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Research Article

Approximate Scattering State Solutions of DKPE and SSE with Hellmann Potential

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We study the approximate scattering state solutions of the Duffin-Kemmer-Petiau equation (DKPE) and the spinless Salpeter equation (SSE) with the Hellmann potential. The eigensolutions, scattering phase shifts, partial-waves transitions, and the total cross section for all the partial waves are obtained and discussed. The dependence of partial-waves transitions on total angular momentum number, angular momentum number, mass combination, and potential parameters was presented in the figures.

1. Introduction

The relativistic and nonrelativistic quantum mechanical study of Hellmann potential is a long-standing and a well-known problem. The Hellmann potential in this study may be written as [1–5]

$$V(r) = -\frac{a}{r} + \frac{b}{r}e^{-\rho r}, \tag{1}$$

where the first part is the Coulomb potential with the strength parameter a and the second part is the screening Coulomb and/or Yukawa potential with the strength parameter b. The parameter ρ is the potential screening parameter which regulates the shape of the potential. The Coulomb potential has been investigated by some authors in all the limits of quantum mechanics due to its importance in atomic physics [6–8]. Both eigenfunctions and eigenvalues and their structures have been presented in the previous work. The bound and scattering states of the screening Coulomb and/or Yukawa potential have been studied by some researchers in various dimensions [9–12]. The energy levels, wave functions, phase shifts, scattering amplitude, and the effect of the screening parameter on quantum systems have been extensively discussed. The importance of these two parts necessitates the study of Hellmann potential in quantum mechanics.

However, most of the recent studies on the quantum mechanical treatment of Hellmann potential focused on the relativistic and nonrelativistic bound state problems [1–5]. Just recently a good number of researchers have explored the study of the scattering states of Hellmann problems so as to obtain new results that will provide a better understanding of quantum systems. In this regard, Yazarloo et al. extended the study to scattering states of Dirac equation with Hellmann potential under the spin and pseudospin symmetries [4]. The Dirac phase shift and normalized wave function for the spin and pseudospin symmetries were reported. Arda and Sever studied the approximate nonrelativistic bound and scattering states with any l values using the PT-/non-PT symmetry and non-Hermitian Hellmann potential [13]. The phase shift was calculated in terms of the angular momentum quantum number. Also, in one of our previous papers, we studied the scattering state solution of Klein-Gordon equation with Hellmann potential [14]. Again, Arda studied the approximate bound state solution of two-body spinless Salpeter equation for Hellmann potential [15]. He obtained energy levels and eigenfunctions in terms of hypergeometric functions. He also treated Yukawa potential and Coulomb potential as special cases. The Hellmann potential finds its applications in nuclear and high energy physics.

The motivation behind this work is to investigate the approximate scattering state solutions of Duffin-Kemmer-Petiau equation (DKPE) and spinless Salpeter equation (SSE) with Hellmann Potential. The SSE explains in detail the dynamics of semirelativistic of and two-body effects particle [see [16] and the references therein] whereas the DKPE explains explicitly the dynamics of relativistic spin 0 and spin 1 particles [see [17–26] and the references therein].

This work is organized as follows: Section 2 presents scattering state solutions of DKPE with Hellmann potential. The scattering state solution of SSE with Hellmann potential is presented in Section 3. In Section 4, we discuss the results and the conclusion are given in Section 5.

2. Scattering States of the Duffin-Kemmer-Petiau Equation (DKPE) with Hellman Potential

The DKP equation with energy $E_{n,J}$, total angular momentum centrifugal term, and the mass m of the particle is given as follows [17–22]:

$$U''_{n,J}(r) - \frac{J(J+1)}{r^2} + \left[\left(E_{n,J} + V_v^0\right)^2 - m^2\right] U_{n,J}(r) = 0, \quad (2)$$

where $U_{n,J}(r)$ is the radial wave function depending on the principal quantum number n and total angular momentum quantum number J and V_v^0 is the vector potential representing Hellmann potential of (1) in this study. The subscript "v" symbolizes vector while the superscript zero ("0") stands for the spin zero for the particle. $E_{n,J}$ is the energy levels of the spin-zero particle.

The effect of total angular momentum centrifugal term in (2) can be subdued using approximation scheme of the type [1–3, 14, 22, 27]

$$\frac{1}{r^2} \approx \frac{\rho^2}{\left(1 - e^{-\rho r}\right)^2}. \quad (3)$$

The above approximation has been reported to be valid for $\rho r \ll 1$ [14, 22, 27]. The approximate schemes to centrifugal terms have been applied by several authors in several important quantum problems. Also, its development is used in treating centrifugal terms by several authors in several important works; [see [28–32] and the references therein]. Substituting (1) and (3) into (2) and transform using mapping function $z = 1 - e^{-\rho r}$ lead to

$$\begin{aligned} z^2(1-z)^2 U''_{n,J}(z) - z^2(1-z) U'_{n,J}(z) \\ + \left[-\beta_1 z^2 + \beta_2 z - \beta_3\right] U_{n,J}(z) = 0, \end{aligned} \quad (4)$$

where we have employed the following parameters for simplicity:

$$\begin{aligned} -\beta_1 &= a\left(a - \frac{2E_{n,J}}{\rho}\right) + b\left(b - \frac{2E_{n,J}}{\rho}\right) - J(J+1) \\ &\quad - \frac{k^2}{\rho^2}, \end{aligned} \quad (5)$$

$$\beta_2 = -\frac{2E_{n,J}}{\rho}(a+b) + 2b(a-b), \quad (6)$$

$$-\beta_3 = J(J+1) - (a-b)^2, \quad (7)$$

and $k = \sqrt{(E_{n,J}^2 - m^2) + a^2\rho^2 - 2a\rho E_{n,J} - J(J+1)\rho^2}$ is the wave propagation constant.

Choosing the trial wave function of the type,

$$U_{n,J}(z) = z^{\gamma}(1-z)^{-i(k/\rho)} u_{n,J}(z), \quad (8)$$

and substituting it into (4), we obtain the hypergeometric type equation [33]

$$\begin{aligned} z(1-z) u''_{n,J}(z) + \left[2\gamma - \left(2\gamma - 2i\frac{k}{\rho} + 1\right) z\right] u'_{n,J}(z) \\ + \left[\left(\gamma - i\frac{k}{\rho}\right)^2 + \beta_1\right] u_{n,J}(z) = 0, \end{aligned} \quad (9)$$

where

$$\gamma = \frac{1}{2} + \sqrt{\left(J + \frac{1}{2}\right)^2 - (a-b)^2}, \quad (10)$$

$$\begin{aligned} \tau_1 &= \gamma - i\frac{k}{\rho} \\ &\quad - \sqrt{a\left(a - \frac{2E_{n,J}}{\rho}\right) + b\left(b - \frac{2E_{n,J}}{\rho}\right) - J(J+1) - \frac{k^2}{\rho^2}}, \end{aligned} \quad (11)$$

$$\begin{aligned} \tau_2 &= \gamma - i\frac{k}{\rho} \\ &\quad + \sqrt{a\left(a - \frac{2E_{n,J}}{\rho}\right) + b\left(b - \frac{2E_{n,J}}{\rho}\right) - J(J+1) - \frac{k^2}{\rho^2}}, \end{aligned} \quad (12)$$

$$\tau_3 = 2\gamma. \quad (13)$$

Therefore, the DKP radial wave functions for any arbitrary $J-$ states may be written as

$$\begin{aligned} U_{n,J}(r) \\ = N_{n,J}\left(1 - e^{-\rho r}\right)^{\gamma} e^{ikr} {}_2F_1\left(\tau_1, \tau_2, \tau_3; 1 - e^{-\rho r}\right), \end{aligned} \quad (14)$$

where $N_{n,J}$ is the normalization factor.

The phase shifts δ_J and normalization factor $N_{n,J}$ can be obtained by applying the analytic-continuation formula [33].

$$\begin{aligned} {}_2F_1(\tau_1, \tau_2, \tau_3; z) &= \frac{\Gamma(\tau_3)\Gamma(\tau_3 - \tau_1 - \tau_2)}{\Gamma(\tau_3 - \tau_1)\Gamma(\tau_3 - \tau_2)} \\ &\quad \cdot {}_2F_1(\tau_1; \tau_2; 1 + \tau_1 + \tau_2 - \tau_3; 1 - z) \\ &\quad + (1-z)^{\tau_3 - \tau_1 - \tau_2} \frac{\Gamma(\tau_3)\Gamma(\tau_1 + \tau_2 - \tau_3)}{\Gamma(\tau_1)\Gamma(\tau_2)} \\ &\quad \cdot {}_2F_1(\tau_3 - \tau_1; \tau_3 - \tau_2; \tau_3 - \tau_1 - \tau_2 + 1; 1 - z). \end{aligned} \quad (15)$$

Considering (15) with the property ${}_2F_1(\tau_1, \tau_2, \tau_3; 0) = 1$, when $r \longrightarrow \infty$, yields

$$ {}_2F_1\left(\tau_1,\tau_2,\tau_3;1-e^{-\rho r}\right)\ \overrightarrow{r\longrightarrow\infty}\ \Gamma\left(\tau_3\right)\left|\frac{\Gamma\left(\tau_3-\tau_1-\tau_2\right)}{\Gamma\left(\tau_3-\tau_1\right)\Gamma\left(\tau_3-\tau_2\right)}+e^{-2ikr}\left|\frac{\Gamma\left(\tau_3-\tau_1-\tau_2\right)}{\Gamma\left(\tau_3-\tau_1\right)\Gamma\left(\tau_3-\tau_2\right)}\right|^{*}\right|. \tag{16} $$

The following relations have been introduced in the process of derivation

$$ \tau_3-\tau_1-\tau_2=\left(\tau_1+\tau_2-\tau_3\right)^{*}=2i\left(\frac{k}{\rho}\right), \tag{17} $$

$$ \begin{aligned} \tau_3-\tau_2&=\gamma+i\frac{k}{\rho}\\ &\quad-\sqrt{a\left(a-\frac{2E_{n,J}}{\rho}\right)+b\left(b-\frac{2E_{n,J}}{\rho}\right)-J\left(J+1\right)-\frac{k^2}{\rho^2}}\\ &=\tau_1{}^{*}, \end{aligned} \tag{18} $$

$$ \begin{aligned} \tau_3-\tau_1&=\gamma+i\frac{k}{\rho}\\ &\quad+\sqrt{a\left(a-\frac{2E_{n,J}}{\rho}\right)+b\left(b-\frac{2E_{n,J}}{\rho}\right)-J\left(J+1\right)-\frac{k^2}{\rho^2}}\\ &=\tau_2{}^{*}. \end{aligned} \tag{19} $$

Now, defining a relation,

$$ \frac{\Gamma\left(\tau_3-\tau_1-\tau_2\right)}{\Gamma\left(\tau_3-\tau_1\right)\Gamma\left(\tau_3-\tau_2\right)}=\left|\frac{\Gamma\left(\tau_3-\tau_1-\tau_2\right)}{\Gamma\left(\tau_3-\tau_1\right)\Gamma\left(\tau_3-\tau_2\right)}\right|e^{i\delta}, \tag{20} $$

and inserting it into (16) yield

$$ {}_2F_1\left(\tau_1,\tau_2,\tau_3;1-e^{-\rho r}\right)\ \overrightarrow{r\longrightarrow\infty}\ \Gamma\left(\tau_3\right)\left[\frac{\Gamma\left(\tau_3-\tau_1-\tau_2\right)}{\Gamma\left(\tau_3-\tau_1\right)\Gamma\left(\tau_3-\tau_2\right)}\right]e^{-ikr}\left[e^{i(kr-\delta)}+e^{-i(kr-\delta)}\right] \tag{21} $$

Thus, we have the asymptotic form of (14) when $r\longrightarrow\infty$ as

$$ U_{n,J}\left(r\right)\ \overrightarrow{r\longrightarrow\infty}\ 2N_{n,J}\Gamma\left(\tau_3\right)\left[\frac{\Gamma\left(\tau_3-\tau_1-\tau_2\right)}{\Gamma\left(\tau_3-\tau_1\right)\Gamma\left(\tau_3-\tau_2\right)}\right]\sin\left(kr+\frac{\pi}{2}+\delta\right). \tag{22} $$

Accordingly, with the appropriate boundary condition imposed by [34], (22) yields

$$ U_{n,J}\left(\infty\right)\longrightarrow 2\sin\left(kr-\frac{l\pi}{2}+\delta_J\right). \tag{23} $$

Comparing (22) and (23), the DKP phase shift and the corresponding normalization factor can be found, respectively, as follows:

$$ \begin{aligned} \delta_J&=\frac{\pi}{2}\left(J+1\right)+\arg\Gamma\left(\tau_3-\tau_1-\tau_2\right)-\arg\Gamma\left(\tau_3-\tau_1\right)\\ &\quad-\arg\Gamma\left(\tau_3-\tau_2\right)\\ &=\frac{\pi}{2}\left(J+1\right)+\arg\Gamma\left(\frac{2ik}{\rho}\right)-\arg\Gamma\left(\tau_2{}^{*}\right)\\ &\quad-\arg\Gamma\left(\tau_1{}^{*}\right) \end{aligned} \tag{24} $$

and

$$ \begin{aligned} N_{n,J}&=\frac{1}{\sqrt{\tau_3}}\left|\frac{\Gamma\left(\tau_3-\tau_2\right)\Gamma\left(\tau_3-\tau_1\right)}{\Gamma\left(\tau_3-\tau_1-\tau_2\right)}\right|\\ &=\frac{1}{\sqrt{\tau_3}}\left|\frac{\Gamma\left(\tau_1{}^{*}\right)\Gamma\left(\tau_2{}^{*}\right)}{\Gamma\left(2i\left(k/\rho\right)\right)}\right|. \end{aligned} \tag{25} $$

The DKP total cross section for the sum of partial-wave cross sections σ_J is defined as [27]

$$ \sigma_{total}=\sum_{l=0}^{\infty}\sigma_J=\frac{\pi}{k^2}\sum_{l=0}^{\infty}\left(2l+1\right)T_J, \tag{26} $$

where $T_J=4\sin^2\delta_J$ defines the DKP partial-wave transitions.

A straightforward substitution of phase shift formula in (24) into (26) yields the total cross section

$$ \begin{aligned} \sigma_{total}&=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}\left(2l+1\right)\sin^2\left[\frac{\pi}{2}\left(J+1\right)+\arg\Gamma\left(\frac{2ik}{\rho}\right)\right.\\ &\quad\left.-\arg\Gamma\left(\tau_2{}^{*}\right)-\arg\Gamma\left(\tau_1{}^{*}\right)\right] \end{aligned} \tag{27} $$

Also, we need to analyze the gamma function $\Gamma(\tau_3-\tau_1)$ [34] from the S-matrix as

$$ \begin{aligned} \tau_3-\tau_1&=\gamma+i\frac{k}{\rho}\\ &\quad+\sqrt{a\left(a-\frac{2E_{n,J}}{\rho}\right)+b\left(b-\frac{2E_{n,J}}{\rho}\right)-J\left(J+1\right)-\frac{k^2}{\rho^2}}. \end{aligned} \tag{28} $$

The first-order poles of $\Gamma(\gamma+i(k/\rho)+\sqrt{a(a-2E_{n,J}/\rho)+b(b-2E_{n,J}/\rho)-J(J+1)-k^2/\rho^2})$ are situated at

$$\Gamma\left(\gamma+i\frac{k}{\rho}+\sqrt{a\left(a-\frac{2E_{n,J}}{\rho}\right)+b\left(b-\frac{2E_{n,J}}{\rho}\right)-J(J+1)-\frac{k^2}{\rho^2}}\right)+n=0 \quad (n=0,1,2,\ldots). \tag{29}$$

By applying algebraic means to (29), we obtain the DKP bound state energy levels equation for the Hellmann potential as follows:

$$k^2=-\rho^2\left[\frac{(n+\gamma)^2+a(2E_{n,J}/\rho-a)+b(2E_{n,J}/\rho-b)-J(J+1)}{2(n+\gamma)}\right]^2. \tag{30}$$

3. Scattering States Solutions of the Spinless Salpeter Equation (SSE) with Hellmann Potential

The spinless Salpeter equation for two different particles interacting in a spherically symmetric potential in the center of mass system is given by [see [35–38] and the references therein]

$$\left[\sum_{i=1,2}\left(\sqrt{-\triangle+m_i^2}-m_i\right)+\left(V(r)-E_{n,l}\right)\right]\chi(r)=0, \tag{31}$$

where $\chi(r)=R_{nl}(r)Y_{lm}(\theta,\varphi)$. Also, using appropriate transformation equation $R_{nl}(r)=\psi_{nl}(r)/r$, the radial component of SSE in the case of heavy interacting particles may be written as [see details in [35–38]]

$$\psi_{nl}''(r)+\left[-\frac{l(l+1)}{r^2}+2\mu\left(E_{n,l}-V(r)\right)+\left(\frac{\mu}{\eta}\right)^3\left(E_{n,l}-V(r)\right)^2\right]\psi_{nl}(r)=0, \tag{32}$$

where

$$\mu=\frac{m_1m_2}{(m_1+m_2)}, \tag{33}$$

$$\left(\frac{\eta}{\mu}\right)^3=\frac{m_1m_2}{(m_1m_2-3\mu^2)}. \tag{34}$$

The units $\hbar=c=1$ have been employed in the process of derivation and $E_{n,l}$ is the semirelativistic energy of the two particles having arbitrary masses m_1 and m_2. μ and η are the reduced mass and mass index, respectively. The solution to (32) becomes nonrelativistic as the term with the mass index tends to zero.

Substituting the potential in (1) and approximation in (3) into (32) and applying the same procedure in Section 2, the radial wave functions for the spinless Salpeter equation with Hellmann potential are obtained as follows:

$$\psi_{nl}(r)=N_{n,l}\left(1-e^{-\rho r}\right)^{\upsilon}e^{ikr}\,{}_2F_1\left(\xi_1,\xi_2,\xi_3;1-e^{-\rho r}\right), \tag{35}$$

having the following useful parameters:

$$k=\sqrt{2\mu\left(E_{n,l}+a\rho\right)+\left(\frac{\mu}{\eta}\right)^3\left(E_{n,l}+a\right)^2-l(l+1)\rho^2}, \tag{36}$$

$$\upsilon=\frac{1}{2}+\sqrt{\left(l+\frac{1}{2}\right)^2-\left(\frac{\mu}{\eta}\right)^3\left(\frac{a}{\rho}-b\right)^2}, \tag{37}$$

$$\xi_1=\upsilon-i\frac{k}{\rho}-\sqrt{\frac{2\mu a}{\rho}+\left(\frac{\mu}{\eta}\right)^3\left[\frac{2E_{n,l}}{\rho}\left(\frac{a}{\rho}-b\right)+\left(\frac{a}{\rho}-b\right)\left(\frac{a}{\rho}+b\right)\right]-l(l+1)-\frac{k^2}{\rho^2}}, \tag{38}$$

$$\xi_2=\upsilon-i\frac{k}{\rho}+\sqrt{\frac{2\mu a}{\rho}+\left(\frac{\mu}{\eta}\right)^3\left[\frac{2E_{n,l}}{\rho}\left(\frac{a}{\rho}-b\right)+\left(\frac{a}{\rho}-b\right)\left(\frac{a}{\rho}+b\right)\right]-l(l+1)-\frac{k^2}{\rho^2}}, \tag{39}$$

$$\xi_3=2\upsilon. \tag{40}$$

The corresponding phase shift for the spinless Salpeter equation containing Hellmann potential is obtained as

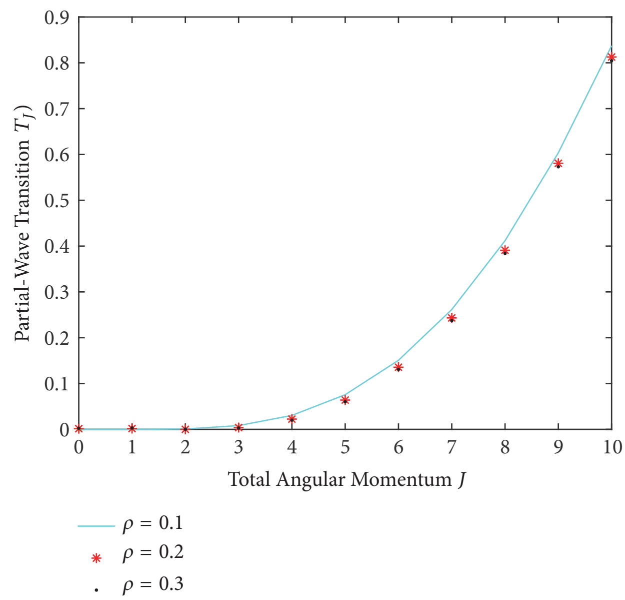

FIGURE 1: DKP partial-wave transition for the Hellman potential as a function of total angular momentum J with $a = b = 0.15$ and $E_{n,J} = m = 1$.

$$\delta_l = \frac{\pi}{2}(l+1) + \arg\Gamma\left(2i\left(\frac{k}{\rho}\right)\right) - \arg\Gamma\left(\xi_2{}^*\right) - \arg\Gamma\left(\xi_1{}^*\right), \tag{41}$$

$$\xi_1{}^* = \xi_3 - \xi_2 = v + i\frac{k}{\rho} - \sqrt{\frac{2\mu a}{\rho} + \left(\frac{\mu}{\eta}\right)^3\left[\frac{2E_{n,l}}{\rho}\left(\frac{a}{\rho} - b\right) + \left(\frac{a}{\rho} - b\right)\left(\frac{a}{\rho} + b\right)\right] - l(l+1) - \frac{k^2}{\rho^2}}, \tag{42}$$

$$\xi_2{}^* = \xi_3 - \xi_1 = v + i\frac{k}{\rho} + \sqrt{\frac{2\mu a}{\rho} + \left(\frac{\mu}{\eta}\right)^3\left[\frac{2E_{n,l}}{\rho}\left(\frac{a}{\rho} - b\right) + \left(\frac{a}{\rho} - b\right)\left(\frac{a}{\rho} + b\right)\right] - l(l+1) - \frac{k^2}{\rho^2}}. \tag{43}$$

and the normalization constant

$$N_{n,l} = \frac{1}{\Gamma(\xi_3)}\left|\frac{\Gamma(\xi_1{}^*)\,\Gamma(\xi_2{}^*)}{\Gamma(2i(k/\rho))}\right|. \tag{44}$$

The energy eigenvalue equation for the spinless Salpeter equation with Hellmann potential is

$$k^2 = -\rho^2\left[\frac{(n+v)^2 - 2\mu a/\rho + (\mu/\eta)^3\left(2bE_{n,l}/\rho - 2aE_{n,l}/\rho^2 - a^2/\rho^2 + b^2\right) + l(l+1)}{2(n+v)}\right]^2. \tag{45}$$

The total scattering cross section for the sum of partial-wave cross sections σ_l is given as

$$\sigma_{tot.} = \sum_{l=0}^{\infty}\sigma_l = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\,T_l, \tag{46}$$

where

$$T_l = 4\sin^2\delta_l \tag{47}$$

which defines the partial-wave transitions for the SSE with Hellmann potential in this present study.

4. Discussion

We have used the units $\hbar = c = 1$ in partial-wave transition illustrations. For equal mass cases, we used $(\mu/\eta)^3 = 1/4$ and $\mu = m_1/2$ while $(\mu/\eta)^3 = 1$ and $\mu = m_1/100$ were used for unequal masses cases. In all the cases, we consider $m_2 = E_{n,l} = 1$ and $m_1 = 1$ for the equal masses case only. For the screening parameters $\rho = 0.1$, $\rho = 0.2$, and $\rho = 0.3$, the DKP partial-waves transitions increase exponentially (see Figure 1). The two-body effect here appears as a shift of the phases of the partial waves. For lower partial-waves, say $l < 5$, the

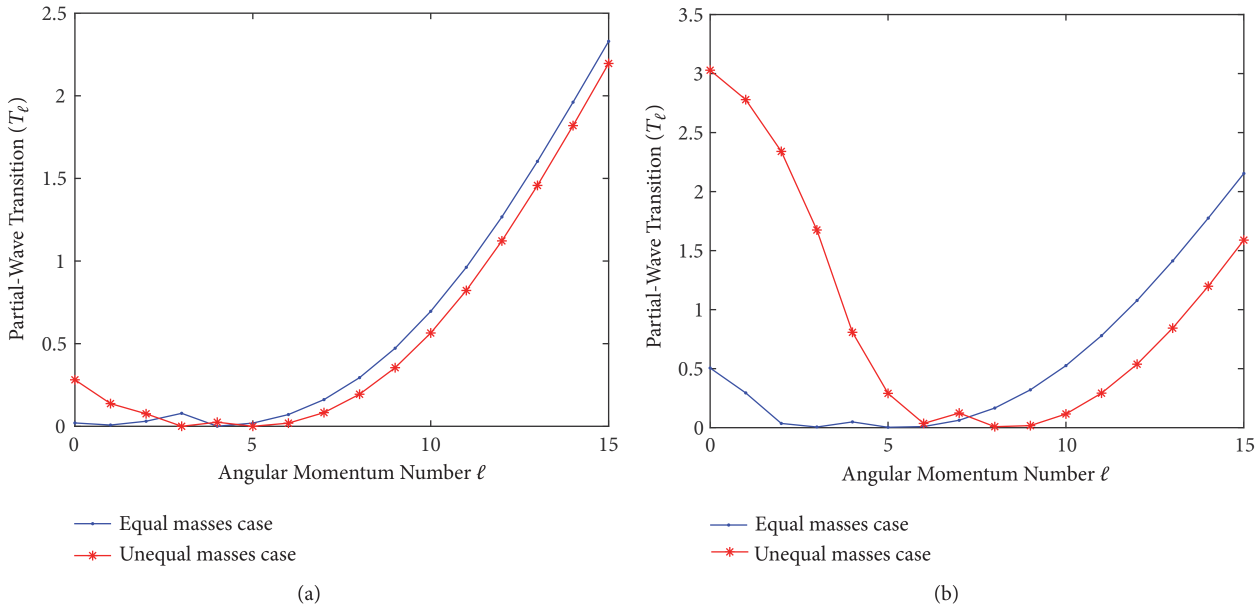

FIGURE 2: (a) Partial-wave transition for the spinless Salpeter equation with the Hellmann potential as a function of angular momentum quantum number l with $a = 0.2, b = -1, E_{n,l} = 1, \rho = 0.5$. (b) Partial-wave transition for the spinless Salpeter equation with the Hellmann potential as a function of angular momentum quantum number l with $a = 2, b = -1, E_{n,l} = 1, \rho = 0.5$.

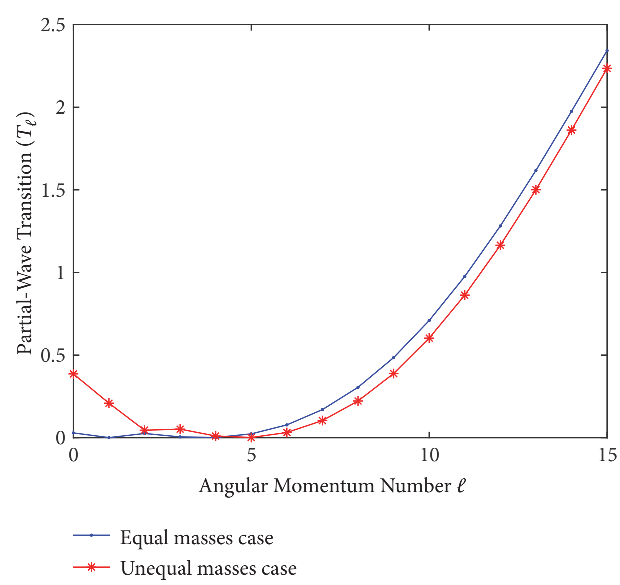

FIGURE 3: Partial-wave transition for the spinless Salpeter equation with the Hellmann potential as a function of angular momentum quantum number l with $a = 0, b = -3, E_{n,l} = 1, \rho = 0.5$.

partial-waves transition decays exponentially whereas, for higher partial waves, say $l > 5$, the partial-waves transition rises exponentially (see Figures 2–4). Also alteration of potential parameters has a serious effect on the partial-wave transition illustrations. Compare Figure 2(a) with Figures 2(b) and 3 and Figure 4(a) with Figure 4(b).

5. Conclusion

We have investigated the approximate scattering state solutions of DKPE and SSE with Hellman potential via analytical method. The approximate DKP and semirelativistic scattering phase shifts, partial-wave transitions, eigenvalues, and

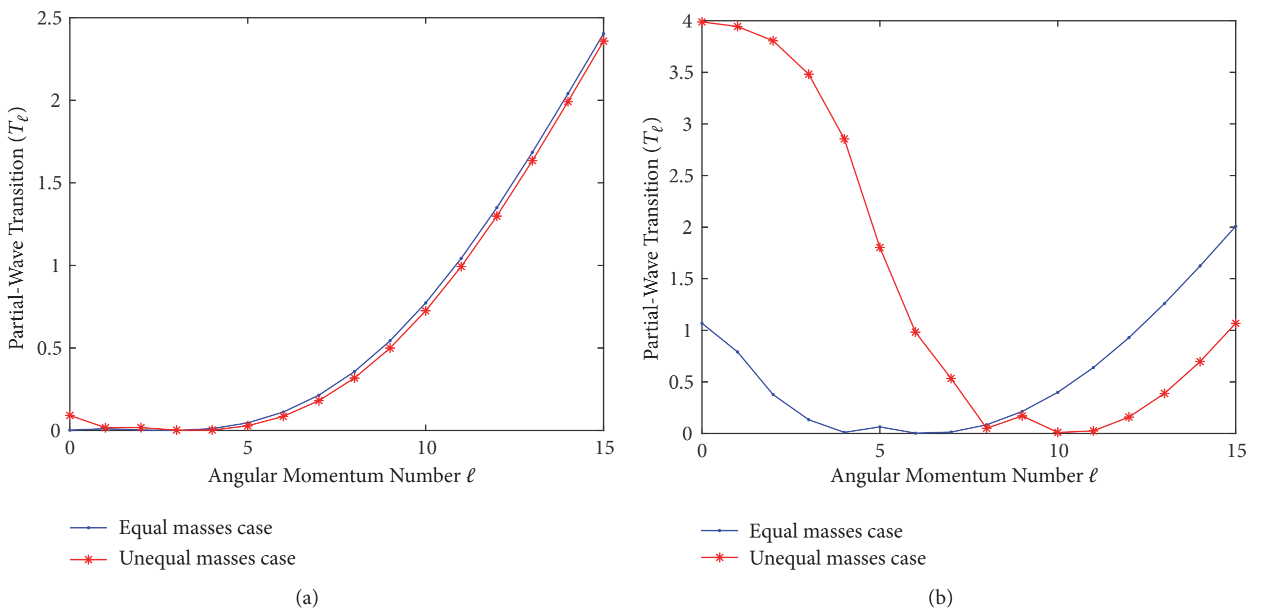

FIGURE 4: (a) Partial-wave transition for the spinless Salpeter equation with the Hellmann potential as a function of angular momentum quantum number l with $a = -2, b = 0, E_{n,l} = 1, \rho = 0.5$. (b) Partial-wave transition for the spinless Salpeter equation with the Hellmann potential as a function of angular momentum quantum number l with $a = 3, b = 0, E_{n,l} = 1, \rho = 0.5$.

normalized eigenfunctions have been obtained. The DKP and semirelativistic partial-wave transition calculations for the Hellmann potential were shown in Figures 1–4, respectively.

It is clearly shown that the total angular momentum number, angular momentum number, and potential parameters contribute significantly to the partial-wave transition and that the two-body effects modify the phases of the partial waves and are usually noticeable for lower partial waves.

Data Availability

The data used to support the findings of this study are included with the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding this paper.

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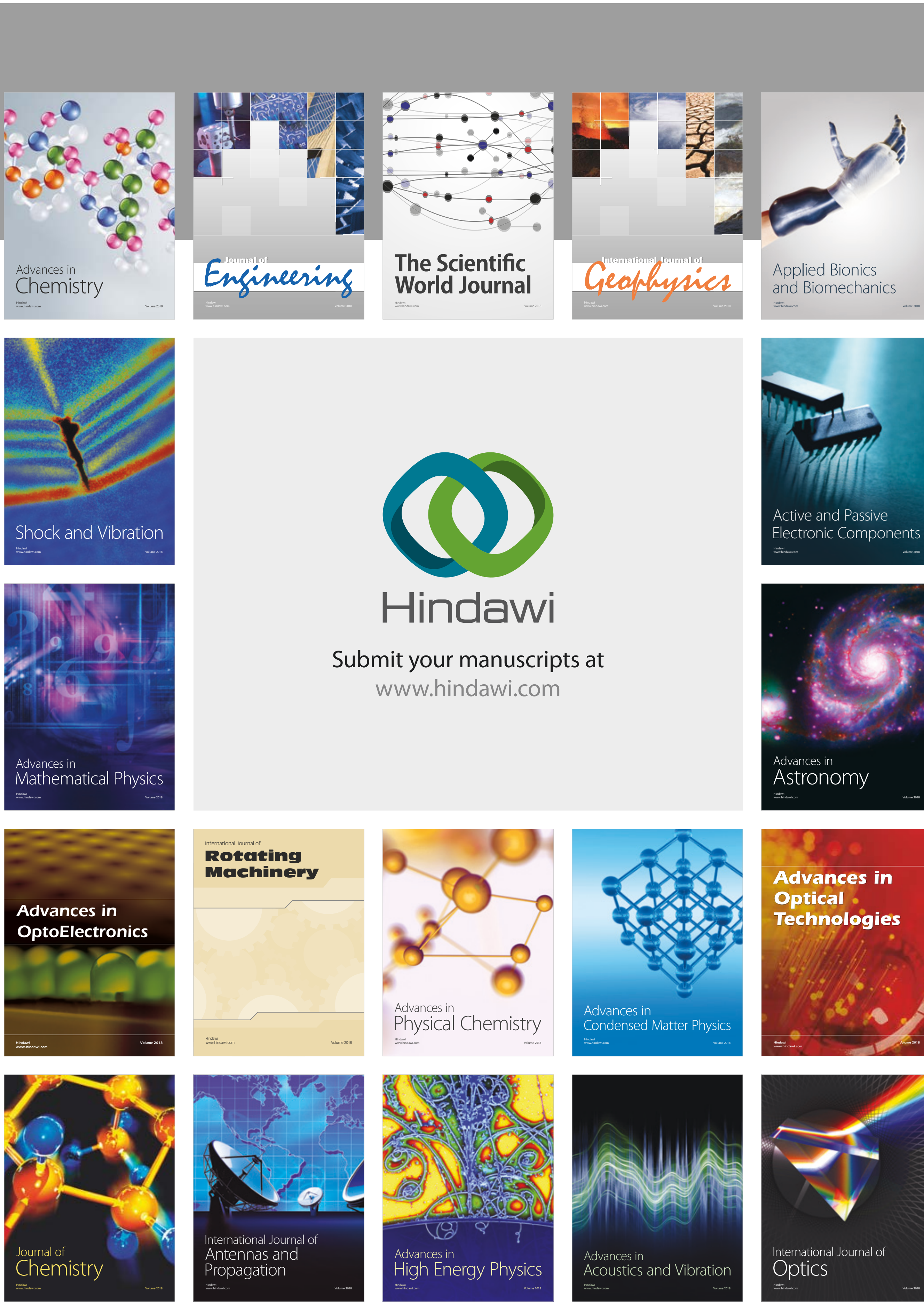

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