Published for SISSA by Springer

Received: May 14, 2021
Accepted: June 9, 2021
Published: June 25, 2021

# Bootstrapping the minimal $\mathcal{N} = 1$ superconformal field theory in three dimensions

**Junchen Rong**[a] **and Ning Su**[b]

[a] *DESY Hamburg, Theory Group, Notkestraße 85, D-22607 Hamburg, Germany*
[b] *Department of Physics, University of Pisa, Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy*

*E-mail:* junchen.rong@desy.de, suning1985@gmail.com

Abstract: We develop the numerical bootstrap technique to study the $2+1$ dimensional $\mathcal{N} = 1$ superconformal field theories (SCFTs). When applied to the minimal $\mathcal{N} = 1$ SCFT, it allows us to determine its critical exponents to high precision. This model was argued in [1] to describe a quantum critical point (QCP) at the boundary of a $3+1$D topological superconductor. More interestingly, this QCP can be reached by tuning a single parameter, where supersymmetry (SUSY) is realized as an emergent symmetry. We show that the emergent SUSY condition also plays an essential role in bootstrapping this SCFT. But performing a "two-sided" Padé re-summation of the large $N$ expansion series, we calculate the critical exponents for Gross-Neveu-Yukawa models at $N$=4 and $N$=8.

Keywords: Conformal Field Theory, Nonperturbative Effects

ArXiv ePrint: 1807.04434



https://doi.org/10.1007/JHEP06(2021)154

JHEP06(2021)154

## Contents



## 1 Introduction and summary

Conformal field theory (CFT) has applications in a wide range of fields in physics, which includes condensed matter physics, statistical mechanics, and string theory. The conformal bootstrap program was initiated in [2, 3]. When applied to the special case of two space-time dimensions, thanks to the Virasoro symmetry, the minimal models can be solved exactly [4]. Since the seminal work of [5], the modern numerical bootstrap has become one of the most powerful tools to study higher dimensional (D>2) conformal field theories. In certain cases, the numerical bootstrap results are the world's most precise predictions of the scaling exponents [6–10]. See [11] for a recent review including many other successful applications.

Supersymmetry is a symmetry between bosons and fermions [12–14]. Even though it was first proposed to solve the long-standing hierarchy problem in particle physics, it has also recently attracted a lot of attention in condensed matter physics due to the possibility of experimental realization [1, 15–21]. Typically in these models supersymmetry is realized as an emergent symmetry at the second-order phase transition point. At such a critical point, the symmetry is described by the so-called superconformal field theory (SCFT), where the usual conformal group is enlarged into the superconformal group. In this work, we develop the numerical bootstrap technique to study the $2+1$ dimensional $\mathcal{N}=1$ superconformal field theories. This allows us to determine the critical exponents of various models.

In this work, we study the minimal superconformal field theory in 2+1 dimensions. We show another example where numerical bootstrap allows us to determine the scaling exponents to unprecedented precision. The Lagrangian of the model is

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\sigma)^2+\bar{\psi}\gamma_\mu\partial^\mu\psi+\frac{\lambda_1}{2}\sigma\bar{\psi}\psi+\frac{\lambda_2^2}{8}\sigma^4. \tag{1.1}$$

Here $\psi$ is a Majorana spinor in three dimensions. This model is sometimes referred as the $\mathcal{N}$=1 Ising model. The theory is invariant under time reversal symmetry (T-parity) under which $\sigma \to -\sigma$ and $\psi \to \gamma^0 \psi$. When $\lambda_1 = \lambda_2$, the model has $\mathcal{N} = 1$ supersymmetry (SUSY) and the Lagrangian can be rewritten into a Wess-Zumino model with superpotential

$$\mathcal{W} = \lambda \Sigma^3. \tag{1.2}$$

Here

$$\Sigma = \sigma + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta\epsilon \tag{1.3}$$

is a real superfield. When $\lambda_1 \neq \lambda_2$, SUSY is broken. However, it is expected that the theory would still flow to the supersymmetric fixed point, and SUSY is realized as an emergent symmetry. It was argued in [1] that this fixed point might be realized as a quantum critical point at the boundary of a $3+1$D topological superconductor. Emergent supersymmetry means that the critical point can be reached by fine-tuning a single physical parameter, which is crucial for experimental realization. This condition is equivalent to saying that the spectrum of the SCFT contains only one relevant scalar operator that is even under time-reversal parity (T-parity). The operator corresponds to the mass term $m^2\sigma^2$ that can be added to the Lagragian (1.1), we need to tune its coefficicent to zero to reach the critical point. We find that when bootstrapping the $\mathcal{N}$=1 Ising model, the physical condition of emergent supersymmetry plays an important role.

To bootstrap this theory, one needs to consider four point correctors $\langle\sigma\sigma\sigma\sigma\rangle$, $\langle\epsilon\epsilon\epsilon\epsilon\rangle$ and $\langle\sigma\sigma\epsilon\epsilon\rangle$, with $\epsilon$ being the superconformal descendant of $\sigma$ ($\epsilon$ is a conformal primary). The operator product expansion (OPE) coefficients in $\sigma\times\sigma$, $\sigma\times\epsilon$ and $\epsilon\times\epsilon$ are related to each other, and the relation is fixed by superysmmetry. This is a generalization of the "long multiplet bootstrap" idea [22] used in two-dimensional superconformal bootstrap. Notice that unlike SCFTs with a higher number of supersymmetry, $\mathcal{N}$=1 SCFTs have no R symmetry and the scaling dimension of $\Phi$ can not be determined exactly by analytic methods. Imposing the emergent supersymmetry condition, we can determine $\Delta_\sigma$ to high precision, providing both an upper and lower bound for its value. Our numerical bootstrap work can be viewed as a numerical "proof" of the emergent supersymmetry of the model (1.1). Furthermore, if we assume that there are only two relevant operators that are T-parity odd in the spectrum, the allow region for $(\Delta_\sigma, \Delta_{\sigma'})$ becomes an isolated island. This helps us determine the critical exponents $\eta_\sigma, \eta_\psi, 1/\nu$ and $\omega$ all together. We also calculate the value of $C_T$, which appears in the two-point function for stress-energy tensor.

The model (1.1) belongs to a family of models called the Gross-Neveu models, whose Lagrangian is given by a four fermion interaction [23], $\mathcal{L} = \sum_{i=1}^N \bar{\psi}_i\gamma_\mu\partial^\mu\psi_i + g(\sum_{i=1}^N \bar{\psi}_i\psi_i)^2$. In D>2, the four fermion interaction is non-renormalizable. The action

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 + \sum_{i=1}^N \bar{\psi}_i\gamma_\mu\partial^\mu\psi_i + \frac{\lambda_1}{2}\sigma\sum_{i=1}^N \bar{\psi}_i\psi_i + \frac{\lambda_2^2}{8}\sigma^4, \tag{1.4}$$

is the UV completion of the Gross-Neveu models in 2<D<4 [24, 25], which are sometimes referred as the Gross-Neveu-Yukawa (GNY) model or the chiral Ising model. Here $N$ counts the number of Majorana fermions. To determine the scaling exponents of the GNY model have been the has been a longtime goal of many theoretical work, either based on $\epsilon$-expansion [26–31], large N method [32–35]. The $N = 4$ and $N = 8$ models are of special interest since they could also be studied using Monte Carlo simulation [27, 36–39]. Also, the $N = 8$ GNY model is known to describe the quantum critical point of the semimetal to charge density wave order transition on graphene [40]. It is a nice surprise that we can use the modern numerical bootstrap method to determine the critical exponents of $N = 1$ special case to high precision. This also allows us to perform a two-sided Padé re-sum the large $N$ critical exponents of (1.4) calculated in [33–35], and get the results for the $N = 4$ and $N = 8$ GNY models. The result is summarised in table 1 and compared with other methods.

## 2 Bootstrap the minimal $\mathcal{N} = 1$ SCFT

Conformal multiplets in $\mathcal{N}$=1 superconformal field theories group themselves into super-multiplets. There are in total four types of multiplets, which we denote as $\mathcal{B}^l_+$, $\mathcal{B}^l_-$, $\mathcal{F}^j_+$ and $\mathcal{F}^j_-$ as in [41]. "$\mathcal{B}/\mathcal{F}$" tells us whether the super-primary field is bosonic or fermionic. A generic super-multiplet contains four conformal primaries, suppose the superconformal primary has spin $l$ and scaling dimension $\Delta_0$, there are two level-1 (super)descendant with $\Delta = \Delta_0 + 1/2$ and spin $l \pm 1$. There is also a level-2 descendant with $\Delta = \Delta_0 + 1$ and spin $l$. The subscritp $+/-$ denote the parity of a bosonic field within the super-multiplet. The super-multiplet $\mathcal{B}^{(l)}_{+/-}$ contain the following component operators (with $l$=integer)

$$[l]^{+/-}_{\Delta} \xrightarrow{Q} [l \pm 1/2]_{\Delta+1/2} \xrightarrow{Q} [l]^{-/+}_{\Delta+1}, \tag{2.1}$$

and the super-multiplet $\mathcal{F}^{(j)}_{+/-}$ contains the following component operators (with $j$=half integer)

$$[j]_{\Delta} \xrightarrow{Q} \begin{matrix} [j-1/2]^{+/-}_{\Delta+1/2} \\ [j+1/2]^{-/+}_{\Delta+1/2} \end{matrix} \xrightarrow{Q} [j+1]_{\Delta+1}. \tag{2.2}$$

Here we denote a conformal primary field with spin $l$, scaling dimension $\Delta$ and even parity to be $[l]^+_{\Delta}$. In our convention, the super-field $\Sigma$, define in (1.3), is a $\mathcal{B}^{(l=0)}_-$ multiplet.

The four point function of the superfield $\Sigma$,

$$\langle \Sigma(x_1, \theta_1)\Sigma(x_2, \theta_2)\Sigma(x_3, \theta_3)\Sigma(x_4, \theta_4)\rangle, \tag{2.3}$$

when expanded in the Grassmann variable $\theta$, contains the four point functions involving not only the superconformal $\sigma$, but also super-descendants $\psi$ and $\epsilon$. For simplicity, in this work, we consider only bosonic fields $\sigma$ and $\epsilon$. In [6], the critical exponents of the three-dimensional Ising model is determined to high precision by studying the set of crossing equation involving the set of four point functions $\{\langle\sigma\sigma\sigma\sigma\rangle, \langle\sigma\sigma\epsilon\epsilon\rangle, \langle\epsilon\epsilon\epsilon\epsilon\rangle\}$. The crossing equations turn out to be

$$\sum_{O^+} \begin{pmatrix} \lambda_{\sigma\sigma O} & \lambda_{\epsilon\epsilon O} \end{pmatrix} \vec{V}_{+,\Delta,\ell} \begin{pmatrix} \lambda_{\sigma\sigma O} \\ \lambda_{\epsilon\epsilon O} \end{pmatrix} + \sum_{O^-} \lambda^2_{\sigma\epsilon O} \vec{V}_{-,\Delta,\ell} = 0, \tag{2.4}$$

The vectors $\vec{V}_{\pm}$ were calculated in [6]. For the reader's convenience, we will note their explicit expressions in appendix A. In previous attempts to determine the critical exponents of (1.1), either based on bootstrapping three-dimensional scalar operators [42], or fermions [43], the only SUSY constrain taken into account was the relation between the scaling dimension of operators, $\Delta_\epsilon = \Delta_\sigma + 1$ and $\Delta_\psi = \Delta_\sigma + 1/2$, since they belong to the same supermultiplet $\Sigma$. Here we however show that OPE relations are much more powerful, therefore are essential in bootstrapping $\mathcal{N}$=1 SCFTs. Suppose $O^+$ and $O^-$ are from the same superconformal multiplet, their OPE coefficients $\lambda_{\sigma\sigma O^+}$, $\lambda_{\epsilon\epsilon O^-}$ and $\lambda_{\sigma\epsilon O^-}$ are proportional to each other. Their ratios are fixed by supersymmetry and can be determined by considering the $\theta$-expansion of three-point functions of superfields $\langle\Sigma\Sigma O\rangle$. $\langle\Sigma\Sigma O\rangle$ is determined by superconformal symmetry and can be worked out based on the results of [44]. Plug these OPE relations into (2.4), and collect blocks for $O^+$ and $O^-$ that belongs to the same multiplet, the crossing equations becomes

$$\sum_{l\in\text{ even}} \lambda^2_{\mathcal{B}_+}\vec{V}^{\mathcal{B}_+}_{\Delta,l} + \sum_{l\in\text{ even}} \lambda^2_{\mathcal{B}_-}\vec{V}^{\mathcal{B}_-}_{\Delta,l} + \sum_{j-1/2\in\text{ even}} \lambda^2_{\mathcal{F}_+}\vec{V}^{\mathcal{F}_+}_{\Delta,j} + \sum_{j-1/2\in\text{ odd}} \lambda^2_{\mathcal{F}_-}\vec{V}^{\mathcal{F}_-}_{\Delta,j} = 0. \tag{2.5}$$

$\Delta$ and $l$ (or $j$) are the scaling dimensions and spins of super-conformal primaries. As will be presented explicitly in appendix A, there are four different types of superconformal blocks $\vec{V}^{\mathcal{B}_+}, \vec{V}^{\mathcal{B}_-}, \vec{V}^{\mathcal{F}_+}$ and $\vec{V}^{\mathcal{F}_-}$. Each corresponds to a type of super-multiplets appearing in $\Sigma\times\Sigma$ OPE. Schematically, the OPE looks like

$$\begin{aligned}\Sigma\times\Sigma \sim\, &\mathcal{B}_+^{\mu_1\cdots\mu_l} + \bar{Q}Q\mathcal{B}_-^{\mu_1\cdots\mu_l}\\ &+ Q^\alpha\mathcal{F}_{+,\alpha}^{\mu_1\cdots\mu_{j-1/2}} + Q^\alpha\mathcal{F}_{-,\beta}^{(\mu_1\cdots\mu}\sigma_\alpha^{\mu_{l+1})\beta}.\end{aligned} \tag{2.6}$$

As mentioned in the introduction, we are going to assume that the $\mathcal{N}$=1 Ising model has emergent supersymmetry. This is equivalent to requiring the spectrum to contain only one relevant T-parity even scalar. In terms of constraints on the super-multiplets, this amounts to imposing the following conditions

- all $\mathcal{B}_+^{(l)}$ multiplets with $l = 0$ have scaling dimension bigger than 3,
- all $\mathcal{B}_-^{(l)}$ multiplets with $l = 0$ (except for $\Sigma$) have scaling dimension bigger than 2 (notice that this multiplet contains a parity even super-descendant with scaling dimension 3),
- all $\mathcal{F}_+^{(j)}$ multiplets with $j = 1/2$ have scaling dimension bigger than 5/2.

Time-reversal symmetry even scalar operators can only appear in the above three channels $\mathcal{B}_+^{(l)}$, $\mathcal{B}_-^{(l)}$ and $\mathcal{F}_+^{(j)}$. Imposing the first and the third conditions, assuming the subleading $\mathcal{B}_-^{(0)}$ super multiplet (which we denote the superfield as $\Sigma'$ and it super-primary as $\sigma'$) to have scaling dimension bigger or equal to $\Delta_{\sigma'}$. We can use numerical bootstrap to carve out the region in $(\Delta_\sigma, \Delta_{\sigma'})$-plane that allows an unitary $\mathcal{N} = 1$ SCFT to exist. The result is shown in figure 1, where the maximum number of derivatives for conformal block approximation is $\Lambda = 13$. For technical details of numerical bootstrap, we refer to [45]. The sharp spike at $\Delta_\sigma \approx 0.584$ indicates an SCFT which we will identify as the 3D $\mathcal{N} = 1$ minimal SCFT. The kink at $\Delta_\sigma \approx 0.96$ appears when the bound meets the line $\Delta_{\sigma'} = \Delta_\sigma$. The identification of this kink to an previously known conformal field theory remains mysterious at the moment. We have checked that if we use higher $\Lambda$ [45]

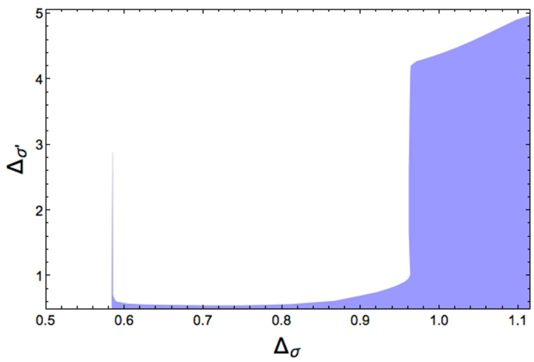


**Figure 1**. Bound on the scaling dimension of the subleading T-parity odd operator. We assume that the spectrum contains only one relevant T-parity even operator. The numerics is performed at $\Lambda = 13$.

to do the numerics, this kink is pushed to $\Delta_\sigma \to 1$. One preliminary guess is that the kink is related to the existence of SCFT with higher number of supersymmetries, since the supermultiplets multiplets of SCFT with $\mathcal{N} > 1$ may contain protected scalar with scaling dimension $\Delta = 1$. For example, the $\mathcal{N} = 2$ flavor current multiplet, when viewed as $\mathcal{N} = 1$ multiplets, branches into a conserved current multiplet, and a real scalar multiplet with the scaling dimension $\Delta = 1$.

If we further assume that $\Sigma$ and $\Sigma'$ are the only two relevant scalar superfields in the spectrum, the allowed region becomes an isolated island. This is shown in figure 2, where we use the parameters $S_{\Lambda=27}$ in [45] for the numerics. The bootstrap island can also be found if one considers only SUSY OPE relations involving scalar operators[1] [46, 47]. Notice that the scaling dimension of $\Sigma'$ is higher than 2, so that the corresponding T-parity even descendant is irrelevant. This bootstrap island can be viewed as a non-perturbative numerical "proof" of the emergent supersymmetry of the model (1.1).

## 3 Scaling exponents and CFT data

From the island, it is easy to get $\Delta_\sigma = 0.584444(30)$, corresponding to the critical exponents

$$\eta_\sigma = \eta_\psi = 0.168888(60), \quad 1/\nu = 1.415556(30). \tag{3.1}$$

$\Sigma'$ contains a super-primary with $\Delta_{\sigma'} = 2.882(9)$ and also a super-descendant which is the lowest dimensional irrelevant T-parity even scalar operator. This helps us determine the critical exponent

$$\omega = 0.882(9). \tag{3.2}$$

The four loop $\epsilon$-expansion of the Gross-Neveu-Yukawa model was calculated in [31], the Padé$_{[3,1]}$ approximation gives $\eta_\sigma = \eta_\psi = 0.170$, $1/\nu = 1.415$ and $\omega = 0.838$. This is consistent with our result and justifies our identification of the island with $\mathcal{N} = 1$ minimal

[1] This bootstrap island was reported in the preprint [arXiv:1807.04434]. Shortly after that, the work of [47] also found the same bootstrap island using SUSY OPE relations involving scalar operators. We should emphasize that to obtain a higher precision result of the critical exponents and to generalize the method to study SCFTs with flavor symmetry, it is essential to include all the SUSY OPE relations.

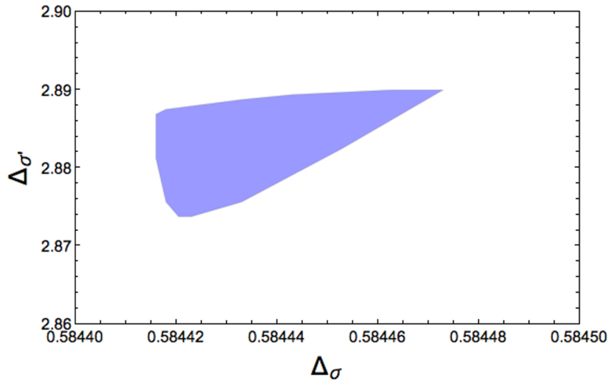


**Figure 2**. The region that allows a unitary $\mathcal{N}=1$ SCFT to exist. We assume that the spectrum contains only one relevant T-parity even operator, and $\sigma$ and $\sigma'$ are the only two relevant T-parity odd scalar operators. The numerics is performed at $\Lambda = 27$.

JHEP06(2021)154

model. We can also compare our results with previous bootstrap determinations of the critical exponents. In [42], it was observed that the SUSY line $\Delta_\epsilon = \Delta_\sigma + 1$ intersects with the region at $\Delta_\sigma = 0.565$, this provides a lower bound for $\eta_\sigma > 0.13$. In [43], the allowed region touches the SUSY line $\Delta_\psi = \Delta_\sigma + 1/2$ at $\Delta_\sigma \approx 0.582$ (though one need to impose the condition $\Delta_{\sigma'} \geq 3$). Our results show that $\mathcal{N}=1$ minimal model is indeed located in this region.

We can also calculate the constant $C_T$ in the stress-tensor two-point function when it is normalized using the Ward identity. By bootstrapping the OPE coefficient $\lambda^2_{\mathcal{F}_-}$, with $\mathcal{F}_-^{\Delta=5/2, j=3/2}$ being the SUSY current multiplet, we get

$$C_T^{\mathcal{N}=1}/C_T^{f.s.} \approx 1.684 \tag{3.3}$$

where $C_T^{f.s.}$ means $C_T$ of a free real scalar. This value is in fair agreement with the value $C_T^{\mathcal{N}=1}/C_T^{f.s.} \approx 1.73$ from one loop $\epsilon$-expansion given in [48].

Since we now know the critical exponents to high precision, we can use this result to perform "two-sided" Padé re-summation of the large-$N$ expansion result of the Gross-Neveu model (see [48] for a "two-sided" Padé resummation of both the $4-\epsilon$ series and the $2+\epsilon$ series), which allows us to estimate the critical exponents of the Gross-Neveu(-Yukawa) models with a higher number of fermions. The critical exponent $\eta_\psi$ is know to $\frac{1}{N^3}$ order [49], while $\eta_\sigma$ and $\nu^{-1}$ are known to $\frac{1}{N^2}$ order [33–35]:

$$\begin{aligned}
\eta_\psi &= \frac{8}{3\pi^2 N} + \frac{1792}{27\pi^4 N^2} \\
&\quad + \frac{64\left(-3402\zeta(3) + 141\pi^2 - 668 + 324\pi^2\log(2)\right)}{243\pi^6 N^3} \\
&\quad + \mathcal{O}(\frac{1}{N^4}), \\
\eta_\sigma &= \frac{1}{2} - \frac{64}{3\pi^2 N} + \frac{2\left(9728 - 864\pi^2\right)}{27\pi^4 N^2} + \mathcal{O}(\frac{1}{N^3}), \\
\nu^{-1} &= 1 - \frac{32}{3\pi^2 N} + \frac{64\left(632 + 27\pi^2\right)}{27\pi^4 N^2} + \mathcal{O}(\frac{1}{N^3}).
\end{aligned} \tag{3.4}$$

| $N = 4$ | $\eta_\psi$ | $\eta_\sigma$ | $\nu^{-1}$ |
|---|---|---|---|
| large-$N$, Padé$_{[2,2]}$ | 0.0942 | – | – |
| large-$N$, Padé$_{[3,1]}$ | 0.1043 | – | – |
| large-$N$, Padé$_{[1,2]}$ | – | 0.570 | 1.017 |
| large-$N$, Padé$_{[2,1]}$ | – | 0.522 | 1.040 |
| $\epsilon$ expansion.[a] | 0.096 | 0.506 | 0.852 |
| Monte Carlo[b] | – | 0.45(2) | 1.30(3) |
| Monte Carlo[c] | – | 0.54(6) | 1.14(2) |
| Monte Carlo[d] | – | 0.275(25) | 1.35(6) |
| $N = 8$ | $\eta_\psi$ | $\eta_\sigma$ | $\nu^{-1}$ |
| large-$N$, Padé$_{[2,2]}$ | 0.0430 | – | – |
| large-$N$, Padé$_{[3,1]}$ | 0.0437 | – | – |
| large-$N$, Padé$_{[1,2]}$ | – | 0.754 | 0.982 |
| large-$N$, Padé$_{[2,1]}$ | – | 0.745 | 0.991 |
| $\epsilon$ expansion.[e] | 0.042 | 0.74 | 0.948 |
| Monte Carlo[f] | – | 0.754(8) | 1.00(4) |
| Monte Carlo[g] | 0.38(1) | 0.62(1) | 1.20(1) |

[a]See [48]. The result is a two sided re-summation of the $2+\epsilon$ expansion [29] and $4-\epsilon$ expansion [27].
[b]See [37].
[c]See [39].
[d]See [38].
[e]See [48]. The result is a two sided re-summation of the $2+\epsilon$ expansion [29] and $4-\epsilon$ expansion [27].
[f]See [27].
[g]See [36].

**Table 1**. $\eta_{\psi\cdot}$, $\eta_{\sigma\cdot}$ and $\nu^{-1}$.

The resumed result is presented in the first two rows of table 1, and compared with the results using other methods. Our $\eta_\psi$ and $\eta_\sigma$ are also compatible with various kinks observed by bootstrapping 3D fermions [50]. We focus on the $N = 4$ and $N = 8$ models due to possible realization using Monte Carlo simulation [27, 36–39]. What's more, the $N = 8$ model corresponds to the quantum critical point of the semimetal to charge density wave order transition on graphene [40].

## 4 Discussion

A Monte Carlo simulation of this $\mathcal{N}=1$ super-Ising models would certainly be interesting. The Lagrangian (1.1) has an anomaly under time-reversal symmetry. One would possibly need to study such a system at the boundary of a 3 + 1 dimensional lattice, which can make the simulation time-consuming. Another interesting direction, as pointed out in [20], is to construct lattice models with non-local interaction. One important lesson that we have learned from this work is that long multiplet bootstrap [22], hence crossing equation involving super-descendant operators (which are conformal primaries), play important role in numerical bootstrap. In other words, it would be interesting to try imposing similar constraints when bootstrapping SCFT's with higher number of supercharges, extending the works of [51, 52].


## Acknowledgments

We are grateful to Ziyang Meng, Yuan Wan, Jeong-Hyuck Park, David Simmons-Duffin and Alessandro Vichi for helpful discussion or comments. J. R. would like to thank the hospitality of Institute of Physics, Chinese Academy of Science while part of the work was finished. N.S.'s research is supported in part by the Key Research Program of Frontier Sciences of CAS under Grant No. QYZDB-SSW-SYS014 and Peng Huanwu center under Grant No. 11747601. The numerical computations in this work are partially supported by HPC Cluster of SKLTP/ITPCAS. To calculate the conformal block functions, we used the code from JuliBootS program [53]. The numerics is solved using SDPB program [45].


## A Superconformal bootstrap equations

To derive the superconoformal bootstrap equations, we first follow the result [44] to consider the three-point function of two $\Sigma$'s and a third superfield,

$$\begin{aligned}&\langle\mathcal{O}^{(l)}(x_1,\theta_1,\eta_1)\Sigma(x_2,\theta_2)\Sigma(x_3,\theta_3)\rangle\\&\qquad=\frac{t(\mathrm{X}_1,\Theta_1,\eta_1)}{x_{12}^{2\Delta_\Phi-\Delta_\mathcal{O}-l}x_{13}^{2\Delta_\Phi-\Delta_\mathcal{O}-l}x_{23}^{\Delta_\mathcal{O}+l}},\end{aligned}\tag{A.1}$$

with

$$\begin{aligned}x_{12}^\mu&=x_1^\mu+x_2^\mu+\mathrm{i}\bar\theta_1\gamma_\mu\theta_2,\\ \mathrm{x}_{12\pm}&=x_{12}^\mu\gamma_\mu\pm\mathrm{i}\frac{1}{2}\bar\theta_{12}\theta_{12},\quad\theta_{12}=\theta_1-\theta_2,\\ \mathrm{X}_1&=\frac{1}{2}(\mathrm{x}_{31+}^{-1}\mathrm{x}_{23-}\mathrm{x}_{21+}^{-1}+\mathrm{x}_{21+}^{-1}\mathrm{x}_{23+}\mathrm{x}_{31-}^{-1}),\\ \Theta_1&=\mathrm{i}(\mathrm{x}_{21+}^{-1}\theta_{21}-\mathrm{x}_{31+}^{-1}\theta_{31}).\end{aligned}\tag{A.2}$$



JHEP06(2021)154

Here $\eta_1$ is an auxiliary two component spinor, valued by commuting numbers. There exist four different $t$-structures

$$
\begin{aligned}
\mathcal{B}_+^{(l)} &: (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^l, \\
\mathcal{B}_-^{(l)} &: \bar{\Theta}_1 \Theta_1 (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^l (\mathrm{tr}[\mathrm{X}_1^2])^{-1/2}, \\
\mathcal{F}_+^{(j)} &: \bar{\eta}_1 \mathrm{X}_1 \Theta_1 (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^{j-1/2} (\mathrm{tr}[\mathrm{X}_1^2])^{-3/4}, \\
\mathcal{F}_-^{(j)} &: \bar{\eta}_1 \Theta_1 (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^{j-1/2} (\mathrm{tr}[\mathrm{X}_1^2])^{-1/4}.
\end{aligned}
$$

Performing a series expansion of the three-point function in terms of the Grassmann variables $\theta_1$, $\theta_2$ and $\theta_3$, we obtain the SUSY OPE relations. It is also necessary to take into account the fact that the operators from $\theta$ expansion are not yet properly normalized. One needs to fix this by expanding the two-point functions

$$
\langle \mathcal{O}^{(l)}(x_1, \theta_1, \eta_1) \mathcal{O}^{(l)}(x_2, \sigma_2, \eta_2) \rangle = \frac{(\bar{\eta}_1 \mathrm{x}_{12+} \eta_2)^{2l}}{(x_{12}^2)^{\Delta+l}}, \tag{A.3}
$$

in Grassmann variables, and then read out the operators' normalization and scaling the OPE relations properly. The convention in this work is the same as [11]. The calculation is in the same spirit of [54], where $\mathcal{N}=1$ superconformal block in four dimensions was calculated.

The superfield $\mathcal{B}_+^{(l)}$ can be expanded in $\theta$ as $\mathcal{B}_+^{\mu_1 \cdots \mu_l} = O_+^{\mu_1 \cdots \mu_l} + \ldots + \bar{\theta}\theta(O_-^{\mu_1 \cdots \mu_l} + \#\epsilon^{(\mu_1|\nu\rho|} P_\nu O_{+\rho}{}^{\mu_2 \cdots \mu_l})$, where the dots denote fermionic operators that we will neglect. $\epsilon^{(\mu_1|\nu\rho|} P_\nu O_{+\rho}{}^{\mu_2 \cdots \mu_l}$ is the conformal descendant of $O_+^{\mu_1 \cdots \mu_l}$. From the $\theta$-expansion of $\langle \mathcal{B}_+^{(l)}(x_1, \theta_1) \Sigma(x_2, \theta_2) \Sigma(x_3, \theta_3) \rangle$, we need to pick the terms promotional to 1, $\bar{\theta}_2\theta_2\bar{\theta}_3\theta_3$ and $\bar{\theta}_1\theta_1\bar{\theta}_3\theta_3$, delete terms that comes from three point function involving $\epsilon P O_+^{(l)}$, and read out the OPE ratios $\lambda_{\sigma\epsilon O_-}/\lambda_{\sigma\sigma O_+}$ and $\lambda_{\epsilon\epsilon O_+}/\lambda_{\sigma\sigma O_+}$. To take care of the normalization, we consider two point function $\langle \mathcal{B}_+^{(l)}(x_1, \theta_1) \mathcal{B}_+^{(l)}(x_2, \theta_2) \rangle$. The term proportional to 1 gives us $\langle O_+ O_+ \rangle$, while the $\bar{\theta}_1\theta_1\bar{\theta}_2\theta_2$ term, after deleting the two point function of descendants $\langle \epsilon P O_+^{(l)} \epsilon P O_+^{(l)} \rangle$, gives us $\langle O_- O_- \rangle$.

The $\theta$-expansion of $\langle \mathcal{B}_-^{(l)} \Sigma\Sigma \rangle$ with $\mathcal{B}_-^{(l)} = O_-^{(l)} + \ldots + \bar{\theta}\theta(O_+ + \#\epsilon P O_-^{(l)})$, after normalization, gives us the OPE ratios $\lambda_{\sigma\epsilon O_-}/\lambda_{\sigma\sigma O_+}$ and $\lambda_{\epsilon\epsilon O_+}/\lambda_{\sigma\sigma O_+}$. This time, we need to study the terms proportional to $\bar{\theta}_1\theta_1$, $\bar{\theta}_3\theta_3$ and $\bar{\theta}_1\theta_1\bar{\theta}_2\theta_2\bar{\theta}_3\theta_3$.

The fermionic superfield $\mathcal{F}_+^{(j)}$ can be expanded in $\theta$ as $\mathcal{F}_+^{(j)} = \ldots + \bar{\eta}\theta \cdot O_+^{(l)} + \bar{\eta}\gamma_\mu\theta \cdot O_-^{(l+1)} + \ldots$, where $l = j - 1/2$. The $\bar{\eta}_1\theta_1$, $\bar{\eta}_1\theta_1\bar{\theta}_2\theta_2\bar{\theta}_3\theta_3$ and $\bar{\eta}_1\gamma_\mu\theta_1\bar{\theta}_3\theta_3$ terms of $\langle \mathcal{F}_+^{(l)}(x_1, \theta_1) \Sigma(x_2, \theta_2) \Sigma(x_3, \theta_3) \rangle$ help us obtain $\lambda_{\sigma\epsilon O_-^{(l+1)}}/\lambda_{\sigma\sigma O_+^{(l)}}$ and $\lambda_{\epsilon\epsilon O_+^{(l)}}/\lambda_{\sigma\sigma O_+^{(l)}}$. To normalise the operators properly, one need to calculate the $\bar{\eta}_1\theta_1\bar{\eta}_2\theta_2$ and $\bar{\eta}_1\gamma_\mu\theta_1\bar{\eta}_2\gamma_\nu\theta_2$ terms of $\langle \mathcal{F}_+(x_1, \theta_1, \eta_1) \mathcal{F}_+(x_2, \theta_2, \eta_2) \rangle$.

A similar calculation can be done for $\mathcal{F}_-^{(j)} = \ldots + \bar{\eta}\theta \cdot O_-^{(l)} + \bar{\eta}\gamma_\mu\theta \cdot O_+^{(l+1)} + \ldots$. This time we need the $\bar{\eta}_1\theta_1\bar{\theta}_3\theta_3$, $\bar{\eta}_1\gamma_\mu\theta_1\bar{\theta}_2\theta_2\bar{\theta}_3\theta_3$ and $\bar{\eta}_1\gamma_\mu\theta_1\bar{\theta}_2$ terms of $\langle \mathcal{F}_-^{(j)} \Sigma\Sigma \rangle$.

Notice an interesting feature in the calculation is that only one conformal primary of the super multiplet appears in the $\sigma \times \sigma$ OPE (or $\sigma \times \epsilon$ OPE).



JHEP06(2021)154

The non-SUSY crossing equation (2.4), studied in [6], is written in terms of the following two vectors,

$$\vec{V}_{-,\Delta,\ell} = \begin{pmatrix} 0 \\ 0 \\ F^{\sigma\epsilon,\sigma\epsilon}_{-,\Delta,\ell}(u,v) \\ (-1)^\ell F^{\epsilon\sigma,\sigma\epsilon}_{-,\Delta,\ell}(u,v) \\ -(-1)^\ell F^{\epsilon\sigma,\sigma\epsilon}_{+,\Delta,\ell}(u,v) \end{pmatrix},$$

$$\vec{V}_{+,\Delta,\ell} = \begin{pmatrix} \begin{pmatrix} F^{\sigma\sigma,\sigma\sigma}_{-,\Delta,\ell}(u,v) & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & F^{\epsilon\epsilon,\epsilon\epsilon}_{-,\Delta,\ell}(u,v) \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{-,\Delta,\ell}(u,v) \\ \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{-,\Delta,\ell}(u,v) & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{+,\Delta,\ell}(u,v) \\ \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{+,\Delta,\ell}(u,v) & 0 \end{pmatrix} \end{pmatrix}. \tag{A.4}$$

As usual, the convoluted conformal bock are defined by

$$F^{ab;cd}_{\pm,\Delta,l}(u,v) = v^{\frac{\Delta_c+\Delta_b}{2}} g^{\Delta_{ab};\Delta_{cd}}_{\Delta,l}(u,v) \pm u \leftrightarrow v.$$

To get the SUSY crossing equation, we simply need to plug in (2.4) the OPE ratios that we have calculated by the expansion of the SUSY three-point functions in Grassmann variables. We get (2.5), with

$$\vec{V}^{\mathcal{B}_+}_{\Delta,l} = \begin{pmatrix} F_{-,\Delta,l}{}^{\sigma\sigma,\sigma\sigma} \\ c_1^2 F_{-,\Delta,l}{}^{\epsilon\epsilon,\epsilon\epsilon} \\ c_2 F_{-,\Delta+1,l}^{\ \sigma\epsilon,\sigma\epsilon} \\ c_1 F_{-,\Delta,l}^{\ \sigma\sigma,\epsilon\epsilon} + c_2(-1)^l F_{-,\Delta+1,l}^{\ \epsilon\sigma,\sigma\epsilon} \\ c_1 F_{+,\Delta,l}^{\ \sigma\sigma,\epsilon\epsilon} - c_2(-1)^l F_{+,\Delta+1,l}^{\ \epsilon\sigma,\sigma\epsilon} \end{pmatrix},$$

$$\vec{V}^{\mathcal{B}_-}_{\Delta,l} = \begin{pmatrix} F_{-,\Delta+1,l}^{\ \sigma\sigma,\sigma\sigma} \\ d_1^2 F_{-,\Delta+1,l}^{\ \epsilon\epsilon,\epsilon\epsilon} \\ d_2 F_{-,\Delta,l}^{\ \sigma\epsilon,\sigma\epsilon} \\ d_1 F_{-,\Delta+1,l}^{\ \sigma\sigma,\epsilon\epsilon} + d_2(-1)^l F_{-,\Delta,l}^{\ \epsilon\sigma,\sigma\epsilon} \\ d_1 F_{+,\Delta+1,l}^{\ \sigma\sigma,\epsilon\epsilon} - d_2(-1)^l F_{+,\Delta,l}^{\ \epsilon\sigma,\sigma\epsilon} \end{pmatrix},$$

$$\vec{V}^{\mathcal{F}_+}_{\Delta,j} = \begin{pmatrix} F^{\sigma\sigma,\sigma\sigma}_{-,\Delta_l,l} \\ f_1^2 F^{\epsilon\epsilon,\epsilon\epsilon}_{-,\Delta_l,l} \\ f_2 F^{\sigma\epsilon,\sigma\epsilon}_{-,\Delta_l,l+1} \\ f_1 F^{\sigma\sigma,\epsilon\epsilon}_{-,\Delta_l,l} + f_2(-1)^{l+1} F^{\epsilon\sigma,\sigma\epsilon}_{-\Delta_l,l+1} \\ f_1 F^{\sigma\sigma,\epsilon\epsilon}_{+,\Delta_l,l} - f_2(-1)^{l+1} F^{\epsilon\sigma,\sigma\epsilon}_{+,\Delta_l,l+1} \end{pmatrix},$$

$$\vec{V}^{\mathcal{F}_-}_{\Delta,j} = \begin{pmatrix} F^{\sigma\sigma,\sigma\sigma}_{-,\Delta_l,l+1} \\ e_1^2 F^{\epsilon\epsilon,\epsilon\epsilon}_{-,\Delta_l,l+1} \\ e_2 F^{\sigma\epsilon,\sigma\epsilon}_{-,\Delta_l,l} \\ e_1 F^{\sigma\sigma,\epsilon\epsilon}_{-,\Delta_l,l+1} + e_2(-1)^{l} F^{\epsilon\sigma,\sigma\epsilon}_{-,\Delta_l,l} \\ e_1 F^{\sigma\sigma,\epsilon\epsilon}_{+,\Delta_l,l+1} - e_2(-1)^{l} F^{\epsilon\sigma,\sigma\epsilon}_{+,\Delta_l,l} \end{pmatrix} \tag{A.5}$$

For the fermionic superblocks, we define $\Delta_l = \Delta + 1/2$, $l = j - 1/2$. The constants with

$$\begin{aligned} c_1 &= \frac{(2\Delta_\sigma - \Delta - l - 1)\,(2\Delta_\sigma - \Delta + l)}{2\Delta_\sigma\,(2\Delta_\sigma - 1)}, \\ c_2 &= \frac{(\Delta - 1)(\Delta - l - 1)(\Delta + l)}{4(2\Delta - 1)\Delta_\sigma\,(2\Delta_\sigma - 1)}, \end{aligned} \tag{A.6}$$

$$\begin{aligned} d_1 &= \frac{(2\Delta_\sigma + \Delta - l - 3)\,(2\Delta_\sigma + \Delta + l - 2)}{2\Delta_\sigma\,(2\Delta_\sigma - 1)}, \\ d_2 &= \frac{(2\Delta - 1)(\Delta - l - 1)(\Delta + l)}{(\Delta - 1)\Delta_\sigma\,(2\Delta_\sigma - 1)}, \end{aligned} \tag{A.7}$$

$$\begin{aligned} f_1 &= \frac{(-2\Delta_\sigma - \Delta_l + l + 4)\,(-2\Delta_\sigma + \Delta_l + l + 1)}{2\Delta_\sigma\,(2\Delta_\sigma - 1)}, \\ f_2 &= \frac{(2l + 1)(\Delta_l - l - 2)(\Delta_l + l)}{2(l + 1)\Delta_\sigma\,(2\Delta_\sigma - 1)}, \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} e_1 &= \frac{(2\Delta_\sigma - \Delta_l + l + 1)\,(2\Delta_\sigma + \Delta_l + l - 2)}{2\Delta_\sigma\,(2\Delta_\sigma - 1)}, \\ e_2 &= \frac{(l + 1)(\Delta_l - l - 2)(\Delta_l + l)}{2(2l + 1)\Delta_\sigma\,(2\Delta_\sigma - 1)}. \end{aligned} \tag{A.9}$$

The OPE coefficients in (2.5) correspond to $\lambda_{\sigma\sigma O}$, with $O$ being the T-parity even operator in the multiplet. It turns out the five crossing equations are not linear independent, when doing numerical bootstrap, we simply delete the second line.

## References


[1] T. Grover, D.N. Sheng and A. Vishwanath, *Emergent Space-Time Supersymmetry at the Boundary of a Topological Phase*, *Science* **344** (2014) 280 [arXiv:1301.7449] [INSPIRE].

[2] A.M. Polyakov, *Nonhamiltonian approach to conformal quantum field theory*, *Zh. Eksp. Teor. Fiz.* **66** (1974) 23 [INSPIRE].

[3] S. Ferrara, A.F. Grillo and R. Gatto, *Tensor representations of conformal algebra and conformally covariant operator product expansion*, *Annals Phys.* **76** (1973) 161 [INSPIRE].

[4] A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, *Infinite Conformal Symmetry in Two-Dimensional Quantum Field Theory*, *Nucl. Phys. B* **241** (1984) 333 [INSPIRE].

[5] R. Rattazzi, V.S. Rychkov, E. Tonni and A. Vichi, *Bounding scalar operator dimensions in 4D CFT*, *JHEP* **12** (2008) 031 [arXiv:0807.0004] [INSPIRE].

[6] F. Kos, D. Poland and D. Simmons-Duffin, *Bootstrapping Mixed Correlators in the 3D Ising Model*, *JHEP* **11** (2014) 109 [arXiv:1406.4858] [INSPIRE].

[7] S. El-Showk, M.F. Paulos, D. Poland, S. Rychkov, D. Simmons-Duffin and A. Vichi, *Solving the 3d Ising Model with the Conformal Bootstrap II. c-Minimization and Precise Critical Exponents*, *J. Stat. Phys.* **157** (2014) 869 [arXiv:1403.4545] [INSPIRE].

[8] F. Kos, D. Poland, D. Simmons-Duffin and A. Vichi, *Precision Islands in the Ising and $O(N)$ Models*, *JHEP* **08** (2016) 036 [arXiv:1603.04436] [INSPIRE].

[9] D. Simmons-Duffin, *The Lightcone Bootstrap and the Spectrum of the 3d Ising CFT*, *JHEP* **03** (2017) 086 [arXiv:1612.08471] [INSPIRE].

[10] S.M. Chester et al., *Carving out OPE space and precise $O(2)$ model critical exponents*, *JHEP* **06** (2020) 142 [arXiv:1912.03324] [INSPIRE].

[11] D. Poland, S. Rychkov and A. Vichi, *The Conformal Bootstrap: Theory, Numerical Techniques, and Applications*, *Rev. Mod. Phys.* **91** (2019) 015002 [arXiv:1805.04405] [INSPIRE].

[12] J.-L. Gervais and B. Sakita, *Field Theory Interpretation of Supergauges in Dual Models*, *Nucl. Phys. B* **34** (1971) 632 [INSPIRE].

[13] P. Ramond, *Dual Theory for Free Fermions*, *Phys. Rev. D* **3** (1971) 2415 [INSPIRE].

[14] J. Wess and B. Zumino, *Supergauge Transformations in Four-Dimensions*, *Nucl. Phys. B* **70** (1974) 39 [INSPIRE].

[15] A. Rahmani, X. Zhu, M. Franz and I. Affleck, *Emergent supersymmetry from strongly interacting majorana zero modes*, *Phys. Rev. Lett.* **115** (2015) 166401 [*Erratum ibid.* **116** (2016) 109901] [arXiv:1504.05192] [INSPIRE].

[16] S.-K. Jian, Y.-F. Jiang and H. Yao, *Emergent spacetime supersymmetry in 3d weyl semimetals and 2d dirac semimetals*, *Phys. Rev. Lett.* **114** (2015) 237001 [arXiv:1407.4497] [INSPIRE].

[17] T.H. Hsieh, G.B. Halász and T. Grover, *All majorana models with translation symmetry are supersymmetric*, *Phys. Rev. Lett.* **117** (2016) 166802 [arXiv:1604.08591] [INSPIRE].

[18] S.-K. Jian, C.-H. Lin, J. Maciejko and H. Yao, *Emergence of supersymmetric quantum electrodynamics*, *Phys. Rev. Lett.* **118** (2017) 166802 [arXiv:1609.02146] [INSPIRE].


JHEP06(2021)154

[19] Z.-X. Li, Y.-F. Jiang and H. Yao, *Edge quantum criticality and emergent supersymmetry in topological phases*, *Phys. Rev. Lett.* **119** (2017) 107202 [arXiv:1610.04616] [INSPIRE].

[20] Z.-X. Li, A. Vaezi, C.B. Mendl and H. Yao, *Observation of Emergent Spacetime Supersymmetry at Superconducting Quantum Criticality*, arXiv:1711.04772 [INSPIRE].

[21] L. Huijse, B. Bauer and E. Berg, *Emergent Supersymmetry at the Ising-Berezinskii-Kosterlitz-Thouless Multicritical Point*, *Phys. Rev. Lett.* **114** (2015) 090404 [arXiv:1403.5565] [INSPIRE].

[22] M. Cornagliotto, M. Lemos and V. Schomerus, *Long Multiplet Bootstrap*, *JHEP* **10** (2017) 119 [arXiv:1702.05101] [INSPIRE].

[23] D.J. Gross and A. Neveu, *Dynamical Symmetry Breaking in Asymptotically Free Field Theories*, *Phys. Rev. D* **10** (1974) 3235 [INSPIRE].

[24] A. Hasenfratz, P. Hasenfratz, K. Jansen, J. Kuti and Y. Shen, *The Equivalence of the top quark condensate and the elementary Higgs field*, *Nucl. Phys. B* **365** (1991) 79 [INSPIRE].

[25] J. Zinn-Justin, *Four fermion interaction near four-dimensions*, *Nucl. Phys. B* **367** (1991) 105 [INSPIRE].

[26] W. Wetzel, *Two Loop $\beta$-function for the Gross-Neveu Model*, *Phys. Lett. B* **153** (1985) 297 [INSPIRE].

[27] L. Karkkainen, R. Lacaze, P. Lacock and B. Petersson, *Critical behavior of the three-dimensional Gross-Neveu and Higgs-Yukawa models*, *Nucl. Phys. B* **415** (1994) 781 [*Erratum ibid.* **438** (1995) 650] [hep-lat/9310020] [INSPIRE].

[28] N.A. Kivel, A.S. Stepanenko and A.N. Vasiliev, *On calculation of $(2+\epsilon)$ RG functions in the Gross-Neveu model from large $N$ expansions of critical exponents*, *Nucl. Phys. B* **424** (1994) 619 [hep-th/9308073] [INSPIRE].

[29] J.A. Gracey, *Four loop MS-bar mass anomalous dimension in the Gross-Neveu model*, *Nucl. Phys. B* **802** (2008) 330 [arXiv:0804.1241] [INSPIRE].

[30] L.N. Mihaila, N. Zerf, B. Ihrig, I.F. Herbut and M.M. Scherer, *Gross-Neveu-Yukawa model at three loops and Ising critical behavior of Dirac systems*, *Phys. Rev. B* **96** (2017) 165133 [arXiv:1703.08801] [INSPIRE].

[31] N. Zerf, L.N. Mihaila, P. Marquard, I.F. Herbut and M.M. Scherer, *Four-loop critical exponents for the Gross-Neveu-Yukawa models*, *Phys. Rev. D* **96** (2017) 096010 [arXiv:1709.05057] [INSPIRE].

[32] J.A. Gracey, *Three loop calculations in the $O(N)$ Gross-Neveu model*, *Nucl. Phys. B* **341** (1990) 403 [INSPIRE].

[33] J.A. Gracey, *Calculation of exponent $\eta$ to $O(1/N^2)$ in the $O(N)$ Gross-Neveu model*, *Int. J. Mod. Phys. A* **6** (1991) 395 [*Erratum ibid.* **6** (1991) 2755] [INSPIRE].

[34] J.A. Gracey, *Anomalous mass dimension at $O(1/N^2)$ in the $O(N)$ Gross-Neveu model*, *Phys. Lett. B* **297** (1992) 293 [INSPIRE].

[35] J.A. Gracey, *Computation of $\beta'(g_c)$ at $O(1/N^2)$ in the $O(N)$ Gross-Neveu model in arbitrary dimensions*, *Int. J. Mod. Phys. A* **9** (1994) 567 [hep-th/9306106] [INSPIRE].

[36] S. Chandrasekharan and A. Li, *Quantum critical behavior in three dimensional lattice Gross-Neveu models*, *Phys. Rev. D* **88** (2013) 021701 [arXiv:1304.7761] [INSPIRE].


JHEP06(2021)154

[37] Z.-X. Li, Y.-F. Jiang and H. Yao, *Fermion-sign-free Majarana-quantum-Monte-Carlo studies of quantum critical phenomena of Dirac fermions in two dimensions*, New J. Phys. **17** (2015) 085003 [arXiv:1411.7383] [INSPIRE].

[38] S. Hesselmann and S. Wessel, *Thermal Ising transitions in the vicinity of two-dimensional quantum critical points*, Phys. Rev. B **93** (2016) 155157 [arXiv:1602.02096] [INSPIRE].

[39] E. Huffman and S. Chandrasekharan, *Fermion bag approach to Hamiltonian lattice field theories in continuous time*, Phys. Rev. D **96** (2017) 114502 [arXiv:1709.03578] [INSPIRE].

[40] I.F. Herbut, *Interactions and phase transitions on graphene's honeycomb lattice*, Phys. Rev. Lett. **97** (2006) 146401 [cond-mat/0606195] [INSPIRE].

[41] C. Cordova, T.T. Dumitrescu and K. Intriligator, *Multiplets of Superconformal Symmetry in Diverse Dimensions*, JHEP **03** (2019) 163 [arXiv:1612.00809] [INSPIRE].

[42] D. Bashkirov, *Bootstrapping the $\mathcal{N} = 1$ SCFT in three dimensions*, arXiv:1310.8255 [INSPIRE].

[43] L. Iliesiu, F. Kos, D. Poland, S.S. Pufu, D. Simmons-Duffin and R. Yacoby, *Bootstrapping 3D Fermions*, JHEP **03** (2016) 120 [arXiv:1508.00012] [INSPIRE].

[44] J.-H. Park, *Superconformal symmetry in three-dimensions*, J. Math. Phys. **41** (2000) 7129 [hep-th/9910199] [INSPIRE].

[45] D. Simmons-Duffin, *A Semidefinite Program Solver for the Conformal Bootstrap*, JHEP **06** (2015) 174 [arXiv:1502.02033] [INSPIRE].

[46] J. Rong and N. Su, *Bootstrapping minimal $\mathcal{N} = 1$ superconformal field theory in three dimensions*, arXiv:1807.04434 [INSPIRE].

[47] A. Atanasov, A. Hillman and D. Poland, *Bootstrapping the Minimal 3D SCFT*, JHEP **11** (2018) 140 [arXiv:1807.05702] [INSPIRE].

[48] L. Fei, S. Giombi, I.R. Klebanov and G. Tarnopolsky, *Yukawa CFTs and Emergent Supersymmetry*, PTEP **2016** (2016) 12C105 [arXiv:1607.05316] [INSPIRE].

[49] J.A. Gracey, *Computation of critical exponent $\eta$ at $O(1/N^3)$ in the four Fermi model in arbitrary dimensions*, Int. J. Mod. Phys. A **9** (1994) 727 [hep-th/9306107] [INSPIRE].

[50] L. Iliesiu, F. Kos, D. Poland, S.S. Pufu and D. Simmons-Duffin, *Bootstrapping 3D Fermions with Global Symmetries*, JHEP **01** (2018) 036 [arXiv:1705.03484] [INSPIRE].

[51] N. Bobev, S. El-Showk, D. Mazac and M.F. Paulos, *Bootstrapping the Three-Dimensional Supersymmetric Ising Model*, Phys. Rev. Lett. **115** (2015) 051601 [arXiv:1502.04124] [INSPIRE].

[52] S.M. Chester, J. Lee, S.S. Pufu and R. Yacoby, *The $\mathcal{N} = 8$ superconformal bootstrap in three dimensions*, JHEP **09** (2014) 143 [arXiv:1406.4814] [INSPIRE].

[53] M.F. Paulos, *JuliBootS: a hands-on guide to the conformal bootstrap*, arXiv:1412.4127 [INSPIRE].

[54] D. Poland and D. Simmons-Duffin, *Bounds on 4D Conformal and Superconformal Field Theories*, JHEP **05** (2011) 017 [arXiv:1009.2087] [INSPIRE].


JHEP06(2021)154