

Research Article

Thermodynamic Limit in High-Multiplicity Proton-Proton Collisions at $\sqrt{s} = 7$ TeV

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An analysis is made of the particle composition in the final state of proton-proton (pp) collisions at 7 TeV as a function of the charged particle multiplicity ($dN_{\text{ch}}/d\eta$). The thermal model is used to determine the chemical freeze-out temperature as well as the radius and strangeness suppression factor γ_s . Three different ensembles are used in the analysis: the grand canonical ensemble, the canonical ensemble with exact strangeness conservation, and the canonical ensemble with exact baryon number, strangeness, and electric charge conservation. It is shown that for the highest multiplicity class the three ensembles lead to the same result. This allows us to conclude that this multiplicity class is close to the thermodynamic limit. It is estimated that the final state in pp collisions could reach the thermodynamic limit when $dN_{\text{ch}}/d\eta$ is larger than twenty per unit of rapidity, corresponding to about 300 particles in the final state when integrated over the full rapidity interval.

1. Introduction

In statistical mechanics the thermodynamic limit is the limit in which the total number of particles N and the volume V become large but the ratio N/V remains finite and results obtained in the microcanonical, canonical, and grand canonical ensembles become equivalent. In this paper we argue that this limit might be reached in high energy pp collisions if the total number of charged hadrons becomes larger than 20 per unit of rapidity in the mid-rapidity region, corresponding to roughly 300 particles in the final state when integrated over the full rapidity interval. For this purpose use is made of the data published by the ALICE Collaboration [1] on the production of multi-strange hadrons in pp collisions as a function of charged particle multiplicity in a one-unit pseudorapidity interval $\langle dN_{\text{ch}}/d\eta \rangle_{|\eta| < 0.5}$. These data have attracted significant attention because they cannot be reproduced by standard Monte Carlo models [2–4].

In high energy collisions applications of the statistical model in the form of the hadron resonance gas model have been successful [5, 6] in describing the composition

of the final state, e.g., the yields of pions, kaons, protons, and other hadrons. In these descriptions use is made of the grand canonical ensemble and the canonical ensemble with exact strangeness conservation. In this paper we consider in addition the use of the canonical ensemble with exact baryon, strangeness, and charge conservation.

The identifying feature of the thermal model is that all the resonances listed in [7] are assumed to be in thermal and chemical equilibrium. This assumption drastically reduces the number of free parameters as this stage is determined by just a few thermodynamic variables, namely, the chemical freeze-out temperature T_{ch} , the various chemical potentials μ determined by the conserved quantum numbers and by the volume V of the system. It has been shown that this description is also the correct one [8–10] for a scaling expansion as first discussed by Bjorken [11]. After integration over p_T these authors have shown that

$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0} \quad (1)$$

where N_i^0 is the particle yield as calculated in a fireball at rest. Hence, in the Bjorken model with longitudinal scaling and radial expansion the effects of hydrodynamic flow cancel out in ratios.

We will show in this paper that the difference between the ensembles used disappears if the final state multiplicity is large. All calculations were done using THERMUS [12].

We compare three different ensembles based on the thermal model.

- (i) Grand canonical ensemble (GCE), the conservation of quantum numbers is implemented using chemical potentials. The quantum numbers are conserved on the average. The partition function depends on thermodynamic quantities and the Hamiltonian describing the system of N hadrons:

$$Z_{GCE} = \text{Tr} \left[e^{-(H-\mu N)/T} \right] \quad (2)$$

which, in the framework of the thermal model considered here, leads to

$$\ln Z_{GCE}(T, \mu, V) = \sum_i g_i V \int \frac{d^3 p}{(2\pi)^3} \exp\left(-\frac{E_i - \mu_i}{T}\right) \quad (3)$$

in the Boltzmann approximation. The yield is given by

$$N_i^{GCE} = V \int \frac{d^3 p}{(2\pi)^3} \exp\left(-\frac{E_i}{T}\right). \quad (4)$$

We have put the chemical potentials equal to zero, as relevant for the beam energies considered here. The decays of resonances have to be added to the final yield

$$N_i^{GCE}(\text{total}) = N_i^{GCE} + \sum_j Br(j \rightarrow i) N_j^{GCE}. \quad (5)$$

- (ii) Canonical ensemble with exact implementation of strangeness conservation, we will refer to this as the strangeness canonical ensemble (SCE). There are chemical potentials for baryon number B and charge Q but not for strangeness:

$$Z_{SCE} = \text{Tr} \left[e^{-(H-\mu N)/T} \delta_{(S, \Sigma_i S_i)} \right] \quad (6)$$

The delta function imposes exact strangeness conservation, requiring overall strangeness to be fixed to the value S , and in this paper we will only consider the case where overall strangeness is zero, $S = 0$. This change leads to [13]

$$Z_{SCE} = \frac{1}{(2\pi)^3} \int_0^{2\pi} d\phi e^{-iS\phi} Z_{GCE}(T, \mu_B, \lambda_S) \quad (7)$$

where the fugacity factor is replaced by

$$\lambda_S = e^{i\phi} \quad (8)$$

$$N_i^{SCE} = V \frac{Z_i^1}{Z_{S=0}^C} \sum_{k,p=-\infty}^{\infty} a_3^p a_2^k a_1^{-2k-3p-s} I_k(x_2) I_p(x_3) I_{-2k-3p-s}(x_1), \quad (9)$$

where $Z_{S=0}^C$ is the canonical partition function

$$Z_{S=0}^C = e^{S_0} \sum_{k,p=-\infty}^{\infty} a_3^p a_2^k a_1^{-2k-3p} I_k(x_2) I_p(x_3) I_{-2k-3p}(x_1), \quad (10)$$

where Z_i^1 is the one-particle partition function calculated for $\mu_S = 0$ in the Boltzmann approximation. The arguments of the Bessel functions $I_s(x)$ and the parameters a_i are introduced as

$$a_s = \sqrt{\frac{S_s}{S_{-s}}}, \quad (11)$$

$$x_s = 2V \sqrt{S_s S_{-s}},$$

where S_s is the sum of all $Z_k^1(\mu_S = 0)$ for particle species k carrying strangeness s . As previously, the decays of resonances have to be added to the final yield

$$N_i^{SCE}(\text{total}) = N_i^{SCE} + \sum_j Br(j \rightarrow i) N_j^{SCE}. \quad (12)$$

- (iii) Canonical ensemble with exact implementation of B , S , and Q conservation, we will refer to this as the full canonical ensemble (FCE). In this ensemble there are no chemical potentials. The partition function is given by

$$Z_{FCE} = \text{Tr} \left[e^{-(H-\mu N)/T} \delta_{(B, \Sigma_i B_i)} \delta_{(Q, \Sigma_i Q_i)} \delta_{(S, \Sigma_i S_i)} \right] \quad (13)$$

$$Z_{FCE} = \frac{1}{(2\pi)^3} \int_0^{2\pi} d\psi e^{-iB\alpha} \int_0^{2\pi} d\phi e^{-iQ\psi} \cdot \int_0^{2\pi} d\alpha e^{-iS\phi} Z_{GCE}(T, \lambda_B, \lambda_Q, \lambda_S) \quad (14)$$

where the fugacity factors have been replaced by

$$\lambda_B = e^{i\alpha},$$

$$\lambda_Q = e^{i\psi}, \quad (15)$$

$$\lambda_S = e^{i\phi}.$$

As before, the decays of resonances have to be added to the final yield

$$N_i^{FCE}(\text{total}) = N_i^{FCE} + \sum_j Br(j \rightarrow i) N_j^{FCE}. \quad (16)$$

A similar analysis was done in [14] for pp collisions at 200 GeV but without the dependence on charged multiplicity.

TABLE I: Comparison between measured and fitted values for pp collisions at 7 TeV for V0M multiplicity class II.

Particle Species	dN/dy (data)	dN/dy (model)		
		Canonical S	Canonical B, S, Q	Grand Canonical
π^+	7.88 ± 0.38	6.78	6.76	6.96
K_S^0	1.04 ± 0.05	1.16	1.16	1.15
p	0.44 ± 0.03	0.50	0.50	0.50
Λ	0.302 ± 0.020	0.259	0.262	0.246
Ξ^-	0.0358 ± 0.0023	0.035	0.035	0.036

In this case the analytic expression becomes very lengthy and we refrain from writing it down here; it is implemented in the THERMUS program [12].

These three ensembles are applied to pp collisions in the central region of rapidity. It is well known that, in this kinematic region, one has particle antiparticle symmetry and therefore there is no net baryon density and also no net strangeness. The GCE implements the conservation of quantum numbers on average; this means that they are not conserved exactly but fluctuations around the average exist. The SCE implements the conservation of the strangeness quantum number exactly. The FCE implements the conservation of the baryon, strangeness, and charge quantum numbers exactly; all three of them are being forced to be zero. Hence the three different ensembles can give different results because of the way they are implemented. Since the yields are measured in a small rapidity window it can be argued that the exact conservation of quantum numbers is never warranted because the full phase space is not covered.

A clear size dependence is present in the results of the ensembles. In the thermodynamic limit they should become equivalent. Clearly there are other ensembles that could be investigated and also other sources of finite volume corrections. We hope to address these in a longer publication in the near future.

A similar analysis was done in [14–16] for pp collisions at 200 GeV but without the dependence on charged multiplicity.

2. Comparison of Different Statistical Ensembles

In Figure 1(a) we show the chemical freeze-out temperature as a function of the multiplicity of hadrons in the final state [1]. The freeze-out temperature has been calculated using three different ensembles. The highest values are obtained using the canonical ensemble with exact conservation of three quantum numbers, baryon number B , strangeness S , and charge Q , all of them being set to zero as is appropriate for the central rapidity region in pp collisions at 7 TeV. In this ensemble the temperature drops very clearly from the lowest to the highest multiplicity intervals. The open symbols in Figure 1 were calculated using as input the yields for $\pi^+ + \pi^-$, $p + \bar{p}$, K_S^0 , $\Lambda + \bar{\Lambda}$, and $\Xi^- + \bar{\Xi}^+$ while the full symbols also include the yields for $\Omega^- + \bar{\Omega}^+$ as given in [1] (The values used in this study were obtained by the ALICE Collaboration and can be found at the url: <https://www.hepdata.net/record/77284>). As

an example we show a comparison between measured and fitted values for the multiplicity class II in Table I.

The lowest values for T_{ch} are obtained when using the grand canonical ensemble; in this case the conserved quantum numbers are again zero but only in an average sense. The results are clearly different from those obtained in the previous ensemble, especially in the low multiplicity intervals. They gradually approach each other and they become equivalent at the highest multiplicities.

For comparison with the previous two cases we also calculated T_{ch} using the canonical ensemble with only strangeness S being exactly conserved using the method presented in [13]. In this case the results are very close to those obtained in the grand canonical ensemble, with the values of T_{ch} always slightly higher than in the grand canonical ensemble. Again for the highest multiplicity interval the results become equivalent. As can be seen in Figure 1(a), even though all the ensembles produce different results, for high multiplicities the results converge to a common value around 160 MeV.

In Figure 1(b) we show results for the strangeness suppression factor γ_s first introduced in [17]. In this case we obtain again quite substantial differences in each one of the three ensembles considered. The highest values are found in the canonical ensemble with exact strangeness conservation. Note that the values of γ_s converge to unity as common value, i.e., full chemical equilibrium.

In Figure 1(c) the radius at chemical freeze-out obtained in the three ensembles is presented. As in the previous figures, the results become independent of the ensemble chosen for the highest multiplicities.

An interesting feature is that the volume at chemical freeze-out increases linearly with the multiplicity in the final state. This means that the density at chemical freeze-out tends to a constant for high multiplicities. Again the three ensembles tend to a common value for the highest multiplicity class. This is shown in Figure 1(d) where the ratio $(dN_{\text{ch}}/d\eta)/(4\pi R^3/3)$ of the system at chemical freeze-out is plotted.

The results in Figure 1 show that there is a strong correlation between some of the parameters. The very high temperature obtained in the canonical BSQ ensemble correlates with the small radius in the same ensemble. Particle yields increase with temperature but a small volume decreases them, hence the correlation between the two parameters.

For completeness we also calculated the energy density ε/T^4 using the three ensembles as this plays a role in many

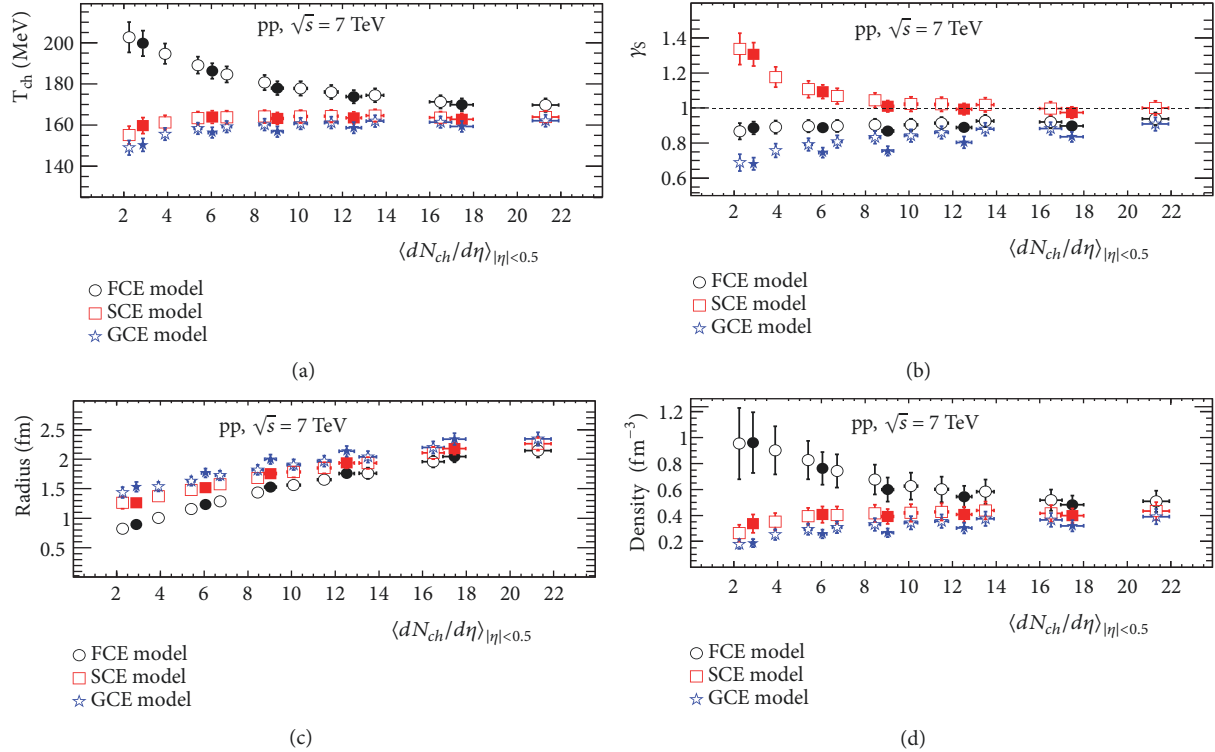


FIGURE 1: The chemical freeze-out temperature T_{ch} obtained for three different ensembles in panel (a). The strangeness suppression factor, γ_s is shown in panel (b). The radius of the system at chemical freeze-out is shown in panel (c). The density is shown in panel (d). The open symbols show results of fitting hadrons yields without Ω whereas solid symbols show fit results including Ω yields.

theoretical considerations. The results obtained are shown in Figure 2 and are in line with those in Figure 1 for the particle density with a convergence to the same energy density for the three different ensembles at the highest multiplicities.

In Figure 3 we show the ratios of particle yields to the pion yields for three different ensembles. Deviations are caused by the known underestimation of pion yield in the thermal models. The comparison of Ω/π ratio data with three different ensembles is shown in Figure 3(e) for the case when Ω is included in the fits.

Table 2 shows the χ^2 values obtained for the three ensembles considered in this paper.

3. Discussion and Conclusions

In this paper we have investigated three different ensembles to analyze the variation of particle yields with the multiplicity of charged particles produced in proton-proton collisions at the center-of-mass energy of $\sqrt{s} = 7$ TeV. It is interesting to note that all three ensembles lead to the same results when the multiplicity of charged particles $dN_{ch}/d\eta$ per unit of the pseudo-rapidity variable η exceeds about 20. This could be interpreted as reaching the thermodynamic limit since the three ensembles lead to the same results. The total number of hadrons in the final state is of the order of 300 for the highest multiplicity class when integrated over the full rapidity interval. Another observation is that the density tends to a constant with increasing multiplicity. It would be

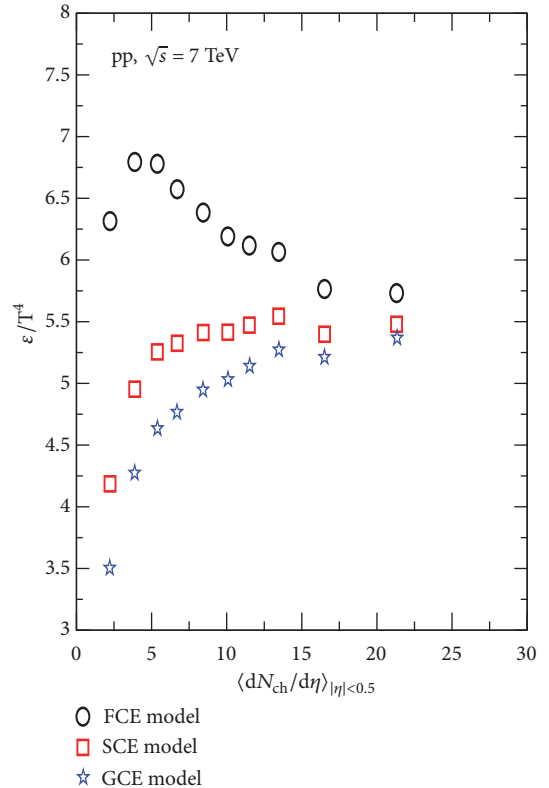


FIGURE 2: The energy density ϵ/T^4 obtained for three different ensembles.

TABLE 2: Values of χ^2/ndf for various fits. The values in the top (bottom) part include (exclude) the Ω yields in the fits.

$\langle dN_{ch}/d\eta \rangle_{ \eta <0.5}$	Canonical S	Canonical B, S, Q	Grand Canonical
2.89	6.04 / 3	24.29 / 3	29.05 / 3
6.06	16.02 / 3	25.89 / 3	32.28 / 3
9.039	21.53 / 3	25.44 / 3	34.58 / 3
12.53	23.83 / 3	25.08 / 3	27.45 / 3
17.47	23.73 / 3	15.93 / 3	11.81 / 3
2.26	3.85 / 2	12.79 / 2	6.45 / 2
3.9	9.15 / 2	20.16 / 2	14.47 / 2
5.4	14.94 / 2	25.46 / 2	20.27 / 2
6.72	16.58 / 2	24.61 / 2	20.09 / 2
8.45	18.71 / 2	24.65 / 2	20.83 / 2
10.08	20.03 / 2	24.45 / 2	21.61 / 2
11.51	20.91 / 2	24.42 / 2	21.80 / 2
13.46	22.25 / 2	24.84 / 2	22.46 / 2
16.51	22.19 / 2	23.52 / 2	22.41 / 2
21.29	21.83 / 2	22.20 / 2	21.55 / 2

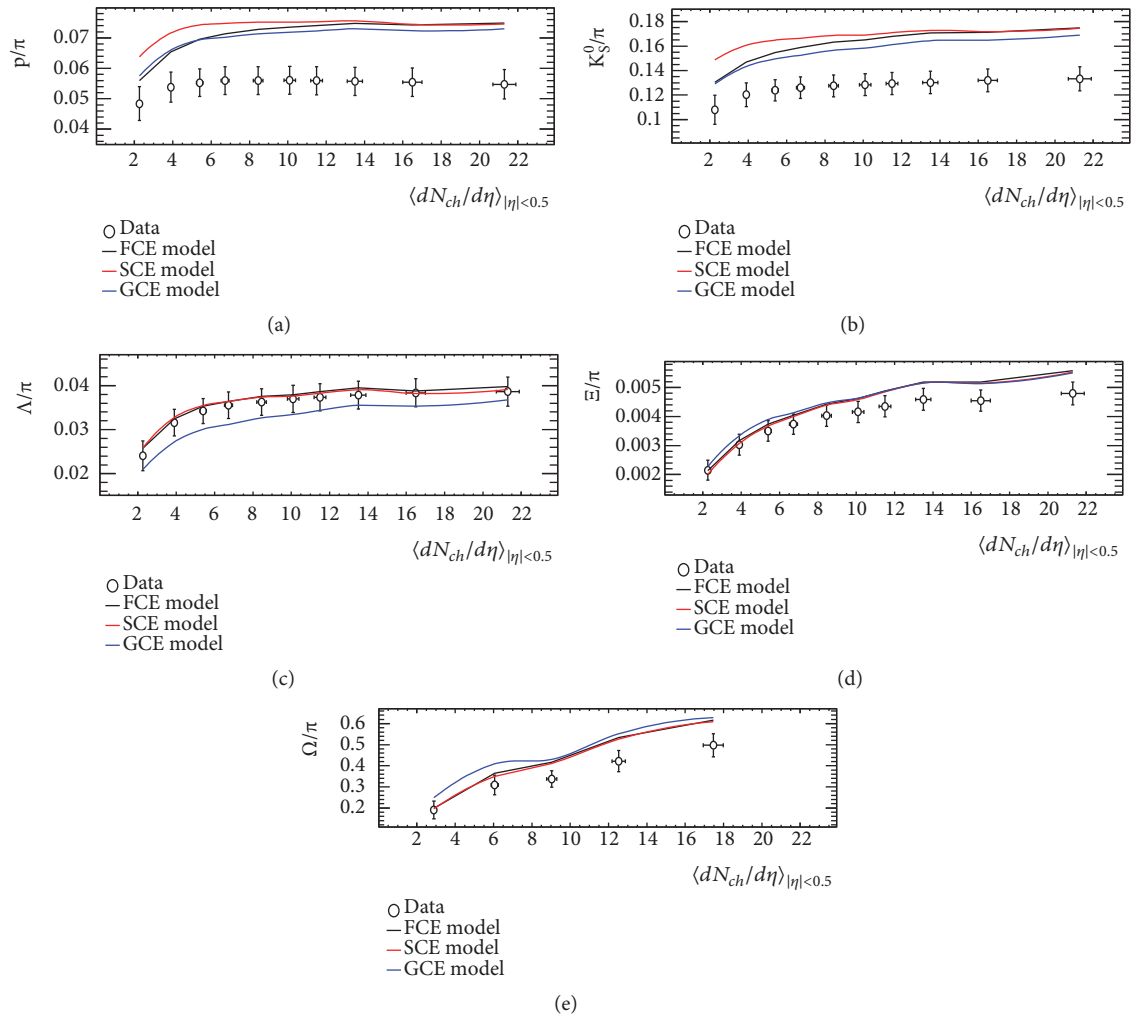


FIGURE 3: Ratios of particle to pion yields as a function of the final-state multiplicity.

of interest to extend this analysis to higher beam energies and higher multiplicity intervals.

Data Availability

The data used in this research paper are available publicly on the webpages of the ALICE collaboration at CERN, Switzerland.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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