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ZZ production at the LHC: Fiducial cross sections and distributions in NNLO QCD

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ABSTRACT

We consider QCD radiative corrections to the production of four charged leptons in the ZZ signal region at the LHC. We report on the complete calculation of the next-to-next-to-leading order (NNLO) corrections to this process in QCD perturbation theory. Numerical results are presented for $\sqrt{s} = 8$ TeV, using typical selection cuts applied by the ATLAS and CMS Collaborations. The NNLO corrections increase the NLO fiducial cross section by about 15%, and they have a relatively small impact on the shape of the considered kinematical distributions. In the case of the $\Delta\Phi$ distribution of the two Z candidates, the computed corrections improve the agreement of the theoretical prediction with the CMS data.

The production of Z-boson pairs at the Large Hadron Collider (LHC) provides an important test of the electroweak (EW) sector of the Standard Model (SM) at the TeV scale. Small deviations in the observed rates or in the kinematical distributions could be a signal of new physics, possibly in the form of anomalous couplings. At the same time, ZZ production is an irreducible background for Higgs boson production and new-physics searches. Particularly important are the off-shell effects below the ZZ threshold, relevant for the Higgs signal region in the four-lepton channel. Various measurements of ZZ hadroproduction have been carried out at the Tevatron and the LHC (for some recent results see Refs. [1–7]).

From the theory side the first NLO predictions for on-shell ZZ production were obtained long ago [8,9]. The leptonic decays of the Z bosons were included, initially neglecting spin correlations in the virtual contributions [10]. The computation of the relevant one-loop helicity amplitudes [11] enabled the first complete NLO calculations [12,13], including spin correlations and off-shell effects. The loop-induced gluon-fusion production channel, which formally contributes only at the next-to-next-to-leading order (NNLO), was computed in Refs. [14,15]. The corresponding leptonic decays were taken into account in Refs. [16–18]. NLO predictions for ZZ production including the gluon-induced contribution, the leptonic decays with spin correlations and off-shell effects were presented in Ref. [19]. The NLO QCD corrections to on-shell $ZZ+\text{jet}$ production were discussed in Refs. [20,21], and approximate NNLO predictions obtained by merging NLO results for ZZ and $ZZ+\text{jet}$ production were presented in Ref. [22]. The EW corrections to ZZ production were obtained in Ref. [23]. A decisive step forward was carried out in Ref. [24] where the inclusive NNLO cross section for on-shell ZZ production was presented. This calculation was based on the evaluation of the two-loop amplitude for on-shell ZZ production. Later, the two-loop helicity amplitudes for all the vector-boson pair production processes were presented [25,26]. This computation paves the way to the consistent inclusion of the leptonic decays and off-shell effects in the NNLO computation.

In this Letter we carry out this step by considering ZZ production at NNLO including the leptonic decays of the vector bosons together with spin correlations and off-shell effects. Contributions from $Z\gamma^*$ and $\gamma^*\gamma^*$ production as well as from $pp \to Z/\gamma^* \to$ 4 leptons topologies are also consistently included with all interference terms. Our calculation allows us to apply arbitrary cuts on the final-state leptons and the associated QCD radiation. Here we present selected numerical results for $pp \to 4$ leptons at the LHC in NNLO QCD, using the typical cuts that are applied in the experimental ZZ analyses.

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Our calculation is performed with the numerical program MATRIX[2], which combines the q_T subtraction [27] and resummation [28] formalisms with the MUNICH Monte Carlo framework [29]. MUNICH provides a fully automated implementation of the Catani–Seymour dipole subtraction method [30,31], an efficient phase-space integration, as well as an interface to the one-loop generator OPENLOOPS [32] to obtain all required (spin- and colour-correlated) tree-level and one-loop amplitudes. For the numerically stable evaluation of tensor integrals, OPENLOOPS relies on the COLLIER library [33], which is based on the Denner–Dittmaier reduction techniques [34,35] and the scalar integrals of [36]. To deal with problematic phase-space points, a rescue system is provided, which employs the quadruple-precision implementation of the OPP method in CUTTOOLS [37] and scalar integrals from ONELOOP [38]. Our implementation of q_T subtraction and resummation for the production of colourless final states is fully general, and it is based on the universality of the hard-collinear coefficients [39] appearing in transverse-momentum resummation. These coefficients were explicitly computed for quark-initiated processes in Refs. [40–42]. For the two-loop helicity amplitudes we use the results of Ref. [26], and of Ref. [43] for Drell–Yan like topologies.

A preliminary version of MATRIX has been employed in the NNLO computations of Refs. [44,24,45,46], and in the resummed calculation of Ref. [47].

We consider pp collisions with $\sqrt{s} = 8$ TeV. As for the EW couplings, we use the so-called G_μ scheme, where the input parameters are G_F, m_W, m_Z. More precisely, consistent with the OPENLOOPS implementation, we use the complex W and Z boson masses to define the EW mixing angle as $\cos\theta_W^2 = (m_W^2 - i\Gamma_W m_W)/(m_Z^2 - i\Gamma_Z m_Z)$. In particular, we use the values $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, $m_W = 80.399$ GeV, $\Gamma_W = 2.1054$ GeV, $m_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV. For the top quark we use $m_t = 173.2$ GeV, $\Gamma_t = 1.4426$ GeV, and for the Higgs boson $m_H = 125$ GeV, $\Gamma_H = 4.07$ MeV. Both the top quark and the Higgs boson only appear in diagrams with closed top-quark loops. Since the two-loop contribution from closed light-fermion loops is extremely small, we estimate that the top-quark diagrams at two-loop order, which are not accounted for in the calculation of Ref. [26], can be safely neglected. We find that also the other diagrams involving a top-quark loop, entering the gluon-fusion and the real-virtual contributions, provide only few per mille of the full NNLO cross section. The Higgs boson contributes less than 1% to the loop-induced $gg \to ZZ$ cross section, whereas its effect on the real-virtual contribution is numerically negligible. We use the NNPDF3.0 [48] sets of parton distributions with $\alpha_S(m_Z) = 0.118$, and the α_S running is evaluated at each corresponding order (i.e., we use $(n+1)$-loop α_S at N^{n}LO, with $n = 0, 1, 2$). We consider $N_f = 5$ massless quark flavours. The central renormalization (μ_R) and factorization (μ_F) scales are set to $\mu_R = \mu_F = m_Z$. This is the same choice adopted for the computation of the inclusive cross section for on-shell ZZ pairs. We believe this choice is justified, since the cuts we are going to consider are not very stringent.

We first consider the ATLAS analysis of Ref. [5] in the three decay channels $e^+e^-e^+e^-$, $\mu^+\mu^-\mu^+\mu^-$, and $e^+e^-\mu^+\mu^-$. The invariant masses of the two reconstructed lepton pairs are required to fulfill the condition 66 GeV $\le m_{ll} \le$ 116 GeV. In the case of two lepton pairs with the same flavours there is a pairing ambiguity, which is resolved by choosing the pairing that makes the sum of the absolute distances from m_Z smaller. The leptons are required to have $p_T \ge 7$ GeV and rapidity $|\eta| \le 2.7$. For any lepton pair we require $\Delta R(l, l') > 0.2$, independently of the flavours and charges of l and l'.

The corresponding cross sections are reported in Table 1, where the ATLAS results are also shown.[3] The uncertainties on our theoretical predictions are obtained by varying the renormalization and factorization scales in the range $0.5m_Z < \mu_R, \mu_F < 2m_Z$ with the constraint $0.5 < \mu_F/\mu_R < 2$. Independently of the leptonic decay channels, the NNLO corrections increase the NLO result by about 15%, similarly to what was found for the inclusive cross section for on-shell ZZ production [24]. This is as expected because the selection cuts are mild and do not significantly alter the impact of radiative corrections. The scale uncertainties are about $\pm 3\%$ at NLO and remain of the same order at NNLO. As noted for the inclusive cross section [24], the NLO scale uncertainty does not cover the NNLO effect. This is not surprizing since the loop-induced gluon-fusion contribution, which provides $\sim 60\%$ of the $\mathcal{O}(\alpha_S^2)$ correction, opens up only at NNLO. The NNLO corrections improve the agreement of the theoretical prediction with the data for the $e^+e^-\mu^+\mu^-$ channel, whereas they deteriorate the agreement in the case of the $4e$ and 4μ channels. We note, however, that the predicted fiducial cross sections are still consistent with the ATLAS measurements at the 1σ level within the statistics-dominated uncertainties.

Secondly, we consider the CMS analysis of Ref. [7]. The fiducial region is defined as follows: all muons are required to fulfill $p_T^\mu > 5$ GeV, $|\eta^\mu| < 2.4$, while all electrons are required to fulfill $p_T^e > 7$ GeV, $|\eta^e| < 2.5$. In addition, the leading- and subleading-lepton transverse momenta must satisfy $p_T^{l,1} > 20$ GeV and $p_T^{l,2} > 10$ GeV, respectively. In the case of two lepton pairs with the same flavours, the pairing ambiguity is resolved by choosing the pair with the smallest distance from m_Z. This pair is called Z_1, the remaining pair is called Z_2. The invariant masses of the two reconstructed lepton pairs are required to fulfill 60 GeV $\le m_{ll} \le$ 120 GeV. We note that in the case of identical flavours this definition of the fiducial region does not prevent the invariant masses of the other two possible lepton pairs from becoming arbitrarily small, giving rise to a collinear $\gamma^* \to l^-l^+$ singularity. To avoid that, we follow CMS and add an additional cut $m_{ll} > 4$ GeV on all lepton pairs of the same flavours and opposite charges.[4] The corresponding fiducial cross sections and scale uncertainties, computed as above, are reported in Table 2. Like for the ATLAS analysis, the NNLO corrections increase the NLO fiducial cross section by about 15%. The scale uncertainties are similar to those reported in Table 1.

CMS does not report the fiducial cross sections corresponding to the above cuts, but only normalized distributions, to which we compare our results. We start with the invariant-mass distribution of the four leptons, which is depicted in Fig. 1. The lower panels show the theory/data comparison, and the NNLO result normalized to the central NLO prediction. We see that the NNLO corrections have a limited impact in the comparison with the data, which still have large uncertainties. The NNLO effects on the normalized distribution are relatively small: they are completely negligible at low

[2] MATRIX is the abbreviation of "MUNICH Automates qT subtraction and Resummation to Integrate X-sections", by M. Grazzini, S. Kallweit, D. Rathlev, M. Wiesemann. In preparation.

[3] Throughout this paper the numerical error on our theoretical predictions corresponds to an estimate of the numerical uncertainties affecting our calculation. Up to NLO this estimate coincides with the statistical error from the numerical integration. At NNLO this estimate is obtained by linearly combining the integration error with the systematic uncertainty of the q_T subtraction method. In practice, the NNLO calculation is performed with a lower bound r_{cut} on the transverse momentum of the four-lepton system normalized to its invariant mass. The final result is obtained by extrapolating r_{cut} to zero. Our results are extremely stable upon variations of this technical cut, and the ensuing uncertainty is estimated to be at the one per mille level, i.e., of the order of the statistical uncertainties.

[4] We thank Alexander Savin for providing us with this information.

Table 1
Fiducial cross sections and scale uncertainties for ATLAS cuts at LO, NLO, and NNLO in the three considered leptonic decay channels. The ATLAS data are also shown.

Channel	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)	σ_{exp} (fb)
$e^+e^-e^+e^-$ $\mu^+\mu^-\mu^+\mu^-$	$3.547(1)^{+2.9\%}_{-3.9\%}$	$5.047(1)^{+2.8\%}_{-2.3\%}$	$5.79(2)^{+3.4\%}_{-2.6\%}$	$4.6^{+0.8}_{-0.7}(\mathrm{stat})^{+0.4}_{-0.4}(\mathrm{syst.})^{+0.1}_{-0.1}(\mathrm{lumi.})$ $5.0^{+0.6}_{-0.5}(\mathrm{stat})^{+0.2}_{-0.2}(\mathrm{syst.})^{+0.2}_{-0.2}(\mathrm{lumi.})$
$e^+e^-\mu^+\mu^-$	$6.950(1)^{+2.9\%}_{-3.9\%}$	$9.864(2)^{+2.8\%}_{-2.3\%}$	$11.31(2)^{+3.2\%}_{-2.5\%}$	$11.1^{+1.0}_{-0.9}(\mathrm{stat})^{+0.5}_{-0.5}(\mathrm{syst.})^{+0.3}_{-0.3}(\mathrm{lumi.})$

Table 2
Fiducial cross sections and scale uncertainties for CMS cuts at LO, NLO, and NNLO in the three considered leptonic decay channels.

Channel	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
$e^+e^-e^+e^-$	$3.149(1)^{+3.0\%}_{-4.0\%}$	$4.493(1)^{+2.8\%}_{-2.3\%}$	$5.16(1)^{+3.3\%}_{-2.6\%}$
$\mu^+\mu^-\mu^+\mu^-$	$2.973(1)^{+3.1\%}_{-4.1\%}$	$4.255(1)^{+2.8\%}_{-2.3\%}$	$4.90(1)^{+3.4\%}_{-2.6\%}$
$e^+e^-\mu^+\mu^-$	$6.179(1)^{+3.1\%}_{-4.0\%}$	$8.822(1)^{+2.8\%}_{-2.3\%}$	$10.15(2)^{+3.3\%}_{-2.6\%}$

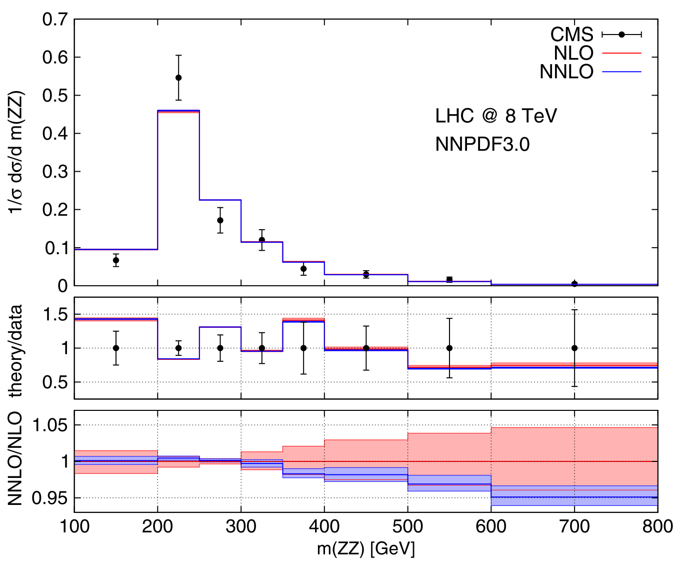

Fig. 1. The four-lepton invariant-mass distribution at NLO and NNLO compared to the CMS data. In the lower panels the ratio of our theoretical results over the data, and the NNLO result normalized to the central NLO prediction are presented. The bands correspond to scale variations as described in the text.

invariant masses, and they increase to −5% in the high mass region. This means that the NNLO corrections make the invariant mass distribution slightly softer. We have checked that this effect is due to the gluon-fusion contribution, whose relative effect decreases at high masses, due to the larger values of Bjorken x that are probed. The NLO (NNLO) scale uncertainties range from about ±2% (±1%) at low m_{ZZ} to ±4% (±2%) at high m_{ZZ}.

In Fig. 2 we show the analogous results for the leading-lepton p_T distribution (left) and the azimuthal separation ($\Delta\Phi$) of the two Z candidates (right). As in Fig. 1, we see that the NNLO effects on the p_T distribution do not change the comparison with the data in a significant way. The NNLO corrections are also relatively small in most of the range of p_T considered, except for the low p_T region, where they increase significantly. This effect is due to the gluon-fusion contribution, whose relative impact increases as p_T decreases. The situation is different for the $\Delta\Phi$ distribution. Here the NNLO corrections improve the agreement with the data, except for the first bin, where the CMS measurement is an order of magnitude below the theoretical NNLO prediction. The larger impact of NNLO corrections in the $\Delta\Phi$ distribution can be understood easily by the observation that at LO the reconstructed Z bosons are always back-to-back, i.e., $\Delta\Phi(Z_1, Z_2) = \pi$. As a consequence, the NNLO calculation is effectively NLO in the region $0 \le \Delta\Phi < \pi$. The NNLO corrections amount to about +25% when $\Delta\Phi \lesssim 1.5$, and decrease as $\Delta\Phi$ increases. We note that this effect is entirely due to the NNLO corrections to the $q\bar{q}$ channel addressed in this paper, since the loop-induced gluon-fusion contribution, which also enters at NNLO, affects the $\Delta\Phi$ distribution only at $\Delta\Phi = \pi$. The NLO scale uncertainties are about ±11%, while at NNLO the uncertainties are about ±5% at low $\Delta\Phi$, and decrease to about ±2% at high $\Delta\Phi$.

We have presented the first complete NNLO QCD calculation for the production of four charged leptons in the ZZ signal region at the LHC. We have studied the impact of NNLO corrections

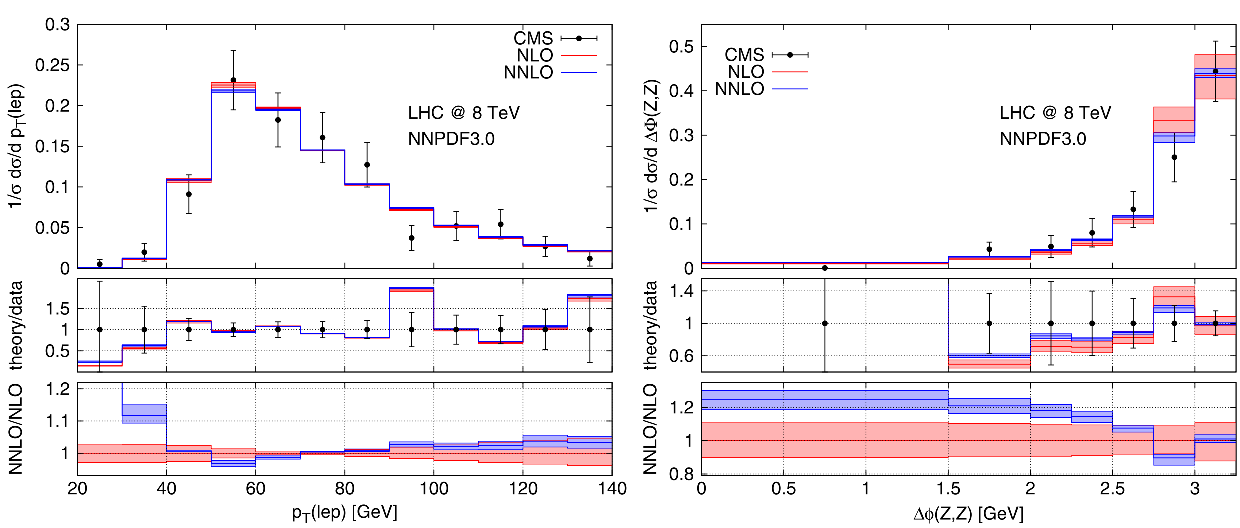

Fig. 2. The leading-lepton p_T (left) and the $\Delta\phi$ (right) distributions at NLO and NNLO compared to the CMS data. In the lower panels the ratio of our theoretical results over the data, and the NNLO result normalized to the central NLO prediction are presented. The bands correspond to scale variations as described in the text.

on the fiducial cross sections and distributions measured by ATLAS and CMS at the LHC. As for the fiducial cross sections, we found about +15% NNLO corrections w.r.t. the NLO prediction, consistent with what was found for the inclusive cross section for on-shell ZZ production [24]. The impact on the normalized distributions we considered is small compared to the experimental uncertainties, but leads to an improved agreement with the data in the case of the $\Delta\Phi$ distribution of the two Z candidates. Our calculation was performed with the numerical program MATRIX, which is able to carry out fully exclusive NNLO computations for a wide class of processes at hadron colliders. We look forward to further applications of our framework to other important LHC processes.

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