Radiative seesaw dark matter

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The singlet Majoron model of seesaw neutrino mass is appended by one dark Majorana fermion singlet χ with $L = 2$ and one dark complex scalar singlet ζ with $L = 1$. This simple setup allows χ to obtain a small radiative mass anchored by the same heavy right-handed neutrinos, whereas the one-loop decay of the standard model Higgs boson to $\chi\chi + \bar{\chi}\bar{\chi}$ provides the freeze-in mechanism for χ to be the light dark matter of the Universe.

I. INTRODUCTION

Neutrino mass [1] and dark matter [2] are two outstanding issues in particle physics and astrophysics. They may be essentially related, such as in scotogenic models [3], where the former requires the existence of the latter. They may also be indirectly related through lepton parity [4] or lepton number [5].

Dark matter is conventionally believed to be a massive particle of order 100 GeV and interacts weakly with matter. Its annihilation cross section × relative velocity at rest should be about 3×10^{-26} cm^3/s, as it freezes out of thermal equilibrium in the early Universe. It should then be observable in direct search experiments in underground laboratories.

An alternative is the freeze-in mechanism [6], where the dark matter interacts very weakly and slowly builds up its relic abundance from the decay of a massive particle, for example, before the latter itself goes out of thermal equilibrium. In this case, the direct detection of dark matter in underground experiments becomes very difficult, which is consistent with the mostly null results obtained so far.

The standard model (SM) of particle interactions conserves baryon number B and L automatically. If a singlet right-handed fermion N_R is added, the term $\bar{N}_R(\nu_L \phi^0 - e_L \phi^+)$ is allowed by gauge invariance; hence, N_R is naturally assigned $L = 1$. On the other hand, gauge invariance also allows the $N_R N_R$ Majorana mass term.

Hence, L naturally breaks to $(-1)^L$, and a seesaw mass for ν_L is obtained as is well known. Instead of letting gauge invariance decide on all the global and discrete symmetries of the Lagrangian, the latter may be imposed as additional inputs. In that case, other choices for N_R are also possible [7].

Returning to the canonical choice of $L = 1$ for N_R, if a new complex neutral scalar singlet η with $L = 2$ is added, then the term $\eta^* N_R N_R$ is allowed, and the Lagrangian with this term replacing the $N_R N_R$ term is invariant under the global symmetry $U(1)_L$. This conscious choice of a new particle with a nonzero lepton number was first made 40 years ago [8], where η is assumed to have a nonzero vacuum expectation value, thereby breaking $U(1)_L$ spontaneously again to $(-1)^L$ but also with the appearance of a massless Goldstone boson called the singlet Majoron. The seesaw mechanism applies as before.

In this paper, the conscious choice of two additional particles with a nonzero lepton number is made. ζ is a neutral complex scalar with $L = 1$, and χ_L is a neutral Majorana fermion with $L = 2$. As expected [4] from $(-1)^{L+2j}$ (j is the intrinsic spin of the particle), they are particles of odd dark parity. It is shown that χ_L obtains a small radiative mass anchored by N_R, and it becomes dark matter through the freeze-in mechanism from Higgs decay.

II. DESCRIPTION OF MODEL

The SM is extended first by three singlet right-handed neutrinos N_R with lepton number $L = 1$ as well as one complex scalar η with $L = 2$, so that the term $\eta^* N_R N_R$ appears in the Lagrangian, which is assumed invariant under $U(1)_L$. However, $\langle \eta \rangle \neq 0$ breaks $U(1)_L$ spontaneously, resulting in a singlet Majoron [8]. This well-known

model is now extended to include one complex scalar ζ with $L = 1$, and one Majorana fermion χ_L with $L = 2$. The Lagrangian of the SM is then expanded, having

$$\mathcal{L}_Y = -\frac{1}{2}f_N \eta^* N_R N_R - f_\nu \bar{N}_R (\nu_L \phi^0 - l_L \phi^+)$$
$$- f_\chi \bar{\chi}_L N_R \zeta + \text{H.c.}, \tag{1}$$

as well as

$$V(\zeta, \eta, \Phi) = -\mu_0^2 \Phi^\dagger \Phi - \mu_2^2 \eta^* \eta + m_1^2 \zeta^* \zeta + \frac{1}{2}\mu_1 \eta^* \zeta^2 + \text{H.c.}$$
$$+ \frac{1}{2}\lambda_0 (\Phi^\dagger \Phi)^2 + \frac{1}{2}\lambda_1 (\zeta^* \zeta)^2 + \frac{1}{2}\lambda_2 (\eta^* \eta)^2$$
$$+ \lambda_{01} (\zeta^* \zeta)(\Phi^\dagger \Phi) + \lambda_{02} (\eta^* \eta)(\Phi^\dagger \Phi)$$
$$+ \lambda_{12} (\zeta^* \zeta)(\eta^* \eta). \tag{2}$$

In the above, lepton number L is conserved, so that χ has no mass, and the η^2 and ζ^2 terms are forbidden. However, it is spontaneously broken to lepton parity $(-1)^L$ by $\langle \eta \rangle$. Note that the f_N term justifies the assignment of $L = 2$ to η. The μ_1 term justifies the assignment of $L = 1$ to ζ, and the f_χ term justifies the assignment of $L = 2$ to χ_L. Without η, the separate assignments of L to ζ and χ_L would have been ambiguous. Let

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ (v + h)/\sqrt{2} \end{pmatrix},$$
$$\eta = \left(u + \frac{\rho}{\sqrt{2}}\right)e^{i\theta/u\sqrt{2}}, \qquad \zeta = \frac{1}{\sqrt{2}}(\zeta_R + i\zeta_I)e^{i\theta/2u\sqrt{2}}, \tag{3}$$

then $\eta^* \zeta^2 + \text{H.c.} = (u + \rho/\sqrt{2})(\zeta_R^2 - \zeta_I^2)$. The minimum of V is determined by

$$\mu_0^2 = \frac{1}{2}\lambda_0 v^2 + \lambda_{02} u^2, \tag{4}$$

$$\mu_2^2 = \lambda_2 u^2 + \frac{1}{2}\lambda_{02} v^2. \tag{5}$$

As a result, the mass-squared matrix spanning h and ρ is given by

$$\mathcal{M}_{h\rho}^2 = \begin{pmatrix} \lambda_0 v^2 & \sqrt{2}\lambda_{02} vu \\ \sqrt{2}\lambda_{02} vu & 2\lambda_2 u^2 \end{pmatrix}. \tag{6}$$

Now, the N_R mass is $m_N = f_N u$; hence, u should be much greater than v. This means that h is predominantly the SM Higgs boson and mixes very little with the very heavy ρ. As for the $\zeta_{R,I}$ masses, they are split by the μ_1 term, i.e.,

$$m_R^2 = m_1^2 + \frac{1}{2}\lambda_{01} v^2 + \lambda_{12} u^2 + \mu_1 u, \tag{7}$$

$$m_I^2 = m_1^2 + \frac{1}{2}\lambda_{01} v^2 + \lambda_{12} u^2 - \mu_1 u. \tag{8}$$

The neutrinos obtain canonical seesaw masses $m_\nu = m_D^2/m_N$, where $m_D = f_\nu v/\sqrt{2}$. Note that ζ is assumed not to have any vacuum expectation value by choosing $m_1^2 > 0$. If it does, then it will mix with Φ and η through the λ_{01} and λ_{12} terms of Eq. (2). This would invalidate the assignment of dark parity that is discussed in the next section.

III. LIGHT DARK MAJORANA FERMION

The $\bar{\chi}_L N_R \zeta$ term is similar to the one found in the prototype renormalizable model [9] of fermionic dark matter χ_L supplemented by a real scalar counterpart S, i.e., $\bar{\chi}_L N_R S$. In that case, odd dark parity is imposed on χ and S. However, if lepton parity L_P is used, i.e., even for χ and odd for S, N, the same dark parity D_P is derived [4], namely, $D_P = L_P(-1)^{2j}$. Note that χ is allowed an arbitrary Majorana mass. This scenario admits either χ or S to be dark matter and has been studied extensively.

In the present model, L is conserved in the Lagrangian; hence, χ (with $L = 2$) is massless at tree level. After L is spontaneously broken by two units, χ then obtains a radiative seesaw mass as shown in Fig. 1. Its structure is analogous to that of the scotogenic model [3], with its finiteness coming from the cancellation of the $\zeta_{R,I}$ contributions. Both χ and $\zeta_{R,I}$ belong to the dark sector, with odd dark parity D_P. The one-loop diagram of Fig. 1 is easily calculated,

$$m_\chi = \frac{f_\chi^2 m_N}{32\pi^2}\left[\frac{m_R^2 \ln(m_R^2/m_N^2)}{m_R^2 - m_N^2} - \frac{m_I^2 \ln(m_I^2/m_N^2)}{m_I^2 - m_N^2}\right]. \tag{9}$$

For $m_R^2 - m_I^2 = 2\mu_1 u \ll m_\zeta^2 = (m_R^2 + m_I^2)/2 \ll m_N^2$, this becomes

$$m_\chi = \frac{f_\chi^2 \mu_1 u}{16\pi^2 m_N}\left[\ln\frac{m_N^2}{m_\zeta^2} - 1\right]. \tag{10}$$

Note that both m_ν and m_χ are of the seesaw form, anchored by m_N. However, m_χ can actually be large [$\mathcal{O}(\text{GeV})$ or
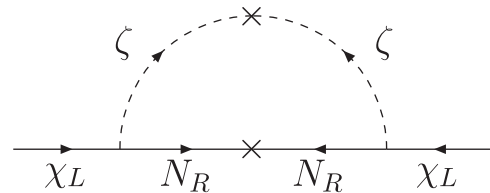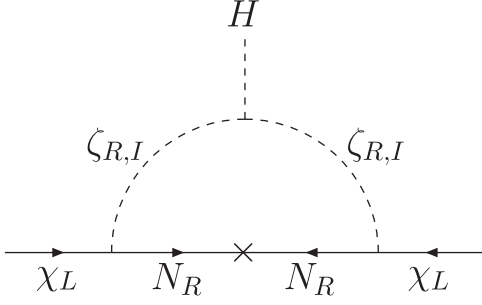

FIG. 1. One-loop radiative Majorana mass for the dark fermion χ.

FIG. 2.   Direct one-loop diagram for Higgs decay to $\chi\chi$.

FIG. 3.   One-loop diagram for Higgs decay to $\chi\chi$ through external m_χ insertion.

larger], as it is also controlled by the free parameter μ_1. A tree-level seesaw for light fermion dark matter is also possible in the context of $U(1)_\chi$ [10,11].

IV. PRODUCTION OF LIGHT FERMION DARK MATTER

The only interaction involving χ is the $f_\chi\bar{\chi}_L N_R\zeta$ term. Assuming that the reheat temperature of the Universe is low enough (say a few TeV), its relic abundance comes from freeze-in through Higgs decay, with the effective interaction

$$\mathcal{L}_{\text{int}} = \frac{1}{2}f_h h(\chi\chi + \bar{\chi}\bar{\chi}), \tag{11}$$

where f_h is given by Fig. 2,

$$f_h = \frac{\lambda_{01} v f_\chi^2 m_N}{32\pi^2}\left[\frac{1}{m_R^2 - m_N^2} - \frac{m_N^2 \ln(m_R^2/m_N^2)}{(m_R^2 - m_N^2)^2} - \frac{1}{m_I^2 - m_N^2}\right.$$
$$\left. + \frac{m_N^2 \ln(m_I^2/m_N^2)}{(m_I^2 - m_N^2)^2}\right]$$
$$= \frac{-\lambda_{01} v f_\chi^2 \mu_1 u}{16\pi^2 m_N m_\zeta^2} = \frac{-\lambda_{01} v m_\chi}{m_\zeta^2}\left[\ln\frac{m_N^2}{m_\zeta^2} - 1\right]^{-1}. \tag{12}$$

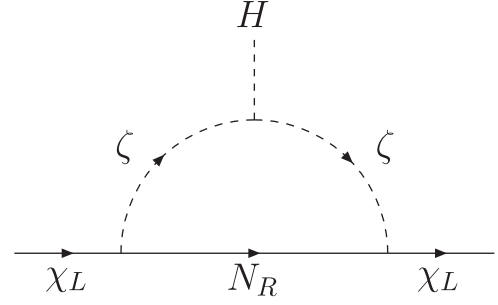The decay rate of h to $\chi\chi + \bar{\chi}\bar{\chi}$ is

$$\Gamma_h = \frac{f_h^2 m_h}{32\pi}\sqrt{1 - 4x^2}(1 - 2x^2), \tag{13}$$

where $x = m_\chi/m_h$.

Another one-loop diagram for h decay to $\chi\chi$ is shown in Fig. 3. The corresponding effective coupling is given by

$$f_h' = \frac{\lambda_{01} v m_\chi f_\chi^2}{16\pi^2 m_N^2}\left[\ln\frac{m_N^2}{m_\zeta^2} - \frac{3}{2}\right]. \tag{14}$$

Hence, $|f_h'| \ll |f_h|$ for $m_\zeta \ll m_N$.

V. PHENOMENOLOGY OF FERMION DARK MATTER

We assume that the reheat temperature of the Universe after inflation is below the decoupling temperature of χ but above m_H, say $T_R \sim 1$–10 TeV. In such a scenario, χ is a feebly interacting massive particle (FIMP), which only production mechanism is freeze-in [6,12] through Higgs decay, before the latter decouples from the thermal bath. (In principle, there could also be $2 \to 2$ scattering of χ with other states in the thermal bath, mediated by the Higgs boson [13,14].) The relic abundance of χ, initially negligible in the early Universe, gradually grows via its feeble coupling f_h. Both N_R and ζ are taken to be much heavier than T_R so that their abundances are Boltzmann suppressed. On the contrary, if m_N or $m_\zeta \lesssim T_R$, new interaction channels would open, and the relic density of χ could excessively increase due to the term $f_\chi\bar{\chi}_L N_R\zeta$, such as $N_R\bar{N}_R \to \chi\bar{\chi}$, $\zeta\zeta \to \chi\chi$, or the decays of ζ. These processes could possibly lead to the thermalization of χ. Even if $m_N(m_\zeta) \gg T_R$, the channel $\nu\bar{\nu} \to \chi\bar{\chi}$ is always present, although it is suppressed by the mixing of N_R with the light active neutrinos. Eventually, we find that this channel never contributes sizeably to the thermalization of χ.

We perform a scan varying randomly the main free parameters which characterize the model, and we rely on public computer tools to 1) implement the model with (FeynRules 2.0 [15] and CalcHEP 3.4 [16]) and 2) numerically compute the relic abundance of χ via freeze-in with (Micromegas 5.2.4 [17]). In the analysis, we require the parameters to comply with the hierarchy of scales $2\mu_1 u \ll m_\zeta^2 \ll m_N^2$ and with light neutrino mass constraints.

Figure 4 shows the values of f_h required to obtain the observed relic density, as a function of m_χ. The green plain line denotes the contour along which $\Omega h^2 = 0.12$ [18]. Solutions in the green shaded area are excluded as they lead to overabundant dark matter. Values in the white area are allowed, but in this case, another dark matter candidate would be required. As expected in a FIMP scenario, for the relic abundance of χ to meet current observations, f_h must be small: $f_h \lesssim 3 \times 10^{-11}$ for $m_\chi \sim 0.1$ GeV. Its smallness is
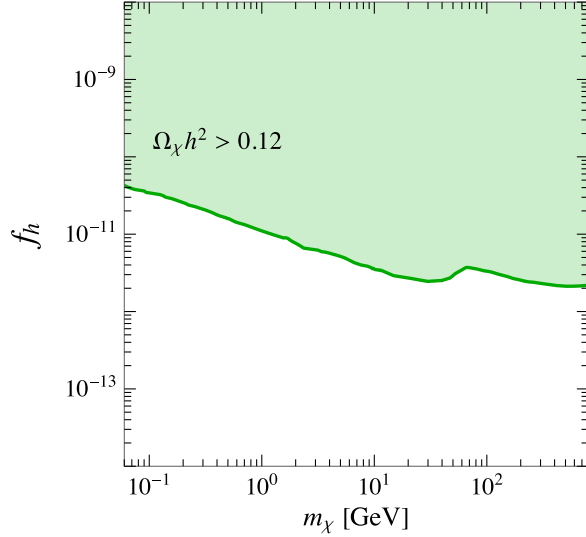
FIG. 4. f_h as a function of m_χ. The green plain line denotes the contour along which $\Omega h^2 = 0.12$ [18]. The region in the green shaded area is excluded as it leads to overabundant dark matter.

FIG. 5. Ωh^2 as a function of m_χ. The dark cyan line denotes the observed value $\Omega h^2 = 0.12$ [18].

here driven by the mass hierarchy $m_\chi \ll v \ll m_\zeta$ and in a minor part by its loop nature. Let us notice that the abundance of χ is insensitive to T_R as long as $T_R \ll m_\zeta, m_{N_R}$. Let us recall that this last assumption is required in order to prevent the thermalization of χ and thus justifies our choice of $T_R \sim 1$–10 TeV for the benchmark values and mass hierarchies here considered.

For the sake of example, we summarize in Table I the values of the relevant parameters corresponding to few benchmark points which lead to a relic density compatible with current observations.

Finally, Fig. 5 depicts the result of a numerical scan of the model parameter space, including the benchmark points collected in Table I. Blue points denote solutions which are compatible with current cosmological observations, while gray points are excluded because of $\Omega h^2 > 0.12$.

VI. CONSTRAINTS ON THE MAJORON

The massless singlet Majoron θ, which couples primarily to the light active neutrinos with a strength suppressed by

TABLE I. Benchmark points which lead to a relic density of χ compatible with current observations.

m_χ [GeV]	m_N [GeV]	m_ζ [GeV]	f_h	f_χ	Ωh^2
0.11	1.4×10^7	7.34×10^4	3.26×10^{-11}	1	0.10
1.13	1.19×10^6	4.47×10^5	1.02×10^{-11}	0.24	0.11
2.42	1.6×10^5	3.57×10^4	7.54×10^{-12}	3.7	0.12
21.14	8.3×10^5	1.21×10^5	2.46×10^{-12}	7.19	0.098

powers of u, may play a role in cosmological and astrophysical environments. A single thermally decoupled massless Majoron can contribute to the effective number of neutrinos, $\Delta N_{\rm eff}$. This extra contribution to the radiation density depends on the Majoron freeze-out temperature. However, because of the large values of the lepton number symmetry breaking scale here considered ($u \gtrsim 10$ TeV) and the low reheat temperature, we find that the massless Majoron never thermalizes with the thermal bath. Still, it could be produced via freeze-in through its linear coupling with the light active neutrinos. Also in this case, given the large values of u, the corresponding neutrino-Majoron couplings lie below current cosmological constraints (see, e.g., [19]). Finally, laboratory searches for nonobserved lepton flavor violating rare decays (e.g., $\ell \to \ell' \theta$) together with stellar cooling bounds further constrain the Majoron dimensionless parameters $m_D^2/vu \lesssim 10^{-5} - 10^{-6}$ [20]. As before, since u is much heavier than v and f_ν can be quite small, the solutions considered in our numerical scan always fall below the current constraints.

VII. CONCLUDING REMARKS

It is proposed that the lepton number may be a key to understanding light freeze-in dark matter. In the context of a spontaneously broken lepton number, first pointed out 40 years ago with a scalar having $L = 2$, it is proposed that two other particles exist with a nonzero lepton number, a scalar ζ with $L = 1$ and a Majorana fermion χ with $L = 2$. It is shown that χ may acquire a small radiative seesaw mass and becomes freeze-in dark matter from the decay of the SM Higgs boson. However, this proposal is similar to others in its practical predictions (see, e.g., [21]), it has the advantage of explaining the smallness of the dark matter mass based on a symmetry, i.e., lepton number, as well as the weakness of its interaction.

ACKNOWLEDGMENTS

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