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Placing of the recently observed bottom strange states $B_{sJ}(6063)$ and $B_{sJ}(6114)$ in bottom spectra

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We employ the heavy quark effective theory (HQET) to give spin-parity quantum numbers for the bottom strange states $B_{sJ}(6063)$ and $B_{sJ}(6114)$ recently observed by the LHCb Collaboration [9]. By exploring flavor-independent parameters $\Delta_F^{(c)} = \Delta_F^{(b)}$ and $\lambda_F^{(c)} = \lambda_F^{(b)}$ appearing in the HQET Lagrangian, we calculate the masses of the experimentally missing bottom strange meson states $2S$, $1P$, $1D$. The parameter Δ_F appears in the HQET Lagrangian and gives the spin-averaged mass splitting between excited-state doublets (F) and ground-state doublets (H). Another parameter λ_F comes from first-order corrections in the HQET Lagrangian and gives hyperfine splittings. We also analyze these bottom strange masses by taking $1/m_Q$ corrections, which lead to modifications of parameter terms of $\Delta_F^{(b)} = \Delta_F^{(c)} + \delta\Delta_F$ and $\lambda_F^{(b)} = \lambda_F^{(c)}\delta\lambda_F$. Further, we analyze their two-body decays, couplings, and branching ratios via the emission of light pseudoscalar mesons. Based on the predicted masses and decay widths, we tentatively identify the states $B_{sJ}(6063)$ as 2^3S_1 and $B_{sJ}(6114)$ as 1^3D_1. Our predictions provide crucial information for future experimental studies.

Subject Index B60, B69

1. Introduction

During recent decades, different experimental facilities, such as LHCb, BABAR, BESIII, FOCUS, SLAC, etc., have been on a discovery spree for stimulating the spectrum of heavy–light mesons. Based on the flavor of the heavy quark, heavy–light mesons can be cataloged into charm and bottom mesons. In the charm meson sector, observations of some ground and excited states like $D_0(2550)$, $D_1^*(2600)$, $D_2(2740)$, $D_3^*(2750)$, $D^0(3000)$, $D_J^*(3000)$ and strange states $D_{s1}^*(2860)$, $D_{sJ}(3040)$, $D_{s0}(2590)$ [1–7] have not only broadened the spectra but also help us in exploring their properties through decay studies. However, experimental growth toward establishing the bottom sector is still lacking. Only ground states $B^{0,\pm}(5279)$, $B^*(5324)$, $B_s(5366)$, $B_s^*(5415)$ and some low-lying states $B_1(5721)$, $B_J^*(5732)$, $B_2^*(5747)$, $B_{s1}(5830)$, $B_{s2}^*(5840)$, $B_{sJ}^*(5850)$, $B_J(5840)$, $B_J(5970)$ are observed experimentally and listed by the Particle Data Group (PDG) [8]. However, apart from these states, full bottom meson spectra are unknown. To fill this gap, experimentalists and theoreticians are trying to predict new states that could fill this gap. During this process, recently, the LHCb Collaboration discovered two new states $B_{sJ}(6063)$ and $B_{sJ}(6114)$ in the B^+K^- mass spectrum [9]. The measured masses and decay widths are given below:

$$M(B_{sJ}(6063)) = 6063.5 \pm 1.2(\text{stat}) \pm 0.8(\text{syst})\,\text{MeV}$$

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$\Gamma(B_{sJ}(6063)) = 26 \pm 4 \pm 4$ MeV/c^2
$M(B_{sJ}(6114)) = 6114 \pm 3(\text{stat}) \pm 5(\text{syst})$ MeV
$\Gamma(B_{sJ}(6114)) = 66 \pm 18 \pm 21$ MeV/c^2.

The successes of the observations of these radially excited states by LHCb have demonstrated that more excited bottom meson states will be discovered in future LHC experiments. The exploration of more highly excited B mesons is no longer occurring. In 1999, LEP (the Large Electron–Positron collider) reported an orbitally excited bottom meson state in the hadronic Z-decay process [10]. This bottom meson's measured mass and decay width are 5937 ± 21(stat) $\pm$ 4(syst) MeV and 50 ± 22(stat) $\pm$ 5(syst) MeV, respectively. However, this state was never reconfirmed by other experimental facilities. After many years, the CDF Collaboration in 2013 observed a new resonance $B(5970)$ in decay modes $B^0\pi^+$ and $B^+\pi^-$ simultaneously [11]. Two years later, four resonances $B_J(5840)^{0,+}$ and $B_J(5960)^{0,+}$, were announced by the LHCb Collaboration in 2015 [12]. Despite these observations of mesons, the spectrum of excited bottom mesons has not been greatly explored. In the strange bottom meson family, only some states, $B_s(5366)$, $B_s^*(5415)$, $B_{s1}(5830)$, $B_{s2}^*(5840)$, are well established and have been collected by the PDG [8]. Among these, $B_s(5366)$, $B_s^*(5415)$ are classified as $1S$ states and $B_{s1}(5830)$, $B_{s2}^*(5840)$ are assigned as $1P$ (1^+, 2^+) states. This shows that experimentalists are continuing to try to establish the bottom strange meson spectrum.

Various theoretical studies have performed different analyses for more highly excited bottom non-strange and bottom strange meson states [13–33]. With the help of theoretical models, $B^{0,\pm}(5279)$, $B^*(5324)$, $B_s(5366)$, $B_s^*(5415)$ are assigned as $1S$ states, matching the experimental data. Further, $B_1(5721)$, $B_2^*(5747)$ are also well established experimentally and are classified as $1P(1^+, 2^+)$ states, respectively. However, theoretically, $B_1(5721)$ is still a disputed candidate because some of the theoretical work with heavy meson effective theory favors it as a $1P(1^+)$ state [25,32], while other work using the relativistic quark model and non-relativistic quark model explained this state as a mixture of $^3P_1$ and $^1P_1$ states [14,16,33]. The J^P state of $B_J(5840)$ is still ambiguous as different models suggest different J^P states for it. The authors of Refs. [16,17] explained $B_J(5840)$ with the quark model and suggested an assignment of 2^1S_0, while Yu and Wang, using a $^3P_0$ decay model analysis, favored the assignment of state $B_J(5840)$ as 2^3S_1 [20]. However, heavy quark effective theory (HQET) explains $B_J(5840)$ as a 1^3D_1 state [28]. The state $B_J(5960)^{0,+}$ is assigned as a 2^3S_1 or 1^3D_3 state, or a 1^3D_1 state with different theoretical models [14–16,22–24,34]. However, its J^P value is still a question mark in work by the PDG, which only mentions its mass and decay width. We discuss here a brief literature survey of these non-strange bottom states. The assignments of these states ($B_1(5721)$, $B_2^*(5747)$, $B_J(5970)$) are also suggested in our previous work [28]. In the case of the strange bottom sector, only a few states have been observed, of which the $B_{s1}(5830)$, $B_{s2}(5840)$ states are well observed by the CDF [11,35], D0 [36], and LHCb [37] Collaborations and are identified as $1P(1^+, 2^+)$ respectively. However, there is ambiguity with the recently observed strange bottom meson states $B_{sJ}(6063)$ and $B_{sJ}(6114)$. The states $B_{sJ}(6063)$ and $B_{sJ}(6114)$ in the non-relativistic quark potential model are identified as 1^3D_1 and 1^3D_3 states, respectively [38,39], while the authors of Ref. [40] assign these states as 1^3D_1 and 2^3S_1 states, respectively. Theoretical analysis for these newly observed states is thus limited in the literature, indicating that it needs more attention. As a continuation of previous work [28], we analyze observed strange bottom meson states $B_{sJ}(6063)$ and $B_{sJ}(6114)$ and give their J^P values within this framework.

The paper is arranged as follows. Section 2 briefly describes the HQET model. Section 3 represents the numerical analysis of $B_{sJ}(6063)$ and $B_{sJ}(6114)$ based on predicted masses and decay widths. Section 4 presents our final conclusion.

2. Theoretical formulation

We apply HQET to assign spin-parity quantum numbers for recently observed heavy–light strange bottom meson states. This theory is simple and powerful, and provides precise calculations of masses and the decay behavior of heavy–light bottom mesons [41]. HQET assumes an infinite mass of heavy quarks ($Q = c, b$) and most of the momentum of the bottom meson is carried by the heavy quark. In this heavy quark limit ($m_Q \to \infty$), the spin of the heavy quark s_Q decouples from the light d.o.f. (degree of freedom), which incorporates the light antiquark and the gluons. The total angular momentum of the light d.o.f. is $s_l = s_q + l$, where $s_q = 1/2$, the spin of the light quark, and l is the total orbital momentum of light quarks. In the heavy quark limit, mesons are categorized in doublets based on the total angular momentum of light quarks. For $l = 0$, $s_l = 1/2$ is associated with the spin of heavy quarks $s_Q = 1/2$ and results in the doublet $(0^-, 1^-)$. This doublet is denoted by (P, P^*). On the other hand, $l = 1$ forms two doublets, given by (P_0^*, P_1') and (P_1, P_2^*) with $J_{s_l}^P = (0^+, 1^+)_{1/2}$ and $J_{s_l}^P = (1^+, 2^+)_{3/2}$ respectively. For $l = 2$, two doublets are expressed by (P_1^*, P_2) and (P_2', P_3^*) with $J_{s_l}^P = (1^-, 2^-)_{3/2}$ and $J_{s_l}^P = (2^-, 3^-)_{5/2}$ respectively. These doublets are introduced in terms of supereffective fields $H_a, S_a, T_a, X_a^\mu, Y_a^{\mu\nu}$ and are described for fields as shown below [25,42]:

$$H_a = \frac{1+\not{v}}{2}\left\{P^*_{a\mu}\gamma^\mu - P_a\gamma_5\right\} \tag{1}$$

$$S_a = \frac{1+\not{v}}{2}\left[P'^{\mu}_{1a}\gamma_\mu\gamma_5 - P^*_{0a}\right] \tag{2}$$

$$T_a^\mu = \frac{1+\not{v}}{2}\left\{P^{*\mu\nu}_{2a}\gamma_\nu - P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{\gamma^\nu(\gamma^\mu - \upsilon^\mu)}{3}\right]\right\} \tag{3}$$

$$X_a^\mu = \frac{1+\not{v}}{2}\left\{P^{\mu\nu}_{2a}\gamma_5\gamma_\nu - P^*_{1a\nu}\sqrt{\frac{3}{2}}\left[g^{\mu\nu} - \frac{\gamma_\nu(\gamma^\mu + v^\mu)}{3}\right]\right\} \tag{4}$$

$$\begin{aligned} Y_a^{\mu\nu} = \frac{1+\not{v}}{2}\Bigg\{&P^{*\mu\nu\sigma}_{3a}\gamma_\sigma - P'^{\alpha\beta}_{2a}\sqrt{\frac{5}{3}}\gamma_5\Bigg[g^\mu_\alpha g^\nu_\beta \\ &- \frac{g^\nu_\beta\gamma_\alpha(\gamma^\mu - v^\mu)}{5} - \frac{g^\mu_\alpha\gamma_\beta(\gamma^\nu - v^\nu)}{5}\Bigg]\Bigg\}. \end{aligned} \tag{5}$$

The field H_a describes S-wave doublets for $J^P = (0^-, 1^-)$. The fields S_a and T_a represent P-wave doublets for $J^P = (0^+, 1^+)$ and $(1^+, 2^+)$ respectively. D-wave doublets for $J^P = (1^-, 2^-)$ and $(2^-, 3^-)$ belong to fields X_a^μ and $Y_a^{\mu\nu}$ respectively. a in the above expressions is the light quark (u, d, s) flavor index. v is the heavy quark velocity, unchanged in strong interactions. The approximate chiral symmetry $SU(3)_L \times SU(3)_R$ is incorporated with fields of pseudoscalar mesons π, K, and η, which are the lightest strongly interacting bosons. They are considered as approximate Goldstone bosons of this chiral symmetry and can be expressed by the matrix field $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$ and $\Sigma = \xi^2$, where $\mathcal{M}$ is given by

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}; \tag{6}$$

f_π is the pion decay constant, and its value is taken 130 MeV. Fields of heavy meson doublets, given in Eqs. (1)–(5), interact with pseudoscalar Goldstone bosons through the covariant derivative $D_{\mu ab} = -\delta_{ab}\partial_\mu + \mathcal{V}_{\mu ab} = -\delta_{ab}\partial_\mu + \frac{1}{2}(\xi^+\partial_\mu\xi + \xi\partial_\mu\xi^+)_{ab}$ and axial vector field $A_{\mu ab} = \frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi)_{ab}$. By including all meson doublet fields and Goldstone fields, the effective Lagrangian is written as:

$$\begin{aligned}\mathcal{L} = {} & iTr\left[\overline{H}_b v^\mu D_{\mu ba} H_a\right] + \frac{f_\pi^2}{8} Tr\left[\partial^\mu \Sigma \partial_\mu \Sigma^+\right] \\ & + Tr\left[\overline{S_b}\left(iv^\mu D_{\mu ba} - \delta_{ba}\Delta_S\right) S_a\right] + Tr\left[\overline{T_b^\alpha}\left(iv^\mu D_{\mu ba} - \delta_{ba}\Delta_T\right) T_{a\alpha}\right] \\ & + Tr\left[\overline{X_b^\alpha}\left(iv^\mu D_{\mu ba} - \delta_{ba}\Delta_X\right) X_{a\alpha}\right] + \\ & Tr\left[\overline{Y_b^{\alpha\beta}}\left(iv^\mu D_{\mu ba} - \delta_{ba}\Delta_Y\right) Y_{a\alpha\beta}\right].\end{aligned} \tag{7}$$

The mass parameter Δ_F in Eq. (7) gives the mass difference between excited-mass doublets (F) and ground-mass doublets (H) in the form of spin-averaged masses of these doublets with the same principal quantum number (n). The mass parameters are described by:

$$\Delta_F = \overline{M_F} - \overline{M_H}, \quad F = S, T, X, Y \tag{8}$$

$$\text{where} \qquad \overline{M_H} = \left(3m^Q_{P^*} + m^Q_P\right)/4 \tag{9}$$

$$\overline{M_S} = \left(3m^Q_{P_1'} + m^Q_{P_0^*}\right)/4 \tag{10}$$

$$\overline{M_T} = \left(5m^Q_{P_2^*} + 3m^Q_{P_1}\right)/8 \tag{11}$$

$$\overline{M_X} = \left(5m^Q_{P_2} + 3m^Q_{P_1^*}\right)/8 \tag{12}$$

$$\overline{M_Y} = \left(7m^Q_{P_3^*} + 5m^Q_{P_2'}\right)/12. \tag{13}$$

The $1/m_Q$ corrections to the heavy quark limit are given by symmetry-breaking terms. The corrections have the following form:

$$\begin{aligned}\mathcal{L}_{1/m_Q} = \frac{1}{2m_Q}\Big[& \lambda_H Tr\left(\overline{H}_a\sigma^{\mu\nu}H_a\sigma_{\mu\nu}\right) - \lambda_S Tr\left(\overline{S}_a\sigma^{\mu\nu}S_a\sigma_{\mu\nu}\right) \\ & + \lambda_T Tr\left(\overline{T}_a^\alpha\sigma^{\mu\nu}T_a^\alpha\sigma_{\mu\nu}\right) - \lambda_X Tr\left(\overline{X}_a^\alpha\sigma^{\mu\nu}X_a^\alpha\sigma_{\mu\nu}\right) \\ & + \lambda_Y Tr\left(\overline{Y}_a^{\alpha\beta}\sigma^{\mu\nu}Y_a^{\alpha\beta}\sigma_{\mu\nu}\right)\Big].\end{aligned} \tag{14}$$

Here the parameters $\lambda_H, \lambda_S, \lambda_T, \lambda_X, \lambda_Y$ are analogous with hyperfine splittings and are expressed in Eqs. (15)–(19). These mass terms in the Lagrangian give only the first order in $1/m_Q$ terms, but higher-order terms may also be present otherwise. We are limited to the first-order corrections in $1/m_Q$:

$$\lambda_H = \frac{1}{8}\left(M^2_{P^*} - M^2_P\right) \tag{15}$$

$$\lambda_S = \frac{1}{8}\left(M^2_{P'_1} - M^2_{P^*_0}\right) \tag{16}$$

$$\lambda_T = \frac{3}{8}\left(M^2_{P^*_2} - M^2_{P_1}\right) \tag{17}$$

$$\lambda_X = \frac{3}{8}\left(M^2_{P_2} - M^2_{P^*_1}\right) \tag{18}$$

$$\lambda_Y = \frac{3}{8}\left(M^2_{P_3} - M^2_{P'^*_2}\right). \tag{19}$$

In HQET, at the scale of 1 GeV, flavor symmetry spontaneously arises for b (bottom quark) and c (charm quark), and hence the beauty of flavor symmetry implies

$$\Delta_F^{(c)} = \Delta_F^{(b)} \tag{20}$$

$$\lambda_F^{(c)} = \lambda_F^{(b)}. \tag{21}$$

This symmetry is broken by the higher-order terms in the HQET Lagrangian involving terms of factor $1/m_Q$ and the parameters Δ_F and λ_F are modified by extra terms $\delta\Delta_F$ and $\delta\lambda_F$. As we know, mass splitting between ground-state vector and pseudoscalar meson doublets originates from chromomagnetic interactions [43] in such ways:

$$\begin{aligned}\frac{\lambda_H^{(b)}}{\lambda_H^{(c)}} &= \frac{m^2_{B^*} - m^2_B}{m^2_{D^*} - m^2_D} \\ &= \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{9/25}\left[1 - \mathcal{O}\left(\frac{\alpha_s}{\pi}\right)\right] + \\ &\Lambda_R\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right),\end{aligned} \tag{22}$$

where Λ_R is a non-perturbative parameter that accounts for higher-order corrections in the heavy quark expansion. However, we take leading-order corrections, and the $1/m_Q$ effect is neglected. QCD corrections change the λ_F relation [44] to

$$\lambda_F^{(b)} = \lambda_F^{(c)}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{9/25} \tag{23}$$

where $\delta\lambda_F = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{9/25}$. Also, we take $m_c = 1180$ MeV and $m_b = 4390$ MeV in our calculations.

The difference of the spin-averaged masses at $1/m_Q$ order modifies the parameter Δ_F by $\delta\Delta_F$ [44], given by

$$\Delta_F^{(b)} = \Delta_F^{(c)} + \delta\Delta_F \tag{24}$$

and

$$\delta\Delta_F = \left(\lambda_1^F - \lambda_1^H\right)\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) \tag{25}$$

where

$$\lambda_1^F - \lambda_1^H = 2m_b m_c\left[\left(\frac{(\overline{M}_F^{b\bar{q}} - \overline{M}_H^{b\bar{q}}) - (\overline{M}_F^{c\bar{q}} - \overline{M}_H^{c\bar{q}})}{m_b - m_c}\right) - 1\right]. \tag{26}$$

The decays $F \to H + M$ ($F = H, S, T, X, Y$, and M represents a light pseudoscalar meson) can be described by effective Lagrangians explained in terms of the fields introduced in

Eqs. (9)–(14) that are valid at leading order in the heavy quark mass and the light meson momentum expansion:

$$L_{HH} = g_{HH} Tr\left\{\overline{H}_a H_b \gamma_\mu \gamma_5 A^\mu_{ba}\right\} \tag{27}$$

$$L_{SH} = g_{SH} Tr\left\{\overline{H}_a S_b \gamma_\mu \gamma_5 A^\mu_{ba}\right\} + \text{h.c.} \tag{28}$$

$$L_{TH} = \frac{g_{TH}}{\Lambda} Tr\left\{\overline{H}_a T^\mu_b \left(iD_\mu \psi A + i\psi DA_\mu\right)_{ba} \gamma_5\right\} + \text{h.c.} \tag{29}$$

$$L_{XH} = \frac{g_{XH}}{\Lambda} Tr\left\{\overline{H}_a X^\mu_b \left(iD_\mu \psi A + i\psi DA_\mu\right)_{ba} \gamma_5\right\} + \text{h.c.} \tag{30}$$

$$\begin{aligned} L_{YH} = \frac{1}{\Lambda^2} Tr\{\overline{H}_a Y^{\mu\nu}_b [k^Y_1 \{D_\mu, D_\nu\} A_\lambda + \\ k^Y_2 \left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)]_{ba} \gamma^\lambda \gamma_5\} + \text{h.c.} \end{aligned} \tag{31}$$

In these equations $D_\mu = \partial_\mu + V_\mu$, $\{D_\mu, D_\nu\} = D_\mu D_\nu + D_\nu D_\mu$, and $\{D_\mu, D_\nu D_\rho\} = D_\mu D_\nu D_\rho + D_\mu D_\rho D_\nu + D_\nu D_\mu D_\rho + D_\nu D_\rho D_\mu + D_\rho D_\mu D_\nu + D_\rho D_\nu D_\mu$. Λ is the chiral symmetry-breaking scale taken as 1 GeV. $g_{HH}, g_{SH}, g_{TH}, g_{YH} = k^Y_1 + k^Y_2$ are the strong coupling constants involved. Using the Lagrangians L_{HH}, L_{SH}, L_{TH}, L_{YH}, the two-body strong decays of $Q\overline{q}$ heavy–light bottom mesons are given as [45–48]:

$(0^-, 1^-) \to (0^-, 1^-) + M$

$$\Gamma(1^- \to 1^-) = C_M \frac{g^2_{HH} M_f p^3_M}{3\pi f^2_\pi M_i} \tag{32}$$

$$\Gamma(1^- \to 0^-) = C_M \frac{g^2_{HH} M_f p^3_M}{6\pi f^2_\pi M_i} \tag{33}$$

$$\Gamma(0^- \to 1^-) = C_M \frac{g^2_{HH} M_f p^3_M}{2\pi f^2_\pi M_i}; \tag{34}$$

$(0^+, 1^+) \to (0^-, 1^-) + M$

$$\Gamma(1^+ \to 1^-) = C_M \frac{g^2_{SH} M_f (p^2_M + m^2_M) p_M}{2\pi f^2_\pi M_i} \tag{35}$$

$$\Gamma(0^+ \to 0^-) = C_M \frac{g^2_{SH} M_f (p^2_M + m^2_M) p_M}{2\pi f^2_\pi M_i}; \tag{36}$$

$(1^+, 2^+) \to (0^-, 1^-) + M$

$$\Gamma(2^+ \to 1^-) = C_M \frac{2g^2_{TH} M_f p^5_M}{5\pi f^2_\pi \Lambda^2 M_i} \tag{37}$$

$$\Gamma(2^+ \to 0^-) = C_M \frac{4g^2_{TH} M_f p^5_M}{15\pi f^2_\pi \Lambda^2 M_i} \tag{38}$$

$$\Gamma(1^+ \to 1^-) = C_M \frac{2g^2_{TH} M_f p^5_M}{3\pi f^2_\pi \Lambda^2 M_i}; \tag{39}$$

$(1^-, 2^-) \to (0^-, 1^-) + M$

$$\Gamma(1^- \to 0^-) = C_M \frac{4g^2_{XH}}{9\pi f^2_\pi \Lambda^2} \frac{M_f}{M_i} [p^3_M (m^2_M + p^2_M)] \tag{40}$$

$$\Gamma(1^- \to 1^-) = C_M \frac{2g^2_{XH}}{9\pi f^2_\pi \Lambda^2} \frac{M_f}{M_i} [p^3_M (m^2_M + p^2_M)] \tag{41}$$

Table 1. Numerical values of the meson masses used in this work [8].

States	B^0	$B^{\pm}$	B^*	B_s	B_s^*
Masses (MeV)	5279.58	5279.25	5325.20	5366.77	5415.40
States	$\pi^{\pm}$	π^0	η	K^+	K^0
Masses (MeV)	139.57	134.97	547.85	493.67	497.61

$$\Gamma(2^- \to 1^-) = C_M \frac{2g_{XH}^2}{3\pi f_\pi^2 \Lambda^2} \frac{M_f}{M_i}[p_M^3(m_M^2 + p_M^2)]; \tag{42}$$

$(2^-, 3^-) \to (0^-, 1^-) + M$

$$\Gamma(2^- \to 1^-) = C_M \frac{4g_{YH}^2}{15\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i}[p_M^7] \tag{43}$$

$$\Gamma(3^- \to 0^-) = C_M \frac{4g_{YH}^2}{35\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i}[p_M^7] \tag{44}$$

$$\Gamma(3^- \to 1^-) = C_M \frac{16g_{YH}^2}{105\pi f_\pi^2 \Lambda^4} \frac{M_f}{M_i}[p_M^7]. \tag{45}$$

Here M_i, M_f give the initial and final momenta and Λ is the chiral symmetry-breaking scale of 1 GeV. p_M, m_M denotes the final momentum and mass of the light pseudoscalar meson. The coupling constant plays a key role in the phenomenological study of heavy–light mesons. These dimensionless coupling constants describe the strength of transition between the H–H field (negative–negative parity), S–H field (positive–negative parity), and T–H field (positive–negative parity). These coupling constants are notated as g_{HH}, g_{SH}, g_{TH}, g_{XH}, g_{YH}, etc. The coefficients C_M for different pseudoscalar particles are: $C_{\pi^\pm}$, $C_{K^\pm}$, C_{K^0}, $C_{\overline{K}^0} = 1$, $C_{\pi^0} = \frac{1}{2}$, and $C_\eta = \frac{2}{3}(c\bar{u}, c\bar{d})$ or $\frac{1}{6}(c\bar{s})$. In our paper, we do not include higher-order corrections of $\frac{1}{m_Q}$ to introduce new couplings. We also expect that higher-order corrections give a small contribution in comparison to the leading-order contributions. The numerical values of various meson masses used in the calculations are listed in Table 1.

3. Numerical analysis

Assigning a particular J^P (spin-parity quantum number) to the experimentally observed excited states is crucial. A specific position of the state in its mass spectra can help in revealing many other important strong interaction parameters such as hadronic coupling constants, branching ratios, spins and mass splittings, decay widths, and many more. Therefore, in this analysis, we aim to suggest a particular J^P state for the recently strange bottom states observed by LHCb. Moreover, we also complete the empty spaces of these strange mass spectra by predicting the masses and other parameters of the missing bottom spectra in the framework of HQET.

In the bottom strange sector, only ground $1S$ $(0^-, 1^-)$ states are confirmed both experimentally and theoretically. The rest of the spectrum is still unknown. Many other theoretical models, such as chiral perturbation theory, the chiral unitary approach, the Regge trajectory, the $^3P_0$ model, the relativistic flux tube model, QCD sum rules, and lattice QCD, etc., have been used in an attempt to fill the gaps but they have not yet been verified experimentally [13–31]. In addition to this, such theoretically calculated masses are unreliable because all these theoretical models depend on certain unknown parameters. The HQET framework chosen by us is

Table 2. Input values used in this work. All values are in units of MeV.

State	J^P	$c\bar{s}$	$b\bar{s}$
1^1S_0	0^-	1968.35 [8]	5366.92 [8]
1^3S_1	1^-	2112.20 [8]	5415.40 [8]
1^3P_0	0^+	2317.80 [8]	–
1^1P_1	1^+	2459.50 [8]	–
1^3P_1	1^+	2535.11 [8]	–
1^3P_2	2^+	2569.10 [8]	–
1^3D_1	1^-	2859.00	–
1^1D_2	2^-	2902.40	–
1^3D_2	2^-	2895.40	–
1^3D_3	3^-	2860.50	–
2^1S_0	0^-	2676.20	–
2^3S_1	1^-	2714.00	–

free from such parameters and uses the most important relations of spin and flavor symmetry present in heavy–light mesons.

3.1. *Mass spectroscopy*

To study the behavior of heavy–light mesons, mass is the prime property that determines many other properties of the mesons. Therefore, we start our calculations by predicting the masses of unavailable strange bottom states using the flavor and spin symmetry property $\Delta_F^{(c)} = \Delta_F^{(b)}$ and $\lambda_F^{(c)} = \lambda_F^{(b)}$. As discussed in Sect. 2, the parameter Δ_F is defined in terms of the spin-averaged mass splittings between the higher-state doublet and ground-state doublet, whereas the other parameter λ_F is the mass splittings between the spin partners of the doublets.

The main aim of our calculation is to incorporate J^P into recently observed strange states of the bottom flavor $B_{sJ}(6063)$ and $B_{sJ}(6114)$ and to fill the gaps near these states so that our calculations can provide motivation and support for the yet-to-be-predicted experimental information. The details of the numerical analysis on the mass spectrum are as follows.

Using the masses of charm and bottom states tabulated in Table 2, the spin-averaged mass splittings $\Delta_F^{(c)}$ and the hyperfine splittings $\lambda_F^{(c)}$ for 1S, 1P, 1D, and 2S come out to be

$$\Delta_{\tilde{H}}^{(c)} = 628.313\,\text{MeV} \tag{46}$$

$$\Delta_{S}^{(c)} = 347.838\,\text{MeV} \tag{47}$$

$$\Delta_{T}^{(c)} = 480.116\,\text{MeV} \tag{48}$$

$$\Delta_{X}^{(c)} = 763.513\,\text{MeV} \tag{49}$$

$$\Delta_{Y}^{(c)} = 798.804\,\text{MeV} \tag{50}$$

$$\lambda_{H}^{(c)} = (270.875)^2\,\text{MeV}^2 \tag{51}$$

$$\lambda_{\tilde{H}}^{(c)} = (159.589)^2\,\text{MeV}^2 \tag{52}$$

Table 3. Predicted values of the radially excited strange $2S$, $1P$, and $1D$ bottom meson states. All masses are in MeV.

	Without corrections	Corrections in λ_F	Corrections in Δ_F	Corrections in both parameters λ_F and Δ_F
$0^-(2^1S_0)$	6018.92	6020.98	6017.74	6019.80
$1^-(2^3S_1)$	6035.82	6035.13	6034.65	6033.96
$0^+(1^3P_0)$	5706.86	5714.07	5705.35	5712.56
$1^+(1^1P_1)$	5765.87	5763.47	5764.37	5761.97
$1^+(1^3P_1)$	5874.18	5875.68	5867.30	5868.80
$2^+(1^3P_2)$	5888.93	5888.03	5882.06	5881.16
$1^-(1^3D_1)$	6175.67	6174.23	6180.17	6178.73
$2^-(1^1D_2)$	6161.47	6162.33	6165.98	6166.84
$2^-(1^3D_2)$	6211.53	6205.10	6210.70	6209.16
$3^-(1^3D_3)$	6195.34	6194.50	6194.51	6195.60

$$\lambda_S^{(c)} = (290.892)^2\ \mathrm{MeV}^2 \tag{53}$$

$$\lambda_T^{(c)} = (180.360)^2\ \mathrm{MeV}^2 \tag{54}$$

$$\lambda_X^{(c)} = (181.228)^2\ \mathrm{MeV}^2 \tag{55}$$

$$\lambda_Y^{(c)} = (204.573)^2\ \mathrm{MeV}^2. \tag{56}$$

The charm mesons with $J^P = 2^-$ for a D-wave with $j = 3/2$ and $j = 5/2$ for 1^1D_2 and 1^3D_2 are experimentally unavailable, so we have taken the average of the theoretical masses [13,49–51] for them.

Masses obtained for B_s mesons with the help of the symmetries are listed in the second column of Table 3. Masses predicted for P-wave $j = 3/2$ strange bottom states 1^3P_1 and 1^3P_2 are in very good agreement with the experimental masses predicted for these states by the LHCb [37], CDF [11,35], and D0 [36] Collaborations. Our calculated masses for 1^3P_1 and 1^3P_2 deviate from their experimental values by only 0.78% and 0.84% respectively. In comparison with predictions of other theoretical models shown in Table 4, our calculated masses are in good agreement. Note that the predictions in Refs. [13,52] are smaller by 100 MeV than our results, while the data in Ref. [40] are larger than our masses by the order of 100 MeV. Also, the calculated mass for $J^P = (1^-, 2^-)$, 3^- belonging to a D-wave with $j = 3/2$, $j = 5/2$ and $J^P = 1^-$ of a radially excited S-wave also match the LHCb states $B_{sJ}(6114)$ and $B_{sJ}(6063)$ respectively. Since the masses of these two states $B_{sJ}(6114)$ and $B_{sJ}(6063)$ deviate only by 61, 47, 81, and 28 MeV from our predicted values, one can easily conclude that the $B_{sJ}(6063)$ state belongs to J^P $2S1^-$, while the other state $B_{sJ}(6114)$ belongs to one of the J^P of the 1 D-wave. The authors of Ref. [40] predicted the J^P for $B_{sJ}(6114)$ as $1D1^-$ for $j = 3/2$. However, to prove this convincingly, we computed the masses by taking higher-order corrections as the splitting parameters (Δ_F and λ_F) can drastically change in the presence of QCD and higher-order ($1/m_Q$) corrections in the HQET Lagrangian. Also, the calculated masses should be able to predict other parameters, such as decay width, which should match the experimental data. Therefore, in the next part,

Table 4. The predicted values of bottom strange meson masses (MeV) compared with some other model predictions.

	Ours	Ref. [52]	Ref. [13]	Ref. [40]
$0^-(2^1S_0)$	6018.92	6003	5985	6025
$1^-(2^3S_1)$	6035.82	6029	6019	6033
$0^+(1^3P_0)$	5706.86	5812	5804	5709
$1^+(1^1P_1)$	5765.87	5828	5805	5768
$1^+(1^3P_1)$	5874.18	5842	5842	5875
$2^+(1^3P_2)$	5888.93	5840	5820	5890
$1^-(1^3D_1)$	6175.67	6119	6127	6247
$2^-(1^1D_2)$	6161.47	6128	6095	6256
$2^-(1^3D_2)$	6211.53	6157	6140	6292
$3^-(1^3D_3)$	6195.34	6172	6103	6297

we present an analysis of such corrections to these splittings and calculate the two-body strong decay widths of these bottom states.

QCD and higher-order ($1/m_Q$) corrections are applied to a scale of $\Lambda_{\mathrm{QCD}}/m_Q$, where they can significantly influence the level of symmetry breaking. The corrections to the Δ_F and λ_F parameters change the heavy quark symmetry relations to $\Delta_F^{(b)} = \Delta_F^{(c)} + \delta\Delta_F$ and $\lambda_F^{(b)} = \lambda_F^{(c)}\delta\lambda_F$. We apply such corrections to these splittings one by one and check the effect of these corrections on the bottom masses followed by a step in which both corrections would be applied simultaneously. In the case of the λ_F parameter, QCD corrections are dominant over $1/m_Q$ corrections because these mass splitting parameters λ_F originate from chromomagnetic interactions. The leading QCD corrections to λ_F are in the form of $\lambda_F^{(b)} = \lambda_F^{(c)}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{9/25}$. The parameters $\alpha_s(m_b)$ and $\alpha_s(m_c)$ for applying the QCD corrections to these splitting parameters are taken as 0.22 and 0.36 [43]. The corrections in the λ_F parameters modify the values to:

$$\lambda_{\tilde{H}}^{(b)} = (133.662)^2\ \mathrm{MeV}^2 \tag{57}$$

$$\lambda_S^{(b)} = (243.632)^2\ \mathrm{MeV}^2 \tag{58}$$

$$\lambda_T^{(b)} = (151.058)^2\ \mathrm{MeV}^2 \tag{59}$$

$$\lambda_X^{(b)} = (151.785)^2\ \mathrm{MeV}^2 \tag{60}$$

$$\lambda_Y^{(b)} = (171.33)^2\ \mathrm{MeV}^2. \tag{61}$$

The B_s masses inherited using these corrections are tabulated in the third column of Table 3. The resulting masses are deflected up to 7.5 MeV from their initial masses and result in a reduction of the gap between our and the LHCb masses. Now we analyze the effect of $1/m_Q$ and QCD corrections to our other parameter Δ_F, which depicts the mass splittings between the higher-mass doublet and the ground-state H field states. The corrections to these parameters are in the form of $\delta\Delta_F$, where $F = S, T, X, Y, \tilde{H}$. To estimate corrections in $\delta\Delta_F$, we vary the $\lambda_1^F - \lambda_1^H$ parameters in an acceptable range, as mentioned in Ref. [44], as the λ_1^F parameter represents the kinetic energy of the heavy–light meson, and we select $\lambda_1^F - \lambda_1^H$ parameters in such a way that the magnitude of these $\lambda_1^F - \lambda_1^H$ parameters should be larger for the excited state than the ground state. The corrections are as follows:

$$\delta\Delta_{\tilde{H}} = 1.687\,\text{MeV} \tag{62}$$

$$\delta\Delta_S = 2.162\,\text{MeV} \tag{63}$$

$$\delta\Delta_T = 9.884\,\text{MeV} \tag{64}$$

$$\delta\Delta_X = 6.487\,\text{MeV} \tag{65}$$

$$\delta\Delta_Y = 1.2\,\text{MeV}. \tag{66}$$

These corrections are for doublets $2S(0^-, 1^-)$, $1P(0^+, 1^+)$, $1P(1^+, 2^+)$, $1D(1^-, 2^-)$, and $1D(2^-, 3^-)$ respectively. Reference [53] follows a similar procedure to calculate the correction in $\delta\Delta_F$. Masses calculated using these corrections are tabulated in the fourth column of Table 3. A comparison of masses concluded that the deviation lies in the range of 0.83–6.90 MeV, which again shows that masses are not greatly affected by corrections, but again resulted in narrowing the gap between our and the experimental masses. Lastly, the masses obtained by applying both corrections simultaneously are listed in the last column of Table 3. It can be summarized that the effect of such corrections is very small in the more highly excited states. Because of the higher angular momentum, such states do not remain in their stable state for long and thus do not explicitly show chromomagnetic effects. However, such corrections have resulted in narrowing the gap between the predicted and experimentally observed mass values but have not adversely affected the masses. To suggest a particular J^P value for the LHCb-observed strange bottom states $B_{sJ}(6063)$ and $B_{sJ}(6114)$, we explore the second most important property of heavy–light mesons, the decay widths.

3.2. *Strong decays*

We apply the effective Lagrangian approach discussed in Sect. 2 to calculate the OZI-allowed two-body strong decay widths and the various branching ratios involved with the bottom states $B_s(2S)$, $B_s(1P)$, and $B_s(1D)$. The numerical values of the partial and total decay widths of these states are given in Tables 5 and 6. Here, we need to emphasize that the calculated total decay widths for the above states do not include the contribution of decays with the emission of vector mesons (ω, ρ, K^*, ϕ) as the contribution of vector mesons to total decay widths is small compared to pseudoscalar mesons. They give a contribution of ± 10 MeV [15] to the total decay widths of the states analyzed above.

To choose the possible J^P for the LHCb bottom state $B_{sJ}(6114)$, we calculated the total decay width for all the possible J^P 1^3D_1, 1^1D_2, and 1^3D_3 in terms of strong coupling constants. The obtained values are

$$\Gamma(1^3D_1) = 6052.34 g_{XH}^2\,\text{MeV} \tag{67}$$

$$\Gamma(1^1D_2) = 4416.36 g_{XH}^2\,\text{MeV} \tag{68}$$

$$\Gamma(1^3D_3) = 933.75 g_{YH}^2\,\text{MeV}. \tag{69}$$

Table 5. Strong decay widths of strange bottom mesons $B_s(1D1^-)$, $B_s(1D2^-)$, $B_s(1D2^-)$, and $B_s(1D3^-)$. The ratio in the fifth column represents $\widehat{\boldsymbol{\Gamma}} = \frac{\Gamma}{\Gamma(B^*_{sJ} \to B^{*0}K^+)}$. The fraction gives the percentage of the partial decay width with respect to the total decay width.

nLs_lJ^P	Decay channel	Decay width (MeV)	Ratio	Fraction
$1D_{s3/2}1^-$	B^0K^0	$1706.81g^2_{XH}$	2.71	28.20
	B^+K^-	$1727.24g^2_{XH}$	2.74	28.53
	$B_s\pi^0$	$863.737g^2_{XH}$	1.37	14.27
	$B_s\eta$	$126.531g^2_{XH}$	0.20	2.09
	$B^{*0}K^0$	$628.449g^2_{XH}$	0.51	5.35
	$B^{*+}K^-$	$635.727g^2_{XH}$	0.06	0.66
	$B^*_s\pi^0$	$323.847g^2_{XH}$	1	10.38
	$B^*_s\eta$	$40.0077g^2_{XH}$	1.01	10.50
	Total	$6052.34g^2_{XH}$		
$1D_{s3/2}2^-$	$B^{*0}K^0$	$1701.43g^2_{XH}$	1	38.52
	$B^{*+}K^-$	$1722.22g^2_{XH}$	1.01	38.99
	$B^*_s\pi^0$	$889.31g^2_{XH}$	0.52	20.13
	$B^*_s\eta$	$103.31g^2_{XH}$	0.06	2.34
	Total	$4416.36g^2_{XH}$		
$1D_{s5/2}2^-$	$B^{*0}K^0$	$290.97g^2_{YH}$	1	33.08
	$B^{*+}K^-$	$298.45g^2_{YH}$	1.02	33.93
	$B^*_s\pi^0$	$251.13g^2_{YH}$	0.868	28.55
	$B^*_s\eta$	$38.85g^2_{YH}$	0.13	4.41
	Total	$879.41g^2_{XH}$		
$1D_{s5/2}3^-$	B^0K^0	$168.95g^2_{YH}$	1.21	18.09
	B^+K^-	$173.47g^2_{YH}$	1.24	18.57
	$B_s\pi^0$	$140.16g^2_{YH}$	18	15.01
	$B_s\eta$	$27.27g^2_{YH}$	0.19	2.92
	$B^{*0}K^0$	$139.01g^2_{YH}$	1	14.88
	$B^{*+}K^-$	$142.78g^2_{YH}$	1.02	15.29
	$B^*_s\pi^0$	$125.11g^2_{YH}$	0.90	13.39
	$B^*_s\eta$	$16.98g^2_{YH}$	0.12	1.81
	Total	$933.75g^2_{YH}$		

On comparing these calculated decay widths with the experimental decay width value of 66 MeV, the coupling constant values come out to be

$$g_{XH} = 0.104 \tag{70}$$

$$g_{XH} = 0.12 \tag{71}$$

$$g_{YH} = 0.26. \tag{72}$$

The available theoretical values of g_{YH} are 0.61 [54], 0.53 [25], and 0.42 [55]. Our computed value of g_{YH} is much smaller, ruling out the 1^3D_3 state from the possible J^P values for the state $B_{sJ}(6114)$. We are therefore left with two available J^P (1^3D_1 and 1^1D_2). In the literature, the available theoretical values of g_{XH} are 0.41 [28], 0.45 [32], 0.53 [56], and 0.19 [25]. In Ref. [25], the coupling constant g_{XH} calculated for the state $B(5970)^0$ is 0.19 ± 0.049. They assigned 1^3D_1 to this state and computed the coupling constant by comparing their theoretical decay width with the experimental value. We followed the same procedure, and our obtained value

Table 6. Strong decay widths of strange bottom mesons $B_s(2S0^-)$, $B_s(2S1^-)$, $B_s(1P0^+)$, $B_s(1P1^+)$, $B_s(1P1^+)$, $B_s(1P2^+)$. The ratio in the fifth column represents the $\widehat{\boldsymbol{\Gamma}} = \frac{\Gamma}{\Gamma(B^*_{sJ} \to B^{*0}K^+)}$. Fraction gives the percentage of the partial decay width with respect to the total decay width.

nLs_lJ^P	Decay channel	Decay width (MeV)	Ratio	Fraction
$2S_{s1/2}0^-$	$B^{*0}K^0$	$785.160\widetilde{g}^2_{HH}$	1	32.64
	$B^{*+}K^-$	$804.921\widetilde{g}^2_{HH}$	1.02	33.46
	$B^*_s\pi^0$	$737.438\widetilde{g}^2_{HH}$	0.93	30.66
	$B^*_s\eta$	$77.419\widetilde{g}^2_{HH}$	0.09	3.21
	Total	$2404.940\widetilde{g}^2_{HH}$		
$2S_{s1/2}1^-$	B^0K^0	$342.716\widetilde{g}^2_{HH}$	0.72	15.87
	B^+K^-	$350.669\widetilde{g}^2_{HH}$	0.73	16.24
	$B_s\pi^0$	$292.676\widetilde{g}^2_{HH}$	0.61	13.55
	$B_s\eta$	$58.29\widetilde{g}^2_{HH}$	0.12	2.70
	$B^{*0}K^0$	$474.96\widetilde{g}^2_{HH}$	1	22.00
	$B^{*+}K^-$	$486.477\widetilde{g}^2_{HH}$	1.2	22.53
	$B^*_s\pi^0$	$466.530\widetilde{g}^2_{HH}$	0.98	21.61
	$B^*_s\eta$	$36.988\widetilde{g}^2_{HH}$	0.07	1.71
	Total	$2158.650\widetilde{g}^2_{HH}$		
$1P_{s1/2}0^+$	$B_s\pi^0$	$147.500g^2_{SH}$	–	100
	$B_s\eta$	–		–
	B^+K^0	–	–	–
	B^-K^+	–	–	–
	Total	$147.500g^2_{SH}$		
$1P_{s1/2}1^+$	$B^*_s\pi^0$	$161.080g^2_{SH}$	–	–
	$B^*_s\eta$	–	–	–
	$B^{*+}K^0$	–	–	–
	$B^{*-}K^+$	–	–	–
	Total	$1161.080g^2_{SH}$		
$1P_{s13/2}1^+$	$B^{*0}K^0$	$5.970g^2_{TH}$	1	6.66
	$B^{*+}K^-$	$7.12g^2_{TH}$	1.19	7.93
	$B^*_s\pi^0$	$76.49g^2_{TH}$	12.81	85.37
	$B^*_s\eta$	–	–	–
	Total	$89.59g^2_{TH}$		
$1P_{s13/2}2^+$	B^0K^0	$18.48g^2_{TH}$	22.67	11.05
	B^+K^-	$20.13g^2_{TH}$	2.91	12.04
	$B_s\pi^0$	$59.590g^2_{TH}$	8.62	35.65
	$B_s\eta$	–	–	–
	$B^{*0}K^0$	$6.92g^2_{TH}$	1	4.13
	$B^{*+}K^-$	$7.92g^2_{TH}$	1.14	4.73
	$B^*_s\pi^0$	$54.09g^2_{TH}$	7.82	32.36
	$B^*_s\eta$	–	–	–
	Total	$167.15g^2_{TH}$		

of g_{XH} enables us to assign the J^P state of $B_{sJ}(6114)$. The coupling values obtained for 1^3D_1 and 1^1D_2 are consistent with $g_{XH} = 0.19$. We suggest these two J^P 1^3D_1 and 1^1D_2 as being the most favorable for the state $B_{sJ}(6114)$. However, $B_{sJ}(6114)$ was observed in BK modes, so it cannot have $J^P = 2^-$ as it does not satisfy the conservation of parity and angular momentum simultaneously for $J^P = 2^-$. So, we assign a particular $J^P = 1^-(1^3D_1)$ to the state $B_{sJ}(6114)$. Further experimental investigations, such as into branching ratios, may provide the necessary conclusions.

3.2.1. *1D state.* The natural parity D states 1^3D_1 and 1^3D_3 are both dominant in the BK decay mode with branching fractions of 56.73% and 36.67% respectively, while the unnatural parity states 1^1D_2 and 1^3D_2 show dominance in the B^*K decay channel with branching fractions of 77.51% and 67.02% respectively. Column 4 of Table 5 gives the ratio of the partial decay widths for $1D$ bottom states with respect to the partial decay width $B^{*+}K^-$. Apart from the decay channels listed in Table 5, these bottom states also decay to P-wave bottom meson states, which occur via D-waves; thus, due to the small phase space, these decay modes are suppressed when compared to decays to ground-state S-wave mesons and hence are not shown in Table 5. The calculated value of $g_{XH} = 0.12$ can be beneficial in finding the total and partial decay widths of the unobserved bottom state 1^3D_1. Thus the calculated total decay width for this state is 87.15 MeV, which deviates by 22.87% from the result of Ref. [39].

3.2.2. *1P state.* We have also analyzed the strong decay widths of $1P$ bottom states and calculated the various branching ratios involved. The calculated values of the partial decay widths for the bottom states 1^3P_0, 1^1P_1, 1^3P_1, and 1^3P_2 are listed in Table 6. The state 1^3P_0 decaying to the BK decay channel is kinematically suppressed as the predicted mass for this state is lower than the BK threshold value. This is a similar situation to that seen in the charm sector for the same J^P state $D^*_{s0}(2317)$, reflecting the flavor symmetry in heavy hadrons. The only kinematically allowed decay channel for this state is $B_s\pi$. Its spin partner 1^1P_1 also decays to the $B^*_s\pi^0$ decay mode only while all other modes are suppressed. Using the available coupling constant value $g_{SH} = 0.56$ [25,32,40], the predicted decay widths for the $(0^+, 1^+)$ doublet are calculated as

$$\Gamma(1^3P_0) = 46.25\,\text{MeV}$$
$$\Gamma(1^1P_1) = 50.51\,\text{MeV}.$$

Our estimated values are greatly overestimated compared to the results of Ref. [40] and underestimated compared to the data from the quark pair creation model and chiral quark model [28,36,40].

The study of the doublet $(1^+, 2^+)$ shows that $B^*_s\pi^0$ and $B_s\pi^0$ are the dominant decay modes for the 1^3P_1 and 1^3P_2 states with branching ratios of 85.33% and 32.30% respectively. The total decay widths, calculated by taking the sum of the partial decay widths listed in Table 6, are

$$\Gamma(1^3P_1) = 14.33\,\text{MeV}$$
$$\Gamma(1^3P_2) = 26.74\,\text{MeV}.$$

These decay values depict these states to be narrower but are still overestimated compared to the experimental values measured by the LHCb and CDF Collaborations [11,37]; however, they are in good agreement with the theoretical data [11,40], where the authors used the mixing angle to calculate the width of the 1^3P_1 state.

3.2.3. *2S state.* For the radially excited ground-state S-wave bottom states, Table 6 reveals the B^*K^+ mode to be the dominant decay mode both for the $\widetilde{B}^*_{1s}$ and $\widetilde{B}_{0s}$ bottom states with branching fractions of 22.53% and 33.46%, respectively. Hence the decay mode B^*K^+ is suitable for the experimental search for the missing strange $2S$ bottom meson states. Using the strong coupling value $\widetilde{g_{HH}} = 0.31$ [55], total decay widths for the bottom state $\widetilde{B}^{*}_{1s}$ and its spin partner $\widetilde{B}_{0s}$ are predicted as 207.44 and 231.11 MeV respectively. These speculated values indicate that these radially excited bottom states have broad resonance, which is in good agreement with the results of Refs. [11,40]. Apart from the partial decay widths mentioned, these bottom states also decay to D-wave bottom mesons. However, these decays are suppressed in our calculations because of their small contributions.

4. Conclusion

In this paper, we have applied HQET to examine the strange bottom mesons $B_{sJ}(6063)$ and $B_{sJ}(6114)$ recently observed by the LHCb Collaboration. In this framework, we have calculated the masses of the strange bottom mesons $2S$, $1P$, $1D$ with the use of available experimental and theoretical data on charm mesons and including non-perturbative parameters (Δ_F and λ_F). These predicted masses for the above-mentioned states matched the other model predictions beautifully. Also, by taking $1/m_Q$ corrections in terms of $\delta\Delta_F$ and $\delta\lambda_F$, we estimated the masses of the strange bottom mesons $2S$, $1P$, $1D$, narrowing the gap between our results and the experimental data. On the basis of the computed masses, we have identified the strange bottom state $B_{sJ}(6063)$ as 2^3S_1 and give three possible J^P values (1^-, 2^-, 3^-) belonging to the D-wave of the $B_{sJ}(6114)$ state. We have analyzed strong decay widths for possible J^P states for $B_{sJ}(6114)$ and concluded that most favorable states for $B_{sJ}(6114)$ are 1^3D_1. In addition to this, we have predicted the branching ratios and the coupling constants for the above states, which can provide crucial information for future experimental searches.

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