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Bounds on quark mixing, $M_{Z'}$ and $Z - Z'$ mixing angle from flavor changing neutral processes in a 3-3-1 model

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ABSTRACT

Meson and anti-meson mixing processes constitute an important source of constraints on models that give tree level contributions to flavor violating neutral processes. In electroweak $SU(3)_L \times U(1)_N$ models, where anomaly cancellation requires that one family of quarks transforms differently from the other ones, processes involving flavor changing neutral currents gain tree level contributions mediated by gauge and scalar fields. Here, we firstly investigate the contributions of the neutral scalar that mimics the standard Higgs to the $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ mixing processes and confront our predictions with experiments. The results will determine the quark mixing matrices $V_L^{u,d}$. In possession of this information we, next, evaluate the contributions of the neutral gauge bosons Z' and Z to the $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ processes. This realistic approach to meson transitions will provide severe bounds on $M_{Z'}$ and $Z - Z'$ mixing angle.

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1. Introduction

Flavor changing neutral processes (FCNP) arise as a natural outcome of the electroweak $SU(3)_L \times U(1)_N$ models [1–4] because anomaly cancellations require at least that one family of quark transforms differently from the others [3,5]. These models predict FCNP mediated by neutral gauge and scalar fields [6–9] and, for example, the meson mixing $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ processes provide an important source of constraint on the parameters of the models [6–18].

Flavor changing neutral processes mediated by Z' have received considerable attention within the $SU(3)_C \times SU(3)_L \times U(1)_N$ (3-3-1) models [6–8,10,15,19–29], but there are few works exploring these processes mediated by neutral scalars [26,29,30]. In the 3-3-1 model with right-handed neutrinos (331RHN) three neutral scalars and one pseudoscalar give contributions at tree level to meson mixing [26,29,30]. In order to address FCNP inside models with complex scalar sector, the first thing to do is to recognize, in the spectrum of the scalar of the model, the neutral scalar that plays the role of the standard Higgs. We then calculate the Higgs contributions to $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ mixing with the aim of determining the unitary matrices, $V_L^{u,d}$, that mix the standard quarks. This is important because with such matrices in hand

we can realistically probe the contributions of the 3-3-1 models to FCNP. This approach gives us the real power of FCNP in constraining the parameters of the 3-3-1 models.

With this in mind, we focus on the 331RHN and recognize the Higgs in the spectrum of scalar of the model and then obtain its Yukawa interactions with the standard physical quarks that lead to FCNP. Next we calculate the contributions of the Higgs to the mass differences associated with $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ transitions. We confront our predictions with experiments and, as result, we determine $V_L^{u,d}$. In possession of $V_L^{u,d}$, we calculate the contributions of Z' to $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ and derive bounds on $M_{Z'}$. Finally, we calculate the contribution of the standard neutral gauge boson to such processes and obtain upper bounds on the $Z - Z'$ mixing angle.

This work is organized as follows. In section 2 we diagonalize the mass matrix of the CP-even scalars, recognize the standard Higgs and obtain their Yukawa interactions. In section 3 we calculate the contributions of the Higgs, Z' and of the standard neutral gauge boson Z to the $\Delta m_{K,B,D}$ and obtain the bounds. In section 5 we summarize our results and present our conclusions.

2. Higgs-quarks Yukawa interactions

We restrict our investigation to the case where the first family of quarks transforms as triplet while the other two transform as anti-triplet. For the fermion representation and content we refer the reader to Ref. [31].

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In the 331RH the Yukawa interactions involving quarks and scalars are composed by the following terms

$$-\mathcal{L}_Y^Q \supset g_{1a} \bar{Q}_{1L} \rho d_{aR} + g_{ia} \bar{Q}_{iL} \eta^* d_{aR} + h_{1a} \bar{Q}_{1L} \eta u_{aR} + h_{ia} \bar{Q}_{iL} \rho^* u_{aR} + \text{H.c.}, \quad (1)$$

where Q_{1L} is the first family of quarks (triplet) and Q_{iL} with $i = 2, 3$ are the second and third ones (anti-triplets).¹

The triplet of scalars η and ρ , together with χ , form the original scalar content of the model. The scalar potential, the shift of the neutral components of the scalar fields that develop VEV and the set of constraint equations that guarantee the model develops a global minimum are found in Refs. [33,34].

We are interested exclusively in the CP-even scalars. More specifically we want to recognize the CP-even scalar that plays the role of the standard Higgs. Then, according to Ref. [33] and considering the basis $(R_{\chi'}, R_\eta, R_\rho)$, these scalars compose the following mass matrix,

$$M_R^2 = \begin{pmatrix} \lambda_1 v_{\chi'}^2 + f v_\eta v_\rho / 4 v_{\chi'} & \lambda_4 v_{\chi'} v_\eta / 2 - f v_\rho / 4 & \lambda_5 v_{\chi'} v_\rho / 2 - f v_\eta / 4 \\ \lambda_4 v_{\chi'} v_\eta / 2 - f v_\rho / 4 & \lambda_2 v_\eta^2 + f v_{\chi'} v_\rho / 4 v_\eta & \lambda_6 v_\eta v_\rho / 2 - f v_{\chi'} / 4 \\ \lambda_5 v_{\chi'} v_\rho / 2 - f v_\eta / 4 & \lambda_6 v_\eta v_\rho / 2 - f v_{\chi'} / 4 & \lambda_3 v_\rho^2 + f v_{\chi'} v_\eta / 4 v_\rho \end{pmatrix}. \quad (2)$$

It is quite impossible to diagonalize such matrix analytically. We then resort to the alignment limit characterized by the conditions, $\lambda_4 = \lambda_5 = \lambda$, $v_\eta = v_\rho = v$ and $f = 2\lambda v_{\chi'}$, where $v = 175$ GeV represents the standard VEV. With this approach M_R^2 get simplified

$$M_R^2 = \begin{pmatrix} \lambda_1 v_{\chi'}^2 + \frac{1}{2} \lambda v^2 & 0 & 0 \\ 0 & \lambda_2 v^2 + \lambda v_{\chi'}^2 / 2 & \lambda_6 v^2 / 2 - \frac{1}{2} \lambda v_{\chi'}^2 \\ 0 & \lambda_6 v^2 / 2 - \frac{1}{2} \lambda v_{\chi'}^2 & \lambda_3 v^2 + \lambda v_{\chi'}^2 / 2 \end{pmatrix}. \quad (3)$$

Diagonalizing analytically this matrix we obtain $H = R_{\chi'}$ whose mass is $m_H^2 \approx \lambda_1 v_{\chi'}^2 + \frac{1}{2} \lambda v^2$. The other eigenvalues are $m_{h_1}^2 \approx \frac{1}{2}(\lambda_2 + \lambda_3 + \lambda_6)v^2$, and $m_{h_2}^2 \approx m_{h_1}^2 - \lambda_6 v^2 + \lambda_1 v_{\chi'}^2$, where $h_1 = \frac{1}{\sqrt{2}}(R_\eta + R_\rho)$, and $h_2 = \frac{1}{\sqrt{2}}(R_\rho - R_\eta)$. We soon recognize that h_1 must play the role of the standard Higgs. The other two neutral scalars, H and h_2 , are heavy particles with their masses scaling with $v_{\chi'}$. In view of this, it is reasonable to assume that the main scalar contribution to the meson transitions is given by h_1 . We follow this approach and, then, obtain the Yukawa interactions among h_1 and the quarks.

Let us consider the standard up quarks. For the basis $u = (u_1, u_2, u_3)$ the Yukawa interactions above provide the following mass matrix

$$M_u = \frac{v}{\sqrt{2}} \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ -h_{21} & -h_{22} & -h_{23} \\ -h_{31} & -h_{32} & -h_{33} \end{pmatrix}, \quad (4)$$

diagonalizing this matrix by a bi-unitary transformation

$$V_L^{u\dagger} M_u V_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad (5)$$

we get the masses of the up quarks. The relation among the basis is given by

¹ For the most general set of Yukawa interactions allowed by the three triplet of scalars and the quarks, see Ref. [32]. All the other terms that appear there mix the standard quarks with the exotic ones. We avoid these terms because they violate lepton number explicitly.

$$\hat{u}_{L,R} = V_{L,R}^{u\dagger} u_{L,R}, \quad (6)$$

with $\hat{u} = (u, c, t)^T$.

For the down quarks we have the mass matrix

$$M_d = \frac{v}{\sqrt{2}} \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix}, \quad (7)$$

diagonalizing this matrix by a bi-unitary transformation

$$V_L^{d\dagger} M_d V_R^d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \quad (8)$$

we get the masses of the down quarks. The relation among the basis is given by

$$\hat{d}_{L,R} = V_{L,R}^{d\dagger} d_{L,R}, \quad (9)$$

with $\hat{d} = (d, s, b)^T$.

Note that $V_R^{d,u}$ act on the quark singlets u_R and d_R which means there is no constraint on them. Consequently, without any loss for the quark phenomenology, we may assume $V_R^{d,u} = I$. In this case Eqs. (4), (5), (7) and (8) give

$$g_{ia} = \sqrt{2}(V_L^d)_{ia} \frac{(m_{down})_a}{v}, \quad g_{3a} = \sqrt{2}(V_L^d)_{3a} \frac{(m_{down})_a}{v}, \\ h_{ia} = -\sqrt{2}(V_L^u)_{ia} \frac{(m_{up})_a}{v}, \quad h_{1a} = \sqrt{2}(V_L^u)_{3a} \frac{(m_{up})_a}{v}, \quad (10)$$

where $i = 2, 3$, $a = 1, 2, 3$, $(m_{down})_a = m_d, m_s, m_b$, and $(m_{up})_a = m_u, m_c, m_t$.

After all this, in the end of the day we obtain the following Yukawa interactions among h_1 and the physical standard quarks

$$\mathcal{L}_Y^{h_1} = \frac{h_1}{\sqrt{2}} \tilde{u}_{bL} \left((V_L^u)_{1a} (V_L^u)_{b1} \frac{(m_{up})_a}{v} + (V_L^u)_{ia} (V_L^u)_{bi} \frac{(m_{up})_a}{v} \right) \hat{u}_{aR} \\ + \frac{h_1}{\sqrt{2}} \tilde{d}_{bL} \left((V_L^d)_{1a} (V_L^d)_{b1} \frac{(m_{down})_a}{v} \right. \\ \left. + (V_L^d)_{ia} (V_L^d)_{bi} \frac{(m_{down})_a}{v} \right) \hat{d}_{aR} + \text{H.c.}, \quad (11)$$

where the subscripts $a, b = 1, 2, 3$ and $i = 2, 3$.

Observe that the interactions in Eq. (11) lead inevitably to processes that violate flavor mediated by h_1 . In this regard, particularly important are its contributions to meson-antimeson mixing $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$. We calculate such contributions with the aim of shedding light in the pattern of quark mixing by determining the elements of $V_L^{u,d}$.

3. Flavor changing neutral current bounds on the parameters of the model

3.1. Higgs contributions

As we can observe in the Eq. (11), the neutral meson-antimeson transitions $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ receive contributions from h_1 by means of the following effective interactions [35]

$$\mathcal{L}_K^{eff} = \frac{1}{2m_{h_1}^2} [\bar{d} (C_K^R P_R - C_K^L P_L) s]^2, \\ \mathcal{L}_B^{eff} = \frac{1}{2m_{h_1}^2} [\bar{b} (C_B^R P_R - C_B^L P_L) d]^2, \\ \mathcal{L}_D^{eff} = \frac{1}{2m_{h_1}^2} [\bar{u} (C_D^R P_R - C_D^L P_L) c]^2, \quad (12)$$

where the coefficients $C_{K,B,D}^{L,R}$ are given by

$$\begin{aligned} C_K^R &= \left[(V_L^d)_{12} (V_L^d)_{11} + (V_L^d)_{i2} (V_L^d)_{i1} \right] \frac{m_s}{v}, \\ C_K^L &= \left[(V_L^d)_{11}^* (V_L^d)_{12}^* + (V_L^d)_{i1}^* (V_L^d)_{i2}^* \right] \frac{m_d}{v}, \\ C_B^R &= \left[(V_L^d)_{11} (V_L^d)_{31} + (V_L^d)_{i1} (V_L^d)_{3i} \right] \frac{m_d}{v}, \\ C_B^L &= \left[(V_L^d)_{31}^* (V_L^d)_{11}^* + (V_L^d)_{3i}^* (V_L^d)_{i1}^* \right] \frac{m_b}{v}, \\ C_D^R &= [(V_L^u)_{12} (V_L^u)_{11} + (V_L^u)_{i2} (V_L^u)_{i1}] \frac{m_c}{v}, \\ C_D^L &= [(V_L^u)_{11}^* (V_L^u)_{12}^* + (V_L^u)_{i1}^* (V_L^u)_{i2}^*] \frac{m_u}{v}, \end{aligned} \quad (13)$$

with $i = 2, 3$.

According to these interactions the contributions of h_1 to Δm_K , Δm_B and Δm_D are given by the expressions [35]

$$\begin{aligned} \Delta m_K &= \frac{m_K B_K f_K^2}{m_{h_1}^2} \left[\frac{5}{24} \text{Re}[(C_K^L)^2] + (C_K^R)^2 \right] \left(\frac{m_K}{m_s + m_d} \right)^2 \\ &\quad + 2 \text{Re}[C_K^L C_K^R] \left(\frac{1}{24} + \frac{1}{4} \left(\frac{m_K}{m_s + m_d} \right)^2 \right), \end{aligned} \quad (14)$$

$$\begin{aligned} \Delta m_B &= \frac{m_B B_B f_B^2}{m_{h_1}^2} \left[\frac{5}{24} \text{Re}[(C_B^L)^2] + (C_B^R)^2 \right] \left(\frac{m_B}{m_d + m_b} \right)^2 \\ &\quad + 2 \text{Re}[C_B^L C_B^R] \left(\frac{1}{24} + \frac{1}{4} \left(\frac{m_B}{m_d + m_b} \right)^2 \right), \end{aligned} \quad (15)$$

$$\begin{aligned} \Delta m_D &= \frac{m_D B_D f_D^2}{m_{h_1}^2} \left[\frac{5}{24} \text{Re}[(C_D^L)^2] + (C_D^R)^2 \right] \left(\frac{m_D}{m_u + m_c} \right)^2 \\ &\quad + 2 \text{Re}[C_D^L C_D^R] \left(\frac{1}{24} + \frac{1}{4} \left(\frac{m_D}{m_u + m_c} \right)^2 \right), \end{aligned} \quad (16)$$

where $m_{K,B,D}$ are the masses of the mesons, $B_{K,B,D}$ are the bag parameters and $f_{K,B,D}$ the decay constants. Observe that the only free parameters involved in these expressions are the elements of the mixing matrix $(V_L^{u,d})_{ij}$ which means that these contributions, in conjunction with the unitary constraint on $V_L^{u,d}$ and the fact that $V_L^u V_L^{u\dagger} = V_{CKM}$, will provide the pattern of mixing among the quarks by determining the elements of $V_L^{u,d}$. As we are going to see, this is very important in determining the contributions of other neutral particles, as Z' , to these transitions.

3.2. Z' contribution

It is important to remember that, from the spectrum of 3-3-1 neutral particles that potentially contribute to meson transitions, we are going to assume that Z' gives the main contribution. That said we have that in the 331RHN the effective lagrangians that characterizes the contributions to Δm_K , Δm_B , and Δm_D mediated by Z' are given by [8,10,22]

$$\mathcal{L}_K^{eff(Z')} = \frac{4G_F c_W^4}{3\sqrt{2}(3-4s_W^2)} \frac{M_Z^2}{M_{Z'}^2} \left[(V_L^d)_{11}^* (V_L^d)_{12} \right]^2 |\bar{d}_L \gamma^\mu s_L|^2, \quad (17)$$

$$\mathcal{L}_B^{eff(Z')} = \frac{4G_F c_W^4}{3\sqrt{2}(3-4s_W^2)} \frac{M_Z^2}{M_{Z'}^2} \left[(V_L^d)_{11}^* (V_L^d)_{13} \right]^2 |\bar{d}_L \gamma^\mu b_L|^2, \quad (18)$$

$$\mathcal{L}_D^{eff(Z')} = \frac{4G_F c_W^4}{3\sqrt{2}(3-4s_W^2)} \frac{M_Z^2}{M_{Z'}^2} \left[(V_L^u)_{11}^* (V_L^u)_{12} \right]^2 |\bar{u}_L \gamma^\mu c_L|^2, \quad (19)$$

which leads to

$$(\Delta m_K)_{Z'} = \frac{4G_F c_W^4}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^d)_{11}^* (V_L^d)_{12} \right]^2 \frac{M_Z^2}{M_{Z'}^2} f_K^2 B_K m_K, \quad (20)$$

$$(\Delta m_B)_{Z'} = \frac{4G_F c_W^4}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^d)_{11}^* (V_L^d)_{13} \right]^2 \frac{M_Z^2}{M_{Z'}^2} f_B^2 B_B m_B, \quad (21)$$

$$(\Delta m_D)_{Z'} = \frac{4G_F c_W^4}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^u)_{11}^* (V_L^u)_{12} \right]^2 \frac{M_Z^2}{M_{Z'}^2} f_D^2 B_D m_D, \quad (22)$$

where G_F is the Fermi constant, M_Z is the mass of standard Z boson, $M_{Z'}$ is the mass of Z' and θ_W is the Weinberg angle. Except by $M_{Z'}$, the values of these parameters are found in Ref. [36]. Observe here that Eqs. (20), (21), (22) involve six free parameters, namely $(V_L^d)_{11}$, $(V_L^d)_{12}$, $(V_L^d)_{13}$, $(V_L^u)_{11}$, $(V_L^u)_{12}$ and $M_{Z'}$. Thus it is imperative to know the elements of $V_L^{u,d}$ to extract any bound on $M_{Z'}$. In previous work on this subject the elements of $V_L^{u,d}$ (the pattern of mixing) were postulated and then bounds on $M_{Z'}$ were obtained, see Refs. [15,26,30]. Here we do the following: we obtain the elements of $V_L^{u,d}$ from h_1 contributions to the meson transitions and use them in the expressions above to extract bound on $M_{Z'}$. Besides of being possible, we think that this approach is much more realistic than the previous one.

3.3. Z contribution

In 3-3-1 models Z mix with Z' to form Z_1 and Z_2 . Due to this mixing Z_1 starts to contribute to meson transitions, too. As we are going to see, such contributions are determined by the mixing angle ϕ . We, then, switch on the Z - Z' mixing, where now $Z_1 = Z \cos \phi - Z' \sin \phi$ play the role of the standard gauge boson. In this case Z_1 contribute to $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $D^0 - \bar{D}^0$ mass difference by means of the following effective lagrangian [10],

$$\mathcal{L}_K^{eff(Z)} = \frac{4G_F c_W^4 \sin^2(\phi)}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^d)_{11}^* (V_L^d)_{12} \right]^2 |\bar{d}_L \gamma^\mu s_L|^2, \quad (23)$$

$$\mathcal{L}_B^{eff(Z)} = \frac{4G_F c_W^4 \sin^2(\phi)}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^d)_{11}^* (V_L^d)_{13} \right]^2 |\bar{d}_L \gamma^\mu b_L|^2, \quad (24)$$

$$\mathcal{L}_D^{eff(Z)} = \frac{4G_F c_W^4 \sin^2(\phi)}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^u)_{11}^* (V_L^u)_{12} \right]^2 |\bar{u}_L \gamma^\mu c_L|^2, \quad (25)$$

which give the following contribution to Δm_K , Δm_B , and Δm_D

$$(\Delta m_K)_Z = \frac{4G_F c_W^4 \sin^2(\phi)}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^d)_{11}^* (V_L^d)_{12} \right]^2 f_K^2 B_K m_K, \quad (26)$$

$$(\Delta m_B)_Z = \frac{4G_F c_W^4 \sin^2(\phi)}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^d)_{11}^* (V_L^d)_{13} \right]^2 f_B^2 B_B m_B, \quad (27)$$

$$(\Delta m_D)_Z = \frac{4G_F c_W^4 \sin^2(\phi)}{3\sqrt{2}(3-4s_W^2)} \left[(V_L^u)_{11}^* (V_L^u)_{12} \right]^2 f_D^2 B_D m_D. \quad (28)$$

These contributions provide information exclusively on the mixing angle ϕ .

In summary, the Higgs contributions to the meson transitions cannot be neglected because they depend directly of $V_L^{u,d}$. As a

Table 1
Input parameters [36,43,44].

Input Parameters			
Quark masses	K^0	D^0	B^0
$m_u = 2.16$ MeV	$m_K = 497.611$ MeV	$m_D = 1864.84$ MeV	$m_B = 5279.66$ MeV
$m_c = 1.27$ GeV	$\sqrt{B_K} f_K = 131$ MeV	$\sqrt{B_D} f_D = 212$ MeV	$\sqrt{B_B} f_B = 210$ MeV
$m_s = 93.4$ MeV			
$m_b = 4.18$ GeV			
$m_d = 4.67$ MeV			

result, such contributions will provide information on $V_L^{u,d}$. In possession of this information, and assuming that the main contribution to the meson transitions from the spectrum of 3-3-1 particles is due to Z' , we extract bound on $m_{Z'}$. From the contribution of Z to the meson transitions we get information on the mixing angle ϕ . We do this analysis in the next section.

4. Numerical analysis

4.1. Higgs contribution

Observe that, according to the coefficients $C_{K,B,D}^{R,L}$ in Eq. (14), the only free parameters in the expressions of the mass differences above are the elements of $V_L^{u,d}$. Thus any bound on these mass differences fall exclusively on the elements of $V_L^{u,d}$. It is in this point that lies the importance of the contributions of the standard Higgs to these processes, namely assuming that such contributions recover the experimental errors, as consequence, we obtain a realistic proposal for the pattern of quark mixing $V_L^{u,d}$.²

In general, neglecting CP phases, unitarity on $V_L^{u,d}$ imposes these mixing matrices be parameterized by three angles each. Moreover they lead to the CKM matrix by means of the relation

$$V_L^u V_L^{d\dagger} = V_{\text{CKM}}, \quad (29)$$

which allows us to eliminate V_L^u in favor of V_L^d in the expression above

$$V_L^u = V_{\text{CKM}} V_L^d. \quad (30)$$

In this way we have that, once V_{CKM} is determined by experiments, in determining V_L^d we automatically obtain V_L^u .

Following the standard parametrization for a 3×3 unitary matrix we write V_L^d as [36,37]

$$V_L^d = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \quad (31)$$

where $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$. The parametrization of V_L^u in terms of s_{ij} and c_{ij} is obtained substituting (31) in (30). With $V_L^{u,d}$ parametrized by the angles θ_{ij} we substitute them in the expressions to the coefficients $C_{K,B,D}^{R,L}$ given above and plug such coefficients in the expressions to $\Delta m_{K,B,D}$. After all this we are ready to obtain constraints on these three angles from Eqs. (14), (15) and (16) by imposing that the Higgs contributions respect the experimental values of $\Delta m_{K,B,D}$. This is reasonable because we have a system involving three equations, namely (14), (15), (16) and three variables (θ_{12} , θ_{23} and θ_{13}). However each equation of the system

behaves likes $(\theta_{ij})^4$ which implies in many set of $(\theta_{13}, \theta_{23}, \theta_{12})$ as solution to the system.

Besides the loop standard model contributions (Δm_{SM}) to these mass differences present a good agreement with experiments [38–40], we have that Δm_{SM} still involves a considerable amount of uncertainty due to errors in QCD corrections [41,42]. We then find reasonable to assume that all the contributions from new physics fall inside the experimental error. This is even more justifiable inside the 3-3-1 model because the model itself provides new contributions at loop level to the mass differences that will increase the uncertainties.

Our input parameters are shown in Table 1, and V_{CKM} is given in the PDG [36]

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97435 & 0.22500 & 0.00369 \\ 0.22486 & 0.97349 & 0.04182 \\ 0.00857 & 0.04110 & 0.999118 \end{pmatrix}. \quad (32)$$

The current experimental value for the mass difference are [45–48]

$$\begin{aligned} \Delta m_K &= (3.484 \pm 0.006) \times 10^{-12} \text{ MeV}, \\ \Delta m_D &= (6.25316_{-2.8962}^{+2.69873}) \times 10^{-12} \text{ MeV}, \\ \Delta m_B &= (3.334 \pm 0.013) \times 10^{-10} \text{ MeV}. \end{aligned} \quad (33)$$

The experimental errors of the mass differences that we use to constrain new physics is,

$$\begin{aligned} \Delta m_K &= 0.006 \times 10^{-12} \text{ MeV}, \\ \Delta m_D &= 2.69 \times 10^{-12} \text{ MeV}, \\ \Delta m_B &= 0.013 \times 10^{-10} \text{ MeV}. \end{aligned} \quad (34)$$

That said, the idea here is to demand that any new physics contribution to these mass differences falls inside these errors. The new contributions we are considering here are due to h_1 , Z and Z' . Then the sum of such contributions must fill the error. In this point of the work we have to make another assumption regarding the amount of contribution of each mediator. We consider two scenarios. Without strong reason, in one scenario we assume that h_1 get in charge of 10% of the error while in the second scenario h_1 get in charge of 80%.

For the case of the Higgs, whose interactions and contributions are presented above, after solving numerically the set of Eqs. (14), (16) and (15) by demanding that the Higgs provides a contribution to the mass differences exactly equal to the 80%, and 10% of the errors in Eq. (34), we obtain the following proposal of solutions to the mixing angles:

For the scenario where h_1 contributes to for 80% of the error, we get as solution,

$$\begin{aligned} \theta_{12} &= 1.580313, \\ \theta_{23} &= 1.59713, \\ \theta_{13} &= -1.845500. \end{aligned} \quad (35)$$

² Of course that we have to have in mind that this is possible based in a certain set of reasonable assumptions as, for example, that the right-handed quarks come in a diagonal basis.

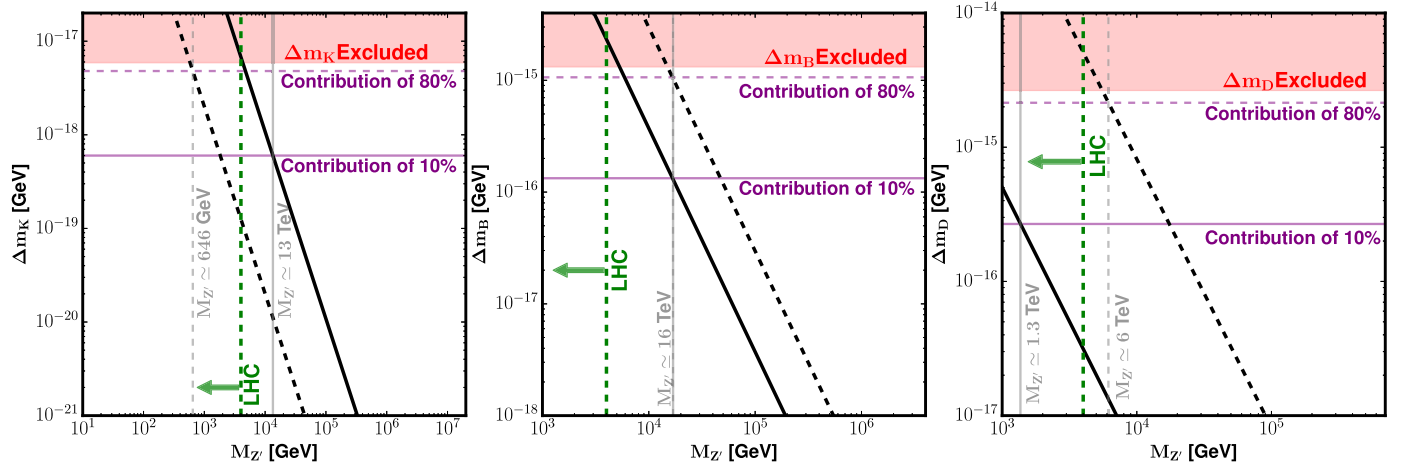


Fig. 1. Evolution of Δm_K , Δm_B , and Δm_D in function of $M_{Z'}$. The excluded red region represents the error of $\Delta m_{K,B,D}$. The continuous (dashed) black lines represent the contribution of Z' for 80% (10%) of the error of difference of meson masses. The continuous (dashed) horizontal purple lines represent the contribution of 10% (80%) of $\Delta m_{K,B,D}$.

These angles imply in the following pattern of quark mixing,

$$V_L^d = \begin{pmatrix} 0.0025814 & -0.271249 & -0.96250 \\ 0.0171729 & 0.962379 & -0.271168 \\ 0.999849 & -0.015829 & 0.00714249 \end{pmatrix}, \quad (36)$$

and by means of $V_L^u = V_{CKM} V_L^d$ we obtain,

$$V_L^u = \begin{pmatrix} 0.010068 & -0.0478151 & -0.998804 \\ 0.0591118 & 0.875211 & -0.480109 \\ 0.999695 & 0.0214141 & -0.0122575 \end{pmatrix} \quad (37)$$

For the scenario where h_1 contributes to 10% of the error, we get as solution,

$$\begin{aligned} \theta_{12} &= 2.047993, \\ \theta_{23} &= 0.978181, \\ \theta_{13} &= 1.555508. \end{aligned} \quad (38)$$

These angles imply in the following pattern of quark mixing,

$$V_L^d = \begin{pmatrix} -0.0070215 & 0.0135799 & 0.999883 \\ -0.115207 & -0.993261 & 0.0126809 \\ 0.993317 & -0.115105 & 0.00853869 \end{pmatrix}, \quad (39)$$

and

$$V_L^u = \begin{pmatrix} -0.0290977 & -0.210677 & 0.977121 \\ -0.0721913 & -0.968689 & 0.237536 \\ 0.987645 & -0.15571 & 0.0176213 \end{pmatrix}. \quad (40)$$

These results are interesting because they offer concrete patterns for the mixtures of the quarks. This allows us to probe concretely the scale of 3-3-1 physics by means of the Z' contribution to such mass differences. We do this in the next section.

Before this, let us check if such patterns of mixing are in agreement with ATLAS bounds on flavor changing neutral process involving the Higgs. We refer to the ATLAS experimental upper limit on the top quark decays $t \rightarrow h_1 c$ and $t \rightarrow h_1 u$ provided by the Yukawa interactions,

$$\mathcal{L}_{h_1}^Y \supset \lambda_{tch_1} \bar{t} h_1 c + \lambda_{tuh_1} \bar{t} h_1 u + \text{H.C.} \quad (41)$$

The ATLAS bounds on these top decays are found in Ref. [49] and impose $\lambda_{tch_1} < 0.13$ and $\lambda_{tuh_1} < 0.13$. In this case we have to verify if the Yukawa interactions in Eq. (11) respect such bounds.

We have that for the two scenarios discussed above when h_1 contributes with 80% of the error and when contributes with 10% of the error we get $\lambda_{tch_1} \sim \mathcal{O}(10^{-4})$ and $\lambda_{tuh_1} \sim \mathcal{O}(10^{-9})$, respectively.

4.2. Z and Z' contributions

Having in hand all this set of information we are ready to obtain the contributions of Z and Z' to $\Delta m_{K,B,D}$ in both scenarios. In the first case, where we assumed that h_1 contributes with 80% of the error, we are going to assume that Z' contributes with 10% of the error, while in the second scenario, where we assumed that h_1 contributes with 10% of the error, we are going to assume that Z' get in charge of 80% of the error. The remaining 10%, in the both scenarios, we assume is filled with the contribution of Z .

In Fig. 1 we show our numerical results for the contributions of Z' to $\Delta m_{K,B,D}$ in both cases. The excluded red region represents the error of $\Delta m_{K,B,D}$ as shown in Eq. (34). The first panel shows the Z' contribution for Δm_K while the second and third panels represent its contributions for Δm_B and Δm_D , respectively. The continuous and dashed black lines represent the evolution of $\Delta m_{K,B,D}$ in function of $M_{Z'}$ as required by Eqs. (20), (21), and (22).

In the three panels of Fig. 1, the continuous black line represents the first case (Z' contributing with 10% of error). The continuous purple horizontal line indicates the value of 10% of $\Delta m_{K,B,D}$. The vertical green line represents the lower limit imposed by LHC [50]. In practical terms this means we are using the pattern quark mixing given in Eqs. (36), and (37). As we can see in these pictures, the three transitions are demanding a heavy Z' . Observe that Δm_B put the strongest bound on $M_{Z'}$. As we can see in the second panel, for Z' contribution constituting 10% of the error of Δm_B transition it demands $M_{Z'} \approx 16$ TeV. This implies that the 3-3-1 model must belong to a scale energy around $v_{\chi'} \approx 40$ TeV.

Concerning the second case, represented by the dashed black lines, where Z' is supposed to contribute with 80% of the error, the strongest bound on $M_{Z'}$ is also given by Δm_B demanding the same $M_{Z'} \approx 16$ TeV since the vertical dashed gray line in the second panel is upon the continuous. In these pictures the dashed purple horizontal lines indicate the value of 80% of $\Delta m_{K,B,D}$ which means the dashed curves represent the pattern of mixing giving in Eqs. (39), and (40).

In general we observe that, in both cases, with Z' contributing with 10% or 80% of the error, the bounds on $M_{Z'}$ constitute the most severe bound on the 331RHN existent in the literature. We stress that this is due to the fact that we are using a pattern

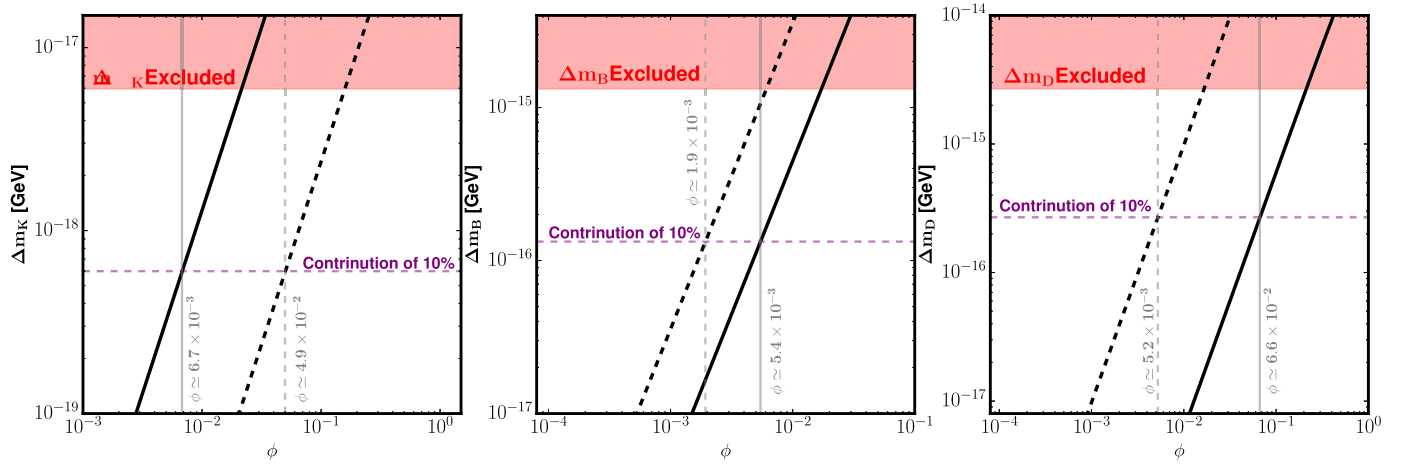


Fig. 2. Evolution of Eq. (26), Eq. (27), and Eq. (28) in function of ϕ (continuous black curves).

of quark mixing allowed by the Higgs contributions to such processes. In Appendix A we display another set of possible patterns for $V_L^{u,d}$ allowed by the Higgs contributions and their respective higher bound on $M_{Z'}$ and ϕ from one of the mass difference $\Delta m_{K,B,D}$. Observe also in these figures that such bound on $M_{Z'}$ is not alleviated by varying the percent of the contribution of Z' to such transitions.

The results of the contributions of Z for the remain 10% of error are shown in Fig. 2, where the first panel represents the contribution of Z to Δm_K in function of ϕ , and the second and third panels represent its contributions for Δm_B , and Δm_D , respectively. The excluded red region represents the error of $\Delta m_{K,B,D}$ as shown in Eq. (34). The purple horizontal dashed lines indicate the value of 10% of $\Delta m_{K,B,D}$, as we required. The black curves represent the evolution of Eq. (26), Eq. (27), and Eq. (28) in function of ϕ . The continuous black curves means Higgs contributing with 80% of the error, and the dashed black curves means Higgs contributing with 10% of error. The contributions of Z to these meson transitions translate exclusively in bounds on the $Z - Z'$ angle mixing. As we can see in these pictures, the most stringent bounds are put by Δm_K and Δm_B where both require $\phi \approx 6 \times 10^{-3}$. Such bounds are close to the previous ones found in the literature, see Refs. [31,51].

We are aware that the bounds on $M_{Z'}$ may change by taking $V_R^{u,d} \neq I$ since, in this case, we may find space to handle $V_L^{u,d}$ to a pattern that could result in alleviation of the bound put here on $M_{Z'}$.³

5. Conclusions

In this work we considered the scalar sector of the 331RHN and obtained the mass matrix of the CP-even scalars. Applying the alignment limit we diagonalized it with the main aim of recognizing the scalar that plays the role of the Higgs, h_1 . Next, we obtained its Yukawa interactions with quarks in a scenario where the first family of quarks transforms as triplet. As expected, these interactions lead to flavor changing neutral processes. We then calculated the contributions of h_1 to the mass differences involved in meson-antimeson transitions. We assumed that any new contribution to the meson transitions cannot be larger than the experimental errors. Our investigation were divided in two scenarios. In the one scenario h_1 get in charge of 80% of the error and in the other scenario it gets in charge of 10%. We stress that the unique variant parameters in the h_1 contributions are the quark mixing matri-

ces $V_L^{u,d}$. The Higgs contributions serve solely to fix the elements of these mixing matrices. This is a very important result because with $V_L^{u,d}$ in hand we may calculate the concrete contributions of the other neutral particles as Z' and Z to the meson transitions.

Throughout the paper we assumed that h_1 , Z and Z' give the dominant contributions to the error of the meson transitions which means the sum of their contribution must practically fill the entire error. We then considered that Z always get in charge of 10% of the error. This means the sum of the contributions of Z' and h_1 must sum 90% of the error. Whatever is the contribution of Z' to the error, it will provide severe constrain on the mass of Z' to values around 16 Tevs. The contribution of 10% of Z to the error translate in bounds on $Z - Z'$ mixing that agree with previous ones.

Our study indicates that FCNC is an important source of constraint on the parameters of the 331RHN. We remark that the parameter space for new physics concerning meson mixing arises from the discrepancy between experimental and theoretical (standard model) predictions but such discrepancy is highly non-perturbative which leads to considerable uncertainty. In view of our results, reducing this uncertainty is very important to the status of the 3-3-1 models.

The results displayed in Fig. 1 and Fig. 2 and in the appendix allow us to conclude that FCNC in the form of meson transitions $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ may be a very efficient way to constrain new physics as is the case of the 331RHN. However, we stress that the weakness behind such analysis is the fact we do not know yet the pattern of quark mixing matrices $V_L^{u,d}$. Determine such mixing from other sources of constraints, as Higgs and quarks violating flavor decays, will be decisive to the efficiency of meson transitions in constraining new physics.

Declaration of competing interest

Vinicius Oliveira reports financial support was provided by CAPES. C. A. de S. Pires reports financial support was provided by CAPES.

Data availability

No data was used for the research described in the article.

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³ Work in progress.

Appendix A

Higgs Contribution of 80%				
θ_{12}	θ_{23}	θ_{13}	$M_{Z'}$ limit	ϕ limit
1.6818	-1.5212	1.612	31 TeV	2×10^{-3}
-3.907867	2.2188	-1.58868	87 TeV	10^{-3}
2.562462	2.402145	-1.586674	90 TeV	10^{-3}
3.128632	-1.86437	0.002682	248 TeV	3.6×10^{-4}
Higgs Contribution of 10%				
θ_{12}	θ_{23}	θ_{13}	$M_{Z'}$ limit	ϕ limit
1.737666	1.6224	4.6562	22 TeV	4×10^{-3}
-1.138012	-1.280638	-1.551463	19 TeV	4×10^{-3}
-1.025063	1.224614	1.525326	56 TeV	1.6×10^{-2}
-0.000419	1.041301	-0.003637	107 TeV	8×10^{-4}

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