{\,2}\right)^2}\ln\left[\left(\frac{-\rho_1}{1-\rho_1}\right)\left(\frac{1-\rho_2}{-\rho_2}\right)\right]-\frac{q_{12}^k}{2\vec{q}_{12}^{\,2}}\ln\left(\frac{\Delta_1}{\Delta_2}\right). \tag{A.9}$$

Please note that in some cases the real part of Δ_1 or Δ_2 will be negative so the previous results can acquire an imaginary part from the imaginary part $\pm i0$ of the arguments.
The last integral in (A.4) can be expressed in terms of the other ones by writing

$$I^{jk} = I_{11}\left(q_{1\perp}^j q_{1\perp}^k\right)+I_{12}\left(q_{1\perp}^j q_{2\perp}^k+q_{2\perp}^j q_{1\perp}^k\right)+I_{22}\left(q_{2\perp}^j q_{2\perp}^k\right), \tag{A.10}$$

with

$$I_{11} = -\frac{1}{2}\frac{\left[\vec{q}_2^{\,2}q_{1\perp k}-\left(\vec{q}_1\cdot\vec{q}_2\right)q_{2\perp k}\right]}{\left[\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\left(\vec{q}_1\cdot\vec{q}_2\right)^2\right]^2} \tag{A.11}$$
$$\times\left[\left(\frac{\vec{q}_1\cdot\vec{q}_2}{\vec{q}_1^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}+\Delta_1}{\Delta_1}\right)q_{1\perp}^k+\left(\vec{q}_2\cdot\vec{q}_{21}\right)I_3^k+\left\{\vec{q}_2^{\,2}\left(\vec{q}_1\cdot\vec{q}_{12}\right)+\Delta_1\vec{q}_2^{\,2}-\Delta_2\left(\vec{q}_1\cdot\vec{q}_2\right)\right\}I_1^k\right],$$

$$I_{12} = \frac{-1}{4\left[\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\left(\vec{q}_1\cdot\vec{q}_2\right)^2\right]}\ln\left(\frac{\vec{q}_1^{\,2}+\Delta_1}{\Delta_1}\right) \tag{A.12}$$
$$+\frac{\vec{q}_2^{\,2}\left(\vec{q}_1\cdot\vec{q}_2\right)}{2\left[\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\left(\vec{q}_1\cdot\vec{q}_2\right)^2\right]^2}\left[\left(\vec{q}_1^{\,2}+\Delta_1\right)\left(q_{1\perp k}I_1^k\right)+\left(q_{1\perp k}I_3^k\right)\right]$$
$$-\frac{\left(\vec{q}_1^{\,2}\vec{q}_2^{\,2}\right)+\left(\vec{q}_1\cdot\vec{q}_2\right)^2}{4\left[\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\left(\vec{q}_1\cdot\vec{q}_2\right)^2\right]^2}\left[\left(\vec{q}_2^{\,2}+\Delta_2\right)\left(q_{1\perp k}I_1^k\right)+\left(q_{1\perp k}I_3^k\right)\right]+(1\leftrightarrow 2),$$

$$I_{22} = I_{11}|_{(1\leftrightarrow 2)}. \tag{A.13}$$

This last expression makes it seem that there is a singularity when $\vec{q}_1$ and $\vec{q}_2$ are collinear or anticollinear. However, this singularity is non-physical and only appears when projecting on the particular basis of 2-dimensional symmetric tensors $(q_1^j q_1^k,\ q_1^j q_2^k+q_2^j q_1^k,\ q_2^j q_2^k)$. One can show that it disappears when projecting on the non-minimal basis $(q_1^j q_1^k,\ q_1^j q_2^k+q_2^j q_1^k,\ q_2^j q_2^k,\ g_\perp^{jk})$. For a further study, the reader is referred to the appendix of ref. [95].

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A.2 Diagram 4

Here the integrals from section A.1 will have the following arguments:

$$\vec{q}_1 = \vec{p}_1 - \left(\frac{x-z}{x}\right)\vec{p}_q, \qquad \vec{q}_2 = \left(\frac{x-z}{x}\right)(x\vec{p}_{\bar{q}} - \bar{x}\vec{p}_q)\,,$$
$$\Delta_1 = (x-z)\,(\bar{x}+z)\,Q^2, \qquad \Delta_2 = -\frac{x\,(\bar{x}+z)}{\bar{x}\,(x-z)}\vec{q}_2^{\,2} - i0\,. \tag{A.14}$$

Let us write the impact factors in terms of these variables. They read:
(longitudinal NLO) × (longitudinal LO) contribution:

$$(\phi_4)_{LL} = -\frac{4\,(x-z)\,(\bar{x}+z)}{z}\left[-\bar{x}\,(x-z)\,(z+1)\,I_2 + q_{2\perp k}\left(2x^2-(2x-z)\,(z+1)\right)I_1^k\right], \tag{A.15}$$

(longitudinal NLO) × (transverse LO) contribution:

$$(\phi_4)^j_{LT} = (1-2x)p_{q1'}{}^j_{\perp}\,(\phi_4)_{LL} - 4(x-z)(\bar{x}+z)(1-2x+z)[(\vec{q}_2\cdot\vec{p}_{q1'})g^j_{\perp k} + q^j_{2\perp}p_{q1'\perp k}]I_1^k\,, \tag{A.16}$$

(transverse NLO) × (longitudinal LO) contribution:

$$\begin{aligned}(\phi_4)^i_{TL} = 2\Big\{&[(x-\bar{x}-z)q^i_{2\perp}q_{1\perp k} + (-8x\bar{x}-6xz+2z^2+3z+1)q^j_{1\perp}q_{2\perp k}]I_1^k\\ &-2[4x^2-x(3z+5)+(z+1)^2]q_{2\perp k}I^{ik} + (x-\bar{x}-z)\,(\vec{q}_2\cdot\vec{q}_1)\,I^i\\ &+I_2[(x-\bar{x}-z)q^i_{2\perp} + \bar{x}(2(x-z)^2-5x+3z+1)q^i_{1\perp}]\\ &-\bar{x}[2(x-z)^2-5x+3z+1]I_3^i\\ &+\frac{x\bar{x}(1-2x)}{z}[2q_{2\perp k}I^{ik} + I_3^i - q^i_{1\perp}(2q_{2\perp k}I_1^k + I_2)]\Big\}\,,\end{aligned} \tag{A.17}$$

(transverse NLO) × (transverse LO) contribution:

$$\begin{aligned}(\phi_4)^{ij}_{TT} = &\left[(x-\bar{x}-2z)(x-\bar{x}-z)(\vec{q}_2\cdot\vec{p}_{q1'})q^i_{1\perp} + (z+1)((\vec{q}_1\cdot\vec{q}_2)\,p^i_{q1'\perp} - (\vec{q}_1\cdot\vec{p}_{q1'})q^i_{2\perp})\right]I_1^j\\ &+2\bar{x}[q_{2\perp k} - (x-z)q_{1\perp k}](p^i_{q1'\perp}I^{jk} - g^{ij}_{\perp}p_{q1'\perp l}I^{kl})\\ &+2(x-z)[(2\bar{x}+z)(\vec{q}_2\cdot\vec{p}_{q1'}) - \bar{x}(\vec{q}_1\cdot\vec{p}_{q1'})]I^{ij}\\ &+[(1-z)((\vec{q}_1\cdot\vec{p}_{q1'})q^j_{2\perp} - (\vec{q}_2\cdot\vec{p}_{q1'})q^j_{1\perp}) - (1-2x)(\bar{x}-x+z)\,(\vec{q}_1\cdot\vec{q}_2)\,p^j_{q1'\perp}]I_1^i\\ &-2\Big[(x-z)(\bar{x}q^j_{1\perp} - (2\bar{x}+z)q^j_{2\perp})p_{q1'\perp k}\\ &+(1-2x)\left(4x^2-(3z+5)x+(z+1)^2\right)q_{2\perp k}p^j_{q1'\perp}\Big]I^{ik}\\ &-\bar{x}\,(\bar{x}-x)\left(2(x-z)^2-5x+3z+1\right)p^j_{q1'\perp}I_3^i\\ &+\bar{x}\,(\bar{x}+z)\,(p^i_{q1'\perp}I_3^j - g^{ij}_{\perp}p_{q1'\perp k}I_3^k)\\ &+I_2\Big[g^{ij}_{\perp}\left((1-z)(\vec{q}_2\cdot\vec{p}_{q1'}) - \bar{x}(1+x-z)(\vec{q}_1\cdot\vec{p}_{q1'})\right)\\ &+((1-z)q^j_{2\perp} - \bar{x}(1+x-z)q^j_{1\perp})p^i_{q1'\perp}\\ &-(\bar{x}-x)\left((\bar{x}-x+z)q^i_{2\perp} - \bar{x}\left(2(x-z)^2-5x+3z+1\right)q^i_{1\perp}\right)p^j_{q1'\perp}\Big]\end{aligned}$$

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$$
\begin{aligned}
&+I_1^k\Big[g_\perp^{ij}\left((x-\bar{x}+z)(\vec{q}_1\cdot\vec{p}_{q1'})q_{2\perp k}+(1-z)(\vec{q}_2\cdot\vec{p}_{q1'})q_{1\perp k}-(z+1)\left(\vec{q}_1\cdot\vec{q}_2\right)p_{q1'\perp k}\right)\\
&+q_{1\perp}^j((x-\bar{x}+z)q_{2\perp k}p_{q1'\perp}^i-(z+1)q_{2\perp}^i p_{q1'\perp k})\\
&+q_{2\perp}^j((x-\bar{x}-2z)(x-\bar{x}-z)q_{1\perp}^i p_{q1'\perp k}+(1-z)q_{1\perp k}p_{q1'\perp}^i)\\
&-(1-2x)((1-2x+z)q_{2\perp}^i q_{1\perp k}-(2z^2+3z-x(8\bar{x}+6z)+1)q_{1\perp}^i q_{2\perp k})p_{q1'\perp}^j\Big]\\
&+\frac{x\bar{x}}{z}\Big[(x-\bar{x})^2 p_{q1'\perp}^j(2q_{2\perp k}I^{ik}+I_3^i-q_{1\perp}^i(I_2+2q_{2\perp k}I_1^k))\\
&+p_{q1'\perp}^i(q_{1\perp}^j(I_2+2q_{2\perp k}I_1^k)-2q_{2\perp k}I^{jk}-I_3^j)\\
&+g_\perp^{ij}((\vec{q}_1\cdot\vec{p}_{q1'})(I_2+2q_{2\perp k}I_1^k)+p_{q1'\perp k}(2q_{2\perp l}I^{kl}+I_3^k))\Big]\,.
\end{aligned}
\tag{A.18}
$$

A.3 Diagram 5

Here the integrals from section A.2 will have the following arguments:

$$
\vec{q}_1=\vec{p}_{q1}-\frac{z}{x}\vec{p}_q,\qquad\qquad \vec{q}_2=\left(\frac{x-z}{x}\right)\vec{p}_3-\frac{z}{x}\vec{p}_1\,,
\tag{A.19}
$$

$$
\Delta_1=\frac{z(x-z)}{x^2\bar{x}}(\vec{p}_{\bar{q}2}^{\,2}+x\bar{x}Q^2),\qquad \Delta_2=(x-z)(\bar{x}+z)Q^2\,,
\tag{A.20}
$$

With such variables, it is easy to see that the argument in the square roots in (A.6) is a full square, so that

$$
\rho_1=-\frac{x\bar{x}Q^2}{\vec{p}_{\bar{q}2}^{\,2}}\,,\quad \rho_2=\frac{x(\bar{x}+z)}{\bar{x}(x-z)}\,.
\tag{A.21}
$$

In terms of the variables in (A.19), the impact factors read:
(longitudinal NLO) $\times$ (longitudinal LO):

$$
(\phi_5)_{LL}=\frac{4(x-z)(-2x(\bar{x}+z)+z^2+z)}{xz}\left[\bar{x}(x-z)I_2-\left(zq_{1\perp k}-x\left(\bar{x}+z\right)q_{2\perp k}\right)I_1^k\right],
\tag{A.22}
$$

(longitudinal NLO) $\times$ (transverse LO):

$$
\begin{aligned}
(\phi_5)_{LT}^j=&\,(\bar{x}-x)p_{q1'\perp}^j\,(\phi_5)_{LL}\\
&+\frac{4(x-z)(x-\bar{x}-z)}{x}\left(zq_{1\perp}^k-x(\bar{x}+z)q_{2\perp}^k\right)p_{q1'\perp l}\left(g_{\perp k}^j I_1^l-g_{\perp k}^l I_1^j\right)\,,
\end{aligned}
\tag{A.23}
$$

(transverse NLO) $\times$ (longitudinal LO):

$$
\begin{aligned}
(\phi_5)_{TL}^i=&\,2\Big[(x-\bar{x}-z)\left(\vec{q}_1\cdot\vec{q}_2\right)-\bar{x}\left(x-z\right)^2Q^2+\left(\frac{z}{x}-x\right)\vec{q}_1^{\,2}\Big]I_1^i\\
&+\frac{2}{x}\left[xq_{2\perp k}\left(-8x\bar{x}-6xz+2z^2+3z+1\right)+2q_{1\perp k}\left(2xz-2x^2+x-z^2\right)\right]q_{1\perp}^i I_1^k\\
&+2q_{2\perp}^i q_{1\perp k}\left(x-\bar{x}-z\right)I_1^k+2\frac{\bar{x}}{x}\left(x\left(8x-3\right)-6xz+2z^2+z\right)I_1^i\\
&+\frac{2}{x}\left[xq_{2\perp}^i\left(x-\bar{x}-z\right)+q_{1\perp}^i\left(8x^3-6x^2\left(z+2\right)+x\left(z+3\right)\left(2z+1\right)-2z^2\right)\right]I_2\\
&-\frac{4}{x}\left[\left(x-z\right)\left(\bar{x}+z\right)q_{1\perp k}+x\left(4x^2-x\left(3z+5\right)+\left(z+1\right)^2\right)q_{2\perp k}\right]I^{ik}\\
&-\frac{4}{z}x\bar{x}\left(x-\bar{x}\right)\left[q_{2\perp k}I^{ik}+I_3^i-q_{1\perp}^i\left(q_{2\perp k}I_1^k+I_2\right)\right]\,,
\end{aligned}
\tag{A.24}
$$

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(transverse NLO) × (transverse LO):

$$
\begin{aligned}
(\phi_5)^{ij}_{TT} = &-2(x-z)\left[\frac{z}{x}(\vec{q}_1\cdot\vec{p}_{q1'}) - (2\bar{x}+z)(\vec{q}_2\cdot\vec{p}_{q1'})\right] I^{ij} \\
&+\Bigg[-\bar{x}(x-z)^2Q^2 p^i_{q1'\perp} + (\bar{x}-x+2z)(\bar{x}-x+z)(\vec{q}_2\cdot\vec{p}_{q1'})q^i_{1\perp} \\
&-\left(\vec{q}_1\cdot\vec{p}_{q1'}\right)\left((z+1)\,q^i_{2\perp} - 2\frac{z}{x}\,(2x-z)\,q^i_{1\perp}\right) \\
&+\left((z+1)\,(\vec{q}_1\cdot\vec{q}_2) - \left(x+\frac{z}{x}\right)\vec{q}_1{}^2\right)p^i_{q1'\perp}\Bigg] I^j_1 \\
&-2\frac{\bar{x}}{x}(xq_{2\perp k} + (x-z)q_{1\perp k})\left(g^{ij}_\perp p_{q1'\perp l} I^{kl} - p^i_{q1'\perp} I^{jk}\right) \\
&+\Big[\bar{x}\,(x-\bar{x})\,(x-z)^2Q^2 p^j_{q1'\perp} - (z-1)(\vec{q}_1\cdot\vec{p}_{q1'})q^j_{2\perp} \\
&+(z-1)(\vec{q}_2\cdot\vec{p}_{q1'})q^j_{1\perp} + \frac{x-\bar{x}}{x}\left((x^2-z)\vec{q}_1{}^2 + x(\bar{x}-x+z)(\vec{q}_1\cdot\vec{q}_2)\right)p^k_{q1'\perp}\Big] I^i_1 \\
&+2\left[\frac{x-\bar{x}}{x}\left(x(4x^2-(3z+5)x+(z+1)^2)q_{2\perp k} + (x-z)(\bar{x}+z)q_{1\perp k}\right)p^j_{q1'\perp}\right. \\
&\left.-\frac{x-z}{x}\left(x(2x-z-2)q^j_{2\perp} + zq^j_{1\perp}\right)p_{q1'\perp k}\right] I^{ik} \\
&+\frac{\bar{x}\,(\bar{x}-x)}{x}\left(2z^2-6xz+z+x(8x-3)\right)p^j_{q1'\perp} I^i_3 \\
&+\Bigg[(x-\bar{x})\left((\bar{x}-x+z)q^i_{2\perp} + \left(6(z+2)x - 8x^2 - (z+3)(2z+1) + 2\frac{z^2}{x}\right)q^i_{1\perp}\right)p^j_{q1'\perp} \\
&+(1-z)(g^{ij}_\perp(\vec{q}_2\cdot\vec{p}_{q1'}) + q^k_{2\perp}p^i_{q1'\perp}) + (2x+z-3)(g^{ik}_\perp(\vec{q}_1\cdot\vec{p}_{q1'}) + q^k_{1\perp}p^i_{q1'\perp})\Bigg] I_2 \\
&+\left(3\bar{x}+z-\frac{z}{x}\right)p^i_{q1'\perp} I^k_3 - \frac{\bar{x}}{x}(3x-z)g^{ij}_\perp p_{q1'\perp k} I^k_3 \\
&+\Bigg[(x-\bar{x})p^j_{q1'\perp}\Big\{(\bar{x}-x+z)q^i_{2\perp}q_{1\perp k} - (2z^2-6xz+3z-8x\bar{x}+1)q_{2\perp k}q^i_{1\perp} \\
&-2(\bar{x}-x+2z-\frac{z^2}{x})q_{1\perp k}q^i_{1\perp}\Big\} + \bar{x}(x-z)^2Q^2 g^{ij}_\perp p_{q1'\perp k} \\
&+(1-z)\,q_{1\perp k}\left(g^{ij}_\perp(\vec{q}_2\cdot\vec{p}_{q1'}) + q^j_{2\perp}p^i_{q1'\perp}\right) \\
&+\left((x-\bar{x}+z)q_{2\perp k} - 2q_{1\perp k}\right)(g^{ij}_\perp(\vec{q}_1\cdot\vec{p}_{q1'}) + q^j_{1\perp}p^i_{q1'\perp}) \\
&+g^{ij}_\perp\left(\left(x+\frac{z}{x}\right)\vec{q}_1{}^2 - (z+1)(\vec{q}_1\cdot\vec{q}_2)\right)p_{q1'\perp k} \\
&+\left((x-\bar{x}-2z)(x-\bar{x}-z)q^i_{1\perp}q^j_{2\perp} - (z+1)q^i_{2\perp}q^j_{1\perp} + 2(2x-z)\frac{z}{x}q^i_{1\perp}q^j_{1\perp}\right)p_{q1'\perp k}\Bigg] I^k_1 \\
&+\frac{2x\bar{x}}{z}\Big[(x-\bar{x})^2 p^j_{q1'\perp}(q_{2\perp k}I^{ik} + I^i_3) - p^i_{q1'\perp}(q_{2\perp k}I^{jk} + I^k_3) + g^{ij}_\perp p_{q1'\perp k}(q_{2\perp l}I^{kl} + I^k_3) \\
&+(I_2 + q_{2\perp k}I^k_1)\left(g^{ij}_\perp(\vec{q}_1\cdot\vec{p}_{q1'}) + q^j_{1\perp}p^i_{q1'\perp} - (1-2x)^2 q^i_{1\perp}p^j_{q1'\perp}\right)\Big]\,. \qquad \text{(A.25)}
\end{aligned}
$$

A.4 Diagram 6

For this diagram we will introduce the variable

$$\vec{q} = \left(\frac{x-z}{x}\right)\vec{p}_3 - \frac{z}{x}\vec{p}_1\,. \qquad \text{(A.26)}$$

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Then the impact factors read:
(longitudinal NLO) × (longitudinal LO):

$$(\phi_6)_{LL} = -4x\bar{x}^2 J_0\,, \tag{A.27}$$

(longitudinal NLO) × (transverse LO):

$$(\phi_6)^j_{LT} = (1-2x)p^j_{q1'\perp}(\phi_6)_{LL}\,, \tag{A.28}$$

(transverse NLO) × (longitudinal LO):

$$(\phi_6)^i_{TL} = 2\bar{x}\left[(1-2x)p^i_{\bar{q}2\perp}J_0 - J^i_{1\perp}\right]\,, \tag{A.29}$$

(transverse NLO) × (transverse LO):

$$\begin{aligned}(\phi_6)^{ij}_{TT} = {}& \bar{x}\left[(x-\bar{x})^2 p^i_{\bar{q}2\perp}p^j_{q1'\perp} - g^{ij}_\perp(\vec{p}_{\bar{q}2}\cdot\vec{p}_{q1'}) - p^i_{q1'\perp}p^j_{\bar{q}2\perp}\right]J_0 \\ & + \bar{x}\left[(x-\bar{x})p^j_{q1'\perp}g^i_{\perp k} - p_{q1'\perp k}g^{ij}_\perp + p^i_{q1'\perp}g^j_{\perp k}\right]J^k_{1\perp}\,.\end{aligned} \tag{A.30}$$

We introduced

$$J^k_{1\perp} = \frac{(x-z)^2}{x^2}\frac{q^k_\perp}{\vec{q}^{\,2}}\ln\left(\frac{\vec{p}^{\,2}_{\bar{q}2}+x\bar{x}Q^2}{\vec{p}^{\,2}_{\bar{q}2}+x\bar{x}Q^2+\frac{x^2\bar{x}}{z(x-z)}\vec{q}^{\,2}}\right), \tag{A.31}$$

and

$$J_0 = \frac{z}{x(\vec{p}^{\,2}_{\bar{q}2}+x\bar{x}Q^2)} - \frac{2x(x-z)+z^2}{xz(\vec{p}^{\,2}_{\bar{q}2}+x\bar{x}Q^2)}\ln\left(\frac{x^2\bar{x}\mu^2}{z(x-z)(\vec{p}^{\,2}_{\bar{q}2}+x\bar{x}Q^2)+x^2\bar{x}\vec{q}^{\,2}}\right). \tag{A.32}$$

B Real correction

Here we present the convoluted impact factors from section 5.

B.1 LL photon transition

$$\begin{aligned}\Phi^+_4(p_{1\perp},p_{2\perp},p_{3\perp})\,\Phi^+_4(p'_{1\perp},p'_{2\perp},p'_{3\perp})^* = {}& \frac{8p_\gamma^{+4}}{z^2\left(\frac{\vec{p}^{\,2}_{\bar{q}2'}}{x_{\bar{q}}(1-x_{\bar{q}})}+Q^2\right)\left(Q^2+\frac{\vec{p}^{\,2}_{q1'}}{x_q}+\frac{\vec{p}^{\,2}_{\bar{q}2'}}{x_{\bar{q}}}+\frac{\vec{p}^{\,2}_{g3'}}{z}\right)} \\ \times\Bigg[& \frac{x_{\bar{q}}\left(dz^2+4x_q\left(x_q+z\right)\right)\left(x_q\vec{p}_{g3}-z\vec{p}_{q1}\right)\left(x_q\vec{p}_{g3'}-z\vec{p}_{q1'}\right)}{x_q\left(x_q+z\right)^2\left(\frac{(\vec{p}_{g3}+\vec{p}_{q1})^2}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{(\vec{p}_{g3}+\vec{p}_{q1})^2}{x_{\bar{q}}}+\frac{\vec{p}^{\,2}_{g3}}{z}+\frac{\vec{p}^{\,2}_{q1}}{x_q}+Q^2\right)} \\ & - \frac{(4x_qx_{\bar{q}}+2z-dz^2)(x_{\bar{q}}\vec{p}_{g3}-z\vec{p}_{\bar{q}2})(x_q\vec{p}_{g3'}-z\vec{p}_{q1'})}{(x_{\bar{q}}+z)\left(x_q+z\right)\left(\frac{(\vec{p}_{\bar{q}2}+\vec{p}_{g3})^2}{x_q(x_{\bar{q}}+z)}+Q^2\right)\left(\frac{(\vec{p}_{\bar{q}2}+\vec{p}_{g3})^2}{x_q}+\frac{\vec{p}^{\,2}_{g3}}{z}+\frac{\vec{p}^{\,2}_{\bar{q}2}}{x_{\bar{q}}}+Q^2\right)}\Bigg] + (q\leftrightarrow\bar{q}).\end{aligned} \tag{B.1}$$

Now $(q\leftrightarrow\bar{q})$ stands for $p_q\leftrightarrow p_{\bar{q}}$, $p_1^{(\prime)}\leftrightarrow p_2^{(\prime)}$, $x_q\leftrightarrow x_{\bar{q}}$.

$$\Phi^+_3(p_{1\perp},p_{2\perp})\,\Phi^+_3(p'_{1\perp},p'_{2\perp})^* = \Phi^+_4(p_{1\perp},p_{2\perp},0)\,\Phi^+_4(p'_{1\perp},p'_{2\perp},0)^* + A^{++} + B^{++}. \tag{B.2}$$

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Here the first term in the r.h.s. is responsible for the emission of the gluon before crossing the shockwave, A describes the emission after the shockwave and B is the interference term. A and B are given by:

$$A^{++} = \frac{8x_{\bar{q}}p_\gamma^{+4}\left(dz^2+4x_q\left(x_q+z\right)\right)}{x_q(\vec{p}_g-\frac{z\vec{p}_q}{x_q})^2\left(\frac{\vec{p}_{\bar{q}2}{}^2}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{\vec{p}_{\bar{q}2'}{}^2}{x_{\bar{q}}(x_q+z)}+Q^2\right)}$$
$$-\frac{8p_\gamma^{+4}\left(2z-dz^2+4x_qx_{\bar{q}}\right)(\vec{p}_g-\frac{z\vec{p}_q}{x_q})(\vec{p}_g-\frac{z\vec{p}_{\bar{q}}}{x_{\bar{q}}})}{(\vec{p}_g-\frac{z\vec{p}_q}{x_q})^2(\vec{p}_g-\frac{z\vec{p}_{\bar{q}}}{x_{\bar{q}}})^2\left(\frac{\vec{p}_{\bar{q}2'}{}^2}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{\vec{p}_{q1}{}^2}{x_q(x_{\bar{q}}+z)}+Q^2\right)}+(q\leftrightarrow\bar{q})\,, \tag{B.3}$$

and

$$B^{++} = \Bigg[\frac{8p_\gamma^{+4}}{z\left(x_q+z\right)\left(\frac{\vec{p}_{\bar{q}2'}^{\,2}}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{\vec{p}_{q1'}^{\,2}}{x_q}+\frac{\vec{p}_{\bar{q}2'}^{\,2}}{x_{\bar{q}}}+\frac{\vec{p}_g^{\,2}}{z}+Q^2\right)}$$
$$\times\Bigg\{\frac{\left(4x_qx_{\bar{q}}+z(2-dz)\right)(\vec{p}_g-\frac{z}{x_{\bar{q}}}\vec{p}_{\bar{q}})(x_q\vec{p}_g-z\vec{p}_{q1'})}{(\vec{p}_g-\frac{z\vec{p}_{\bar{q}}}{x_{\bar{q}}})^2\left(\frac{\vec{p}_{q1}^{\,2}}{x_q(x_{\bar{q}}+z)}+Q^2\right)}$$
$$-\frac{x_{\bar{q}}\left(dz^2+4x_q\left(x_q+z\right)\right)(\vec{p}_g-\frac{z}{x_q}\vec{p}_q)(\vec{p}_g-\frac{z}{x_q}\vec{p}_{q1'})}{(\vec{p}_g-\frac{z\vec{p}_q}{x_q})^2\left(\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}(x_q+z)}+Q^2\right)}\Bigg\}+(q\leftrightarrow\bar{q})\Bigg]$$
$$+(1\leftrightarrow1',2\leftrightarrow2'). \tag{B.4}$$

On one hand in the collinear region (7.1) only the first term of A^{++} in (B.3) gives a nonvanishing contribution in the small cone approximation. Using the variables defined in (7.4), (7.10), (7.11), the first line of A^{++} becomes

$$\frac{8x_{\bar{q}}p_\gamma^{+4}\left(dz^2+4x_q\left(x_q+z\right)\right)}{x_q(\vec{p}_g-\frac{z\vec{p}_q}{x_q})^2\left(\frac{\vec{p}_{\bar{q}2}{}^2}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{\vec{p}_{\bar{q}2'}{}^2}{x_{\bar{q}}(x_q+z)}+Q^2\right)}=\frac{8(x_j-z)(1-x_j)p_\gamma^{+4}\left(dz^2+4x_j\left(x_j-z\right)\right)}{x_j^2\Delta_q^2\left(\frac{\vec{p}_{\bar{j}2}{}^2}{x_j(1-x_j)}+Q^2\right)\left(\frac{\vec{p}_{\bar{j}2'}{}^2}{x_j(1-x_j)}+Q^2\right)}$$
$$=\Phi_0^+\Phi_0^{+*}\frac{1}{4}\frac{\left(x_j-z\right)\left(dz^2+4x_j\left(x_j-z\right)\right)}{x_j^3\Delta_q^2}, \tag{B.5}$$

which coincides with the integrand for n_j defined in (7.9).
On the other hand if we use the soft gluon approximation by renaming

$$\vec{p}_g=z\,\vec{u} \tag{B.6}$$

and taking $z\to0$, we obtain

$$\Phi_3^+(p_{1\perp},p_{2\perp})\,\Phi_3^+(p'_{1\perp},p'_{2\perp})^*|_{z\to0}$$
$$=\frac{1}{z^2}\Phi_0^+(p_{1\perp},p_{2\perp})\,\Phi_0^+(p'_{1\perp},p'_{2\perp})^*\frac{\left(\frac{\vec{p}_q}{x_q}-\frac{\vec{p}_{\bar{q}}}{x_{\bar{q}}}\right)^2}{\left(\vec{u}-\frac{\vec{p}_q}{x_q}\right)^2\left(\vec{u}-\frac{\vec{p}_{\bar{q}}}{x_{\bar{q}}}\right)^2}+O(z^{-1}), \tag{B.7}$$

which is the soft gluon emission factor.

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B.2 TL photon transition

$$\Phi_4^i(p_{1\perp},p_{2\perp},p_{3\perp})\,\Phi_4^+(p'_{1\perp},p'_{2\perp},p'_{3\perp})^* = \frac{-4p_\gamma^{+3}}{\left(Q^2+\frac{\vec{p}_{g3}^{\,2}}{z}+\frac{\vec{p}_{q1}^{\,2}}{x_q}+\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}}\right)\left(Q^2+\frac{\vec{p}_{g3'}^{\,2}}{z}+\frac{\vec{p}_{q1'}^{\,2}}{x_q}+\frac{\vec{p}_{\bar{q}2'}^{\,2}}{x_{\bar{q}}}\right)}$$
$$\times\left(\frac{z\left((\vec{P}\vec{p}_{q1})G^i_\perp-(\vec{G}\vec{p}_{q1})P^i_\perp\right)(dz+4x_q-4)-(\vec{G}\vec{P})p^i_{q1\perp}(2x_q-1)\left(4(x_q-1)x_{\bar{q}}-dz^2\right)}{z^2x_{\bar{q}}(z+x_{\bar{q}})^3\left(Q^2+\frac{\vec{p}_{q1}^{\,2}}{x_q(z+x_{\bar{q}})}\right)\left(Q^2+\frac{\vec{p}_{q1'}^{\,2}}{x_q(z+x_{\bar{q}})}\right)}\right.$$
$$+\frac{z\left((\vec{P}\vec{p}_{q1})H^i_\perp-(\vec{H}\vec{p}_{q1})P^i_\perp\right)(dz+4x_q-2)-(\vec{H}\vec{P})p^i_{q1\perp}(2x_q-1)(z(2-dz)+4x_qx_{\bar{q}})}{z^2x_q(z+x_q)(z+x_{\bar{q}})^2\left(Q^2+\frac{\vec{p}_{\bar{q}2'}^{\,2}}{(z+x_q)x_{\bar{q}}}\right)\left(Q^2+\frac{\vec{p}_{q1}^{\,2}}{x_q(z+x_{\bar{q}})}\right)}$$
$$\left.+\frac{H^i_\perp\left(z(zd+d-2)+x_q(2-4x_{\bar{q}})\right)x_{\bar{q}}}{z(z+x_q)^2(z+x_{\bar{q}})\left(Q^2+\frac{\vec{p}_{\bar{q}2'}^{\,2}}{(z+x_q)x_{\bar{q}}}\right)}\right)+(q\leftrightarrow\bar{q}). \tag{B.8}$$

Here

$$G^i_\perp=x_{\bar{q}}p^i_{g3'\perp}-zp^i_{\bar{q}2'\perp},\quad H^i_\perp=x_qp^i_{g3'\perp}-zp^i_{q1'\perp},\quad P^i_\perp=x_{\bar{q}}p^i_{g3\perp}-zp^i_{\bar{q}2\perp}. \tag{B.9}$$

Similarly to the longitudinal to longitudinal photon transition, we write

$$\Phi_3^i(p_{1\perp},p_{2\perp})\,\Phi_3^+(p'_{1\perp},p'_{2\perp})^*=\Phi_4^i(p_{1\perp},p_{2\perp},0)\,\Phi_4^+(p'_{1\perp},p'_{2\perp},0)^*+A^{i+}+B^{i+}, \tag{B.10}$$

where A and B are now given by

$$A^{i+}=\frac{4p_\gamma^{+3}x_q}{\vec{\Delta}_q^2\vec{\Delta}_{\bar{q}}^2(x_q+z)^2(x_{\bar{q}}+z)\left(\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{\vec{p}_{q1'}^{\,2}}{x_q(x_{\bar{q}}+z)}+Q^2\right)}$$
$$\times\Big[z(4x_{\bar{q}}+dz-2)\left(\Delta^i_{q\perp}(\vec{p}_{\bar{q}2}\vec{\Delta}_{\bar{q}})-\Delta^i_{\bar{q}\perp}(\vec{p}_{\bar{q}2}\vec{\Delta}_q)\right)$$
$$+(2x_{\bar{q}}-1)(\vec{\Delta}_q\vec{\Delta}_{\bar{q}})p^i_{\bar{q}2\perp}(4x_qx_{\bar{q}}+z(2-dz))\Big]$$
$$-\frac{4p_\gamma^{+3}x_q(2x_{\bar{q}}-1)\left(dz^2+4x_q(x_q+z)\right)p^i_{\bar{q}2\perp}}{\vec{\Delta}_q^2(x_q+z)^3\left(\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{\vec{p}_{\bar{q}2'}^{\,2}}{x_{\bar{q}}(x_q+z)}+Q^2\right)}+(q\leftrightarrow\bar{q}) \tag{B.11}$$

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and

$$
\begin{aligned}
B^{i+} = 4p_\gamma^{+3} \Bigg(& \frac{\Delta_{q\perp}^i x_q x_{\bar q}\left(dz^2+dz-2z+2x_q-4x_q x_{\bar q}\right)}{\vec\Delta_q^2\,(z+x_q)^{\,2}\,(z+x_{\bar q})\left(Q^2+\frac{\vec p_g^{\,2}}{z}+\frac{\vec p_{q1}^{\,2}}{x_q}+\frac{\vec p_{\bar q2}^{\,2}}{x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{\bar q2'}^{\,2}}{(z+x_q)x_{\bar q}}\right)} \\
& - \frac{(\vec J\vec\Delta_q)p_{\bar q2\perp}^i\left(dz^2+4x_q\,(z+x_q)\right)(1-2x_{\bar q})+z\left((\vec J\vec p_{\bar q2})\Delta_{q\perp}^i-(\vec p_{\bar q2}\vec\Delta_q)J_\perp^i\right)(dz+4x_{\bar q}-4)}{z\,(z+x_q)^{\,3}\vec\Delta_q^2\left(Q^2+\frac{\vec p_g^{\,2}}{z}+\frac{\vec p_{q1'}^{\,2}}{x_q}+\frac{\vec p_{\bar q2'}^{\,2}}{x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{\bar q2}^{\,2}}{(z+x_q)x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{\bar q2'}^{\,2}}{(z+x_q)x_{\bar q}}\right)} \\
& - \frac{x_q\Big(z\Big((\vec K\vec p_{\bar q2})\Delta_{q\perp}^i-(\vec p_{\bar q2}\vec\Delta_q)K_\perp^i\Big)(dz+4x_{\bar q}-2)+(\vec K\vec\Delta_q)p_{\bar q2\perp}^i(1-2x_{\bar q})(z(dz-2)-4x_q x_{\bar q})\Big)}{z\,(z+x_q)^{\,2}x_{\bar q}\,(z+x_{\bar q})\,\vec\Delta_q^2\left(Q^2+\frac{\vec p_g^{\,2}}{z}+\frac{\vec p_{q1'}^{\,2}}{x_q}+\frac{\vec p_{\bar q2'}^{\,2}}{x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{\bar q2}^{\,2}}{(z+x_q)x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{q1'}^{\,2}}{x_q(z+x_{\bar q})}\right)} \\
& - \frac{z\Big((\vec p_{q1}\vec\Delta_q)X_\perp^i-(\vec X\vec p_{q1})\Delta_{q\perp}^i\Big)(dz+4x_q-2)+(\vec X\vec\Delta_q)p_{q1\perp}^i\,(1-2x_q)\,(z(dz-2)-4x_q x_{\bar q})}{z\vec\Delta_q^2\,(z+x_q)\,(z+x_{\bar q})^{\,2}\left(Q^2+\frac{\vec p_g^{\,2}}{z}+\frac{\vec p_{q1}^{\,2}}{x_q}+\frac{\vec p_{\bar q2}^{\,2}}{x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{\bar q2'}^{\,2}}{(z+x_q)x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{q1}^{\,2}}{x_q(z+x_{\bar q})}\right)} \\
& + \frac{z\left((\vec X\vec p_{q1})\Delta_{\bar q\perp}^i-(\vec p_{q1}\vec\Delta_{\bar q})X_\perp^i\right)(dz+4x_q-4)-(\vec X\vec\Delta_{\bar q})p_{q1\perp}^i\,(2x_q-1)\big(4\,(x_q-1)\,x_{\bar q}-dz^2\big)}{z\,(z+x_{\bar q})^{\,3}\vec\Delta_{\bar q}^2\left(Q^2+\frac{\vec p_g^{\,2}}{z}+\frac{\vec p_{q1}^{\,2}}{x_q}+\frac{\vec p_{\bar q2}^{\,2}}{x_{\bar q}}\right)\left(Q^2+\frac{\vec p_{q1}^{\,2}}{x_q(z+x_{\bar q})}\right)\left(Q^2+\frac{\vec p_{q1'}^{\,2}}{x_q(z+x_{\bar q})}\right)}\Bigg) \\
& + (q\leftrightarrow\bar q).
\end{aligned}
\tag{B.12}
$$

Here we used the following variables:

$$
\begin{aligned}
& X_\perp^i = x_{\bar q}p_{g\perp}^i - zp_{\bar q2\perp}^i = P_\perp^i|_{p_3=0}, \quad J_\perp^i = x_q p_{g\perp}^i - zp_{q1'\perp}^i = H_\perp^i|_{p_3'=0}, \\
& K_\perp^i = x_{\bar q}p_{g\perp}^i - zp_{\bar q2'\perp}^i = G_\perp^i|_{p_3'=0}.
\end{aligned}
\tag{B.13}
$$

Similarly to the case of a longitudinal photon, in the collinear region i.e. when $\Delta_q \to 0$ only the last line of A^{i+} gives a divergent contribution, which will be proportional to the square of the Born impact factor. Using the jet variables (7.10), (7.11) we write

$$
\Phi_3^i(p_{1\perp},p_{2\perp})\Phi_3^k(p_{1\perp}',p_{2\perp}')^* = \Phi_0^i\Phi_0^{k*}\frac{1}{4}\frac{(x_j-z)\left(dz^2+4x_j\,(x_j-z)\right)}{x_j^3\Delta_q^2}+O\left(\Delta_q^{-1}\right). \tag{B.14}
$$

Again the integrand of n_j appears as in (7.9). In the soft gluon region we can also write, using (B.6):

$$
\Phi_3^i\left(p_{1\perp},p_{2\perp}\right)\Phi_3^+\left(p_{1\perp}',p_{2\perp}'\right)^*|_{z\to 0} = \frac{1}{z^2}\Phi_0^i\Phi_0^{+*}\frac{\left(\frac{\vec p_q}{x_q}-\frac{\vec p_{\bar q}}{x_{\bar q}}\right)^2}{\left(\vec u-\frac{\vec p_q}{x_q}\right)^2\left(\vec u-\frac{\vec p_{\bar q}}{x_{\bar q}}\right)^2}+O(z^{-1}), \tag{B.15}
$$

which is the soft gluon emission factor, coming from A^{i+}.

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B.3 TT photon transition

$$\Phi_4^i(p_{1\perp},p_{2\perp},p_{3\perp})\Phi_4^k(p'_{1\perp},p'_{2\perp},p'_{3\perp})^* = \Bigg(\frac{p_\gamma^{+2}}{\left(Q^2+\frac{\vec{p}_{g3}^{\,2}}{z}+\frac{\vec{p}_{q1}^{\,2}}{x_q}+\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}}\right)\left(Q^2+\frac{\vec{p}_{g3'}^{\,2}}{z}+\frac{\vec{p}_{q1'}^{\,2}}{x_q}+\frac{\vec{p}_{\bar{q}2'}^{\,2}}{x_{\bar{q}}}\right)}$$

$$\times\Bigg[-\frac{g_\perp^{ik}x_qx_{\bar{q}}\,(zd+d-2+2x_{\bar{q}})}{(z+x_q)^{\,2}\,(z+x_{\bar{q}})}-\frac{2P_\perp^k p_{q1\perp}^i\,(1-2x_q)}{z\,(z+x_{\bar{q}})^{\,2}\left(Q^2+\frac{\vec{p}_{q1}^{\,2}}{x_q(z+x_{\bar{q}})}\right)}\left(\frac{(d-2)z-2x_{\bar{q}}}{z+x_{\bar{q}}}+\frac{dz+2x_{\bar{q}}}{z+x_q}\right)$$

$$-\frac{2\left(g_\perp^{ik}(\vec{P}\vec{p}_{q1})+P_\perp^i p_{q1\perp}^k\right)}{z\,(z+x_{\bar{q}})^{\,2}\left(Q^2+\frac{\vec{p}_{q1}^{\,2}}{x_q(z+x_{\bar{q}})}\right)}\left(\frac{(d-4)z-2x_{\bar{q}}}{z+x_q}+\frac{(d-2)z-2x_{\bar{q}}}{z+x_{\bar{q}}}\right)$$

$$-\frac{1}{z^2x_q\,(z+x_q)^{\,2}x_{\bar{q}}\,(z+x_{\bar{q}})^{\,2}\left(Q^2+\frac{\vec{p}_{\bar{q}2'}^{\,2}}{(z+x_q)x_{\bar{q}}}\right)\left(Q^2+\frac{\vec{p}_{q1}^{\,2}}{x_q(z+x_{\bar{q}})}\right)}\Big\{(\vec{H}\vec{P})\left[p_{q1\perp}^i p_{\bar{q}2'\perp}^k\,(1-2x_q)\right.$$

$$\times\,(1-2x_{\bar{q}})\,(z(2-dz)+4x_qx_{\bar{q}})+(g_\perp^{ik}(\vec{p}_{q1}\vec{p}_{\bar{q}2'})+p_{q1\perp}^k p_{\bar{q}2'\perp}^i)\,(z(2-(d-4)z)+4x_qx_{\bar{q}})\Big]$$

$$+\,((d-4)z-2)\left[z(\vec{H}\vec{p}_{\bar{q}2'})(g_\perp^{ik}(\vec{P}\vec{p}_{q1})+P_\perp^i p_{q1\perp}^k)+zH_\perp^k\left((\vec{P}\vec{p}_{q1})p_{\bar{q}2'\perp}^i-(\vec{p}_{q1}\vec{p}_{\bar{q}2'})P_\perp^i\right)\right]$$

$$+\,((d-4)z+2)\left[zH^i\left((\vec{P}\vec{p}_{\bar{q}2'})p_{q1\perp}^k-(\vec{p}_{q1}\vec{p}_{\bar{q}2'})P_\perp^k\right)+z(\vec{H}\vec{p}_{q1})(g_\perp^{ik}(\vec{P}\vec{p}_{\bar{q}2'})+P_\perp^k p_{\bar{q}2'\perp}^i)\right]$$

$$+\,2z\left((\vec{H}\vec{p}_{\bar{q}2'})P_\perp^k-(\vec{P}\vec{p}_{\bar{q}2'})H_\perp^k\right)p_{q1\perp}^i\,(1-2x_q)\,(dz+4x_{\bar{q}}-2)\Big\}$$

$$-\frac{1}{z^2x_qx_{\bar{q}}\,(z+x_{\bar{q}})^{\,4}\left(Q^2+\frac{\vec{p}_{q1}^{\,2}}{x_q(z+x_{\bar{q}})}\right)\left(Q^2+\frac{\vec{p}_{q1'}^{\,2}}{x_q(z+x_{\bar{q}})}\right)}\Big\{z\,((d-4)z-4x_{\bar{q}})$$

$$\times\Big[g_\perp^{ik}\left((\vec{G}\vec{p}_{q1'})(\vec{P}\vec{p}_{q1})-(\vec{G}\vec{p}_{q1})(\vec{P}\vec{p}_{q1'})\right)+(\vec{p}_{q1}\vec{p}_{q1'})\left(G_\perp^iP_\perp^k-G_\perp^kP_\perp^i\right)$$

$$+\,2(\vec{G}\vec{p}_{q1'})\left(P_\perp^ip_{q1\perp}^k+P_\perp^kp_{q1\perp}^i\,(1-2x_q)\right)-2(\vec{G}\vec{p}_{q1})\left(P_\perp^kp_{q1'\perp}^i+P_\perp^ip_{q1'\perp}^k\,(1-2x_q)\right)\Big]$$

$$+\,(\vec{G}\vec{P})\left[p_{q1\perp}^kp_{q1'\perp}^i-p_{q1\perp}^ip_{q1'\perp}^k\,(1-2x_q)^2+g_\perp^{ik}\,(\vec{p}_{q1}\vec{p}_{q1'})\right]\left(dz^2+4x_{\bar{q}}\,(z+x_{\bar{q}})\right)\Big\}\Bigg]$$

$$+\,(1\leftrightarrow1',2\leftrightarrow2',3\leftrightarrow3',i\leftrightarrow k)\Bigg)+(q\leftrightarrow\bar{q}). \tag{B.16}$$

Once more we write

$$\Phi_3^i(p_{1\perp},p_{2\perp})\Phi_3^k(p'_{1\perp},p'_{2\perp})^* = \Phi_4^i(p_{1\perp},p_{2\perp},0)\Phi_4^k(p'_{1\perp},p'_{2\perp},0)^* + A^{ik}+B^{ik}. \tag{B.17}$$

Then

$$A^{ik} = \frac{-2p_\gamma^{+2}}{\vec{\Delta}_q^2\vec{\Delta}_{\bar{q}}^2\,(x_q+z)^{\,2}\,(x_{\bar{q}}+z)^{\,2}\left(\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}(x_q+z)}+Q^2\right)\left(\frac{\vec{p}_{q1'}^{\,2}}{x_q(x_{\bar{q}}+z)}+Q^2\right)}\Big\{z((d-4)z-2)$$

$$\times\Big[(\vec{p}_{q1'}\vec{\Delta}_{\bar{q}})\left((\vec{p}_{\bar{q}2}\vec{\Delta}_q)g_\perp^{ik}+\Delta_{q\perp}^ip_{\bar{q}2\perp}^k\right)-(\vec{\Delta}_q\vec{\Delta}_{\bar{q}})\left((\vec{p}_{q1'}\vec{p}_{\bar{q}2})g_\perp^{ik}+p_{q1'\perp}^ip_{\bar{q}2\perp}^k\right)$$

$$+\,\Delta_{\bar{q}\perp}^kp_{q1'\perp}^i(\vec{p}_{\bar{q}2}\vec{\Delta}_q)-\Delta_{q\perp}^i\Delta_{\bar{q}\perp}^k(\vec{p}_{q1'}\vec{p}_{\bar{q}2})\Big]+(\vec{\Delta}_q\vec{\Delta}_{\bar{q}})$$

$$\times\Big[(2x_q-1)\,(2x_{\bar{q}}-1)\,p_{q1'\perp}^kp_{\bar{q}2\perp}^i\,(4x_qx_{\bar{q}}+z(2-dz))+4x_qx_{\bar{q}}\left((\vec{p}_{q1'}\vec{p}_{\bar{q}2})g_\perp^{ik}+p_{q1'\perp}^ip_{\bar{q}2\perp}^k\right)\Big]$$

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$$
\begin{aligned}
&+\left((\vec{p}_{q1'}\vec{\Delta})\left((\vec{p}_{\bar{q}2}\vec{\Delta}_{\bar{q}})g_{\perp}^{ik}+\Delta_{\bar{q}\perp}^{i}p_{\bar{q}2\perp}^{k}\right)+\Delta_{q\perp}^{k}p_{q1'\perp}^{i}(\vec{p}_{\bar{q}2}\vec{\Delta}_{\bar{q}})-\Delta_{q\perp}^{k}\Delta_{\bar{q}\perp}^{i}(\vec{p}_{q1'}\vec{p}_{\bar{q}2})\right)\\
&\times z((d-4)z+2)+z\left(2x_{\bar{q}}-1\right)\left(dz+4x_{q}-2\right)p_{\bar{q}2\perp}^{i}\left(\Delta_{\bar{q}\perp}^{k}(\vec{p}_{q1'}\vec{\Delta}_{q})-\Delta_{q\perp}^{k}(\vec{p}_{q1'}\vec{\Delta}_{\bar{q}})\right)\\
&+z\left(2x_{q}-1\right)p_{q1'\perp}^{k}\left(4x_{\bar{q}}+dz-2\right)\left(\Delta_{q\perp}^{i}(\vec{p}_{\bar{q}2}\vec{\Delta}_{\bar{q}})-\Delta_{\bar{q}\perp}^{i}(\vec{p}_{\bar{q}2}\vec{\Delta}_{q})\right)\Big\}\\
&-\frac{2x_{q}p_{\gamma}^{+2}\left(dz^{2}+4x_{q}\left(x_{q}+z\right)\right)\left((\vec{p}_{\bar{q}2}\vec{p}_{\bar{q}2'})g_{\perp}^{ik}-(1-2x_{\bar{q}})^{2}p_{\bar{q}2\perp}^{i}p_{\bar{q}2'\perp}^{k}+p_{\bar{q}2'\perp}^{i}p_{\bar{q}2\perp}^{k}\right)}{x_{\bar{q}}\vec{\Delta}_{q}^{2}\left(x_{q}+z\right)^{4}\left(\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}(x_{q}+z)}+Q^{2}\right)\left(\frac{\vec{p}_{\bar{q}2'}^{\,2}}{x_{\bar{q}}(x_{q}+z)}+Q^{2}\right)}\\
&+(q\leftrightarrow\bar{q}),
\end{aligned}
\tag{B.18}
$$

and

$$
\begin{aligned}
B^{ik}=&\left(\frac{2p_{\gamma}^{+2}}{\vec{\Delta}_{q}^{2}\left(Q^{2}+\frac{\vec{p}_{g}^{\,2}}{z}+\frac{\vec{p}_{q1}^{\,2}}{x_{q}}+\frac{\vec{p}_{\bar{q}2}^{\,2}}{x_{\bar{q}}}\right)\left(Q^{2}+\frac{\vec{p}_{\bar{q}2'}^{\,2}}{(z+x_{q})x_{\bar{q}}}\right)}\right.\\
&\times\left[\frac{((d-2)z-2x_{q})\,x_{q}}{(z+x_{q})^{3}}\left(g_{\perp}^{ik}(\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})+p_{\bar{q}2'\perp}^{i}\Delta_{q\perp}^{k}+p_{\bar{q}2'\perp}^{k}\Delta_{q\perp}^{i}\left(1-2x_{\bar{q}}\right)\right)\right.\\
&+\frac{x_{q}\left(((d-4)z-2x_{q})\left(g_{\perp}^{ik}(\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})+p_{\bar{q}2'\perp}^{i}\Delta_{q\perp}^{k}\right)+p_{\bar{q}2'\perp}^{k}\Delta_{q\perp}^{i}\left(dz+2x_{q}\right)\left(1-2x_{\bar{q}}\right)\right)}{(z+x_{q})^{2}(z+x_{\bar{q}})}\\
&-\frac{1}{z\,(z+x_{q})^{2}x_{\bar{q}}\,(z+x_{\bar{q}})^{2}\left(Q^{2}+\frac{\vec{p}_{q1}^{\,2}}{x_{q}(z+x_{\bar{q}})}\right)}\Big\{z((d-4)z+2)\\
&\times\left[p_{q1\perp}^{i}\left((\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})X_{\perp}^{k}-(\vec{X}\vec{p}_{\bar{q}2'})\Delta_{q\perp}^{k}\right)(2x_{q}-1)-(\vec{X}\vec{p}_{\bar{q}2'})\left(g_{\perp}^{ik}(\vec{p}_{q1}\vec{\Delta}_{q})+p_{q1\perp}^{k}\Delta_{q\perp}^{i}\right)\right.\\
&\left.-X_{\perp}^{k}\left((\vec{p}_{q1}\vec{\Delta}_{q})p_{\bar{q}2'\perp}^{i}-(\vec{p}_{q1}\vec{p}_{\bar{q}2'})\Delta_{q\perp}^{i}\right)\right]+4x_{q}z\left(1-2x_{q}\right)p_{q1\perp}^{i}\left((\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})X_{\perp}^{k}-(\vec{X}\vec{p}_{\bar{q}2'})\Delta_{q\perp}^{k}\right)\\
&+z\left(1-2x_{\bar{q}}\right)\left(dz+4x_{q}-2\right)p_{\bar{q}2'\perp}^{k}\left((\vec{p}_{q1}\vec{\Delta}_{q})X_{\perp}^{i}-(\vec{X}\vec{p}_{q1})\Delta_{q\perp}^{i}\right)-z((d-4)z-2)\\
&\times\left[\left(g_{\perp}^{ik}(\vec{X}\vec{p}_{q1})+X_{\perp}^{i}p_{q1\perp}^{k}\right)(\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})+\left((\vec{X}\vec{p}_{q1})p_{\bar{q}2'\perp}^{i}-(\vec{p}_{q1}\vec{p}_{\bar{q}2'})X_{\perp}^{i}\right)\Delta_{q\perp}^{k}\right]\\
&+(\vec{X}\vec{\Delta}_{q})p_{q1\perp}^{i}p_{\bar{q}2'\perp}^{k}\left(1-2x_{q}\right)\left(1-2x_{\bar{q}}\right)\left(z(dz-2)-4x_{q}x_{\bar{q}}\right)\\
&-(\vec{X}\vec{\Delta}_{q})\left(g_{\perp}^{ik}(\vec{p}_{q1}\vec{p}_{\bar{q}2'})+p_{q1\perp}^{k}p_{\bar{q}2'\perp}^{i}\right)\left(z(2-(d-4)z)+4x_{q}x_{\bar{q}}\right)\Big\}\\
&-\frac{1}{z\,(z+x_{q})^{4}\left(Q^{2}+\frac{\vec{p}_{\bar{q}2}^{\,2}}{(z+x_{q})x_{\bar{q}}}\right)x_{\bar{q}}}\Big\{z\left(dz+4x_{\bar{q}}-4\right)\Big[\left(1-2x_{\bar{q}}\right)\\
&\times\left(p_{\bar{q}2'\perp}^{k}\left((\vec{p}_{\bar{q}2}\vec{\Delta}_{q})V_{\perp}^{i}-(\vec{V}\vec{p}_{\bar{q}2})\Delta_{q\perp}^{i}\right)+p_{\bar{q}2\perp}^{i}\left((\vec{V}\vec{p}_{\bar{q}2'})\Delta_{q\perp}^{k}-(\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})V_{\perp}^{k}\right)\right)\\
&+V_{\perp}^{k}\left((\vec{p}_{\bar{q}2}\vec{\Delta}_{q})p_{\bar{q}2'\perp}^{i}-(\vec{p}_{\bar{q}2}\vec{p}_{\bar{q}2'})\Delta_{q\perp}^{i}\right)+\left((\vec{p}_{\bar{q}2}\vec{p}_{\bar{q}2'})V_{\perp}^{i}-(\vec{V}\vec{p}_{\bar{q}2})p_{\bar{q}2'\perp}^{i}\right)\Delta_{q\perp}^{k}\\
&+g_{\perp}^{ik}\left((\vec{V}\vec{p}_{\bar{q}2'})(\vec{p}_{\bar{q}2}\vec{\Delta}_{q})-(\vec{V}\vec{p}_{\bar{q}2})(\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})\right)+p_{\bar{q}2\perp}^{k}\left((\vec{V}\vec{p}_{\bar{q}2'})\Delta_{q\perp}^{i}-(\vec{p}_{\bar{q}2'}\vec{\Delta}_{q})V_{\perp}^{i}\right)\Big]\\
&+(\vec{V}\vec{\Delta}_{q})\left(p_{\bar{q}2\perp}^{i}p_{\bar{q}2'\perp}^{k}\left(1-2x_{\bar{q}}\right)^{2}-g_{\perp}^{ik}(\vec{p}_{\bar{q}2}\vec{p}_{\bar{q}2'})-p_{\bar{q}2\perp}^{k}p_{\bar{q}2'\perp}^{i}\right)\left(dz^{2}-4x_{q}\left(x_{\bar{q}}-1\right)\right)\Big\}\Big]\\
&\left.+\ (1\leftrightarrow1',2\leftrightarrow2',i\leftrightarrow k)\right)+(q\leftrightarrow\bar{q}).
\end{aligned}
\tag{B.19}
$$

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Here we introduced

$$V_\perp^i = x_q p_{g\perp}^i - z p_{q1\perp}^i. \tag{B.20}$$

Once again in the collinear region, i.e. when $\Delta_q \to 0$, only the last line of A^{ik} gives a divergent contribution, which is proportional to the square of the Born impact factors. Using the jet variables (7.10), (7.11) we obtain

$$\Phi_3^i(p_{1\perp}, p_{2\perp})\Phi_3^k(p'_{1\perp}, p'_{2\perp})^* = \Phi_0^i \Phi_0^{k*} \frac{1}{4} \frac{(x_j - z)\left(dz^2 + 4x_j(x_j - z)\right)}{x_j^3 \Delta_q^2} + O\left(\Delta_q^{-1}\right). \tag{B.21}$$

Again the integrand of n_j appears as in (7.9).
In the soft gluon region we can also write, using (B.6):

$$\Phi_3^i\left(p_{1\perp}, p_{2\perp}\right)\Phi_3^k\left(p'_{1\perp}, p'_{2\perp}\right)^* |_{z\to 0} = \frac{1}{z^2}\Phi_0^i\Phi_0^{k*} \frac{\left(\frac{\vec{p}_q}{x_q} - \frac{\vec{p}_{\bar{q}}}{x_{\bar{q}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_q}{x_q}\right)^2 \left(\vec{u} - \frac{\vec{p}_{\bar{q}}}{x_{\bar{q}}}\right)^2} + O\left(z^{-1}\right), \tag{B.22}$$

which is the soft gluon emission factor.

C Integral $I(R, E)$

Here we will consider the integral (7.21)

$$I\left(R, E\right) \equiv -\int_0^{\frac{2E}{p_\gamma^+}} \frac{dz}{z} \int_{\left\{\vec{u}^{\,2} > (p_\gamma^+)^2\left(\frac{2E}{z p_\gamma^+} - 1\right)\right\} \cap \Omega_{nc}} \frac{d\vec{u}}{\pi} \frac{\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 \left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2}, \tag{C.1}$$

where

$$\Omega_{nc} = \left\{\left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2 > \frac{R^2 \vec{p}_j^{\,2}}{x_j^2}\right\} \cap \left\{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 > \frac{R^2 \vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}^2}\right\}, \tag{C.2}$$

see eq. (7.18). Introducing the variable $y = \frac{2E}{z p_\gamma^+} - 1$, one gets

$$I(R, E) = -\int_0^{+\infty} \frac{dy}{y+1} \int_{\{\vec{u}^{\,2} > y(p_\gamma^+)^2\} \cap \Omega_{nc}} \frac{d\vec{u}}{\pi} \frac{\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 \left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2}. \tag{C.3}$$

The change of the integration order plus integration w.r.t. y yields

$$I(R, E) = -\int_{\Omega_{nc}} \frac{d\vec{u}}{\pi} \frac{\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 \left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2} \ln\left(1 + \frac{\vec{u}^{\,2}}{(p_\gamma^+)^2}\right). \tag{C.4}$$

Next, introducing

$$v = \max\left(\left|\frac{\vec{p}_j}{x_j}\right|, \left|\frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right|\right), \tag{C.5}$$

one can split the integration area into 2 parts

$$\Omega_{nc} = \Omega_{nc_1} \cup \Omega_{nc_2}, \quad \Omega_{nc_1} = \{\vec{u}^{\,2} < (2v)^2\} \cap \Omega_{nc}, \quad \Omega_{nc_2} = \{\vec{u}^{\,2} > (2v)^2\}. \tag{C.6}$$

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In the first domain $\frac{\vec{u}^{\,2}}{(p_\gamma^+)^2} < \frac{(2v)^2}{(p_\gamma^+)^2} = O\left(\frac{1}{s}\right)$. Hence one can estimate this integral by putting this maximal value into the logarithm and expanding it

$$\int_{\Omega_{nc_1}} \frac{d\vec{u}}{\pi} \frac{\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 \left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2} \ln\left(1 + \frac{\vec{u}^{\,2}}{(p_\gamma^+)^2}\right) < O\left(\frac{1}{s}\right) \int_{\Omega_{nc_1}} \frac{d\vec{u}}{\pi} \frac{\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 \left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2}. \tag{C.7}$$

Although the integral in the r.h.s. of this inequality contains $\ln R$, we neglect terms $\sim O\left(\frac{\ln R}{s}\right)$ in the Regge limit. In the second domain one can write

$$\begin{aligned} &\int_{\Omega_{nc_2}} \frac{d\vec{u}}{\pi} \frac{\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{\left(\vec{u} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2 \left(\vec{u} - \frac{\vec{p}_j}{x_j}\right)^2} \ln\left(1 + \frac{\vec{u}^{\,2}}{(p_\gamma^+)^2}\right) \\ &< \int_{\Omega_{nc_2}} \frac{d\vec{u}}{\pi} \frac{\left(\frac{\vec{p}_j}{x_j} - \frac{\vec{p}_{\bar{j}}}{x_{\bar{j}}}\right)^2}{(u-v)^4} \ln\left(1 + \frac{\vec{u}^{\,2}}{(p_\gamma^+)^2}\right) \sim O\left(\frac{v^{\,2}}{(p_\gamma^+)^2}\right) = O\left(\frac{1}{s}\right), \end{aligned} \tag{C.8}$$

where the latter integral is easy to take. Finally,

$$I(R, E) = O\left(\frac{\ln R}{s}\right). \tag{C.9}$$

Therefore we neglect this contribution.

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