

Intermediate inflation in a generalized induced-gravity scenario

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Abstract An intermediate inflationary Universe model in the context of non-minimal coupling to the scalar curvature is analyzed. We will conduct our analysis in the slow-roll approximation of the inflationary dynamics and the cosmological perturbations considering a coupling of the form $F(\varphi) = \kappa + \xi_n \varphi^n$. Considering the trajectories in the r - n_s plane from the Planck data, we find the constraints on the parameter space in our model.

1 Introduction

It is well known that the inflationary era, or simply inflation, has been a fundamental contributor to our understanding of the early Universe. In this sense, inflation has been successful in explaining some of cosmological puzzles, i.e., the horizon and flatness problems etc. [1–3]. However, the most important element in this scenario is that inflation gives us a theoretical framework within which to describe the large-scale structure (LSS) [4–9], as well as the anisotropy of the cosmic microwave background (CMB) radiation from the early Universe [10–12]. From an observational point of view, the Planck satellite [13] together with the LSS experiments [14–17] [in particular considering the baryon acoustic oscillations (BAO) data], have been fundamental to our understanding of the CMB anisotropies of the Universe.

It is thought that the inflationary epoch is driven by a scalar field, or simply inflaton, that can interact fundamentally with other fields, and also with gravity. In this sense, it is natural to consider the non-minimal coupling between the scalar field and gravitation. Historically, the non-minimal coupling with gravity, and in particular with the Ricci scalar, was originally studied in radiation problems [18]. Within the context of the renormalization of the quantum fields in curved backgrounds it was considered in Refs. [19–22]. From the point of view of

cosmology this non-minimal coupling of the scalar field was originally analyzed in Ref. [23], and also by Brans and Dicke [24]; see also Refs. [25, 26]. In the literature of the 1980s, cosmic inflation in the framework of the induced-gravity scalar-tensor theory has been studied in Refs. [27–29]. In particular, a description of the inflationary models considering the non-minimal coupling between the scalar field and the gravitation has been developed in Refs. [30–39]. Particularly, in Ref. [34] was studied the chaotic model considering this coupling. In Ref. [40] was assumed the effective chaotic potential $V \approx \varphi^n$ in which $n > 4$, considering one large scale for the field φ within the framework to this coupling. Here, the authors obtained different constraints on the parameter coupling ξ ; also see Refs. [41–44]. As regards the relation between the tensor-to-scalar ratio and the scalar spectral index the consistency relation was studied in Refs. [14, 45] and applied to the chaotic inflation model, and a global stability analysis also was studied in Ref. [46]. Recently, the frame-independent classification of single-field inflationary models was considered in Ref. [47], and the important case of Higgs inflation in the non-minimally coupled inflation sector was studied in Ref. [48].

In the context of the dynamics background for the inflationary model, exact solutions within the framework of general relativity (GR) can be found when the scale factor expands exponentially from a constant potential, the de Sitter inflationary model [2]. Similarly, an expansion of the power-law type, in which we have the scale factor $a(t) \propto t^p$ with $p > 1$, can be obtained from an exponential potential, giving exact solutions [49]. Nevertheless, intermediate inflation is another kind of exact solution, in which the expansion of the scale factor is slower than de Sitter expansion, but faster than power-law type. During intermediate expansion the scalar factor $a(t)$ expands as

$$a(t) = \exp[A t^f], \quad (1)$$

where A and f are two constants, in which $A > 0$ and $0 < f < 1$ [50]. As mentioned previously, this inflation-

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ary model was originally studied in order to find an exact solution to the background equations. Nevertheless from the observational point of view, intermediate inflation is more effectively motivated in the slow-roll approximation [51–60]. Here, we consider the slow-roll approximation; it is feasible to find a scalar spectral index $n_s \sim 1$, and this kind of spectrum is favored by the current CMB data. In particular, for the specific value of $f = 2/3$ in which the scale factor varies as $a(t) \propto t^{2/3}$, the scalar spectral index becomes $n_s = 1$, i.e., we have the Harrison–Zel’dovich spectrum. However, within the framework of GR this model presents a fundamental problem due to the fact that the tensor-to-scalar ratio $r > 0.1$, wherewith the model is disfavored by observational data, as a result of the Planck data’s establishment of an upper bound for the ratio r at the pivot scale $k_* = 0.002 \text{ Mpc}^{-1}$, in which $r_{0.002} < 0.1$. In this form, the model of intermediate inflation in the framework of GR does not work.

In this paper we would like to study the possible actualization of an expanding intermediate inflation within the framework of a non-minimal coupling with the curvature. We will explore the dynamics from the slow-roll approximation in this theory considering a function $F(\varphi) = \kappa + \xi_n \varphi^n$. In this context, we will find the cosmological perturbations – scalar perturbations and tensor perturbations – and the trajectories in the r – n_s plane, and we will establish whether or not the model works. In our analysis, we shall resort to Planck satellite data [13] in order to constrain the parameters in our model.

The outline of the article is as follows. The next section presents the background dynamics and we find the slow-roll solutions for our model. In Sect. 3 we determine the corresponding cosmological perturbations. Finally, in Sect. 4 we summarize our findings. We chose units so that $c = \hbar = 1$.

2 Intermediate inflation: background equations

We consider a generalized induced-gravity action in the Jordan frame given by

$$S = \int d^4x \sqrt{-g} \left[\frac{F(\varphi)}{2} R - \frac{1}{2} g^{\mu\nu} \varphi_{;\mu} \varphi_{;\nu} - V(\varphi) \right], \quad (2)$$

where $F(\varphi)$ can be an arbitrary function of the scalar field in the Jordan frame, R is the Ricci scalar and $V(\varphi)$ is the effective potential associated to the scalar field φ .

Different types of functions $F(\varphi)$ have been studied in the literature. In particular, we have the case that the function $F(\varphi) = (1 - \xi \varphi^2)$ coincides with the non-minimal coupling action; see Ref. [61]. Also, the special case in which the function $F(\varphi) \propto \varphi^2$ was studied in [62]. In the following, we will assume that the function $F(\varphi)$ is defined as

$$F(\varphi) = \kappa + \xi_n \varphi^n, \quad (3)$$

where $\kappa = \frac{1}{8\pi G} = \frac{M_p^2}{8\pi}$, with M_p the reduced Planck mass and the quantities n (dimensionless) and ξ_n (with units of M_p^{2-n}) being two constants. The particular case in which the exponent of the function corresponds to $n = 2$ ($\xi_{n=2} = \xi$), together with $\kappa = 0$, coincides with the corresponding induced-gravity model studied in Ref. [63].

From the action given by Eq. (2), the cosmological equations in a spatially flat Friedmann–Robertson–Walker (FRW) cosmological model in the Jordan frame are given by

$$H^2 = \frac{1}{6F} \left[\dot{\varphi}^2 + 2V - 6H\dot{F} \right], \quad (4)$$

$$\dot{H} = \frac{1}{2F} \left[-\dot{\varphi}^2 + H\dot{F} - \ddot{F} \right], \quad (5)$$

and

$$V_{,\varphi} = 3F_{,\varphi}(\dot{H} + 2H^2) - 3H\dot{\varphi} - \ddot{\varphi}, \quad (6)$$

where $H = \dot{a}/a$ is the Hubble parameter and a corresponds to the scale factor. Here, the dots mean derivatives with respect to time and the subscript $(, \varphi)$ means derivative with respect to the scalar field φ .

We introduce the slow-roll parameters defined by

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_2 \equiv \frac{\ddot{\varphi}}{H\dot{\varphi}}, \quad \epsilon_3 \equiv \frac{\ddot{F}}{2HF},$$

$$\text{and } \epsilon_4 \equiv \frac{\dot{E}}{2HE}, \quad (7)$$

in which $\epsilon_i \ll 1$ and its evolution $\dot{\epsilon}_i \simeq 0$ during inflation. Here, the quantity E is defined as $E = F(1 + \frac{3\dot{F}^2}{2F\varphi^2})$.

Considering the slow-roll parameter ϵ_3 , we have $\frac{\ddot{F}}{2F} = H\dot{\epsilon}_3$; then we can neglect the last right side term in Eq. (5); thus, during the slow-roll scenario we get

$$2F\dot{H} + \dot{\varphi}^2 - H\dot{F} \simeq 0. \quad (8)$$

Now combining Eqs. (1), (3) and (8) we find that the differential equation for the scalar field is given by

$$2Af(f-1)t^{f-2}(\kappa + \xi_n \varphi^n) + \dot{\varphi}^2 - Afn\xi_n t^{f-1} \varphi^{n-1} \dot{\varphi} = 0. \quad (9)$$

The solution of Eq. (9) for the scalar field can be written as

$$\varphi(t) = c t^m, \quad (10)$$

where m and c are two constants. In order to satisfy the power-law solution of the scalar field given by Eq. (10), we find that we need the constraints

$$m = \frac{f}{2}, \quad c^2 = \frac{8\kappa A(1-f)}{f}, \quad \text{and} \quad n = 4 \left(1 - \frac{1}{f}\right).$$

As we have mentioned previously, the parameter f lies between $0 < f < 1$, in order to obtain an acceleration of the Universe; then the parameter $n < 0$.

By using the slow-roll approximation in Eq. (4) and combining Eqs. (1) and (10), the effective potential in terms of the scalar field results:

$$V(\varphi) \approx 3[H^2 F + H\dot{F}] = 3Afc^{-n} \times \left[Af\kappa + Af\xi_n\varphi^n + \xi_n n m c^2 \varphi^{n-2}\right] \varphi^n. \quad (11)$$

Here, we note that considering the special case in which $\xi_n = 0$ (or equivalently $F = \text{const.}$), the effective potential given by Eq. (11) coincides with that corresponding to the standard intermediate inflation in general relativity, where $V(\varphi) \propto \varphi^{4(1-f^{-1})}$; see Refs. [51–60].

An important quantity during the background dynamics is the number of e-folds N at the end of inflation and it becomes

$$N = \int_{t_1}^{t_2} H dt = \int_{\varphi_1}^{\varphi_2} \frac{H(\varphi)}{\dot{\varphi}} d\varphi = \frac{f}{8\kappa(1-f)} \{\varphi_2^2 - \varphi_1^2\}, \quad (12)$$

between two different values of cosmological times t_1 and t_2 or between the values φ_1 and φ_2 of the scalar field.

On the other hand, the parameter ϵ_1 from Eqs. (7) and (10) can be written as

$$\epsilon_1 = \frac{8\kappa(1-f)^2}{f^2} \frac{1}{\varphi^2}. \quad (13)$$

Here, we note that the slow-roll parameter ϵ_1 diverges when the scalar field $\varphi \rightarrow 0$ and then $\epsilon_1 \gg 1$ (or equivalently $\ddot{a} \ll 0$, deceleration). In this limit, when the field approaches zero, the effective potential given by Eq. (11) also diverges (since the exponent $n < 0$). Now, for large values of the scalar field, we note that asymptotically the effective potential and the slow-roll parameter ϵ_1 tend to zero, in which $\epsilon_1 < 1$. In this sense, since during the evolution of the Universe the effective potential decreases, we consider the inflation scenario to begin at the earliest possible epoch, in which $\epsilon_1 = 1$, where the scalar field $\varphi_1 = \varphi(t_1)$ results:

$$\varphi_1 = \sqrt{8\kappa} \frac{(1-f)}{f}. \quad (14)$$

Also, we note that the inflationary epoch takes place when $\epsilon_1 < 1$ (or equivalently $\ddot{a} > 0$); then the scalar field satisfies the condition $\varphi > \frac{\sqrt{8\kappa}(1-f)}{f}$.

In this way, we find that the value of the scalar field $\varphi_2 = \varphi(t = t_2)$ in terms of the number of e-folds N can be written as

$$\varphi_2^2 = \frac{8\kappa(1-f)(1+f(N-1))}{f^2}. \quad (15)$$

Here, we have used Eqs. (12) and (14).

3 Cosmological perturbations

In this section we will study the cosmological perturbations in our model of intermediate inflation in a generalized induced-gravity scenario. In this framework, the perturbation metric around the flat background can be written as

$$ds^2 = -(1 + 2\Phi)dt^2 + 2a(t)\Theta_{,i}dx^i dt + a^2(t) \times [(1 - 2\psi)\delta_{ij} + 2E_{,i,j} + 2h_{ij}]dx^i dx^j, \quad (16)$$

where the quantities Φ , Θ , ψ and E correspond to the scalar-type metric perturbations, and h_{ij} denotes the transverse traceless tensor perturbation. Also the perturbation in the field φ is given by $\varphi(t, \mathbf{x}) = \varphi(t) + \delta\varphi(t, \mathbf{x})$, in which $\delta\varphi(t, \mathbf{x})$ is a small perturbation that corresponds to small fluctuations of the scalar field. Thus, introducing the comoving curvature perturbations, \mathcal{R} defined as $\mathcal{R} = \Psi + \mathcal{H} \frac{\delta\varphi}{\dot{\varphi}}$ [64], where the new Hubble parameter is given by $\mathcal{H} \equiv \frac{a'}{a}$ (a prime corresponds to the derivative with respect to a conformal time, $d\eta = a^{-1}dt$), the scalar density perturbation \mathcal{P}_S in the Jordan frame can be written as [65,66]

$$\mathcal{P}_S \equiv \frac{k^3}{2\pi^2} |\mathcal{R}|^2 = A_S^2 \left[\frac{k|\eta|}{2}\right]^{3-2\nu_s}, \quad (17)$$

where the amplitude $A_S^2 \equiv \frac{1}{Q_s} \left(\frac{H}{2\pi}\right)^2 \left(\frac{1}{aH|\eta|}\right)^2 \left[\frac{\Gamma(\nu_s)}{\Gamma(3/2)}\right]^2$ and the quantities Q_s and ν_s are defined as

$$Q_s = \frac{\varphi'^2 \left[1 + \frac{3\xi_n n^2 \varphi^{n-2}}{2}\right]}{\left[\mathcal{H} + \frac{n\varphi'}{2\varphi}\right]^2}, \quad \text{and} \quad \nu_s = \sqrt{\gamma_s + 1/4},$$

respectively. Here, the parameter γ_s is given by $\gamma_s = \frac{(1+\delta_s)(2-\epsilon_1+\delta_s)}{(1-\epsilon_1)^2}$, in which $\delta_s = \frac{\dot{Q}_s}{2HQ_s}$.

On the other hand, the scalar spectral index n_s is defined by $n_s = 1 + \frac{d \ln \mathcal{P}_S}{d \ln k}$. Thus, from Eq. (17) and considering the slow-roll approximations, the scalar spectral index n_s in terms of the slow-roll parameters can be written as [65]

$$n_s \simeq 1 - 2(2\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4). \quad (18)$$

By considering the slow-roll parameters from Eq. (7), the scalar spectral index n_s as a function of the scalar field, i.e., $n_s = n_s(\varphi)$, results:

$$n_s \simeq 1 - C_1\varphi^{-2} - C_2\varphi^{n-2} \times \left[\frac{1 + 3\xi_n n(n-1)\varphi^{n-2}}{\kappa + \xi_n\varphi^n + \frac{3}{2}\xi_n^2 n^2 \varphi^{2(n-1)}} - \frac{1}{\kappa + \xi_n\varphi^n}\right], \quad (19)$$

where the constants C_1 and C_2 are defined as

$$C_1 = \frac{16\kappa(1-f)(1-\frac{3}{2}f)}{f^2}, \quad \text{and} \quad C_2 = \frac{4\xi_n n \kappa(1-f)}{f}.$$

From Eq. (19) we observe that in the limit $\xi_n \rightarrow 0$ the spectral index n_s agrees with the appropriate standard intermediate inflationary model, in which $n_s = 1 - C\varphi^{-2}$ with $C = 8\kappa(1-f)(2-3f)/f^2$; see Refs. [51–60]. Also we note that for the specific value $f = 2/3$ we clearly find from Eq. (19) that $n_s \neq 1$, contrary to standard intermediate inflation, where for $f = 2/3$ the index $n_s = 1$, i.e., we have the so-called Harrison–Zel’dovich spectrum [51–60]. In this form, we note from Eq. (19) that the spectral index includes two free parameters (f, ξ_n) , unlike what occurs in the standard intermediate inflation, where n_s has only one free parameter, namely f . In relation to the constants, we note that for values of $f \leq 2/3$ the constant $C_1 \geq 0$. Similarly, we observe that as the parameter $n < 0$, then the constant $C_2 \geq 0$ when the parameter $\xi_n \leq 0$.

Also, we find that the spectral index n_s in terms of the number of e-folds, N , becomes

$$n_s \simeq 1 - \frac{C_1}{\kappa\beta^2} - \frac{C_2}{\kappa^{(2-n)/2}\beta^{2-n}} \times \left[\frac{1 + 3\xi_n n(n-1)\kappa^{(n-2)/2}\beta^{n-2}}{\kappa + \xi_n \kappa^{n/2}\beta^n + \frac{3}{2}\xi_n^2 n^2 \kappa^{(n-1)}\beta^{2(n-1)}} - \frac{1}{\kappa + \xi_n \kappa^{n/2}\beta^n} \right], \quad (20)$$

where the quantity β is given by $\beta = \frac{1}{\sqrt{8(1-f)(1+f[N-1])}}$.

Numerically from Eq. (20) we find a constraint for the parameter ξ_n . Certainly, we can find the value of the parameter ξ_n giving a specific value of the parameter f , when the number of e-folds N and the index n_s are given. Particularly, for the values $n_s = 0.967$, $N = 60$ and $f = 0.9$ (or equivalently $n \simeq -0.44$), we obtain from Eq. (20) that the real solution for $\xi_{n=-0.44}$ corresponds to $\xi_{n=-0.44} \simeq -5.84M_p^{2.44}$. For the case in which $f = 2/3$ corresponds to $\xi_{n=-2} \simeq -7.42M_p^4$, when $f = 0.5$ the parameter $\xi_{n=-4}$ results: $\xi_{n=-4} \simeq -70.01M_p^6$, and for the case where $f = 0.46$ the parameter $\xi_{n=-4.70}$ is given by $\xi_{n=-4.70} \simeq -170.82M_p^{6.70}$.

On the other hand, the generation of tensor perturbations during inflation would produce gravitational waves, where its power spectrum \mathcal{P}_T , following Ref. [66], can be written as

$$\mathcal{P}_T \equiv A_T^2 \left[\frac{k|\eta|}{2} \right]^{3-2\nu_T}, \quad (21)$$

where now the tensor amplitude $A_T^2 = \frac{8}{F} \left(\frac{H}{2\pi} \right)^2 \left(\frac{1}{aH|\eta|} \right)^2 \left[\frac{\Gamma(\nu_T)}{\Gamma(3/2)} \right]^2$, and the quantities ν_T , γ_T and δ_T are defined as

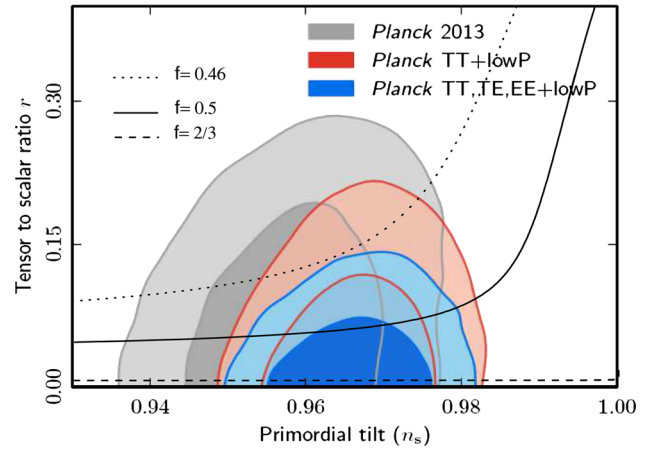


Fig. 1 The tensor-to-scalar ratio r versus the spectral index n_s . Here, from Planck data, two dimensional marginalized constraints on the ratio r and n_s (at 1σ confidence level, i.e., 68% and at 2σ confidence level, i.e., 95%). In this plot, the dotted, solid, and dashed lines correspond to the values of $f = 0.46$ together with the value $\xi_{n=-4.7} = -170.82M_p^{6.7}$, ($f = 0.5, \xi_{n=-4} = -70.01M_p^6$ and ($f = 2/3, \xi_{n=-2} = -7.42M_p^4$), respectively. Here, we have used the value $M_p = 1$

$$\nu_T \equiv \sqrt{\gamma_T + 1/4}, \quad \gamma_T = \frac{(1 + \delta_T)(2 - \epsilon_1 + \delta_T)}{(1 - \epsilon_1)^2},$$

$$\text{and } \delta_T = \frac{n\dot{\phi}}{2H\varphi}.$$

In this context, an important observational quantity is the tensor-to-scalar ratio r , given by $r = \mathcal{P}_T/\mathcal{P}_S$. This ratio can be written in terms of the slow-roll parameters results [65],

$$r \simeq | -13.8(\epsilon_1 + \epsilon_3) |. \quad (22)$$

In this form, considering the slow parameters given by Eq. (7), we write the tensor-to-scalar ratio as

$$r \simeq | \frac{-13.8}{2} \left[C_1 \frac{(1-f)}{(1-3f/2)} + C_2 \frac{\varphi^n}{\kappa + \xi_n \varphi^n} \right] \varphi^{-2} |, \quad (23)$$

and in terms of the number of e-folds, N , we have

$$r \simeq | -\frac{13.8}{2} \kappa \beta^2 \left[\frac{C_1(1-f)}{(1-3f/2)} + C_2 \frac{\beta^n \kappa^{n/2}}{\kappa + \xi_n \kappa^{n/2} \beta^n} \right] |, \quad (24)$$

here, we have used Eqs. (12) and (22).

In Fig. 1 shows the contour plot for the tensor-to-scalar ratio r versus the spectral index n_s , for distinct values of the parameter f associated to intermediate expansion of the scale factor. From Ref. [67], we have two dimensional marginalized constraints on the ratio $r_{0.002}$ and index n_s (at 1σ confidence level i.e., 68% and at 2σ confidence level i.e., 95%). Here, the dotted, solid, and dashed lines correspond to the values of $f = 0.46$ together with the value

$\xi_{n=-4.7} = -170, 82M_p^{6.7}$, ($f = 0.5, \xi_{n=-4} = -70.01M_p^6$) and ($f = 2/3, \xi_{n=-2} = -7.42M_p^4$), respectively. In order to write down the ratio r on the spectral index n_s , we use Eqs. (20) and (24) and we numerically obtain the parametric plot for the relation $r = r(n_s)$. We observe that for the values of $f \gtrsim 0.46$ (or equivalently $n \gtrsim -4.7$) and $\xi_{n=-4.7} \gtrsim -171M_p^{6.7}$ the model is well corroborated by the Planck data, as visualized by this figure. For the values of the parameter $f < 0.46$ and $\xi_n < -171M_p^{2-n}$ the model becomes disfavored by the Planck data, because the tensor-to-scalar ratio $r_{0.002} > 0.1$. Also, we observe that, for values of the parameter $f \sim 1$, the tensor-to-scalar ratio r tends to zero. In this way, we find that the constraints for the parameter f associated to intermediate expansion of the scale factor is given by $0.46 \lesssim f < 1$ and for the parameter $-171M_p^{2-n} \lesssim \xi_n < 0$, from the Planck data.

On the other hand, an analysis of all data taken by the BICEP2 and Keck Array CMB polarization experiments together with the Planck data, were analyzed in Ref. [68]. Here, combining the results from BICEP2 and Keck Array with the constraints from Planck analysis of the CMB temperature, plus BAO and other data, a combined limit is obtained for the tensor-to-scalar ratio r at pivot scale $k_* = 0.05\text{Mpc}^{-1}$, given by $r_{0.05} < 0.07$ at 95% (2σ confidence level). In this sense, the value of $f = 0.46$ together with the value $\xi_{n=-4.7} = -171M_p^{6.7}$ (see Fig. 1) are disfavored by Ref. [68], since the tensor-to-scalar ratio $r > 0.7$. In this form, considering Ref. [68] we see that the constraints for the quantity f associated with intermediate expansion of the scale factor is given by $0.46 < f < 1$ and for the parameter $-171M_p^{2-n} < \xi_n < 0$.

This suggests that the function $F(\varphi)$ can be written as $F(\varphi) = \kappa - |\xi_n|\varphi^{-|n|}$, and assuming that the function $F(\varphi) > 0$, then the range for the scalar field during intermediate inflation in this framework satisfies $\varphi > (|\xi_n|/\kappa)^{(1/|n|)}$. Also, we note that in the limit $\varphi \rightarrow \infty$, the function $F(\varphi)$ takes the asymptotic value $F(\varphi)_{\varphi \rightarrow \infty} \rightarrow \kappa = (8\pi G)^{-1}$. Thus, from the observational point of view (in particular from the consistency relation $r = r(n_s)$), we found that the intermediate inflation in a generalized induced-gravity scenario is less limited than the standard intermediate inflation in which GR is utilized, due to the incorporation of a new parameter, i.e., ξ_n .

In the following we will mention some constraints on the coupling parameter ξ_n obtained in the literature, in order to compare with our results. We assume the framework of induced-gravity inflation where the function $F(\varphi) = \xi\varphi^2$ was studied in Ref. [62]. Here, for the chaotic potential case we found the constraint by the coupling $\xi_{n=2} = \xi \geq 10^{-3}$, and for the new inflation case $\xi \leq 4 \times 10^{-3}$, assuming the constraint from scalar spectral index n_s found in [62]. For the case in which the function $F(\varphi) = 1 + \xi\varphi^2$ with

φ^4 self-interaction, it was found that $\xi \geq 4 \times 10^{-3}$ [62]. For the same coupling function and analyzing the potentials φ^p and exponential, the constraints on ξ considering the Wilkinson Microwave Anisotropy Probe (WMAP) data were obtained in Ref. [69]. Here, for the value $p = 4$ it was found that $\xi \leq -0.17$ and $\xi \geq 0.01$, and for the exponential potential $0.271 \leq \xi \leq 0.791$. In Ref. [66] the function $F(\varphi) = (1 - \xi\kappa^2\varphi^2)/\kappa^2$ was analyzed together with the potential $V(\varphi) \propto \varphi^p$. The constraint obtained for the parameter ξ from the $r-n_s$ plane, for the quadratic potential ($p = 2$) is $\xi > -1.1 \times 10^{-2}$ (2σ bound) and for the case in which $p = 4, \xi < -3.0 \times 10^{-4}$ (2σ bound) [66]. In virtue of these results, our constraint indicates that in order to have values of the coupling $|\xi_n|/M_p^{2-n} \mathcal{O}(1)$, necessarily the parameter f tends to one, wherewith the tensor-to-scalar ratio $r \sim 0$.

4 Conclusions

In this paper we have studied the intermediate inflationary model in the context of generalized induced-gravity scenario. From the function $F(\varphi) = \kappa + \xi_n\varphi^n$, we have found solutions to the background dynamics under the slow-roll approximation. Also, we have found expressions for the scalar and tensor power spectrum, scalar spectral index, and tensor-to-scalar ratio. In this sense, we have found the constraints on some parameters from the Planck data. In this context, we have found from Eqs. (20) and (24) that the trajectories in the $r-n_s$ plane are well supported by the data (see Fig. 1), and we have obtained the constraints on the parameters f and ξ_n given by $0.46 \lesssim f < 1$ and $-171M_p^{2-n} \lesssim \xi_n < 0$, respectively. However, considering the combined limit of Ref. [68] in which $r_{0.05} < 0.7$ at 2σ confidence level, i.e., 95%, we have found the constraints on the parameters f and ξ_n to be given by $0.46 < f < 1$ and $-171M_p^{2-n} < \xi_n < 0$, respectively.

Also, we have noted that intermediate inflation in a generalized induced-gravity scenario is less restricted than the standard intermediate inflation, due to the incorporation of a new parameter, i.e., ξ_n . Thus, the inclusion of this parameter from the function $F(\varphi)$ permits us freedom on the consistency relation $r = r(n_s)$.

Finally, we should mention that the effective potential given by Eq. (11) does not have a minimum. In this form, the scalar field does not oscillate around this minimum [70–75], and therefore is a problem for the standard mechanism of reheating in these types of intermediate models [76]. We hope to return to the subject of reheating in the near future.

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