CALCULATION OF BRANCHING FRACTION AND CP VIOLATION IN $B^- \to D_s^- D^0$ DECAY*

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The most precise measurement of the CP asymmetry in the $B^- \to D_s^- D^0$ decay has been reported by the LHCb Collaboration with the value of $(-0.4 \pm 0.5 \pm 0.5)\%$. In this study, the CP violation in the $B^- \to D_s^- D^0$ decay has been calculated under the factorization approach. This decay mode includes the current–current tree, and penguin diagrams and their amplitudes are considered separately. In each of the tree and penguin amplitudes, the strong and weak phases have been introduced. The CP asymmetry has been calculated in this work to be $(-0.35 \pm 0.03)\%$. Finally, from the sum of the amplitudes, we have calculated the total amplitude and obtained comparable results with the experimental value for the branching ratio of $B^- \to D_s^- D^0$ decay.

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1. Introduction

In the Standard Model (SM), the primary importance of studying the nonleptonic two-body decays of B mesons is to explore CP violation and flavour parameters. The different weak- and strong-interaction phases that arise from the interference of several competing amplitudes play an important role in CP violation. The weak complex phases are obtained from the argument of quark mixing matrix elements entering each amplitude vertex, which follow the unitarity triangle relation, $V_{tb}^* V_{uq} + V_{cb}^* V_{cq} + V_{tb}^* V_{tq} = 0$ $(q = d, s)$. All measured CP asymmetries are in the SM related through this unitarity condition. The weak phases are introduced in a standard convention as $\phi_1 = -\arg(V_{tq})$ and $\phi_2 = \arg(V_{tb}^*)$, the strong interaction phase differences between tree and penguin amplitudes, δ, is another phase that is necessary for CP violation to occur.

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In some charm decays of B mesons, such as the $B^- \to D_s^- D^0$ decay, the decay rates are obtained for combined particles and antiparticles, while such decays are affected by CP violation. It is in such cases that the weak- and strong-interaction phases induce the violation that has occurred for the charge and parity in reality.

The LHCb Collaboration has reported on the first measurement of CP asymmetry in the $B^- \to D_s^- D^0$ decay [1]. The value was measured to be $(-0.4 \pm 0.5 \pm 0.5)\%$. There are also several experimental results and some of their averages for the CP-averaged branching ratio of the $B^- \to D_s^- D^0$ decay are shown in Table 1.

Table 1. Some experimental results and their averages of CP-averaged branching ratio for $B^- \to D_s^- D^0$ decay (in units of 10^{-3}).

Refs.	$\mathcal{B}(B^- \to D_s^- D^0)$
BABAR [2]	$13.3 \pm 1.8 \pm 3.2$
Belle [3]	9.5 ± 0.2
LHCb [4]	$8.6 \pm 0.2 \pm 0.4 \pm 1.0$
HFLAV [5]	13.3 ± 3.7
PDG(2020) Avg. [6]	9.0 ± 0.9

In this work, we have calculated the CP-violation and CP-averaged branching ratio for the $B^- \to D_s^- D^0$ decay. First, we have drawn all the contributions of decay diagrams in accordance with Feynman's rules. Using the factorization approaches, the matrix elements of effective Hamiltonian have been evaluated. In these approaches, the simple factorization of the elements of the hadron matrix appears as the product of two matrices. One of these matrices arises from the transition between meson B and one of the final mesons (form factor). The other matrix is created by the residual end state due to the vacuum (decay constant). In the studied decay, both matrices are for pseudoscalar (spin 0), so the form factor becomes $\langle B^- \to D^0 \rangle$, and the decay constant takes the form of $\langle 0 \to D_s^- \rangle$. The main purpose of obtaining hadron matrix elements is to estimate the amplitude. Then, the decay rate, branching ratio, and CP violation are obtained from it. The branching fraction is obtained to be $\mathcal{B}(B^- \to D_s^- D^0) = (9.33 \pm 1.17) \times 10^{-3}$ at $\mu = m_b/2$ scale. This value is well compatible with the value of $\mathcal{B}(B^- \to D_s^- D^0) = (9.00 \pm 0.90) \times 10^{-3}$ that was reported by PDG(2020) Avg. [6]. A value, $(-0.35 \pm 0.03)\%$, comparable to the experimental result of LHCb [1], $(-0.4 \pm 0.5 \pm 0.5)\%$, is obtained for CP violation.

2. Branching fraction and CP asymmetry in $B^- \to D_s^- D^0$ decay

In this section, we calculate the CP asymmetry and the branching ratio for comparison with experimentally measured results. In the study of CP violation in B decays, it turned out to be useful to make a classification of CP violating effects that is more transparent than the division into the indirect and direct CP violation. Generally, complex phases may enter the particle–antiparticle mixing and the decay processes through the complex elements of the CKM matrix. As the phases in mixing and decay are convention-dependent, the CP violating effects depend only on the differences of these phases. Three types of CP violation are: CP violation in mixing, CP violation in decay, CP violation in the interference of mixing and decay [7].

In this study, the CP violation in decay may occur for $B^- \to D_s^- D^0$ decay. CP violation in decay may be zero due to very close values of particle and antiparticle decay rates. In order for this symmetry not to be zero, two different contributions in the amplitude with the strong (δ_i) and weak (ϕ_i) phases are needed. These could be, for instance, two tree diagrams, two penguin diagrams, or just a diagram of each of them. Considering the process of $B^- \to D_s^- D^0$ decay (and $B^+ \to D_s^+ \bar{D}^0$ decay), this decay mode can proceed through two different elementary amplitudes $\mathcal{A}_1$ and $\mathcal{A}_2$, this means that the decay can proceed by two different paths: tree ($\mathcal{A}_1$) and penguin ($\mathcal{A}_2$) diagrams. For the total decay amplitude, we can write [8]

$$\mathcal{A}\left(B^- \to D_s^- D^0\right) = |\mathcal{A}_1| \mathrm{e}^{i\delta_1} \mathrm{e}^{i\phi_1} + |\mathcal{A}_2| \mathrm{e}^{i\delta_2} \mathrm{e}^{i\phi_2} \,, \tag{1}$$

where $|\mathcal{A}_1|$ and $|\mathcal{A}_2|$ represent $|\mathcal{A}_1\left(B^- \to D_s^- D^0\right)|$ and $|\mathcal{A}_2\left(B^- \to D_s^- D^0\right)|$. To obtain the antiparticle amplitude ($B^+ \to D_s^+ \bar{D}^0$), in Eq. (1), the weak phases (ϕ_i) become its complex conjugate and the strong phases (δ_i) remain unchanged.

For the decay examined in this study, $B^- \to D_s^- D^0$ decay, Feynman's main diagrams are the tree-level diagram that has the largest contribution in the amplitude, and the penguin level diagram, which is significantly smaller than the tree-level diagram. The Feynman diagrams of $B^- \to D_s^- D^0$ decay are shown in Fig. 1 and then, the decay amplitude reads

$$\begin{aligned}&\mathcal{A}\left(B^- \to D_s^- D^0\right) = \\ &\frac{iG_{\mathrm{F}}}{\sqrt{2}} f_{D_s} F_0^{B\to D}\left(m_{D_s}^2\right)\left(V_{cb}V_{cs}^* a_1 - V_{tb}V_{ts}^*\left(a_4 + a_{10} + \xi(a_6 + a_8)\right)\right) \,,\end{aligned} \tag{2}$$

where the tree-and penguin-level amplitudes are as follows, respectively:

$$\mathcal{A}_1\left(B^- \to D_s^- D^0\right) = \frac{iG_{\mathrm{F}}}{\sqrt{2}} f_{D_s} F_0^{B\to D}\left(m_{D_s}^2\right) V_{cb}V_{cs}^* a_1 \,, \tag{3}$$

and

$$\mathcal{A}_2\left(B^- \to D_s^- D^0\right) = \frac{iG_\mathrm{F}}{\sqrt{2}} f_{D_s} F_0^{B\to D}\left(m_{D_s}^2\right) V_{tb}V_{ts}^*\left(a_4 + a_{10} + \xi\left(a_6 + a_8\right)\right) . \tag{4}$$

ξ depends on properties of the final-state mesons involved and is defined as [9]

$$\xi = \frac{2m_{D_s}^2}{(m_b - m_c)(m_c + m_s)} . \tag{5}$$

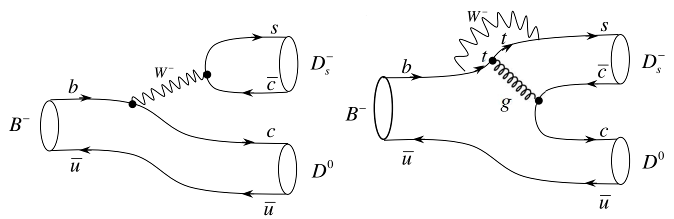

Fig. 1. Feynman diagrams contributing to $B^- \to D_s^- D^0$ decay.

To calculate the form factor F_0, we take the form [10]

$$f\left(q^2\right) = \frac{f(0)}{1 - \sigma_1 q^2/m_\mathrm{P}^2 + \sigma_2 q^4/m_\mathrm{P}^4} , \tag{6}$$

where $q^2 = P_{B^-}^2 - P_{D^0}^2 = P_{D_s}^2$. The value of the parameter m_P (pole mass), which is equal to the lowest resonance mass, is fixed to its physical value for proper choice of the quark-model parameters and for the reliability of the calculations. With this description, the m_P is the m_{B_c} for $B \to D$ transition. The values of $f(0)$, σ_1 and σ_2 are as follows [10]

$$F_0^{B\to D}: \qquad f(0) = 0.67 , \qquad \sigma_1 = 0.78, \qquad \sigma_2 = 0 . \tag{7}$$

The quantities a_i (i = 1, ..., 10) are the following combinations of the effective Wilson coefficients

$$a_{2i-1} = c_{2i-1} + \frac{1}{3}c_{2i} , \qquad a_{2i} = c_{2i} + \frac{1}{3}c_{2i-1} , \qquad i = 1,2,3,4,5 . \tag{8}$$

The Wilson coefficients, c_i, in the effective weak Hamiltonian have been reliably evaluated by the next-to-leading logarithmic order. To proceed, we use the following numerical values at three different choices of μ scale, which have been obtained in the NDR scheme and are shown in Table 2. The meson

Table 2. Wilson coefficients c_i in the NDR scheme ($\alpha = 1/129$) [11].

NLO	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$
c_1	1.137	1.081	1.045
c_2	−0.295	−0.190	−0.113
c_3	0.021	0.014	0.009
c_4	−0.051	−0.036	−0.025
c_5	0.010	0.009	0.007
c_6	−0.065	−0.042	−0.027
c_7/α	−0.024	−0.011	0.011
c_8/α	0.096	0.060	0.039
c_9/α	−1.325	−1.254	−1.195
c_{10}/α	0.331	0.223	0.144

and quark masses and decay constants needed in our calculations are taken as (in units of MeV) [6]

$$\begin{aligned} m_{B_c} &= 6274.9 \pm 0.8\,, & m_{D_s^-} &= 1968.35 \pm 0.07\,, \\ m_{B^\pm} &= 5279.34 \pm 0.12\,, & m_{D^0} &= 1864.84 \pm 0.05\,, \\ m_b &= 4180^{+40}_{-30}\,, & m_s &= 93^{+11}_{-5}\,, \\ m_c &= 1270 \pm 20\,, & f_{D_s} &= 241 \pm 3\,[12]\,. \end{aligned} \tag{9}$$

The decay rates corresponding to the $\mathcal{A}\left(B^- \to D_s^- D^0\right)$ and $\mathcal{A}\left(B^+ \to D_s^+ \bar{D}^0\right)$ amplitudes can be then written as [13]

$$\begin{aligned} \varGamma\left(B^- \to D_s^- D^0\right) &= \left(|\mathcal{A}_1|\mathrm{e} + |\mathcal{A}_2|\mathrm{e}^{i(\delta_2+\phi_2)}\right)^2, \\ \varGamma\left(B^+ \to D_s^+ \bar{D}^0\right) &= \left(|\mathcal{A}_1|\mathrm{e}^{i(\delta_1-\phi_1)} + |\mathcal{A}_2|\mathrm{e}^{i(\delta_2-\phi_2)}\right)^2. \end{aligned} \tag{10}$$

The branching fraction for the $B^- \to D_s^- D^0$ decay is given by

$$\mathcal{B}\left(B^- \to D_s^- D^0\right) = \frac{\varGamma\left(B^- \to D_s^- D^0\right)}{\varGamma_{\mathrm{tot}}}\,, \tag{11}$$

where the $\varGamma_{\mathrm{tot}}$ for charged B meson is $(4.02 \pm 0.01) \times 10^{-13}$ GeV. By dividing the difference between these two decay rates by their sum, the CP asymmetry in decay rates is given by [14]

$$\begin{aligned} \mathcal{A}_{\mathrm{CP}} &= \frac{\varGamma\left(B^- \to D_s^- D^0\right) - \varGamma\left(B^+ \to D_s^+ \bar{D}^0\right)}{\varGamma\left(B^- \to D_s^- D^0\right) + \varGamma\left(B^+ \to D_s^+ \bar{D}^0\right)} \\ &= \frac{2|\mathcal{A}_2/\mathcal{A}_1|\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)}{1 + |\mathcal{A}_2/\mathcal{A}_1|^2 + 2|\mathcal{A}_2/\mathcal{A}_1|\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)}\,. \end{aligned} \tag{12}$$

It should be noted that the numerical value of the difference between the tree and penguin amplitudes, $\mathcal{A}_1\left(B^- \to D_s^- D^0\right) - \mathcal{A}_2\left(B^- \to D_s^- D^0\right)$, is obtained as a complex number. The strong phase δ_1–δ_2 arises from the ratio of the imaginary part to the real part. In fact, the argument of the difference of two amplitudes gives the strong phase, the estimated value is δ_1–$\delta_2 = 89.94°$.

As mentioned in the introduction section, the weak phase ϕ_1 is calculated from the argument of the complex element of V_{ts} in the CKM matrix, the result is achieved to be $\phi_1 = 1.10°$. Considering that the element V_{tb} in the CKM matrix is a real number and the weak phase ϕ_2 comes from the argument of this element, so we set $\phi_2 = 0$.

In the Standard Model, the Cabibbo–Kobayashi–Maskawa (CKM) quark-mixing matrix is a unitary matrix, in which an expansion is introduced by the small value of λ. The CKM matrix at he order of λ^5 can be parameterized as [15]

$$V = \begin{pmatrix} 1-1/2\lambda^2-1/8\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+1/2A^2\lambda^5[1-2(\rho+i\eta)] & 1-1/2\lambda^2-1/8\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3[1-\left(1-1/2\lambda^2\right)(\rho+i\eta)] & -A\lambda^2+1/2A\lambda^4[1-2(\rho+i\eta)] & 1-1/2A^2\lambda^4 \end{pmatrix} . \tag{13}$$

Recent Particle Data Group (PDG) average values for the Wolfenstein parameters are [6]

$$\begin{aligned} \lambda &= 0.22650 \pm 0.00048\,, \quad A = 0.790^{+0.017}_{-0.012}\,, \\ \bar\rho &= 0.141^{+0.016}_{-0.017}\,, \qquad\quad \bar\eta = 0.357 \pm 0.011\,, \end{aligned} \tag{14}$$

where $\bar\rho = \rho(1-1/2\lambda^2)$ and $\bar\eta = \eta(1-1/2\lambda^2)$ [16]. The CKM matrix elements used in this work are obtained by the above calculations as follows (in units of 10^{-3}):

$$\begin{aligned} V_{cb} &= 40.529\,, \quad V_{cs} = 973.198\,, \\ V_{tb} &= 999.179\,, \quad V_{ts} = -39.790 - 0.762i\,. \end{aligned} \tag{15}$$

3. Numerical results and conclusion

The numerical results of the CP-violation and CP-averaged branching ratio for $B^- \to D_s^- D^0$ are presented in Table 3. In this paper, we have analyzed the decay of B meson into two pseudoscalar mesons. We have drawn Feynman diagrams completely for the $B^- \to D_s^- D^0$ decay based on the Standard Model. This decay can violate CP symmetry. In general, CP asymmetry can be calculated from the difference between the particle and the antiparticles decay rates relative to their sum and it becomes

Table 3. The numerical results of CP-violation and CP-averaged branching ratio for $B^- \to D_s^- D^0$ decay at three different choices of μ scale.

$B^- \to D_s^- D^0$	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$	Exp.
$\mathcal{A}_{\rm CP}(\%)$	-0.96 ± 0.09	-0.35 ± 0.03	-0.11 ± 0.01	$-0.4 \pm 0.5 \pm 0.5$ [4]
$\mathcal{B}(\times 10^{-3})$	9.33 ± 1.17	10.12 ± 1.31	10.75 ± 1.41	9.00 ± 0.90 [6]

non-zero for the mentioned decay. Here, we have obtained the CP violation by calculating the amplitude of the current–current tree and penguin diagrams that are considered separately and using the strong (δ_i) and weak (ϕ_i) phases. The weak and strong phases have been obtained by complex elements of the CKM matrix and from current–current tree and penguin amplitude differences, respectively. We have estimated the CP violation as $\mathcal{A}_{\rm CP}(B^- \to D_s^- D^0) = (-0.35 \pm 0.03)\%$. Also, from the sum of the amplitudes, we have calculated the total amplitude and obtained comparable results with experimental values for the branching ratio as: $\mathcal{B}(B^- \to D_s^- D^0) = (9.33 \pm 1.17) \times 10^{-3}$ at $\mu = m_b/2$ scale.

Theoretical uncertainties in our calculations are due to the uncertainties in the form factors, decay constants, meson masses and the uncertainties of the input parameters in CKM elements.

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