

Anisotropy in Dijet Production in Exclusive and Inclusive Processes

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We investigate the effect of soft gluon radiations on the azimuthal angle correlation between the total and relative momenta of two jets in inclusive and exclusive dijet processes. We show that the final state effect induces a sizable $\cos(2\phi)$ anisotropy due to gluon emissions near the jet cones. The phenomenological consequences of this observation are discussed for various collider experiments, including diffractive processes in ultraperipheral pA and AA collisions, inclusive and diffractive dijet production at the EIC, and inclusive dijet in pp and AA collisions at the LHC.

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Introduction.—Dijet productions are the most abundant events in hadronic collisions and have been under intensive investigations from both experiment and theory sides. Typically the two final state jets are produced in the so-called correlation configuration, namely, back-to-back in the transverse plane with nearly balanced transverse momenta [1–8]. Deviations from the perfect back-to-back configuration inform important strong interaction physics, and, in particular, the nonperturbative structure of the nucleon and nucleus [9–22]. It is expected to reveal the medium property by studying decorrelation of dijet in heavy ion collisions as well [23–30].

Let $\vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$ be the total transverse momentum of the two jets. We also define $\vec{P}_\perp = (\vec{k}_{1\perp} - \vec{k}_{2\perp})/2$ as the leading jet transverse momentum, see Fig. 1. The correlation limit is defined by the condition $q_\perp \ll P_\perp \sim k_{1\perp} \sim k_{2\perp}$. In general, the azimuthal angular correlation between \vec{q}_\perp and \vec{P}_\perp is isotropic. However, anisotropic distribution can be generated from nontrivial correlations in the transverse momentum dependent parton distribution (TMD) associated with the incoming hadrons. This provides a unique probe to the novel tomography imaging of the nucleon and nucleus in dijet production.

Among the proposed observables, of particular interest is the $\cos(2\phi)$ anisotropic correlation in exclusive diffractive dijet production [20] where ϕ is the angle between \vec{P}_\perp

and the target recoil momentum $\vec{\Delta}_\perp$. Such a correlation provides a unique access to the so-called elliptic gluon Wigner distribution of the target [20,31,32]. Partly motivated by this observation (see, also, [33]), very recently, the CMS Collaboration has reported a preliminary measurement of this anisotropy in ultraperipheral collisions at the LHC [34]. Using q_\perp as a proxy for Δ_\perp , CMS reported a significant $\cos(2\phi)$ asymmetry. Intrigued by this result, in this Letter, we will perform a detailed analysis of this anisotropic distribution, focusing mainly on the soft gluon radiation contributions.

Soft gluon radiation contributions to the azimuthal angular anisotropy of $\cos(2\phi)$ and their resummation for various processes have been investigated in Refs. [35–42]. For the jet-related processes, they were first studied in Refs. [41,42] and it was found that their contributions are

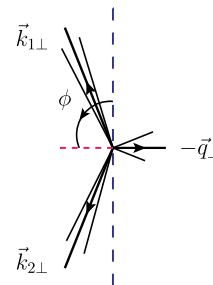


FIG. 1. Dijet in transverse plane perpendicular to the beam direction at hadron colliders. Their total transverse momentum $\vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp}$ is much smaller than the individual jet momentum $\vec{P}_\perp = (\vec{k}_{1\perp} - \vec{k}_{2\perp})/2$. Angular distribution between q_\perp and P_\perp has an anisotropy due to soft gluon radiation associated with the final state jet with a nonzero $\langle \cos(2\phi) \rangle$.

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not power suppressed and can be resummed to all orders in perturbation theory. In this Letter, we will follow these developments and apply to dijet production processes. For azimuthally symmetric distributions, the soft gluon resummation has been derived in Refs. [43–51].

The physical picture is as follows. Soft gluons emitted from the final state jets tend to be aligned with the jet directions. Since gluons emitted too close to the jet axis become part of the jet, what matters is the emissions slightly outside the jet cones, and \vec{q}_\perp is essentially the recoil momentum against these gluons. This means that \vec{q}_\perp also tends to point to jet directions on average, resulting in a positive $\langle \cos(2\phi) \rangle$. When $q_\perp \ll P_\perp$, one needs to perform the resummation of logarithms $\alpha_s^n (\ln P_\perp/q_\perp)^m$ which can be done in the TMD framework. We shall demonstrate that this simple picture can explain, at least partly, the observation by the CMS Collaboration.

Our result in this Letter will have a broad impact on the tomographic study of nucleons and nuclei at the future electron-ion collider (EIC) [52,53]. Various anisotropy observables have been proposed to study the novel structure of nucleon and nucleus [13–22]. The results in these previous works have to be reexamined.

Diffractive photoproduction of dijet.—We first study the diffractive photoproduction of dijets, $\gamma A \rightarrow q\bar{q} + A$. The photon fluctuates into a quark-antiquark pair which then scatters off the nucleon and nucleus target and forms a final state dijet with momentum k_1 and k_2 . This process can be studied, for example, in ultraperipheral pA and AA collisions and the planned EIC in the future.

Let us define the soft factor $S_J(q_\perp)$ as the probability for the dijet to emit total transverse momentum q_\perp outside the jet cones. Implicitly, it should also depend on the dijet relative transverse momentum $P_\perp = \frac{1}{2}(k_{1\perp} - k_{2\perp})$. The dijet cross section then reads

$$\frac{d\sigma}{d^2P_\perp d^2q_\perp} = \int d^2q'_\perp \frac{d\sigma_0}{d^2P_\perp d^2q'_\perp} S_J(q_\perp - q'_\perp), \quad (1)$$

where σ_0 is the cross section without soft radiations. We assume that the two jets are back to back in azimuth, and consider the soft regime $|q_\perp| \ll |P_\perp|$. To lowest nontrivial order, S_J is given by the standard eikonal formula

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2} \int dy_g \left(\frac{k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g} \right)_{\vec{q}_\perp = -\vec{k}_{g\perp}}, \quad (2)$$

where k_g is the soft gluon momentum emitted from the dijet and y_g is its rapidity. A power-counting argument shows that $S_J(q_\perp) \sim 1/q_\perp^2$ at small q_\perp , and we neglect power corrections $(q_\perp/P_\perp)^n$ to this leading behavior such as coming from gluon emissions from the t -channel Pomeron (see, e.g., [54]). We shall be particularly interested in the ϕ distribution where ϕ is the azimuthal angle between q_\perp and P_\perp . In general one can expand

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2 \cos(2\phi) S_{J2}(|q_\perp|) + \dots, \quad (3)$$

where for simplicity higher harmonics are neglected in this work. To $\mathcal{O}(\alpha_s)$, $S_{J0, J2}$ can be obtained by calculating the Fourier coefficients

$$\int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dy_g \frac{k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g} \{1, \cos(2\phi)\}. \quad (4)$$

There are collinear divergences when k_g is parallel to either k_1 or k_2 . They are factorized into the jet functions associated with the two final state jets. In practice, we introduce the θ -function constraints

$$\Theta[2 \cosh(y_1 - y_g) - 2 \cos \phi - R^2] \approx 1 - \Theta[R^2 - (y_1 - y_g)^2 - \phi^2], \quad (5)$$

where R is the jet radius, and similarly for the second jet. In the narrow jet approximation $R \ll 1$ [55,56], we obtain

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi} \frac{1}{q_\perp^2}, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi} \frac{1}{q_\perp^2}, \quad (6)$$

where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2 \ln \frac{a_0}{R^2}, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2 \ln \frac{a_2}{R^2}. \quad (7)$$

a_0 depends on the rapidity difference $\Delta y_{12} = |y_1 - y_2|$ as $a_0 = 2 + 2 \cosh(\Delta y_{12})$. A closed analytic expression is not available for a_2 except in the two limiting cases $\Delta y_{12} = 0, \infty$, but it has very mild dependence on Δy_{12} . It increases slightly from $a_2 = 1/4$ for $\Delta y_{12} = 0$ to $a_2 = 1/e$ for $\Delta y_{12} \rightarrow \infty$.

We observe that the $\cos(2\phi)$ term is not power suppressed, which is consistent with the finding in Ref. [42]. In the small- R limit, the ratio α_2/α_0 is essentially unity, and the ϕ distribution reduces to two delta functions around 0 and π . However, for realistic values of R , typically $\alpha_2/\alpha_0 \lesssim 0.2$ because $a_0 \gg a_2$.

To have a reliable prediction, we need to perform the resummation of higher order contributions to S_J [49]. The constraint $q_\perp = -k_{1g\perp} - k_{2g\perp} - \dots$ from multiple gluon emissions in momentum space can be conveniently deconvoluted by Fourier transforming to b_\perp space,

$$\begin{aligned} \tilde{S}_J(b_\perp) &= \int d^2q_\perp e^{i\vec{q}_\perp \cdot \vec{b}_\perp} S_J(q_\perp) \\ &= \tilde{S}_{J0}(|b_\perp|) - 2 \cos(2\phi_b) \tilde{S}_{J2}(|b_\perp|) + \dots, \end{aligned} \quad (8)$$

where ϕ_b represents the angle between \vec{b}_\perp and \vec{P}_\perp . Again we have only kept the leading two harmonics. To one loop, we find

$$\tilde{S}_{J_0}(b_\perp) = 1 + \alpha_0 \ln(\mu_b^2/P_\perp^2), \quad \tilde{S}_{J_2}(b_\perp) = \alpha_2, \quad (9)$$

where $\mu_b = 2e^{-\gamma_E}/b_\perp$ with γ_E being the Euler constant. In the above equation, we have taken into account the virtual diagram contribution to S_{J_0} whose natural scale is the jet momentum P_\perp . Note that in the S_{J_2} term, the Fourier transformation does not introduce IR divergence. We extend (9) to all orders by exponentiating to the Sudakov form factor,

$$\tilde{S}_{J_0}(b_\perp) = e^{-\Gamma_0(b_\perp)}, \quad \tilde{S}_{J_2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)}, \quad (10)$$

where $\Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} (d\mu^2/\mu^2)\alpha_0$. Here and in what follows, we neglect the nonglobal logarithms [44,57] which are not significant in the kinematics of our interest.

In the fixed coupling case, the Fourier transform of (10) can be obtained analytically, and we find that $\langle \cos(2\phi) \rangle = S_{J_2}(q_\perp)/S_{J_0}(q_\perp)$ remains a constant independent of q_\perp , but its value is reduced by a factor of $(1 - \alpha_0)$ compared to the leading order result α_2/α_0 . In the running coupling case, we proceed numerically and adopt the b_* prescription [58]: $\Gamma_0(b_\perp) \Rightarrow \Gamma_0(b_*) + g_\Lambda b_\perp^2$, where $b_* = b_\perp/\sqrt{1 + b_\perp^2/b_{\max}^2}$ with $b_{\max} = 1.5 \text{ GeV}^{-1}$. The parameter g_Λ is applied to include nonperturbative effects at large b_\perp , which should be order Λ_{QCD}^2 . In the numerical calculations, we take three different values $g_\Lambda = 0.05, 0.1, 0.2 \text{ GeV}^2$. In Fig. 2, we show our prediction for $\langle \cos(2\phi) \rangle$ for diffractive dijet production $\gamma A \rightarrow q\bar{q} + A$, with $P_\perp = 35 \text{ GeV}$, $R = 0.4$, and $\Delta y_{12} = 0$. For this jet size, the narrow jet approximation used in the above derivations is reasonably good [55,56]. We have explicitly verified this by comparing Eq. (7) with the exact numerical results. In the figure, we have also plotted the leading order result $\alpha_2/\alpha_0 \approx 0.14$ for comparison. We see that the anisotropy

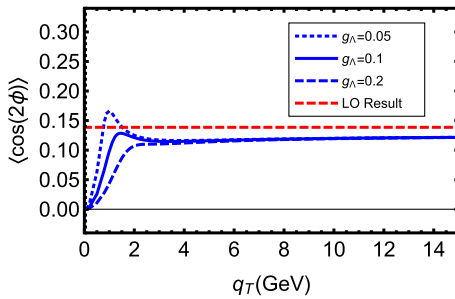


FIG. 2. Anisotropy in diffractive dijet production $\gamma + A \rightarrow q\bar{q} + A$ in ultraperipheral heavy ion collisions at the LHC. The kinematics correspond to the CMS measurements [34] with the leading jet $P_\perp = 35 \text{ GeV}$, the two jets are at the same rapidity $\Delta y_{12} = 0$ and the jet size $R = 0.4$. The figure shows $\langle \cos(2\phi) \rangle$ as a function of q_\perp , where ϕ is the azimuthal angle between q_\perp and P_\perp . The leading order result (LO) represents the ratio α_2/α_0 of Eq. (7), whereas full resummation depends on a nonperturbative parameter g_Λ (see the main text for the description).

does not change dramatically with q_\perp in most kinematics except in the very small q_\perp region, where the prediction is sensitive to the nonperturbative contributions as expected.

In the above result, we have neglected the possible contribution from the elliptic gluon Wigner distribution [20]. The anisotropy $\langle \cos(2\phi) \rangle$ due to the elliptic Wigner is at most a few percent [21,31], which is further suppressed after convoluted with the symmetric part S_{J_0} [49]. It is thus negligible compared to the result in Fig. 2.

In Ref. [22], a significant $\cos(2\phi)$ anisotropy was also found from a leading order color-glass-condensate calculation in the kinematics of $q_\perp \gg P_\perp$, which is opposite to the correlation limit considered in this Letter. Comparison of these $\cos(2\phi)$ anisotropies will help us to understand the underlying physics in dijet processes.

Inclusive dijet in γp collisions.—Next we turn to inclusive dijet photoproduction $\gamma p \rightarrow jjX$. We shall show that, in contrast to the diffractive case, the resummation of initial state radiations in the TMD of the incoming proton strongly affects the azimuthal distribution of dijets. In experiments such as at the EIC, photoproduction contains both direct photon and resolved photon contributions. Here, we focus on the direct photon contribution where the leading order partonic subprocesses are $\gamma g \rightarrow q\bar{q}$ and $\gamma q \rightarrow qg$. We take the example of the $\gamma g \rightarrow q\bar{q}$ channel. The other channel $\gamma q \rightarrow qg$ can be treated similarly.

In $\gamma g \rightarrow q\bar{q}$, we will have contributions from the soft gluon radiation associated with the incoming gluon. To lowest nontrivial order, the differential cross section can be written as

$$\frac{d^4\sigma}{d\Omega} = \sigma_0 x_g f_g(x_g) \frac{1}{q_\perp^2} \left[\frac{\alpha_0^{\gamma g}}{\pi} + \frac{\alpha_2^{\gamma g}}{\pi} 2 \cos(2\phi) \right], \quad (11)$$

where σ_0 represents the leading order cross section, $d\Omega = dy_1 dy_2 dP_\perp^2 d^2q_\perp$ for the phase space, and $f_g(x_g)$ is the gluon distribution with $x_g = P_\perp(e^{y_1} + e^{y_2})/\sqrt{S_{\gamma p}}$ is momentum fraction of the nucleon carried by the gluon. The coefficient $\alpha_{0,2}$ can be derived from the soft gluon radiation amplitude of [46],

$$\alpha_0^{\gamma g} = \frac{\alpha_s}{2\pi} \left[C_A \ln \frac{P_\perp^2}{q_\perp^2} + 2C_F \ln \frac{a_0}{R^2} \right],$$

$$\alpha_2^{\gamma g} = \frac{\alpha_s}{2\pi} \left[C_A \ln \frac{a_1}{R^2} - \frac{1}{N_c} \ln \frac{a_2}{R^2} \right], \quad (12)$$

where $a_1 = 1/e$ and a_2 is the same as in the previous section. The emission from the incoming gluon is isotropic, and gives rise to the double logarithmic term $(1/q_\perp^2) \ln q_\perp^2$.

We now perform the resummation of double logarithms in the standard TMD framework,

$$\frac{d^4\sigma}{d\Omega} = \sum_{ab} \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} [\tilde{W}_0^{\gamma p}(|b_\perp|) - 2\cos(2\phi_b) \tilde{W}_2^{\gamma p}(|b_\perp|)], \quad (13)$$

where the azimuthal symmetric term can be written as

$$\tilde{W}_0^{\gamma p}(b_\perp) = x_g f_g(x_g, \mu_b) e^{-S^{\gamma p}(P_\perp, b_\perp)}. \quad (14)$$

We separate the Sudakov form factor $S(P_\perp, b_\perp)$ into perturbative and nonperturbative parts: $S(P_\perp, b_\perp) = S_{\text{pert}}(P_\perp, b_\perp) + S_{\text{NP}}(P_\perp, b_\perp)$ with the perturbative part at one-loop order is defined,

$$S_{\text{pert}}^{\gamma p} = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s C_A}{2\pi} \left[\ln \frac{P_\perp^2}{\mu^2} - 2\beta_0 + \frac{2C_F}{C_A} \ln \frac{a_0}{R^2} \right], \quad (15)$$

where $\beta_0 = 11/12 - N_f/18$. For the nonperturbative part, we follow those for the TMD quark distributions in Refs. [59,60] with an appropriate color factor,

$$S_{\text{NP}}^{\gamma p} = \frac{C_A}{C_F} \left[0.106b_\perp^2 + 0.42 \ln \frac{P_\perp}{Q_0} \ln \frac{b_\perp}{b_*} \right], \quad (16)$$

with $Q_0^2 = 2.4 \text{ GeV}^2$. For the $\cos(2\phi)$ term, we have

$$\tilde{W}_2^{\gamma p}(b_\perp) = \alpha_2^{\gamma p} \tilde{W}_0^{\gamma p}(b_\perp). \quad (17)$$

In Fig. 3, we show our numerical results for $\langle \cos(2\phi) \rangle$ as a function of q_\perp in the leading order [from (12)] and resummed [from (13)] cases. We take the typical kinematics of inclusive dijet production at the EIC with $\sqrt{S_{\gamma p}} = 100 \text{ GeV}$, $P_\perp \sim 15 \text{ GeV}$ and the two jets are at the same rapidity. Though it is not manifest in the plot, the leading order (red) curve goes to zero as $q_\perp \rightarrow 0$, as it should. Compared to the results in the diffractive case, the impact of resummation is more pronounced. This is because the TMD resummation suppresses the region

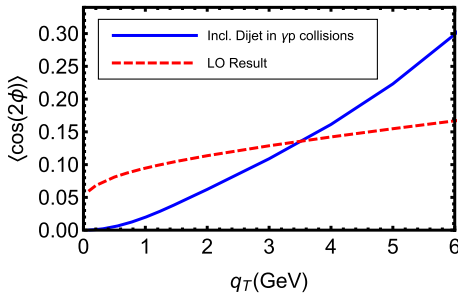


FIG. 3. Anisotropy of inclusive dijet production in γp collisions at the future EIC for the typical kinematics: the leading jet $P_\perp = 15 \text{ GeV}$ and both jets are at the same rapidity. Here, we plot $\langle \cos(2\phi) \rangle$ as function and q_\perp , where ϕ is the azimuthal angle between q_\perp and P_\perp . The leading order (LO) and full results are calculated from (12) and (13), respectively.

$q_\perp \sim 0$ and shifts the distribution of q_\perp to larger values. This affects the angular dependent and independent parts of the cross section differently, and therefore their ratio $\langle \cos 2\phi \rangle$ has a stronger q_\perp dependence. In contrast, in the diffractive case where the resummation is single logarithmic, the effect largely cancels in the ratio.

The $\cos(2\phi)$ asymmetry in inclusive dijet production in deep-inelastic scattering ($Q^2 \neq 0$) has been proposed to study the so-called linearly polarized gluon distribution in the nucleon [13–15,17]. As we have seen, the final state radiation from the dijet system contributes to exactly the same $\cos(2\phi)$ asymmetry already at leading order. Moreover, at higher orders the effect becomes sensitive to the initial state radiation through the TMD resummation. It is important to carefully subtract these radiative contributions in order to unambiguously extract the linearly polarized gluon distribution from the experimental data. We will publish the relevant results in a separate paper.

Inclusive dijet in pp collisions.—We can also extend the above studies to inclusive dijet production in pp collisions. Again we have to take into account the soft gluon radiation associated with incoming partons. In the following, we take the example of the $gg \rightarrow gg$ channel, which is also the dominant contribution for dijet production in the typical kinematics at the LHC. The differential cross section and the resummation formula follow the similar expressions as Eqs. (11), (13), and (14). For the soft gluon radiation contribution, we find the following results:

$$\alpha_2^{gg} = \frac{\alpha_s C_A}{2\pi} \left[2 \ln \frac{a_1}{R^2} + \frac{\hat{s}^2 + \hat{u}^2}{2(\hat{s}^2 - \hat{t}\hat{u})} \ln \frac{a_2}{a_1} \right], \quad (18)$$

and the associated α_0^{gg} has been derived in Ref. [48], where \hat{s} , \hat{t} , and \hat{u} are usual Mandelstam variables in the partonic process. For simplicity, we take the leading logarithmic approximation, for example, for the azimuthal symmetric one,

$$\tilde{W}_0^{gg}(b_\perp) = x_1 f_g(x_1, \mu_b) x_2 f_g(x_2, \mu_b) e^{-S^{gg}(P_\perp, b_\perp)}, \quad (19)$$

where $f_{a,b}(x, \mu_b)$ are parton distributions for the incoming partons a and b , $x_{1,2} = P_\perp (e^{\pm y_1} + e^{\pm y_2}) / \sqrt{S}$ are momentum fractions of the incoming hadrons carried by the partons. The perturbative part is defined as one-loop order,

$$S_{\text{pert}}^{gg} = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s C_A}{\pi} \left[\ln \left(\frac{P_\perp^2}{\mu^2} \right) - 2\beta_0 + \ln \frac{a_0}{R^2} \right]. \quad (20)$$

For the $\cos(2\phi)$ term, we have

$$\tilde{W}_2^{gg}(b_\perp) = \alpha_2^{gg} \tilde{W}_0^{gg}(b_\perp). \quad (21)$$

In Fig. 4, we show the $\langle \cos(2\phi) \rangle$ as a function of q_\perp for dijet production through the $gg \rightarrow gg$ channel. In the numeric calculations, we take the typical kinematics at

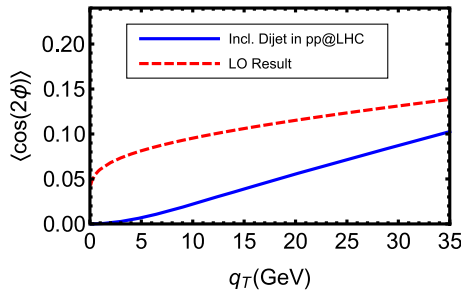


FIG. 4. Anisotropy of inclusive dijet production from the $gg \rightarrow gg$ channel in pp collisions at the LHC at $\sqrt{S_{pp}} = 7$ TeV for the typical kinematics: the leading jet $P_{\perp} = 100$ GeV and both jets are at midrapidity. Here, we plot $\langle \cos(2\phi) \rangle$ as function and q_{\perp} , where ϕ is the azimuthal angle between q_{\perp} and P_{\perp} .

the LHC with $\sqrt{S_{pp}} = 7$ TeV, leading jet $P_{\perp} \sim 100$ GeV, and the two jets are both at midrapidity.

In heavy ion collisions, the final state jet suffers multiple interactions with the hot QCD matter when it traverses through the medium. It also generates medium induced soft gluon radiations. Taking into account the above two effects, we have the following modifications on the Fourier transform \tilde{W} ,

$$\tilde{W}_0 \Rightarrow \tilde{W}_0^{\text{med}} = \tilde{W}_0 + Q_s^2 b_{\perp}^2 / 4, \quad (22)$$

$$\tilde{W}_2 \Rightarrow (\alpha_2 + \alpha_2^m) \tilde{W}_0^{\text{med}}, \quad (23)$$

where $Q_s^2 = \langle \hat{q}L \rangle$ represents the medium p_T -broadening effects with the medium transport coefficient \hat{q} and jet traverse length L and α_2^m for the medium induced soft gluon radiation contribution to the $\cos(2\phi)$ anisotropy. Because the medium induced radiation also has a collimation associated with the jet [61], this contribution will be similar to what was discussed in previous sections. Therefore, we may utilize the modification of the $\cos(2\phi)$ anisotropy as a measure of the medium induced soft gluon radiation in quark-gluon plasma in heavy ion collisions.

Conclusions.—In summary, we have studied the soft gluon radiation contribution to the dijet production in various hadronic collisions, focusing on the anisotropy of the total transverse momentum with respect to the leading jet. Our results have shown that these soft gluon radiation contributions lead to characteristic behaviors of $\langle \cos(2\phi) \rangle$ as functions of the total transverse momentum q_{\perp} . Experimental studies, as shown in a preliminary analysis of the diffractive dijet production in ultraperipheral AA collisions at the LHC [34], will provide a unique opportunity to investigate the final state radiation associated with the jet. Our finding has far-reaching consequences in extracting the linearly polarized gluon distribution and the elliptic Wigner distribution from the measurement of angular anisotropies in inclusive and exclusive dijet processes.

A number of further developments shall follow. In this Letter we focused on the $\cos(2\phi)$ anisotropy. Following the examples of Ref. [42], other harmonics such as the $\cos(4\phi)$ anisotropy can be derived as well. Second, we have made the narrow jet approximation in the derivations. The finite- R corrections, together with the subeikonal corrections which might be relevant at large q_{\perp} , should be taken into account when we compare to the precise experimental data from the LHC and future EIC. Third, more rigorous derivation of the resummation formulas needs to be carried out along the lines of Refs. [41,42,62].

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