Study on possible molecular states composed of $\Lambda_c\bar{D}$ ($\Lambda_b B$) and $\Sigma_c\bar{D}$ ($\Sigma_b B$) within the Bethe-Salpeter framework

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$P_c(4312)$ observed by the LHCb collaboration is confirmed as a pentaquark and its structure, production, and decay behaviors attract great attention from theorists and experimentalists. Since its mass is very close to sum of Σ_c and $\bar{D}$ masses, it is naturally tempted to be considered as a molecular state composed of Σ_c and $\bar{D}$. Moreover, $P_c(4312)$ is observed in the channel with $J/\psi p$ final state, requiring that isospin conservation $P_c(4312)$ is an isospin-1/2 eigenstate. In the literature, several groups used various models to estimate its spectrum. We systematically study the pentaquarks within the framework of the Bethe-Salpeter equation; thus $P_c(4312)$ is an excellent target because of the available data. We calculate the spectrum of $P_c(4312)$ in terms of the Bethe-Salpter equations and further study its decay modes. Some predictions on other possible pentaquark states that can be tested in future experiments are made.

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I. INTRODUCTION

Because of the innovation of experimental techniques and facilities as well as the advances in theory of recent years, several exotic states have been experimentally observed and theoretically studied. Indeed, more constituents would cause more ambiguities, unlike the simplest $q\bar{q}$ for mesons and qqq for baryons. The inner structures of the exotic states are still not clear yet; those discoveries stir up large numbers of discussions [1]. Indeed the theoretical exploration is crucial for getting a better understanding of the quark model and obtaining valuable information about nonperturbative physics. Definitely, to complete the theoretical job achieving more accurate data would compose the key.

Some hidden charm or bottom states were measured in two-meson final states [2–11]. They are regarded as tetraquark states or meson-meson molecular states. In 2003 a baryon was measured by LEPS [12] that was conjectured as a pentaquark; however later the allegation was negated by further more accurate experiments. Breaking the frustration on the existence of the pentaquark that was predicted by Gell-Mann in his first paper on the quark model, the LHCb collaboration reported two pentaquark states observed in Λ_b decays where peaks appear at the $J/\psi p$ final states [13].

Recently another narrow pentaquark state $P_c(4312)$ [14] has also been observed in the $J/\psi p$ mass spectrum. Its mass and width are $4311.9 \pm 0.7^{+6.8}_{-0.6}$ and $9.8 \pm 2.7^{+3.7}_{-4.5}$ MeV, respectively. Since its mass is very close to the sum of Σ_c and $\bar{D}$ masses, it is natural to regard it as a molecular state of $\Sigma_c\bar{D}$ [15–26]. Furthermore, its width is rather wide in accordance with the property of molecular states, so the phenomenon further supports the proposal of its molecular structure. Some other theorists conjecture $P_c(4312)$ as a compact pentaquark [26,27] instead. In Ref. [28] the authors think the interaction between Σ_c and $\bar{D}$ is too weak to bind them into a bound state. It is worth deeper explorations about whether the molecule picture is reasonable. In this work we calculate the mass spectrum of $P_c(4312)$ based on the assumption that it is a stable bound state of Σ_c and $\bar{D}$. Additionally we also study other possible bound states of $\Lambda_c\bar{D}$, $\Lambda_b B$, and $\Sigma_b B$ and see if they can be formed.

We employ the Bethe-Salpeter (B-S) equation to study the possible bound state, which consists of a baryon and a meson. The B-S equation is a relativistic equation to deal with the bound state and established on the basis of quantum field theory [29]. Initially, people used the B-S equation to study the bound state of two fermions [30,31] and the system of one-fermion-one-boson [32]. In Refs. [33,34] the authors employed the Bethe-Salpeter equation to study the $K\bar{K}$ and $B\bar{K}$ molecular states and their decays. With the same approach we studied the

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molecular state of $B\pi$ [35], $D^{(*)}D^{(*)}$, and $B^{(*)}B^{(*)}$ [36]. Recently the approach was extended to explore double charmed baryons [37,38] and pentaquarks, which are assumed to be two-body bound systems. In Ref. [39] the authors studied possible bound states of Λ (Σ) and $\bar{K}$. In this work we employ a similar approach to study the possible bound states of $\Sigma_c\bar{D}$, $\Lambda_c\bar{D}$, $\Lambda_b B$, and $\Sigma_b B$.

At present, pentaquark states $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$ have been measured in decays of Λ_b where the pentaquark states peak up at the invariant mass spectrum of $J/\psi p$, so their isospin is $\frac{1}{2}$ because of isospin conservation. Thus we require that the two hadron constituents reside in an isospin eigenstate. Instead, for the $\Lambda_c\bar{D}$ (as well $\Lambda_b B$) system its isospin must be $\frac{1}{2}$ but the $\Sigma_c\bar{D}$ (or $\Sigma_b B$) system may reside in either isospin $\frac{1}{2}$ or $\frac{3}{2}$. Certainly, for a bound system with spin parity $\frac{1}{2}^-$ the two constituents are in the S wave.

For carrying on our calculation the interactions between two constituents are needed. According to the quantum field theory two particles interact via exchanging certain mediate particles. Since two constituents in a pentaquark are color-singlet hadrons the exchanged particles are some light hadrons such as ρ or (and) ω etc. The effective interactions are deduced from the chiral Lagrangian [40–42], which we list in the Appendix A. With the effective interactions we obtain the kernel and establish the corresponding B-S equation.

With a reasonable parameter set, the B-S equation is solved. For a spin-isospin eigenstate, if the equation does not possess a solution, then we would conclude that the corresponding bound state should not exist in nature; on the contrary, a solution of the B-S equation implies the bound state being formed. At the same time the B-S wave function is obtained and we are able to use the corresponding formula for calculating the rates of strong decay $P_c(4312) \to \text{proton} + \mathcal{V}$ (vector), which can be compared with the data.

This paper is organized as follows: after this introduction we derive the B-S equations related to possible bound states composed of a baryon and a meson and the formula for its strong decays. Then in Sec. III we solve the B-S equation numerically and present our results by figures and tables. Section IV is devoted to a brief summary.

II. THE BOUND STATES OF $\Lambda_c\bar{D}$ AND $\Sigma_c\bar{D}$

Since the newly found pentaquarks $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$ are all hadrons containing hidden charms (or hidden bottoms) and their masses are close to the sums of the masses of several real hadrons, we focus on the molecular structures composed of one charmed (bottomed) baryon and an anticharmed (antibottomed) meson. Concretely, in this paper we study $\Lambda_c\bar{D}$, $\Sigma_c\bar{D}$, $\Lambda_b B$, and $\Sigma_b B$ systems whose spin parity is $\frac{1}{2}^-$; i.e., the spatial wave function is in S wave. In this section as an example we only formulate the corresponding quantities for $\Lambda_c\bar{D}$ and $\Sigma_c\bar{D}$ systems. These formulas can be equally applied to $\Lambda_b B$ and $\Sigma_b B$ systems.

A. The isospin states of $\Lambda_c\bar{D}$ and $\Sigma_c\bar{D}$

The isospin structure of the possible bound state of $\Lambda_c\bar{D}$ is

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = |\Lambda_c\bar{D}^0\rangle. \tag{1}$$

We use $P'_{c(\frac{1}{2},\frac{1}{2})}$ to denote this resonance.

Instead, the possible bound states of $\Sigma_c\bar{D}$ should be in three isospin assignments; i.e., $|I,I_3\rangle$ are $|\frac{1}{2},\pm\frac{1}{2}\rangle$, $|\frac{3}{2},\pm\frac{1}{2}\rangle$, and $|\frac{3}{2},\pm\frac{3}{2}\rangle$. Let us work out the explicit isospin states,

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}|\Sigma_c^{++}D^-\rangle - \sqrt{\frac{1}{3}}|\Sigma_c^{+}\bar{D}^0\rangle, \tag{2}$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}|\Sigma_c^{++}D^-\rangle + \sqrt{\frac{2}{3}}|\Sigma_c^{+}\bar{D}^0\rangle, \tag{3}$$

and

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = |\Sigma_c^{++}\bar{D}^0\rangle. \tag{4}$$

The states $|\frac{1}{2},-\frac{1}{2}\rangle$, $|\frac{3}{2},-\frac{1}{2}\rangle$, and $|\frac{3}{2},-\frac{3}{2}\rangle$ are just the charge conjugate states of $|\frac{1}{2},\frac{1}{2}\rangle$, $|\frac{3}{2},\frac{1}{2}\rangle$, and $|\frac{3}{2},\frac{1}{2}\rangle$; therefore, their hadronic properties are the same. We use $P_{c(\frac{1}{2},\frac{1}{2})}$, $P_{c(\frac{3}{2},\frac{1}{2})}$, and $P_{c(\frac{3}{2},\frac{3}{2})}$ to denote the three isospin states of $\Sigma_c\bar{D}$: $|\frac{1}{2},\frac{1}{2}\rangle$, $|\frac{3}{2},\frac{1}{2}\rangle$, and $|\frac{3}{2},\frac{3}{2}\rangle$, respectively, for latter discussions.

In order to discuss the Isospin factors in the B-S equation we define the fields of baryons and mesons in the expressions [39]

$$\begin{aligned}
\mathcal{B}_1(x) &= \int \frac{d^4q}{(2\pi)^4\sqrt{2m_{\mathcal{B}^{++}}}}(a_{\mathcal{B}^{--}}e^{-iqx} + a^\dagger_{\mathcal{B}^{++}}e^{iqx}),\\
\mathcal{B}_2(x) &= \int \frac{d^4q}{(2\pi)^4\sqrt{2m_{\mathcal{B}^{+}}}}(a_{\mathcal{B}^{-}}e^{-iqx} + a^\dagger_{\mathcal{B}^{+}}e^{iqx}),\\
\mathcal{M}_1(x) &= \int \frac{d^4q}{(2\pi)^4\sqrt{2m_{\mathcal{M}^{+}}}}(a_{\mathcal{M}^{+}}e^{-iqx} + a^\dagger_{\mathcal{M}^{-}}e^{iqx}),\\
\mathcal{M}_2(x) &= \int \frac{d^4q}{(2\pi)^4\sqrt{2m_{\mathcal{M}^{0}}}}(a_{\mathcal{M}^{0}}e^{-iqx} + a^\dagger_{\bar{\mathcal{M}}^{0}}e^{iqx}),
\end{aligned} \tag{5}$$

where $\mathcal{B}$ represents Λ_c or Σ_c and $\mathcal{M}$ denotes D.

B. The B-S equation for $\frac{1}{2}^-$ molecular state

In the effective theory a meson and a baryon can interact via exchanging hadrons. The Feynman diagram at the leading order is depicted in Fig. 1. (It is noted that the diagram where the exchanged hadron is a heavy baryon is

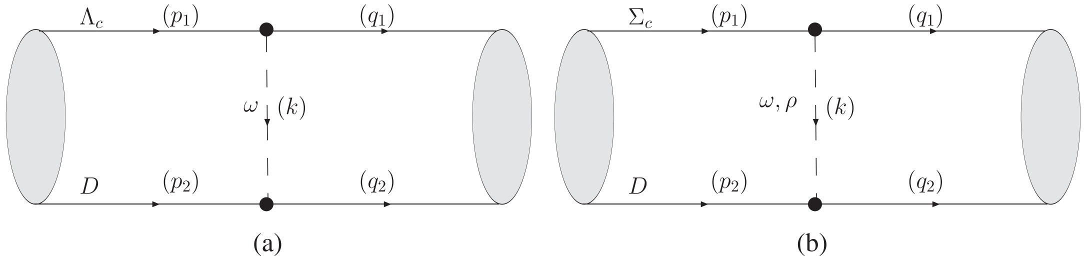

FIG. 1. the bound states of $\Lambda_c\bar{D}$ (a) and $\Sigma_c\bar{D}$ (b) formed by exchanging light vector meson(s).

ignored at the leading order.) The relative and total momenta of the bound state in the equations are defined as

$$p = \eta_2 p_1 - \eta_1 p_2, \qquad q = \eta_2 q_1 - \eta_1 q_2,$$
$$P = p_1 + p_2 = q_1 + q_2, \tag{6}$$

where p and q are the relative momenta at the two sides of the effective vertex, p_1 (q_1) and p_2 (q_2) are those momenta of the constituents, P is the total momentum of the bound state, k is the momentum of the exchanged meson, $\eta_i = m_i/(m_1 + m_2)$, and $m_i (i = 1, 2)$ is the mass of the ith constituent meson.

The bound state composed of a baryon and a meson can be written as

$$\chi_P(x_1, x_2) = \langle 0|T\mathcal{B}(x_1)\mathcal{M}(x_2)|P\rangle. \tag{7}$$

The B-S wave function is a Fourier transformation of that in momentum space,

$$\chi_P(x_1, x_2) = e^{-iPX} \int \frac{d^4q}{(2\pi)^4} \chi_P(p). \tag{8}$$

By the so-called ladder approximation the corresponding B-S equation was deduced in earlier references as

$$\chi_P(p) = S_B(p_1) \int \frac{d^4q}{(2\pi)^4} K(P, p, q)\chi_P(q)S_M(p_2), \tag{9}$$

where $S_B(p_1)$ is the propagator of the baryon (Λ_c or Σ_c), $S_M(p_2)$ is that of the meson ($\bar{D}$), and $K(P, p, q)$ is the kernel that can be obtained by calculating the Feynman diagram in Fig. 1. For later convenience the relative momentum p is decomposed into the longitudinal p_l ($\equiv p \cdot v$) and transverse projection p_t^μ ($\equiv p^\mu - p_l v^\mu$) $= (0, \mathbf{p}_T)$ according to the momentum of the bound state P ($v = \frac{P}{M}$).

$$S_B(\eta_1 P + p) = \frac{i[(\eta_1 M + p_l)\not{v} + \not{p}t + m_1]}{(\eta_1 M + p_l + \omega_l - i\epsilon)(\eta_1 M + p_l - \omega_l + i\epsilon)}, \tag{10}$$

$$S_M(\eta_2 P - p) = \frac{i}{(\eta_2 M - p_l + \omega_2 - i\epsilon)(\eta_2 M - p_l - \omega_2 + i\epsilon)}, \tag{11}$$

where M is the total energy of the bound state, $\omega_i = \sqrt{{p_l}^2 + m_i^2}$, and m_1 (m_2) is the mass of the baryon (meson).

By the Feynman diagram the kernel $K(P, p, q)$ is written as

$$K(P, p, q) = -C_{I,I_z} g_{\mathcal{M}\mathcal{M}\mathcal{V}} g_{\mathcal{B}\mathcal{B}\mathcal{V}} \left(\gamma^\alpha - \frac{\kappa_{\mathcal{B}\mathcal{B}\rho}}{2m_{\mathcal{B}}} \sigma^{\alpha\beta} k_\beta\right) \times (p_2 + q_2)^\mu \Delta_{\alpha\mu}(k, m_V) F^2(k), \tag{12}$$

where m_V is the mass of the exchanged meson, g_{MMV}, g_{BBV}, and $\kappa_{BB\rho}$ are the concerned coupling constants, C_{I,I_z} is the isospin coefficient that is given in Appendix B, and $\Delta_{\alpha\mu}(k, m_V) = (-g_{\alpha\mu} + k_\alpha k_\mu/m_v^2)/(k^2 - m_v^2)$. Apparently the contribution of the tensor term is much smaller than that of the first term; thus we can ignore it in practical computations. Indeed, a numerical estimate verifies this allegation.

Since the constituents of the molecule (meson and baryon) are not point particles, a form factor at each effective vertex should be introduced. The form factor suggested by many researchers is of the form

$$F(\mathbf{k}, m_{\mathrm{V}}^2) = \frac{\Lambda^2 - m_{\mathrm{V}}^2}{\Lambda^2 + \mathbf{k}^2}, \qquad \mathbf{k} = \mathbf{p} - \mathbf{p}', \tag{13}$$

where Λ is a cutoff parameter. Since the form factor is not derived from a fundamental principle, the concerned cutoff parameter is neither determined theoretically; thus until now we have known little about the cutoff parameter Λ. In some references [43–46] the form factor is parametrized as $\lambda\Lambda_{\mathrm{QCD}} + m_V$ with $\Lambda_{\mathrm{QCD}} = 220$ MeV and the dimensionless parameter λ is of order unit. We employ the expression $\Lambda = \lambda\Lambda_{\mathrm{QCD}} + m_V$ in our calculation.

The three-dimensional B-S wave function is obtained after integrating over p_l,

$$\chi_P(p_t) = \int \frac{dp_l}{2\pi} \chi_P(p). \tag{14}$$

For the S-wave system, the spatial wave function can be easily derived [37–39],

$$\chi_P(p_t) = [f_1(|\mathbf{p}_T|) + f_2(|\mathbf{p}_T|)\not{p}t]u(v, s), \tag{15}$$

where $f_1(|\mathbf{p}_T|)$ and $f_2(|\mathbf{p}_T|)$ are the radial wave functions, and $u(v,s)$, v, and s are the spinor, velocity, and total spin of the pentaquark, respectively.

Substituting Eq. (12) into Eq. (9) and employing the so-called covariant instantaneous approximation where $q_l = p_l$, i.e., p_l takes the place of q_l in the kernel $K(P,p,q)$, $K(P,p,q)$ no longer depends on q_1. Then we perform a series of manipulations: integrate over q_l on the right side of Eq. (9); multiply $\int \frac{dp_l}{(2\pi)}$ on both sides of Eq. (9), and integrate over p_l on the left side using Eq. (9). Finally, substituting Eq. (15) we obtain

$$\begin{aligned}
&[f_1(|\mathbf{p}_T|) + f_2(|\mathbf{p}_T|)\not{p}t]u(v,s)\\
&= -\int \frac{dp_l}{(2\pi)} \int \frac{d^3\mathbf{q}_T}{(2\pi)^3} \frac{iC_{I,I_z} g_{MMV} g_{BBV}[(\eta_1 M + p_l)\not{v} + \not{p}t + m_1]}{[(\eta_1 M + p_l)^2 - \omega_l^2 + i\epsilon][(\eta_1 M - p_l)^2 - \omega_2^2 + i\epsilon)]}\\
&\quad\times\left\{-\frac{\kappa[2(\eta_2 M - p_l)\not{v} - \not{p}t - \not{q}t](\not{p}t - \not{q}t) - (\not{p}t - \not{q}t)[2(\eta_2 M - p_l)\not{v} - \not{p}t - \not{q}t]}{4m_B[-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2]}\right.\\
&\quad\left.+\frac{2(\eta_2 M - p_l)\not{v} - \not{p}t - \not{q}t - (\not{p}t - \not{q}t)(\mathbf{p}_T^2 - \mathbf{q}_T^2)/m_V^2}{-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2}\right\} F^2(k, m_V)[f_1(|\mathbf{q}_T|) + f_2(|\mathbf{q}_T|)\not{q}t]u(v,s). \qquad (16)
\end{aligned}$$

Now let us finally fix the expressions of $f_1(|\mathbf{p}_T|)$ and $f_2(|\mathbf{p}_T|)$. Multiplying $\bar{u}(v,s)$ on both sides of Eq. (16), we get an expression that only contains f_1, whereas multiplying $\bar{u}(v,s)\not{p}t$ to the expression, f_2 is obtained; then by taking a trace, the resultant formulas are

$$\begin{aligned}
f_1(|\mathbf{p}_T|) = &-\int \frac{dp_l}{(2\pi)} \int \frac{d^3\mathbf{q}_T}{(2\pi)^3} \frac{iC_{I,I_z} g_{MMV} g_{BBV} F^2(k, m_V)}{[(\eta_1 M + p_l)^2 - \omega_l^2 + i\epsilon][(\eta_1 M - p_l)^2 - \omega_2^2 + i\epsilon]}\\
&\times\left\{\frac{\mathbf{p}_T\cdot\mathbf{q}_T + {\mathbf{p}_T}^2 + 2(m_1 + p_l + M\eta_1)(M\eta_2 - p_l) + ({\mathbf{p}_T}^2 - \mathbf{p}_T\cdot\mathbf{q}_T)({\mathbf{p}_T}^2 - {\mathbf{q}_T}^2)/{m_V}^2}{-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2} f_1(|\mathbf{q}_T|)\right.\\
&+\frac{\frac{({\mathbf{p}_T}^2 - {\mathbf{q}_T}^2)(\mathbf{p}_T\cdot\mathbf{q}_T - {\mathbf{q}_T}^2)(m_1 + p_l + M\eta_1)}{{m_V}^2} + (m_1 + M\eta_1 + p_l)(\mathbf{p}_T\cdot\mathbf{q}_T + {\mathbf{q}_T}^2) + 2(M\eta_2 - p_l)\mathbf{p}_T\cdot\mathbf{q}_T}{-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2} f_2(|\mathbf{q}_T|)\\
&+\frac{\kappa}{m_B}\frac{[\mathbf{p}_T\cdot{\mathbf{q}_T}^2 - {\mathbf{p}_T}^2{\mathbf{q}_T}^2 + (m_1 + p_l + M\eta_1)({\mathbf{q}_T}^2 - \mathbf{p}_T\cdot\mathbf{q}_T)(p_l - M\eta_2)]f_2(|\mathbf{q}_T|)}{[-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2]}\\
&\left.-\frac{\kappa}{m_B}\frac{(\mathbf{p}_T\cdot\mathbf{q}_T - {\mathbf{p}_T}^2)(p_l - M\eta_2)f_1(|\mathbf{q}_T|)}{[-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2]}\right\}. \qquad (17)
\end{aligned}$$

$$\begin{aligned}
f_2(|\mathbf{p}_T|){\mathbf{p}_T}^2 = &-\int \frac{dp_l}{(2\pi)} \int \frac{d\mathbf{q}_T}{(2\pi)^3} \frac{-iC_{I,I_z} g_{MMV} g_{BBV} F^2(k, m_V)}{[(\eta_1 M + p_l)^2 - \omega_l^2 + i\epsilon][(\eta_1 M - p_l)^2 - \omega_2^2 + i\epsilon)}\\
&\times\left\{\frac{-{\mathbf{p}_T}^2(\mathbf{p}_T\cdot\mathbf{q}_T + {\mathbf{q}_T}^2) + 2\mathbf{p}_T\cdot\mathbf{q}_T(m_1 - p_l - M\eta_1)(M\eta_2 - p_l) + {\mathbf{p}_T}^2\frac{({\mathbf{q}_T}^2 - \mathbf{p}_T\cdot\mathbf{q}_T)({\mathbf{p}_T}^2 - {\mathbf{q}_T}^2)}{{m_V}^2}}{-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2} f_2(|\mathbf{q}_T|)\right.\\
&+\frac{\frac{({\mathbf{p}_T}^2 - {\mathbf{q}_T}^2)(\mathbf{p}_T\cdot\mathbf{q}_T - {\mathbf{p}_T}^2)(-m_1 + p_l + M\eta_1)}{{m_V}^2} + (m_1 - M\eta_1 + p_l)(\mathbf{p}_T\cdot\mathbf{q}_T + {\mathbf{p}_T}^2) - 2M\eta_2{\mathbf{p}_T}^2 - 2p_l\mathbf{p}_T\cdot\mathbf{q}_T}{-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2} f_1(|\mathbf{q}_T|)\\
&-\frac{\kappa}{m_B}\frac{[(M\eta_2 - pl)\mathbf{p}_T\cdot\mathbf{q}_T{\mathbf{p}_T}^2 - \mathbf{p}_T\cdot{\mathbf{q}_T}^2(m_1 - p_l - M\eta_1) + {\mathbf{p}_T}^2{\mathbf{q}_T}^2(m_1 - M)]f_2(|\mathbf{q}_T|)}{[-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2]}\\
&\left.-\frac{\kappa}{m_B}\frac{(\mathbf{p}_T\cdot\mathbf{q}_T - {\mathbf{p}_T}^2)(p_l - M\eta_2)(m_1 - p_l - M\eta_1)f_1(|\mathbf{q}_T|)}{[-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2]}\right\}. \qquad (18)
\end{aligned}$$

To extract $f_1(|\mathbf{p}_T|)$ and $f_2(|\mathbf{p}_T|)$ from the above equations, instead of the procedure adopted in earlier works, we multiply $\bar{u}(v)$ from the right side of the equation and sum over the spin projections of $u(v)$; then taking a trace of the modified equation, the job is done. The advantage of this procedure is to keep the equation of motion $v\not{u}(v,s) = u(v,s)$.

Now we perform an integral over p_l on the right side of Eqs. (16) and (18) where four poles exist at $-\eta_1 M - \omega_1 + i\epsilon$, $-\eta_1 M + \omega_1 - i\epsilon$, $\eta_2 M + \omega_2 - i\epsilon$, and $\eta_2 M - \omega_2 + i\epsilon$. By choosing an appropriate contour (16) and (18) we calculate the residuals at $p_l = -\eta_1 M - \omega_1 + i\epsilon$ and $p_l = \eta_2 M - \omega_2 + i\epsilon$. The coupled equations after the contour integrations are collected in the Appendix [Eqs. (C1) and (C2)]. Then one can carry out the azimuthal integration and reduce Eqs. (C1) and (C2) to one-dimensional integral equations,

$$
\begin{aligned}
f_1(|\mathbf{p}_T|) &= \int d|\mathbf{q}_T|[A_{11}(|\mathbf{q}_T|,|\mathbf{p}_T|)f_1(|\mathbf{q}_T|) + A_{12}(|\mathbf{q}_T|,|\mathbf{p}_T|)f_2(|\mathbf{q}_T|)],\\
f_2(|\mathbf{p}_T|) &= \int d|\mathbf{q}_T|[A_{21}(|\mathbf{q}_T|,|\mathbf{p}_T|)f_1(|\mathbf{q}_T|) + A_{22}(|\mathbf{q}_T|,|\mathbf{p}_T|)f_2(|\mathbf{q}_T|)],
\end{aligned}
\tag{19}
$$

where A_{11}, A_{12}, A_{21}, and A_{22} are presented in the Appendix [see Eqs. (C6)–(C9)].

C. The normalization condition for the B-S wave function

The normalization condition for the B-S wave function of a bound state is [33,37]

$$
i\int \frac{d^4p d^4q}{(2\pi)^8}\bar{\chi}_P(p)\frac{\partial}{\partial P_0}[I(P,p,q) + K(P,p,q)]\chi_P(q) = 1,
\tag{20}
$$

where P_0 is the energy of the bound state and the spinor relation $\sum_s u(v,s)\bar{u}(v,s) = \frac{\not{v}+1}{2}$ is used. $I(P,p,q)$ is the reciprocal of the four-point propagator

$$
I(P,p,q) = \frac{\delta^4(p-q)}{(2\pi)^4}[S_{\mathcal{B}}(p_1)]^{-1}[S_{\mathcal{M}}(p_2)]^{-1}.
\tag{21}
$$

For the molecular sates composed of two mesons the second term in the normalization condition is several orders smaller than the first term [35,36]; thus we have every reason to believe that the rule also applies to the case where the molecule is composed of a baryon and a meson; consequently the term $\frac{\partial}{\partial P_0}K(P,p,q)$ can be ignored and then

$$
\begin{aligned}
&-\int \frac{d^4p}{(2\pi)^4}\bar{\chi}_P(p)\eta_1\not{v}[S_{\mathcal{M}}(p_2)]^{-1}\chi_P(q)\\
&-\int \frac{d^4p}{(2\pi)^4}\bar{\chi}_P(p)2\eta_2 p_2\cdot v[S_{\mathcal{B}}(p_1)]^{-1}\chi_P(q) = 1.
\end{aligned}
\tag{22}
$$

Let us define the transverse projections of the B-S wave function as follows:

$$
\begin{aligned}
\alpha_P(p) &= -i[S_{\mathcal{B}}(p_1)]^{-1}\chi_P(q)[S_{\mathcal{M}}(p_2)]^{-1},\\
\beta_P(p) &= -i[S_{\mathcal{M}}(p_2)]^{-1}\bar{\chi}_P(q)[S_{\mathcal{B}}(p_1)]^{-1};
\end{aligned}
\tag{23}
$$

the normalization condition is

$$
\begin{aligned}
&-\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\alpha_P(p)\beta_P(p)S_{\mathcal{B}}(p_1)\eta_1\not{v}S_{\mathcal{B}}(p_1)S_{\mathcal{M}}(p_2)]\\
&-\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\alpha_P(p)\beta_P(p)2\eta_2 p_2\cdot v S_{\mathcal{B}}(p_1)S_{\mathcal{M}}(p_1)S_{\mathcal{M}}(p_1)] = 1.
\end{aligned}
\tag{24}
$$

Substituting the expression $\bar{\chi}_P(p)$ [Eq. (9)] into Eqs. (23) under the covariant instantaneous approximation, one can obtain the expressions of $\alpha_P(p)$ and $\beta_P(p)$, for example,

$$
\begin{aligned}
\alpha_P(p) = -\int \frac{d^3\mathbf{q}_T}{(2\pi)^3}C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}\Bigg\{&\frac{2(\eta_2 M - p_l)\not{v} - \not{p}t - \not{q}t - (\not{p}t - \not{q}t)(\mathbf{p}_T^2 - \mathbf{q}_T^2)/m_V^2}{-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2}\\
&-\frac{\kappa[2(\eta_2 M - p_l)\not{v} - \not{p}t - \not{q}t](\not{p}t - \not{q}t) - (\not{p}t - \not{q}t)[2(\eta_2 M - p_l)\not{v} - \not{p}t - \not{q}t]}{4m_B[-(\mathbf{p}_T - \mathbf{q}_T)^2 - m_V^2]}\Bigg\}\\
&\times F^2(k,m_V)[f_1(|\mathbf{q}_T|) + f_2(|\mathbf{q}_T|)\not{q}t]u(v,s),
\end{aligned}
\tag{25}
$$

and $\alpha_P(p)$ and $\beta_P(p)$ can be parametrized into

$$\alpha_P(p) = [h_1(|\mathbf{p}_T|) + h_2(|\mathbf{p}_T|)\not{p}t]u(v,s),$$
$$\beta_P(p) = \bar{u}(v,s)[h_1(|\mathbf{p}_T|) + h_2(|\mathbf{p}_T|)\not{p}t], \tag{26}$$

with

$$h_1(|\mathbf{p}_T|) = -\int \frac{d^3\mathbf{q}_T}{(2\pi)^3}\left\{\frac{2f_1(\mathbf{q}_T)(M\eta_2 - p_l) + f_2(|\mathbf{q}_T|)[\mathbf{q}_T^2 + \mathbf{p}_T\cdot\mathbf{q}_T + \frac{(\mathbf{p}_T^2-\mathbf{q}_T^2)(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{q}_T^2)}{m_V{}^2}]}{-(\mathbf{p}_T-\mathbf{q}_T)^2 - m_V^2}\right.$$
$$\left. + \frac{4\kappa[f_2(|\mathbf{q}_T|)((M\eta_2 - p_l)(\mathbf{p}_T\cdot\mathbf{q}_T - \mathbf{q}_T^2)]}{4m_B[-(\mathbf{p}_T-\mathbf{q}_T)^2 - m_V^2]}\right\} C_{I,I_z} g_{\mathcal{MMV}} g_{\mathcal{BBV}} F^2(k,m_V),$$
$$h_2(|\mathbf{p}_T|) = -\int \frac{d^3\mathbf{q}_T}{(2\pi)^3}\left\{\frac{f_1(\mathbf{q}_T)(\frac{(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{p}_T^2)(\mathbf{p}_T^2-\mathbf{q}_T^2)}{m_V{}^2} - \mathbf{p}_T\cdot\mathbf{q}_T - \mathbf{p}_T^2) + 2f_2(\mathbf{q}_T)(p_l - M\eta_2)\mathbf{p}_T\cdot\mathbf{q}_T}{\mathbf{p}_T^2[-(\mathbf{p}_T-\mathbf{q}_T)^2 - m_V^2]}\right.$$
$$\left. + \frac{4\kappa([f_2(\mathbf{q}_T)(\mathbf{p}_T^2\mathbf{q}_T^2 - \mathbf{p}_T\cdot\mathbf{q}_T{}^2) + f_1(\mathbf{q}_T)(\mathbf{p}_T\cdot\mathbf{q}_T - \mathbf{p}_T^2)(p_l - M\eta_2)]}{4m_B\mathbf{p}_T^2[-(\mathbf{p}_T-\mathbf{q}_T)^2 - m_V^2]}\right\} C_{I,I_z} g_{\mathcal{MMV}} g_{\mathcal{BBV}} F^2(k,m_V). \tag{27}$$

Substituting Eqs. (10) and (11) and equation group (26) into Eq. (24) we obtain

$$i\int \frac{d^4p}{(2\pi)^4} 2\{h_1{}^2[m_1{}^2 + p_l{}^2 + \mathbf{p}_T^2 + 2Mp_l\eta_1 + M^2\eta_1{}^2 + 2m_1(p_l + M\eta_1)]$$
$$+ h_2{}^2\mathbf{p}_T^2(m_1{}^2 + p_l{}^2 + \mathbf{p}_T^2 + 2Mp_l\eta_1 + M^2\eta_1{}^2 - 2m_1(p_l + M\eta_1)) - 4h_1h_2\mathbf{p}_T^2(p_l + M\eta_1)\}$$
$$/\{[(\eta_1 M + p_l)^2 - \omega_l^2 + i\epsilon]^2[(\lambda_1 M - p_l)^2 - \omega_2^2 + i\epsilon)]\}$$
$$+ i\int \frac{d^4p}{(2\pi)^4} 2[h_1{}^2(m_1 + p_l + M\eta_1) - 2h_1h_2\mathbf{p}_T^2 + h_2{}^2\mathbf{p}_T^2(p_l - m_1 + M\eta_1)]$$
$$/\{[(\eta_1 M + p_l)^2 - \omega_l^2 + i\epsilon]^2[(\lambda_1 M - p_l)^2 - \omega_2^2 + i\epsilon)]\} = 1. \tag{28}$$

After the contour integration on p_l and the azimuthal integration the normalization condition can be calculated numerically and the values of $f_1(|\mathbf{p}_T|)$ and $f_2(|\mathbf{p}_T|)$ are fixed at the same time.

D. The decay of $P_c \to \mathcal{V}$+proton

Now we investigate the strong decays of P_c in terms of the framework formulated above.

The amplitudes corresponding to the two diagrams in Fig. 2 are

$$\mathcal{A}_a = C_I g_{\mathcal{BB}'D} g_{DD\mathcal{V}} \int \frac{d^4p}{(2\pi)^4} \bar{U}_{\mathcal{B}'}\gamma^5\chi_P(p)(k - p_2)_\nu \epsilon^\nu$$
$$\times \frac{1}{k^2 - M_D^2} F^2(k, m_D), \tag{29}$$

$$\mathcal{A}_b = 2C_I g_{\mathcal{BB}'D^*} g_{DD^*\mathcal{V}} \int \frac{d^4p}{(2\pi)^4} \bar{U}_{\mathcal{B}'}\left(\gamma^\sigma - \frac{\kappa_{BBD^*}}{2m_B}\sigma^{\sigma\omega}k_\omega\right)$$
$$\times \chi_P(p)\varepsilon^{\alpha\beta\mu\nu}k_\mu q_{2\alpha}\epsilon_\nu \tag{30}$$

$$\frac{g_{\sigma\beta} - k_\beta k_\sigma/M_{D^*}^2}{k^2 - M_{D^*}^2} F^2(k, m_{D^*}), \tag{31}$$

where C_I is the isospin coefficient of the transition, $k = p - (\eta_2 q_1 - \eta_1 q_2)$, and $\mathcal{B}$ denotes the charmed baryon in the molecular state: Σ_c or Λ_c; ϵ is the polarization vector of $\mathcal{V}$ and $\mathcal{B}'$ represents the proton. We take the approximation $k_0 = 0$ to carry out the calculation.

The total amplitude is

$$\mathcal{A} = \mathcal{A}_a + \mathcal{A}_b = \bar{u}_{\mathcal{B}'}[\gamma^5 g_1\gamma^\mu + i\gamma^5 g_2\sigma^{\mu\nu}q_{2\nu}$$
$$+ ig_3\gamma_\nu\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_{2\beta}]u(v)\epsilon_\mu. \tag{32}$$

The factors g_1, g_2, and g_3 can be extracted from the expressions of $\mathcal{A}_1$ and $\mathcal{A}_2$.

Then the partial width is expressed as

$$d\Gamma = \frac{1}{32\pi^2}|\mathcal{A}|^2\frac{|q_2|}{M^2}d\Omega. \tag{33}$$

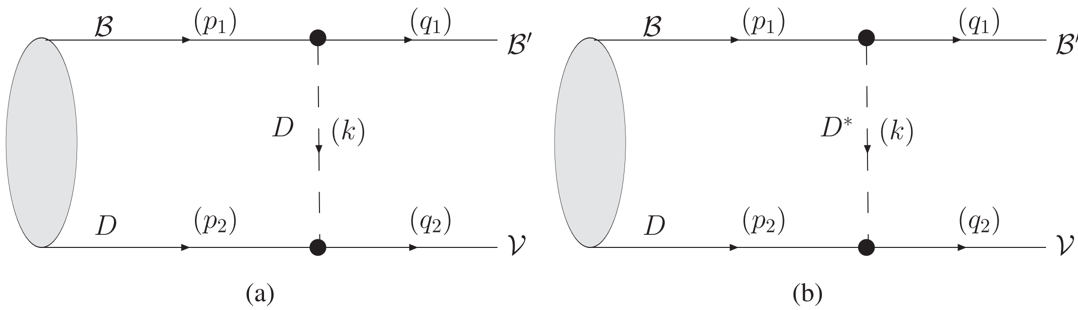

FIG. 2. the decay of P_c by exchanging mesons.

III. NUMERICAL RESULTS

A. The numerical results

In order to solve the B-S equation numerically some parameters are needed. The mass m_{Λ_c}, m_{Σ_c}, m_D, m_ω, m_ρ is taken from the databook [47]. Following Refs. [40,48], we set the coupling constants $g_{DD\omega} = g_{DD\rho} = 3.02$, $g_{\Lambda_c\Lambda_c\omega}{=}8.125$, $g_{\Sigma_c\Sigma_c\omega}{=}g_{\Sigma_c\Sigma_c\rho}{=}7.475$, $f_{\Sigma_c\Sigma_c\omega}{=}\kappa g_{\Sigma_c\Sigma_c\omega}{=}9.9125$, $f_{\Sigma_c\Sigma_c\rho} = \kappa g_{\Sigma_c\Sigma_c\rho} = 9.9125$.

With these parameters and the corresponding isospin factors, a complete B-S equation [the coupled equations (19)] is established. These coupled equations are complicated integral equations; thus to numerically solve them, the standard way is to discretize them; namely, we would convert them into algebraic equations. Concretely, we set a reasonable finite range for $|\mathbf{p_T}|$ and $|\mathbf{q_T}|$, and let the variables take n ($n = 129$ in our calculation) discrete values $Q_1, Q_2, \ldots Q_n$, which distribute with equal gap from $Q_1 = 0.001$ GeV to $Q_n = 2$ GeV. The gap between two adjacent values is $\Delta|\mathbf{p_T}| = (1.999/128)$ GeV. For clarity, we let n values of $f_1(|\mathbf{p_T}|)$ and n values of $f_2(|\mathbf{p_T}|)$ constitute a column matrix with $2n$ rows and the $2n$ elements $f_1(|\mathbf{q_T}|)$, $f_2(|\mathbf{q_T}|)$ construct another column matrix residing on the right side of the equation as shown below. The column matrix composed of $f_1(|\mathbf{p_T}|)$ and $f_2(|\mathbf{p_T}|)$ is associated with the right column matrix of $f_1(|\mathbf{q_T}|)$ and $f_2(|\mathbf{q_T}|)$ by a $2n \times 2n$ matrix whose elements are the coefficients given in Eq. (19). The standard way to treat the equation is to let $|\mathbf{p_T}|$ and $|\mathbf{q_T}|$ take the same sequential values $Q_1, Q_2, \ldots Q_n$ for discretizing the integral equation.

$$\begin{pmatrix} f_1(Q_1) \\ \cdots \\ f_1(Q_{129}) \\ f_2(Q_1) \\ \cdots \\ f_2(Q_{129}) \end{pmatrix} = A(\Delta E, \lambda) \begin{pmatrix} f_1(Q_1) \\ \cdots \\ f_1(Q_{129}) \\ f_2(Q_1) \\ \cdots \\ f_2(Q_{129}) \end{pmatrix}.$$

As a matter of fact, it is a homogeneous linear equation group. If it possesses nontrivial solutions, the necessary and sufficient condition is the coefficient determinant to be 0. In our case, it is $|A(\Delta E, \lambda) - I| = 0$ (I is the unit matrix) where $A(\Delta E, \lambda)$. Now we calculate that the determinant of $|A(\Delta E, \lambda) - I|$ is a function of the binding energy $\Delta E = m_1 + m_2 - M$ and parameter λ. Our strategy is the following: we arbitrarily vary ΔE within a possible range; by requiring $|A(\Delta E, \lambda) - I| = 0$, we obtain a corresponding λ. In Ref. [43] λ was fixed to be 3. In our earlier paper [46] we change the value of λ from 1 to 3 to explore possible dependence of the results on it; it seems that a value of λ within the range of $0 \sim 4$ is reasonable for forming a bound

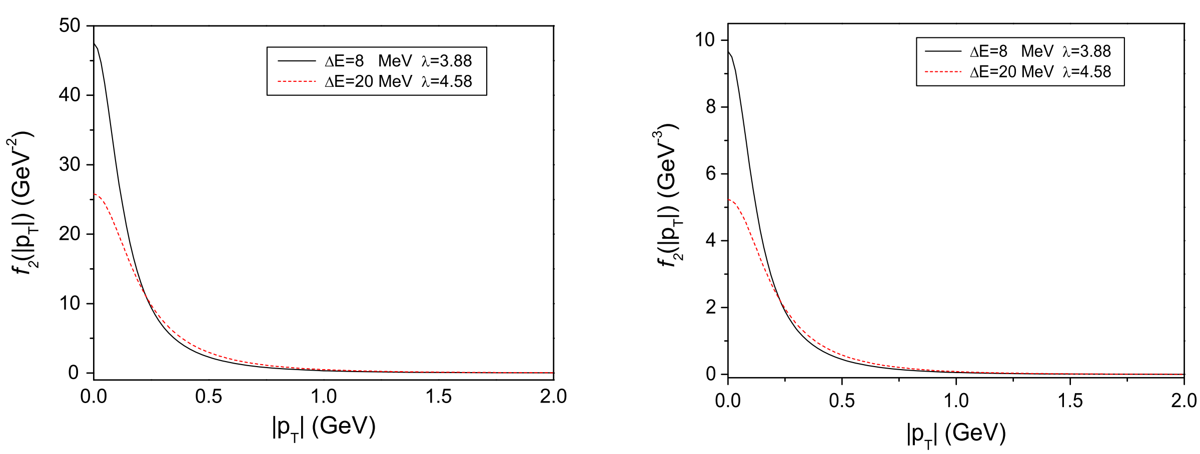

FIG. 3. The normalized wave function $f_1(|\mathbf{p}_T|)$ and $f_2(|\mathbf{p}_T|)$ for $P_{c(\frac{1}{2},\frac{1}{2})}$.

state of two hadrons. Consequently, if the obtained λ is much beyond the range, we would conclude that the resonance cannot exist.

To get the wave function $T(f_1(Q_1), f_1(Q-2)\ldots, f_2(Q_1)\ldots f_2(Q_{129})$, we adopt a special method. Namely, we suppose a matrix equation $(A(\Delta E,\lambda)_{ij})(f(j)) = \beta(f(i))$, which is just an eigenequation. In terms of the standard software, we can find all the possible "eigenvalues" β, and among them only $\beta = 1$ is the solution we expect; then the corresponding wave function is gained (see Fig. 3), which is just the solution of the B-S equation.

For $|A(\Delta E,\lambda) - I| = 0$, inputting some binding energies, we would check whether we can obtain reasonable values for λ. If yes, we substitute the values of λ and the binding energy into the matrix equation to obtain the B-S wave functions. With this strategy, we investigate the molecular structure of Λ_c and $\bar{D}$ as well as that of Σ_c and $\bar{D}$.

If the exchanging particles are limited to the light vector meson, only ω and ρ can be exchanged between charmed baryons and D. Of course, exchanging two ρ mesons between Λ_c (Σ_c) and $\bar{D}$ can also induce a potential, but it undergoes a loop suppression; therefore, we do not consider that contribution.

As the first trial, let us study a simple compound; namely we explore the possible bound states of Λ_c and $\bar{D}$, which is an $I = \frac{1}{2}$ state; therefore only ω can be exchanged between Λ_c and $\bar{D}$. We find that there is no solution for the B-S equation; therefore we would conclude that the interaction induced by the single ω exchange is repulsive.

With the same procedure, we study a molecule composed of Σ_c and $\bar{D}$ whose isospin could be either 1/2 or 3/2 and the coefficient is $C_{\frac{1}{2},\frac{1}{2}} = 1$. Since $P_C(4312)$ is observed in the $J/\psi p$ portal, it is confirmed to be a state of $I = 1/2$. In this case both ω and ρ exchanges between the two ingredients are allowed. The isospin factor for the ρ exchange is -2; namely it plays an opposite role to the ω exchange. We try to solve the equation $|A(\Delta E,\Lambda) - I| = 0$ for some chosen ΔE and find a solution for $\Sigma_c\bar{D}$ with the quantum number $I(J) = \frac{1}{2}(\frac{1}{2})$ where the factor λ can span a large range.

The result indicates that although the ω exchange contributes a repulsive interaction, for the $\Sigma_c\bar{D}$ molecule, the total interaction can be attractive due to a larger contribution from the ρ exchange. Numerically, the obtained values of λ and corresponding ΔE for the $\Sigma_c\bar{D}$ system are presented in Table I. Our numerical computation also confirms that the tensor coupling in the $\mathcal{L}_{\mathcal{BBV}}$ has little effect on the results. For example, setting $\Delta E = 8$ MeV one can fix $\lambda = 3.77$ and 3.88 with and without the tensor contribution and the obtained wave functions are very close to each other so we can safely ignore the tensor coupling in the vertex $\mathcal{L}_{\mathcal{BBV}}$. Apparently when ΔE is very small the obtained λ is smaller than 4, so Σ_c and $\bar{D}$ should form a weak bound state. At present the pentaquark $P_c(4312)$ has been experimentally observed in the $\Lambda_b \to J/\psi pK$ portal, which is peaked at the invariant mass spectrum of $J/\psi p$ and has the invariant mass of about 4312 MeV. Apparently its isospin is $\frac{1}{2}$, and the majority of authors [15,19–22,25,26] regarded this pentaquark as a bound state of Σ_c and $\bar{D}$ and we agree with it.

Using the normalized wave functions the transition $P_{c(\frac{1}{2},\frac{1}{2})} \to J/\psi + p$ is calculable. The form factors defined in Eq. (32) with the coupling constants are evaluated: $g_{\mathcal{BB'}D} = 2.7$, $g_{\mathcal{BB'}D^*} = 3.0$, $g_{DD\psi} = 7.4$, $g_{DD^*\psi} = 2.5\ \mathrm{GeV}^{-1}$ [41]. We obtain $g_1 = 0.396$ GeV, $g_2 = 0.270$, $g_3 = 0.00632\ \mathrm{GeV}^{-1}$, and the decay width $\Gamma[P_{c(\frac{1}{2},\frac{1}{2})} \to J/\psi p] = 3.66$ MeV. If the binding energy is 20 MeV, $g_1 = 0.412$ GeV, $g_2 = 0.282$, $g_3 = 0.00923\ \mathrm{GeV}^{-1}$ can be obtained and the estimated decay width is $\Gamma[P_{c(\frac{1}{2},\frac{1}{2})} \to J/\psi p] = 3.90$ MeV. We notice that our results are close to that of Refs. [20,25], but the results given in Ref. [24] are 1–3 orders smaller than ours where different ultraviolet regulators are employed.

By our observation given above, for the state with $I = \frac{3}{2}$ the isospin factor is 1 for exchanging either ω or ρ; therefore the total interaction is repulsive; it means that Σ_c and $\bar{D}$ cannot form a bound state with $I = \frac{3}{2}$.

B. Predictions about pentaquark P_b

The isospin of the $\Lambda_b B^+$ system is

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = |\Lambda_b B^+\rangle. \tag{34}$$

The isospin of the $\Sigma_b B$ system can be $|\frac{1}{2},\pm\frac{1}{2}\rangle$, $|\frac{3}{2},\pm\frac{1}{2}\rangle$, and $|\frac{3}{2},\pm\frac{3}{2}\rangle$. Let us work on the isospin states

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}|\Sigma_b^+ B^0\rangle - \sqrt{\frac{1}{3}}|\Sigma_b^0 B^+\rangle, \tag{35}$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}|\Sigma_b^+ B^0\rangle + \sqrt{\frac{2}{3}}|\Sigma_b^0 B^+\rangle, \tag{36}$$

and

TABLE I. The cutoff parameter λ and the corresponding binding energy ΔE for the bound state $\Sigma_c\bar{D}$ with $I = \frac{1}{2}$ and $I_z = \frac{1}{2}$.

ΔE MeV	2	8	20	30	40
λ	3.31	3.88	4.58	5.04	5.44

TABLE II. The cutoff parameter λ and the corresponding binding energy ΔE for the bound state $\Sigma_b B$ with $I = \frac{1}{2}$ and $I_z = \frac{1}{2}$.

ΔE	10	20	30	40	50
λ	2.13	2.51	2.82	3.09	3.35

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = |\Sigma_b^+ B^+\rangle. \tag{37}$$

Using the masses of Λ_b, Σ_b, and B presented in Ref. [47] and other parameters listed in previous sections, we solve those B-S equations. It is found that only the equation for the $\Sigma_b B$ system with $I=\frac{1}{2}$ has a solution. The binding energy and corresponding λ values are presented in Table II. That implies that the bound state with $I=\frac{1}{2}$ can exist in nature. Under the heavy quark symmetry we suppose that the couplings are unchanged when b hadrons replace c hadrons. We turn to study the transition $P_{b(\frac{1}{2},\frac{1}{2})} \to \Upsilon p$. We obtain $g_1 = 0.00346$ GeV, $g_2 = 0.252$, $g_3 = 0.0000911\ \mathrm{GeV}^{-1}$ and predict the decay width $\Gamma[P_{b(\frac{1}{2},\frac{1}{2})} \to \Upsilon p] = 0.690$ keV as the binding energy is 10 MeV. If $\Delta E = 20$ MeV the decay width $\Gamma[P_{b(\frac{1}{2},\frac{1}{2})} \to \Upsilon p] = 1.09$ keV and $g_1 = 0.00435$ GeV, $g_2 = 0.318$, $g_3 = 0.000149\ \mathrm{GeV}^{-1}$.

IV. CONCLUSION AND DISCUSSION

Within the B-S framework we explore several bound states that are composed of a baryon and a meson. Their total spin and parity is $\frac{1}{2}^-$; i.e., the orbital angular momentum $L=0$ (S wave). We try to solve the B-S equation for getting possible spatial wave functions for $\Lambda_c\bar{D}$, $\Sigma_c\bar{D}$, $\Lambda_b B$, and $\Sigma_b B$ systems. If the B-S equation for a supposed molecular structure has a stable solution, we would conclude that the concerned pentaquark could exist in the nature; oppositely, no solution means the supposed pentaquark cannot appear as a resonance or the molecular state is not an appropriate structure. The solution can apply as a criterion for the structures of the pentaquark states which have already been or will be experimentally observed. In this scenario, the two constituents interact by exchanging light vector mesons. For the $\Lambda_c\bar{D}$ ($\Lambda_b B$) system only ω is the exchanged mediate meson, while for the $\Sigma_c\bar{D}$ system ($\Sigma_b B$) both ω and ρ contribute. The chiral interaction determines if those molecular states can be formed.

For $\frac{1}{2}^-$ baryon (S wave), the B-S wave function possesses two scalar functions $f_1(|\mathbf{p_T}|)$ and $f_2(|\mathbf{p_T}|)$, which should be solved numerically. Discretizing the integral equations, we simplify the B-S equation into two coupled algebraic equations about $f_1(|\mathbf{p_T}|)$ and $f_2(|\mathbf{p_T}|)$.

As $|\mathbf{p_T}|$ ($i=1,2$) takes n discrete values the two coupled equations are converted into a matrix equation, which can be easily solved numerically in terms of available softwares. When all known parameters are input there still is one undetermined parameter λ. Our strategy is inputting binding energies within a range and then fixing λ by solving the matrix equation. If λ is located in a reasonable range one can expect the bound state to exist. We find the B-S equation of the state $\Lambda_c\bar{D}$ system has no solution for λ when the binding energy takes experimentally allowed values. For the $\Sigma_c\bar{D}$ system there are three isospin eigenstates. Because of the isospin factors, the B-S equations for $P_{c(\frac{1}{2},\frac{1}{2})}$, $P_{c(\frac{3}{2},\frac{1}{2})}$, and $P_{c(\frac{3}{2},\frac{3}{2})}$ are set. We find that the equation for $|\frac{1}{2},\frac{1}{2}\rangle$ has a solution for λ falling into the reasonable range. It means that $P_c(4312)$ is maybe a molecular state of $\Sigma_c\bar{D}$. The decay width of $P_{c(\frac{1}{2},\frac{1}{2})} \to J/\psi p$ is calculated within this framework and we obtain it as about 3.66 MeV.

It is noted that we ignore the couple channel effects in the Bethe-Salpeter framework. We also note that if the couple channel interaction between $\Lambda_c\bar{D}$ and $\Sigma_c\bar{D}$ is taken into account, just as the authors of Ref. [48] did, a bound state of $\Lambda_c\bar{D}$ may exist via the coupled channel with $\Sigma_c\bar{D}$ ($I=\frac{1}{2}$). In other words, there is a $\Lambda_c\bar{D}$ component in the physical state of $P_{c(\frac{1}{2},\frac{1}{2})}$.

In this work, we study $\Lambda_b B$ and $\Sigma_b B$ systems and solve the B-S equations for $\Sigma_b B$ and $\Lambda_b B$. Our conclusion is that the bound state $P_{b(\frac{1}{2},\frac{1}{2})}$ is still a promising pentaquark. The partial width $P_{b(\frac{1}{2},\frac{1}{2})} \to \Upsilon p$ is about 1.06 keV at $\Delta E = 20$ MeV, which will be checked by future experiments.

Within the B-S framework, we systematically investigate the molecular structure of pentaquarks. We pay special attention to $P_c(4312)$ because it is experimentally well measured. From that study, we have accumulated valuable knowledge on probable molecular structure of pentaquarks, which can be applied to future research. Definitely, the discovery of pentaquarks opens a window for understanding the quark model established by Gell-Mann and several other predecessors. Deeper study on their structure and concerned effective interaction that binds the ingredients to form a molecule would greatly enrich our theoretical asset. So we continue to do research along this line.

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APPENDIX A: THE EFFECTIVE INTERACTIONS

The effective interactions can be found in [40–42]

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\rho} = ig_{\mathcal{P}\mathcal{P}\rho}\phi_{\mathcal{P}}\rho^\mu \cdot \tau\partial_\mu\phi_{\mathcal{P}} + \mathrm{c.c.}, \tag{A1}$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathcal{V}} = ig_{\mathcal{P}\mathcal{P}\mathcal{V}}\phi(x)_{\mathcal{P}}\partial_\mu\phi(x)_{\mathcal{P}}\phi(x)^\mu_{\mathcal{V}} + \mathrm{c.c.}, \tag{A2}$$

$$\mathcal{L}_{\mathcal{B}\mathcal{B}\rho} = -g_{\mathcal{B}\mathcal{B}\rho}\bar{\psi}_{\mathcal{B}}\left(\gamma^\mu - \frac{\kappa_{\mathcal{B}\mathcal{B}\rho}}{2m_{\mathcal{B}}}\sigma^{\mu\nu}\partial_\nu\right)\rho^\mu \cdot \tau\psi_{\mathcal{B}}, \tag{A3}$$

$$\mathcal{L}_{\mathcal{B}\mathcal{B}\omega} = -g_{\mathcal{B}\mathcal{B}\omega}\bar{\psi}_{\mathcal{B}}\left(\gamma^\mu - \frac{\kappa_{\mathcal{B}\mathcal{B}\omega}}{2m_{\mathcal{B}}}\sigma^{\mu\nu}\partial_\nu\right)\omega^\mu\psi_{\mathcal{B}}, \tag{A4}$$

$$\mathcal{L}_{\mathcal{V}\mathcal{V}\mathcal{P}} = ig_{\mathcal{V}\mathcal{V}\mathcal{P}}\varepsilon_{\mu,\nu,\alpha,\beta}\partial^\mu\phi_{\mathcal{V}}(x)^\nu\partial_\alpha\phi_{\mathcal{V}}(x)^\beta\phi_{\mathcal{P}}(x) + \mathrm{c.c.}, \tag{A5}$$

$$\mathcal{L}_{\mathcal{BBP}} = ig_{\mathcal{BBP}}\bar{\psi}_{\mathcal{B}}\gamma^5\psi_{\mathcal{B}}, \tag{A6}$$

where c.c. is the complex conjugate term, τ is the Pauli matrix for $I=\frac{1}{2}$ and

$$\tau_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\tau_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \text{and} \quad \tau_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

for $I=1$. When $\phi_{\mathcal{P}} = \binom{D^0}{D^+}$ and

$$\psi_{\mathcal{B}} = \begin{pmatrix} \Sigma_c^{++} \\ \Sigma_c^{+} \\ \Sigma_c^{0} \end{pmatrix}$$

the effective interactions are consistent with those in Ref. [23].

APPENDIX B: THE ISOSPIN FACTORS IN THE KERNEL

To gain the characteristic hadronic property of the pentaquark, one needs to project the bound states on the vacuum via the field operators $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{M}_1$, and $\mathcal{M}_2$, and

$$\langle 0|T\mathcal{B}_i(x_1)\mathcal{M}_j(x_2)|P\rangle_{I,I_3} = C^{ij}_{(I,I_3)}\chi^I_P(x_1,x_2), \tag{B1}$$

where $\chi^I_P(x_1,x_2)$ is the B-S wave function for the bound state with isospin I. The isospin coefficients $C^{22}_{(\frac{1}{2},\frac{1}{2})}$ for $\Lambda_c D$ bound state are 1; the isospin coefficients for $\Sigma_c D$ bound states are

$$C^{11}_{(\frac{1}{2},\frac{1}{2})} = \sqrt{\frac{2}{3}}, \qquad C^{22}_{(\frac{1}{2},\frac{1}{2})} = -\sqrt{\frac{1}{3}},$$
$$C^{11}_{(\frac{3}{2},\frac{1}{2})} = \sqrt{\frac{2}{3}}, \qquad C^{22}_{(\frac{3}{2},\frac{1}{2})} = \sqrt{\frac{1}{3}}, \qquad C^{12}_{(\frac{3}{2},\frac{3}{2})} = 1. \tag{B2}$$

Then corresponding B-S equation was deduced in Ref. [39] as

$$C^{ij}_{(I,I_3)}\chi^I_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}K^{ij,lk}(P,p,q) \times C^{lk}_{(I,I_3)}\chi^I_P(q)S_{\mathcal{M}}(p_2), \tag{B3}$$

where $K^{ij,lk}(P,p,q)$ is still the kernel and its superscripts ij and lk denote the initial and final components.

For $\Lambda_c\bar{D}$

$$\chi_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}K^{22,22}\chi_P(q)S_{\mathcal{M}}(p_2). \tag{B4}$$

For $I=\frac{1}{2}$, $I_z=\frac{1}{2}$ state $\Sigma_c\bar{D}$ if the components are $\Sigma_c^{++}D^-$

$$\chi_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}\left(-K^{11,11} - \frac{1}{\sqrt{2}}K^{11,22}\right) \times\chi_P(q)S_{\mathcal{M}}(p_2), \tag{B5}$$

if the components are $\Sigma_c^{+}\bar{D}^0$

$$\chi_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}(K^{22,22} - \sqrt{2}K^{22,11})\chi_P(q)S_{\mathcal{M}}(p_2), \tag{B6}$$

so

$$\chi_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}\left(-\frac{2}{3}K^{11,11} - \frac{\sqrt{2}}{3}K^{11,22} + \frac{1}{3}K^{22,22} - \frac{\sqrt{2}}{3}K^{22,11}\right)\chi_P(q)S_{\mathcal{M}}(p_2). \tag{B7}$$

For $I=\frac{3}{2}$, $I_z=\frac{1}{2}$ $\Sigma_c\bar{D}$ state if the components are $\Sigma_c^{++}D^-$

$$\chi_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}(-K^{11,11} + \sqrt{2}K^{11,22}) \times\chi_P(q)S_{\mathcal{M}}(p_2), \tag{B8}$$

if the components are $\Sigma_c^{+}\bar{D}^0$

$$\chi_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}\left(K^{22,22} + \frac{1}{\sqrt{2}}K^{22,11}\right) \times\chi_P(q)S_{\mathcal{M}}(p_2), \tag{B9}$$

so

$$\chi_P(p) = S_{\mathcal{B}}(p_1)\int\frac{d^4q}{(2\pi)^3}\left(-\frac{1}{3}K^{11,11} + \frac{\sqrt{2}}{3}K^{11,22} + \frac{2}{3}K^{22,22} + \frac{\sqrt{2}}{3}K^{22,11}\right)\chi_P(q)S_{\mathcal{M}}(p_2). \tag{B10}$$

The sign—before $K^{11,11}$ in Eqs. (B5) and (B8) comes from the interactions in Appendix A. For $\Lambda_c\bar{D}$ state the two components interact only by exchanging ω. However, ω and ρ can contribute to the $\Sigma_c\bar{D}$ state. One also has $K^{11,11}(\omega) = K^{22,22}(\omega)$, $K^{11,22}(\omega) = K^{22,11}(\omega) = 0$, $K^{11,22}(\rho) = K^{22,11}(\rho) = \sqrt{2}K^{11,11}(\rho)$ and $K^{22,22}(\rho) = 0$. In Eqs. (B7) and (B10) $K^{11,11}$, $K^{11,22}$, and $K^{22,11}$ can be changed into $K^{11,11}$ and then the coefficient of $K^{11,11}$ is just the isospin factor C_{I,I_z}: $C_{\frac{1}{2},\frac{1}{2}} = 1, -2$ for ω and ρ, $C_{\frac{3}{2},\frac{1}{2}} = C_{\frac{3}{2},\frac{3}{2}} = 1, 1$ for ω and ρ.

APPENDIX C: THE COUPLED EQUATION OF $f_1(|\mathbf{p}_T|)$ AND $f_2(|\mathbf{p}_T|)$ AFTER INTEGRATING OVER p_l AND SOME FORMULAS FOR AZIMUTHAL INTEGRATION

$$
\begin{aligned}
f_1(|\mathbf{p}_T|) = &-\int \frac{d^3\mathbf{q}_T}{(2\pi)^3}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}F^2(k,m_V)}{2\omega_1(M+\omega_1+\omega_2)(M+\omega_1-\omega_2)}\\
&\times\left\{\frac{\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{p}_T{}^2+2(m_1-\omega_1)(M+\omega_1)+(\mathbf{p}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T)(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)/m_V{}^2}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\right.\\
&+\frac{\frac{(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{q}_T{}^2)(m_1-\omega_l)}{m_V{}^2}+(m_1-\omega_1)(\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{q}_T{}^2)+2(M+\omega_1)\mathbf{p}_T\cdot\mathbf{q}_T}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\\
&-\frac{\kappa}{4m_B}\frac{-4[\mathbf{p}_T\cdot\mathbf{q}_T{}^2-\mathbf{p}_T{}^2\mathbf{q}_T{}^2+(m_1-\omega_1)(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{q}_T{}^2)(\omega_1+M)]}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\\
&\left.-\frac{\kappa}{4m_B}\frac{4(\mathbf{p}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T)(M+\omega_1)}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\right\}\\
&+\int \frac{d^3\mathbf{q}_T}{(2\pi)^3}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}F^2(k,m_V)}{2\omega_2(M+\omega_1-\omega_2)(M-\omega_1-\omega_2)}\\
&\times\left\{\frac{\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{p}_T{}^2+2(m_1-\omega_2+M)\omega_2+(\mathbf{p}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T)(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)/m_V{}^2}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\right.\\
&+\frac{\frac{(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{q}_T{}^2)(m_1-\omega_2+M)}{m_V{}^2}+(m_1+M-\omega_2)(\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{q}_T{}^2)+2\omega_2\mathbf{p}_T\cdot\mathbf{q}_T}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\\
&-\frac{\kappa}{4m_B}\frac{-4[\mathbf{p}_T\cdot\mathbf{q}_T{}^2-\mathbf{p}_T{}^2\mathbf{q}_T{}^2+(m_1-\omega_2+M)(\mathbf{q}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T)(-\omega_2)]}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\\
&\left.-\frac{\kappa}{4m_B}\frac{4(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{p}_T{}^2)(-\omega_2)}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\right\}.
\end{aligned}
\tag{C1}
$$

$$
\begin{aligned}
f_2(|\mathbf{p}_T|)\mathbf{p}_T{}^2 = &-\int \frac{d^3\mathbf{q}_T}{(2\pi)^3}\frac{-C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}F^2(k,m_V)}{2\omega_1(M+\omega_1+\omega_2)(M+\omega_1-\omega_2)}\\
&\times\left\{\frac{-\mathbf{p}_T{}^2(\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{q}_T{}^2)+2\mathbf{p}_T\cdot\mathbf{q}_T(m_1+\omega_1)(M+\omega_1)+\mathbf{p}_T{}^2\frac{(\mathbf{q}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T)(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)}{m_V{}^2}}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\right.\\
&+\frac{\frac{(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{p}_T{}^2)(-m_1-\omega_1)}{m_V{}^2}+(m_1-\omega_1)(\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{p}_T{}^2)-2M\mathbf{p}_T{}^2+2\omega_1\mathbf{p}_T\cdot\mathbf{q}_T}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\\
&-\frac{\kappa}{4m_B}\frac{4[(M+\omega_1)\mathbf{p}_T\cdot\mathbf{q}_T\mathbf{p}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T{}^2(m_1+\omega_1)+\mathbf{p}_T{}^2\mathbf{q}_T{}^2(m_1-M)]}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\\
&\left.-\frac{\kappa}{4m_B}\frac{4(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{p}_T{}^2)(-\omega_1-M)(m_1+\omega_1)}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\right\}\\
&+\int \frac{d^3\mathbf{q}_T}{(2\pi)^3}\frac{-C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}F^2(k,m_V)}{2\omega_2(M+\omega_1-\omega_2)(M-\omega_1-\omega_2)}\\
&\times\left\{\frac{-\mathbf{p}_T{}^2(\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{q}_T{}^2)+2\mathbf{p}_T\cdot\mathbf{q}_T(m_1+\omega_2-M)\omega_2+\mathbf{p}_T{}^2\frac{(\mathbf{q}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T)(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)}{m_V{}^2}}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\right.\\
&+\frac{\frac{(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{p}_T{}^2)(M-m_1-\omega_2)}{m_V{}^2}+(m_1-M-\omega_2)(\mathbf{p}_T\cdot\mathbf{q}_T+\mathbf{p}_T{}^2)+2\omega_2\mathbf{p}_T\cdot\mathbf{q}_T}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\\
&-\frac{\kappa}{4m_B}\frac{4[\omega_2\mathbf{p}_T\cdot\mathbf{q}_T\mathbf{p}_T{}^2-\mathbf{p}_T\cdot\mathbf{q}_T{}^2(m_1+\omega_2-M)+\mathbf{p}_T{}^2\mathbf{q}_T{}^2(m_1-M)]}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_2(|\mathbf{q}_T|)\\
&\left.-\frac{\kappa}{4m_B}\frac{4(\mathbf{p}_T\cdot\mathbf{q}_T-\mathbf{p}_T{}^2)\omega_2(M-m_1-\omega_2)}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}f_1(|\mathbf{q}_T|)\right\}.
\end{aligned}
\tag{C2}
$$

Since $d^3\mathbf{q}_T = \mathbf{q}_T^2 \sin(\theta)d|\mathbf{q}_T|d\theta d\phi$ and $\mathbf{p}_T \cdot \mathbf{q}_T = |\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)$, one can carry out the azimuthal integration for Eqs. (C1) and (C2) analytically. Some useful integrations are defined as follows:

$$\begin{aligned}
J_0 &\equiv \int_0^\pi \sin(\theta)d\theta \frac{1}{-(\mathbf{p}_T-\mathbf{q}_T)^2 - m_V^2}\left[\frac{\Lambda^2-m_V^2}{\Lambda^2-(\mathbf{p}_T-\mathbf{q}_T)^2}\right]^2 \\
&= \int_0^\pi \frac{\sin(\theta)d\theta}{-[\mathbf{p}_T^2+\mathbf{q}_T^2-2|\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)]-m_V^2}\left\{\frac{\Lambda^2-m_V^2}{\Lambda^2-[\mathbf{p}_T^2+\mathbf{q}_T^2-2|\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)]}\right\}^2 \\
&= -\frac{2(m_V^2-\Lambda^2)}{[(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+\Lambda^2][(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+\Lambda^2]} \\
&\quad + \frac{1}{2|\mathbf{p}_T||\mathbf{q}_T|}\left\{\mathrm{Ln}\left[\frac{(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+\Lambda^2}{(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+\Lambda^2}\right] - \mathrm{Ln}\left[\frac{(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+m_V^2}{(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+m_V^2}\right]\right\},
\end{aligned} \tag{C3}$$

$$\begin{aligned}
J_1 &\equiv \int_0^\pi \sin(\theta)d\theta \frac{\mathbf{p}_T\cdot\mathbf{q}_T}{-(\mathbf{p}_T-\mathbf{q}_T)^2 - m_V^2}\left[\frac{\Lambda^2-m_V^2}{\Lambda^2-(\mathbf{p}_T-\mathbf{q}_T)^2}\right]^2 \\
&= \int_0^\pi \frac{|\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)\sin(\theta)d\theta}{-[\mathbf{p}_T^2+\mathbf{q}_T^2-2|\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)]-m_V^2}\left\{\frac{\Lambda^2-m_V^2}{\Lambda^2-[\mathbf{p}_T^2+\mathbf{q}_T^2-2|\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)]}\right\}^2 \\
&= -\frac{(m_V^2-\Lambda^2)(|\mathbf{p}_T|^2+|\mathbf{q}_T|^2+\Lambda^2)}{[(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+\Lambda^2][(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+\Lambda^2]} \\
&\quad + \frac{(|\mathbf{p}_T|^2+|\mathbf{q}_T|^2+m_V^2)}{4|\mathbf{p}_T||\mathbf{q}_T|}\left\{\mathrm{Ln}\left[\frac{(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+\Lambda^2}{(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+\Lambda^2}\right] - \mathrm{Ln}\left[\frac{(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+m_V^2}{(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+m_V^2}\right]\right\},
\end{aligned} \tag{C4}$$

$$\begin{aligned}
J_2 &\equiv \int_0^\pi \sin(\theta)d\theta \frac{(\mathbf{p}_T\cdot\mathbf{q}_T)^2}{-(\mathbf{p}_T-\mathbf{q}_T)^2 - m_V^2}\left[\frac{\Lambda^2-m_V^2}{\Lambda^2-(\mathbf{p}_T-\mathbf{q}_T)^2}\right]^2 \\
&= \int_0^\pi \frac{|\mathbf{p}_T|^2|\mathbf{q}_T|^2\cos^2(\theta)\sin(\theta)d\theta}{-[\mathbf{p}_T^2+\mathbf{q}_T^2-2|\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)]-m_V^2}\left\{\frac{\Lambda^2-m_V^2}{\Lambda^2-[\mathbf{p}_T^2+\mathbf{q}_T^2-2|\mathbf{p}_T||\mathbf{q}_T|\cos(\theta)]}\right\}^2 \\
&= -\frac{(m_V^2-\Lambda^2)(|\mathbf{p}_T|^2+|\mathbf{q}_T|^2+\Lambda^2)^2}{2[(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+\Lambda^2][(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+\Lambda^2]} \\
&\quad + \frac{1}{8|\mathbf{p}_T||\mathbf{q}_T|}\left\{(|\mathbf{p}_T|^2+|\mathbf{q}_T|^2+2m_V^2-\Lambda^2)(|\mathbf{p}_T|^2+|\mathbf{q}_T|^2+\Lambda^2)\mathrm{Ln}\left[\frac{(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+\Lambda^2}{(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+\Lambda^2}\right]\right. \\
&\quad \left. - (|\mathbf{p}_T|^2+|\mathbf{q}_T|^2+m_V^2)^2\mathrm{Ln}\left[\frac{(|\mathbf{p}_T|+|\mathbf{q}_T|)^2+m_V^2}{(|\mathbf{p}_T|-|\mathbf{q}_T|)^2+m_V^2}\right]\right\}.
\end{aligned} \tag{C5}$$

$$\begin{aligned}
A_{11}(\mathbf{p_T},\mathbf{q_T}) &= \frac{-\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_1(M+\omega_1+\omega_2)(M+\omega_1-\omega_2)} \\
&\quad \times\left\{[J_1+\mathbf{p}_T{}^2J_0+2(m_1-\omega_1)(M+\omega_1)J_0+(\mathbf{p}_T{}^2J_0-J_1)(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)/m_V{}^2]f_1(|\mathbf{q}_T|)\right. \\
&\quad \left. -\frac{\kappa}{4m_B}[4(\mathbf{p}_T{}^2J_0-J_1)(M\eta_2+\omega_1)]f_1(|\mathbf{q}_T|)\right\} \\
&\quad +\frac{\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_2(M+\omega_1-\omega_2)(M-\omega_1-\omega_2)} \\
&\quad \times\left\{[J_1+\mathbf{p}_T{}^2J_0+2(m_1-\omega_2+M)\omega_2J_0+(\mathbf{p}_T{}^2J_0-J_1)(\mathbf{p}_T{}^2-\mathbf{q}_T{}^2)/m_V{}^2]f_1(|\mathbf{q}_T|)\right. \\
&\quad \left. -\frac{\kappa}{4m_B}[4(J_1-\mathbf{p}_T{}^2J_0)(-\omega_2)]f_1(|\mathbf{q}_T|)\right\}.
\end{aligned} \tag{C6}$$

$$
\begin{aligned}
A_{12}(\mathbf{p_T},\mathbf{q_T}) = {} & \frac{-\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_1(M+\omega_1+\omega_2)(M+\omega_1-\omega_2)} \\
& \times\left\{\left[\frac{({\mathbf{p}_T}^2-{\mathbf{q}_T}^2)(J_1-{\mathbf{q}_T}^2J_0)(m_1-\omega_l)}{{m_V}^2}+(m_1-\omega_1)(J_1+{\mathbf{q}_T}^2J_0)+2(M+\omega_1)J_1\right]f_2(|\mathbf{q}_T|)\right. \\
& \left.+\frac{\kappa}{m_B}[J_2-{\mathbf{p}_T}^2{\mathbf{q}_T}^2J_0+(m_1-\omega_1)(J_1-{\mathbf{q}_T}^2J_0)(\omega_1+M)]f_2(|\mathbf{q}_T|)\right\} \\
& +\frac{\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_2(M+\omega_1-\omega_2)(M-\omega_1-\omega_2)} \\
& \times\left\{\left[\frac{({\mathbf{p}_T}^2-{\mathbf{q}_T}^2)(J_1-{\mathbf{q}_T}^2J_0)(m_1-\omega_2+M)}{{m_V}^2}+(m_1+M-\omega_2)(J_1+{\mathbf{q}_T}^2J_0)+2\omega_2J_1\right]f_2(|\mathbf{q}_T|)\right. \\
& \left.+\frac{\kappa}{m_B}[J_2-{\mathbf{p}_T}^2{\mathbf{q}_T}^2J_0+(m_1-\omega_2+M)({\mathbf{q}_T}^2J_0-J_1)(-\omega_2)]f_2(|\mathbf{q}_T|)\right\}. \qquad \text{(C7)}
\end{aligned}
$$

$$
\begin{aligned}
A_{21}(\mathbf{p_T},\mathbf{q_T}){\mathbf{p}_T}^2 = {} & \frac{\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_1(M+\omega_1+\omega_2)(M+\omega_1-\omega_2)} \\
& \times\left\{\left[\frac{({\mathbf{p}_T}^2-{\mathbf{q}_T}^2)(J_1-{\mathbf{p}_T}^2J_0)(-m_1-\omega_1)}{{m_V}^2}+(m_1-\omega_1)(J_1+{\mathbf{p}_T}^2J_0)-2M{\mathbf{p}_T}^2J_0+2\omega_1J_1\right]f_1(|\mathbf{q}_T|)\right. \\
& \left.-\frac{\kappa}{4m_B}\left[\frac{4(J_1-{\mathbf{p}_T}^2J_0)(-\omega_1-M)(m_1+\omega_1)}{-(\mathbf{p}_T-\mathbf{q}_T)^2-m_V^2}\right]f_1(|\mathbf{q}_T|)\right\} \\
& +\frac{\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_2(M+\omega_1-\omega_2)(M-\omega_1-\omega_2)} \\
& \times\left\{\left[\frac{({\mathbf{p}_T}^2-{\mathbf{q}_T}^2)(J_1-{\mathbf{p}_T}^2J_0)(M-m_1-\omega_2)}{{m_V}^2}+(m_1-M-\omega_2)(J_1+{\mathbf{p}_T}^2J_0)+2\omega_2J_1\right]f_1(|\mathbf{q}_T|)\right. \\
& \left.-\frac{\kappa}{4m_B}[4(J_1-{\mathbf{p}_T}^2J_0)\omega_2(M-m_1-\omega_2)]f_1(|\mathbf{q}_T|)\right\}. \qquad \text{(C8)}
\end{aligned}
$$

$$
\begin{aligned}
A_{22}(\mathbf{p_T},\mathbf{q_T}){\mathbf{p}_T}^2 = {} & \frac{\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_1(M+\omega_1+\omega_2)(M+\omega_1-\omega_2)} \\
& \times\left\{\left[-{\mathbf{p}_T}^2(J_1+{\mathbf{q}_T}^2J_0)+2J_1(m_1+\omega_1)(M+\omega_1)+{\mathbf{p}_T}^2\frac{({\mathbf{q}_T}^2J_0-J_1)({\mathbf{p}_T}^2-{\mathbf{q}_T}^2)}{{m_V}^2}\right]f_2(|\mathbf{q}_T|)\right. \\
& \left.-\frac{\kappa}{4m_B}4[(M+\omega_1){\mathbf{p}_T}^2J_1-(m_1+\omega_1)J_2+{\mathbf{p}_T}^2{\mathbf{q}_T}^2(m_1-M)J_0]f_2(|\mathbf{q}_T|)\right\} \\
& +\frac{\mathbf{q}_T^2}{(2\pi)^2}\frac{C_{I,I_z}g_{\mathcal{MMV}}g_{\mathcal{BBV}}}{2\omega_2(M+\omega_1-\omega_2)(M-\omega_1-\omega_2)} \\
& \times\left\{\left[-{\mathbf{p}_T}^2(J_1+{\mathbf{q}_T}^2J_0)+2(m_1+\omega_2-M)\omega_2J_1+{\mathbf{p}_T}^2\frac{({\mathbf{q}_T}^2J_0-J_1)({\mathbf{p}_T}^2-{\mathbf{q}_T}^2)}{{m_V}^2}\right]f_2(|\mathbf{q}_T|)\right. \\
& \left.-\frac{\kappa}{4m_B}4[\omega_2{\mathbf{p}_T}^2J_1-(m_1+\omega_2-M)J_2+{\mathbf{p}_T}^2{\mathbf{q}_T}^2(m_1-M)J_0]f_2(|\mathbf{q}_T|)\right\}. \qquad \text{(C9)}
\end{aligned}
$$

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