



Holographic paramagnetism–ferromagnetism phase transition in the Born–Infeld electrostatics



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ARTICLE INFO

Article history:

Received 2 June 2016

Received in revised form 4 July 2016

Accepted 11 July 2016

Available online 15 July 2016

Editor: N. Lambert

Keywords:

AdS/CFT correspondence

Holographic ferromagnetism

Born–Infeld electrostatics

ABSTRACT

In the probe limit, we investigate the effects of the Born–Infeld electrostatics on the holographic paramagnetism–ferromagnetism phase transition in the background of a Schwarzschild–AdS black hole spacetime. We find that the presence of Born–Infeld scale parameter b decreases the critical temperature and makes the magnetic moment harder to form in the case of without external field. Furthermore, the increase of b will result in extending the period of the external magnetic field.

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1. Introduction

For the past few years, as one of the most significant developments in fundamental physics, the AdS/CFT duality [1–4], which provides an exact correspondence between a weakly coupled gravity theory in $(d + 1)$ -dimensional anti-de Sitter (AdS) space and a strongly coupled conformal field theory (CFT) living on d dimensions, has been extensively applied in order to study various phenomena in condensed matter physics, including high-Tc superconductivity [5–11]. Recently, some efforts have been made to generalize the correspondence to magnetism. The authors of Ref. [12] introduced the holographic paramagnetism–ferromagnetism model in a dyonic Reissner–Nordström–AdS black brane and observed that the properties of a $(2 + 1)$ -dimensional ferromagnetism can indeed be realized in this simple model. Since then, a large number of the holographic dual models have been constructed and some interesting behaviors have been found, for reviews, see Refs. [13–20] and references therein.

All of the above mentioned models are carried out in the framework of usual Maxwell electrostatics. Along with the conventional Maxwell electrodynamic theory, nonlinear electrodynamic theories, which correspond to the higher derivative corrections to the Abelian gauge fields, have also become interesting topics of research in the past several decades. And it turned out that the

higher derivative corrections of the gauge field carry more plentiful information than the usual Maxwell electrostatics [21–26]. One of the most popular nonlinear electromagnetic theories is Born–Infeld electrostatics [27] whose the primary motivation is to avoid the infinite self energies for point-like charged particles. Jing and Chen introduced holographic dual model in Born–Infeld electrostatics at first and observed that the nonlinear Born–Infeld corrections make it harder for the scalar condensation to form [28], decrease the critical temperature, and change the condensation gap. Moreover, the dependence of the condensation gap and the critical temperature on the Born–Infeld scale parameter is similar to that on the Gauss–Bonnet term (corresponds to the higher derivative correction to gravitation) in the holographic superconductor. Along this direction, there has been accumulated interest to study various holographic dual models with the nonlinear electrostatics [29–40]. At the same time, similar to the case of Born–Infeld nonlinear electrostatics, other types of nonlinear electrostatics in the context of gravitational field have been introduced, which can also remove the divergence arising in Maxwell theory at the origin. Two well known nonlinear Lagrangians for electrostatics are logarithmic (LEN) [41,42] and exponential (ENE) [43,44] Lagrangian. And considering three types of typical nonlinear electrostatics, the authors of Ref. [45] observed that the exponential form of nonlinear electrostatics has stronger effect on the condensation formation and conductivity for the holographic conductors in the backgrounds of AdS black hole. However, in the AdS soliton background the critical chemical potentials are independent of the explicit form of the nonlinear elec-

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rodynamics, i.e., the ENE, BINE and LNE correction do not effect on the critical potentials. In this paper, therefore, it is interesting to try to investigate the effects of the nonlinear electrodynamics on the holographic paramagnetism–ferromagnetism phase transition, particularly, in the presence of Born–Infeld correction.

This paper is organized as follows. In section 2, we introduce the basic field equations of holographic ferromagnetism model with Born–Infeld electrodynamics in the Schwarzschild–AdS black hole. In section 3 by the numerical method we obtain the critical temperature and study the magnetic moment. Magnetic susceptibility density and hysteresis loop will be shown in section 4. Finally in the last section we will include our summary and discussion.

2. Holographic model

In this paper, the model we are considering is just general relativity with a negative cosmological constant $\Lambda = -3/L^2$, a U(1) field A_μ and a massive 2-form field $M_{\mu\nu}$ in 4-dimension spacetime. The ghost free action

$$S = \frac{1}{2\kappa^2} \int d^4 \sqrt{-g} (L_1 + \lambda^2 L_2), \quad (1)$$

$$L_1 = R + 6/L^2 + L_{BI},$$

$$L_2 = \frac{1}{12} (dM)^2 + \frac{m^2}{4} M_{\mu\nu} M^{\mu\nu} + \frac{1}{2} M^{\mu\nu} F_{\mu\nu} + \frac{J}{8} V(M),$$

where dM is the exterior differential of 2-form field $M_{\mu\nu}$, m^2 is the squared mass of 2-form field $M_{\mu\nu}$ being greater than zero (see Ref. [16] for detail), λ and J are two real model parameters with $J < 0$ for producing the spontaneous magnetization, λ^2 characterizes the backreaction of the 2-form field $M_{\mu\nu}$ to the background geometry and to the Maxwell field strength, and $V(M)$ is a nonlinear potential of the 2-form field describing the self-interaction of the polarization tensor. For simplicity, we take the form of $V(M)$ as follows

$$V(M) = (*M_{\mu\nu} M^{\mu\nu})^2 = [(M \wedge M)]^2, \quad (2)$$

where $*$ is the Hodge-star operator. As shown in Ref. [16], this potential shows a global minimum at some nonzero value of ρ . Meanwhile, L_{BI} is Lagrangian density of the Born–Infeld electrodynamics

$$L_{BI} = \frac{1}{b} (1 - \sqrt{1 + 2bF}). \quad (3)$$

Here, $F \equiv F_{\mu\nu} F^{\mu\nu}$ and $F_{\mu\nu}$ is the nonlinear electromagnetic tensor. The scale parameter b indicates the difference between Born–Infeld and Maxwell electrodynamics. As b tends to zero, the Lagrangian L_{BI} approaches to $F_{\mu\nu} F^{\mu\nu}$.

By varying action (1), we can get the equations of motion for matter fields

$$\nabla^\tau (dM)_{\tau\mu\nu} - m^2 M_{\mu\nu} - J (*M_{\tau\sigma} M^{\tau\sigma}) (*M_{\mu\nu}) = F_{\mu\nu},$$

$$\nabla^\mu \left(\frac{F_{\mu\nu}}{\sqrt{1 + 2bF}} + \frac{\lambda^2}{4} M_{\mu\nu} \right) = 0. \quad (4)$$

In what follows, we will work in the probe limit and the background is a 4-dimensional planner Schwarzschild–AdS black hole

$$ds^2 = L^2 (-r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 (dx^2 + dy^2)), \quad (5)$$

with

$$f(r) = 1 - \frac{r_+^3}{r^3}, \quad (6)$$

where the r_+ is the event horizon of the black hole and the Hawking temperature is

$$T = \frac{3r_+}{4\pi}. \quad (7)$$

In order to study systematically the effects of the b on the holographic ferromagnetic phase transition, we take the following self-consistent ansatz with matter fields,

$$M_{\mu\nu} = -p(r) dt \wedge dr + \rho(r) dx \wedge dy,$$

$$A_\mu = \phi(r) dt + B x dy, \quad (8)$$

where B is a constant magnetic field viewed as the external magnetic field in the boundary field theory. Thus nontrivial equations of motion read

$$\rho'' + \frac{f'}{f} \rho' - \frac{1}{r^2 f} [m^2 + 4Jp^2] \rho + \frac{B}{r^2 f} = 0,$$

$$(m^2 - \frac{4J\rho^2}{r^4}) p - \phi' = 0, \quad (9)$$

$$\phi'' + \frac{1}{16B^2 b r + 4r^5} (16B^2 b + 8r^4 - 32br^4 \phi'^2) \phi'$$

$$- \frac{r^3 \lambda^2 (2p + p'r)}{16B^2 b + 4r^4} (1 + \frac{4B^2 b}{r^4} - 4b\phi'^2)^{3/2} = 0,$$

here a prime denotes the derivative with respect to r . Obviously, the equation of motion for gauge field $\phi(r)$ is more complicated than that in usual Maxwell theory due to the presence of BI correction. In order to solve the nonlinear equations (9) numerically, we should first solve the equation of ϕ' and put it into the equation of ϕ'' and get the equation of p . And then we need to seek the boundary condition for ρ , ϕ and p near the black hole horizon $r \rightarrow r_+$ and at the spatial infinite $r \rightarrow \infty$. The regularity condition for $\rho(r_+)$ at the horizon gives the boundary condition $\phi(r_+) = 0$. Near the boundary $r \rightarrow \infty$, the nonlinear equations give the following asymptotic solution for matter fields

$$\rho = \rho_+ r^{\Delta_+} + \rho_- r^{\Delta_-} + \dots + \frac{B}{m^2},$$

$$\phi = \mu - \frac{\sigma}{r} + \dots, \quad p = \frac{\sigma}{m^2 r^2} + \dots, \quad (10)$$

where ρ_\pm , μ and σ are all constants and $\Delta_\pm = \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4m^2}$. The Breitenlohner–Freedman (BF) bound requires $m^2 > -1/4$. From the dual field theory, the constants μ and σ can be interpreted as the chemical potential and charge density, respectively. The coefficient ρ_+ and ρ_- correspond to the source and vacuum expectation value of dual operator in the boundary field theory when $B = 0$. Therefore one should set $\rho_+ = 0$ since one wants the condensation to happen spontaneously below a critical temperature. When $B \neq 0$, the asymptotic behavior is governed by external magnetic field B .

3. Spontaneous magnetization

In this paper we work in the grand canonical ensemble where the chemical potential μ will be fixed. And the expression of magnetic moment as

$$N = -\lambda^2 \int \frac{\rho}{2r^2} dr. \quad (11)$$

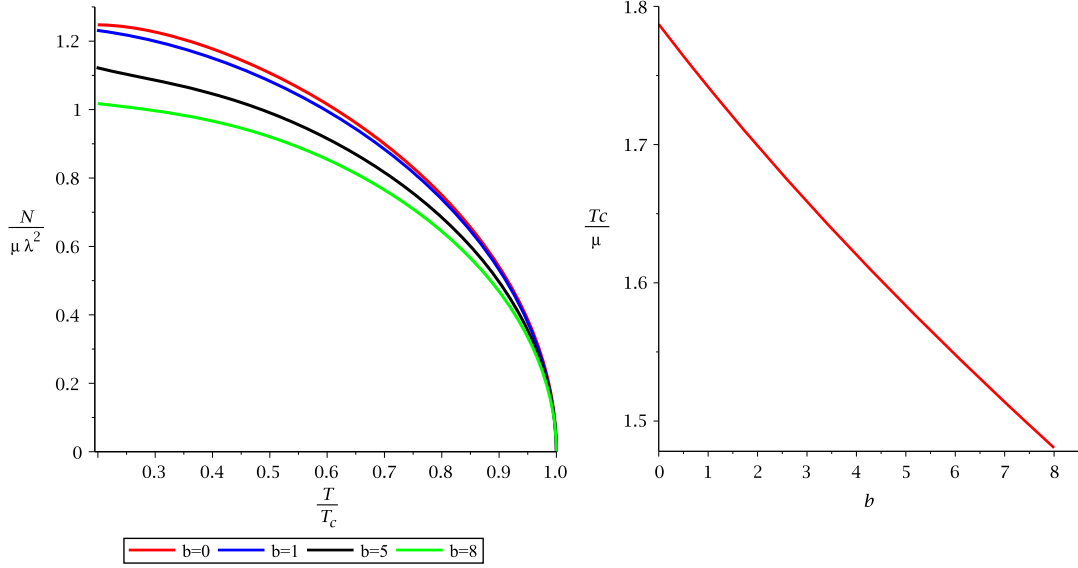


Fig. 1. Left: the magnetic moment N as a function of temperature with the Born-Infeld scale parameter b . Right: the relation between the critical temperature T_c and parameter b .

Table 1
The critical temperature T_c and magnetic moment N with different values of b .

b	0	1	5	8
T_c/μ	1.7870	1.7417	1.5834	1.4806
$N/\lambda^2\mu$	$2.9409(1 - T/T_c)^{1/2}$	$2.8514(1 - T/T_c)^{1/2}$	$2.4835(1 - T/T_c)^{1/2}$	$2.2130(1 - T/T_c)^{1/2}$

As a typical example, we take $J = -1/8$, $m^2 = 1/8$ and $\lambda = 1/2$ which can capture the basic features of the model. Changing the Born-Infeld scale parameter b , we present in the left panel of Fig. 1 the magnetic moment as a function of temperature in $d = 4$ dimension. It will be found that the spontaneous condensate of ρ (corresponding to the magnetic moment) in the bulk in the absence of external magnetic field appears and has similar behavior for different b when the temperature is lower than critical temperature T_c . Meanwhile, by fitting this curve in the vicinity of critical temperature, we find that the phase transition is a second order one with behavior $N \propto \sqrt{1 - T/T_c}$ for all cases calculated above. The results are still consistent with one in the mean field theory and have been shown in Table 1. In other words, there exists the holographic paramagnetism-ferromagnetism phase transition when the mass 2-form field couples a Born-Infeld electromagnetic field in the Schwarzschild-AdS black hole.

From the left panel of Fig. 1, we observe that the increasing value of the nonlinearity parameter b makes the magnetic moment smaller. It means that the condensation is more harder to be formed in the Born-Infeld electrodynamics, which agrees well with the results given in [25,38]. In Table 1 we present the critical temperature T_c for the magnetic moment and list the behavior of these condensation curves near $T \sim T_c$. It is easy to find that as b increases the critical temperature decreases, which is exhibited in the right panel of Fig. 1 and agrees well with the finding in the left panel of Fig. 1. This behavior has been seen for the holographic superconductor in the background of a Schwarzschild-AdS black hole, where the Born-Infeld electrodynamics make scalar condensation harder to form [28]. At the same time, the dependence of the magnetic moment and the critical temperature on the Born-Infeld scale parameter is similar to that on the Gauss-Bonnet term in the holographic superconductor.

4. The response to the external magnetic field

Let us turn on the external field to examine the response to magnetic moment N . This can be described by magnetic susceptibility density χ , defined as

$$\chi = \lim_{B \rightarrow 0} \frac{\partial N}{\partial B}. \tag{12}$$

In the high temperature region $T > T_c$, the ferromagnetic material is in a paramagnetic phase whose magnetic moments are randomly distributed. So the susceptibility obeys the Curie-Weiss law

$$\chi = \frac{C}{T + \theta}, T > T_c, \theta < 0, \tag{13}$$

where C and θ are two constants. Note that a significant difference between the antiferromagnetism and paramagnetism can be seen from the magnetic susceptibility. In the paramagnetic phase of antiferromagnetic material and paramagnetic material, the magnetic susceptibility also obeys the Curie-Weiss law, but the constant θ in Eq. (13) is positive and zero, respectively. Fig. 2 shows the magnetic susceptibility as a function of temperature. In the paramagnetic phase, we observe that the magnetic susceptibility increases when the temperature is lowered for the fixed nonlinearly parameter b . Moreover, the magnetic susceptibility satisfies the Curie-Weiss law of the ferromagnetism near the critical temperature whether $b = 0$ or not. Concretely, the results have been presented in Table 2 for the chosen model parameters. It is easy to see that coefficient in front of $\frac{T}{T_c}$ for $\frac{1}{\chi}$ increases with the increasing b , which meets well with the discovery in Fig. 2. However, the absolute value of $\frac{\theta}{\mu}$ will decrease when the Born-Infeld scale parameter b increases. In the plot of Fig. 3, we show that the magnetic moment with respect to external field B in region of

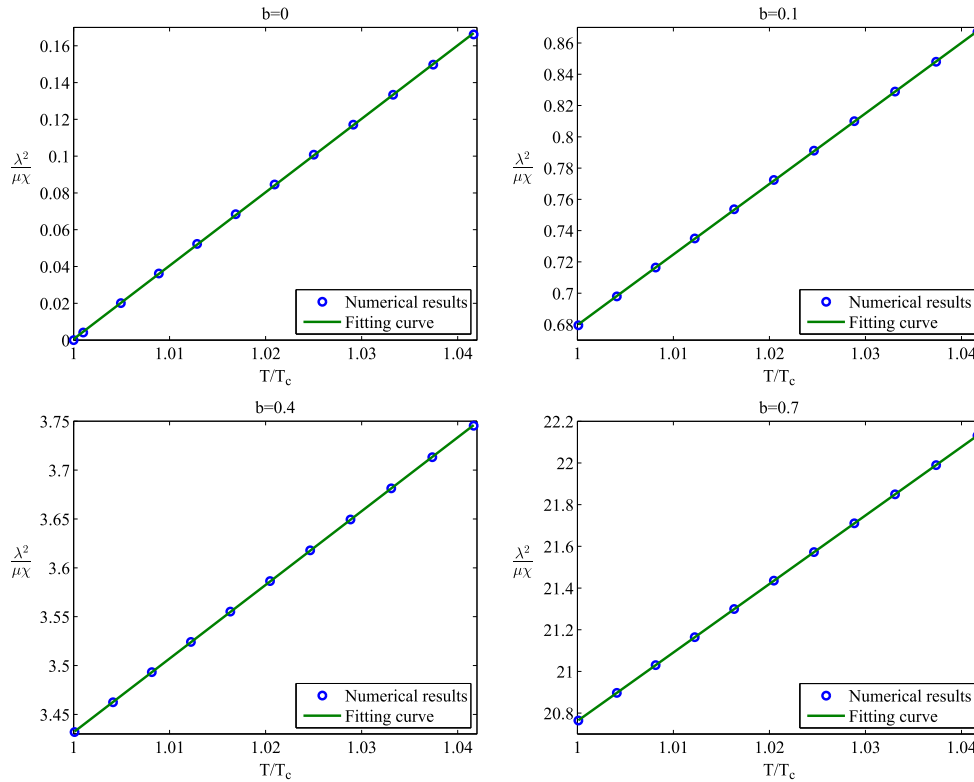


Fig. 2. Magnetic susceptibility as a function of temperature with different b .

Table 2
The magnetic susceptibility χ with different values of b .

b	0	0.1	0.4	0.7
$\lambda^2/\chi\mu$	$3.9906(T/T_c - 1)$	$4.5130(T/T_c - 0.85)$	$7.5416(T/T_c - 0.54)$	$32.8633(T/T_c - 0.37)$
θ/μ	-1.7871	-1.5150	-0.9550	-0.6494

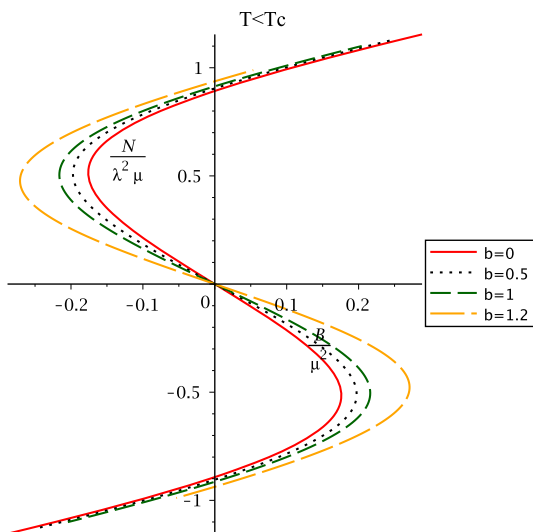


Fig. 3. The magnetic moment with respect to external magnetic field B in lower temperature.

$T < T_c$ (i.e., $T = 0.89T_c$) with different parameter b . And from each line in Fig. 3, we see that the magnetic moment is not single valued when the external magnetic field continuously changes between $-B_{max}$ and B_{max} periodically. Thus a hysteresis loop in the single magnetic domain will be obtained and the Born–Infeld scale

parameter b has an effect on it quantitatively. Along the horizontal direction (the magnetic moment has been taken a same value), one need a larger external field as the Born–Infeld scale parameter b increases. In other words, the Born–Infeld electrodynamics makes the periodicity of hysteresis loop bigger which is different from the effect of Lifshitz dynamical exponent z on it. However, all the curves will overlap once the value of the magnetic field exceeds the maximum that corresponding to the case of $b = 1.2$, which can be seen from Fig. 3.

5. Summary and discussion

In this letter we have studied holographic paramagnetism–ferromagnetism phase transition in the presence of Born–Infeld correction to Maxwell electrodynamics in 4-dimensional Schwarzschild–AdS black hole spacetime, and obtained the effect of the Born–Infeld scale parameter b on the holographic paramagnetism–ferromagnetism phase transition. Adopting the probe limit, we have found that the improving of the nonlinear parameter b results in the decrease of critical temperature T_c and magnetic moment in the case without external field. It means that the phase transition becomes difficult, and the magnetic moment is harder to form. This behavior is similar to that seen for the holographic superconductor in the background of a Schwarzschild–AdS black hole, where the Born–Infeld electrodynamics makes scalar condensation harder to form. In the vicinity of the critical point, however, the behavior of the magnetic moment is always as $(1 - T/T_c)^{1/2}$, regardless of the values of b , which is

in agreement with the result from mean field theory. Moreover, in the presence of the external magnetic field, the inverse magnetic susceptibility as $T \sim T_c$ behaves as $C/(T + \theta)$, ($\theta < 0$) in all cases, which satisfies the Curie–Weiss law. Yet both the constant C and the absolute value of θ decrease with the increasing Born–Infeld scale parameter b . Furthermore, we have observed the hysteresis loop in the single magnetic domain when the external field continuously changes between the maximum and minimum values periodically with b . The increase of the nonlinear parameter b could result in extending the period of the external magnetic field. Note that in this paper we just investigate the influence of Born–Infeld correction on paramagnetism/ferromagnetism phase transition. It would be of interest to generalize our study to other nonlinear electrodynamic theories. Work in this direction will be reported in the future.

Acknowledgements

We would like to thank Prof. R.G. Cai and Dr. R.Q. Yang for their helpful discussions and comments. This work is supported by the National Natural Science Foundation of China (Grant Nos. 11175077, 11575075), and the Joint Fund Project of Science and Technology Department in Guizhou Province (No. Qian Ke He LH Zi[2015]7709).

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