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Semi-inclusive back-to-back production of a hadron pair and a single hadron in e^+e^- annihilation

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Abstract: Inclusive hadron production in e^+e^- annihilation has long been used to study both single hadron fragmentation functions (FF) and dihadron fragmentation functions (DiFF). In particular, the polarized DiFFs can be accessed in electron-positron annihilation by measuring azimuthal correlations between two back-to-back pairs of hadrons in the center of mass system, where the relevant structure functions can be expressed as convolutions of two (polarized) DiFFs. Here we explore the advantages of measuring the inclusive back-to-back production of a single hadron on one side against a hadron pair on the opposite side of the detector in two jet events. The leading twist cross section for this process contains convolutions of the corresponding single hadron FFs on one side and the DiFFs for the hadron pair on the other side, which furnishes several interesting new opportunities. A measurement of the unpolarized cross section with a number of different types of observed hadrons will help in untangling the quark flavor dependence of the unpolarized DiFFs, when the results are analyzed together with the inclusive measurements of dihadron pairs, such as those recently performed by the BELLE collaboration. Even more interesting, with a polarized hyperon on one side we can study the quark spin-dependent DiFFs of an unpolarized hadron pair on the other side. This, in turn, will allow us to test the universality of the spin-dependent DiFFs entering the cross sections of electron-positron annihilation and semi-inclusive deep inelastic scattering processes.

Keywords: Deep Inelastic Scattering (Phenomenology), QCD Phenomenology

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1 Introduction

The study of the hadronization process, which is quantified by various fragmentation functions, has gained a great deal of attention in recent years [1]. This has been motivated by the ability of the new experiments to measure various azimuthal correlations in the deep inelastic scattering processes that involve polarized FFs and DiFFs. For example, the measurements of the quark transverse polarization dependent Collins FF and the so-called interference DiFF (IFF) in electron-positron annihilation in BELLE [2–5] and BaBar [6] experiments, have allowed the extraction of the quark transversity parton distribution function (PDF) [7–10] using semi-inclusive deep inelastic scattering (SIDIS) measurements with one and two detected final state hadrons by the HERMES [11, 12] and COMPASS [13, 14] collaborations. It is worth mentioning that the cross section for hadron production in e^+e^- annihilation involves a summation over all intermediate quark-antiquark pairs, thus requiring additional input in order to access the quark flavor dependence of the FFs and DiFFs.

The key for such a combined analysis is the universality of the FFs and DiFFs entering the cross section of both electron-positron annihilation and SIDIS processes, which was proven explicitly for the transverse-momentum dependent FFs [15, 16], while similar arguments should apply in the case of DiFFs [17]. This is in contrast to the prediction of the process dependence of the naive-time-reversal-odd (T-odd) Sivers PDF, which is predicted to change sign between SIDIS and Drell-Yan annihilation [18]. An experimental test of such universality for the naive-time-reversal-odd FFs has been proposed in ref. [19], by exploring the so-called "polarizing" FF $D_{1T}^{\perp}$ of a spin 1/2 hyperon. The "polarizing" FF describes the correlation of the transverse polarization of a produced hadron with its

own transverse momentum in the quark fragmentation process and is a chiral-even function. Thus, in SIDIS with a final state hyperon detected, it couples to the well-determined unpolarized PDF, while in e^+e^- annihilation with back-to-back hyperon and unpolarized hadron production, it couples to the corresponding unpolarized FF.

Recently, we proposed two new measurements [20] of the quark helicity dependent DiFF $G_1^\perp$, both in SIDIS with two final state detected hadrons and two back-to-back hadron pair production in e^+e^- annihilation. The chiral-even nature of $G_1^\perp$ entails that in SIDIS it couples to the well-determined quark unpolarized and helicity PDFs [21]. In e^+e^- annihilation though, the $G_1^\perp$ from one pair couples to that for the second pair on the other side [22, 23], making it not possible (or at least very hard) to determine the sign of this function. Thus, such measurements allow one to test only for the agreement between the magnitudes of $G_1^\perp$ extracted from the two measurements.

In this work we propose a new measurement in e^+e^- annihilation, exploiting the production of a single inclusive hadron back-to-back to a hadron pair, where the relevant cross section should involve convolutions of FFs for the single hadron in one jet and the DiFFs for the hadron pair produced in the opposite jet. The purpose of such a measurement is two-fold, and will leverage our knowledge of the single hadron FFs. First, the absolute cross section measurements will provide a wider basis for extracting the quark flavor dependence of the DiFFs, especially when analyzed together with the inclusive hadron pair measurements in the same jet [5]. Secondly, by studying various azimuthal asymmetries we can better determine the polarized DiFFs, and also access their sign.

This paper is organized in the following way. In the next section we detail the derivation of the cross section for the proposed new process. In section 3, we explore the relevant unpolarized measurements and the azimuthal asymmetries for accessing the DiFFs. In section 4, we examine the particular case where one detects a polarized Λ^0 hyperon. Finally, we present the summary of our findings and conclusions in section 5.

2 The cross section calculation

In this section we detail the derivation of the leading twist cross section for the process $e^+e^- \to h_1h_2 + \Lambda + X$, where the electron and positron with momenta l and l' annihilate into an intermediate virtual photon with momentum $q = l + l'$. In the final state, we detect a hadron Λ with momentum P_Λ, produced back-to-back to the unpolarized hadron pair h_1, h_2 of momenta P_1 and P_2, as illustrated in figure 1. Here Λ denotes either an unpolarized hadron (for example π, K, etc) hadron, or a spin 1/2 baryon withe a polarization vector $\boldsymbol{S}_\Lambda$. We restrict our consideration to the case where the center-of-mass energy of the electron-positron pair is far below the mass of the Z boson. We use the conventional framework for the inclusive hadron production in e^+e^- annihilation [22–25]. In the next subsection we first describe the kinematics of the process and then detail the calculation of the cross section in the following subsection.

2.1 Kinematics

For our calculations we choose the e^+e^- center-of-mass coordinate system, where the $\hat{z}$ axis is chosen to point opposite to the Λ's 3-momentum $\boldsymbol{P}_\Lambda$. We denote the components

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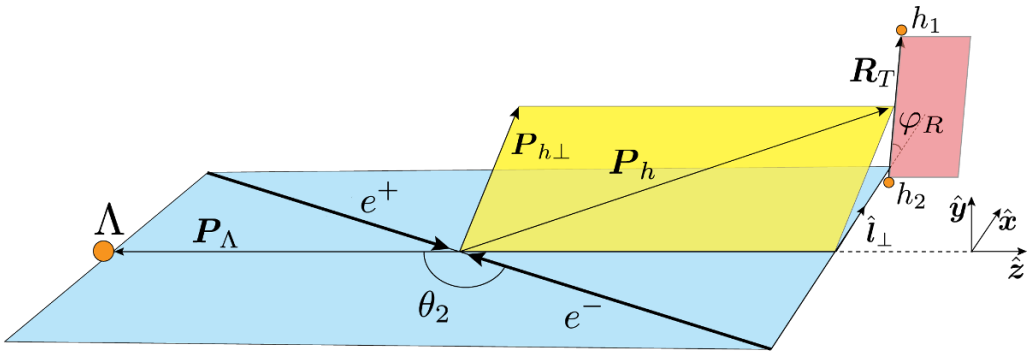

Figure 1. The kinematics of $e^+e^- \to h_1h_2 + \Lambda + X$ process.

of 3-vectors transverse to $\hat{z}$ axis with subscript $_\perp$. The $\hat{x}$ axis is then taken along the transverse component $\boldsymbol{l}_\perp$ of the electron momentum. Together, the $\hat{z}$ and $\hat{x}$ axes span the lepton plane. In dihadron studies it is useful to replace P_1, P_2 by the total and the relative hadron momenta, defined as

$$P_h = P_1 + P_2, \tag{2.1}$$

$$R = \frac{1}{2}(P_1 - P_2), \tag{2.2}$$

and the invariant mass of the pair is defined by $P_h^2 = M_h^2$.

We consider the kinematic regime where the virtuality of the time like momentum of the virtual photon is much larger than the typical hadronic mass scales, thus defining $Q^2 \equiv q^2$ we can ignore any contributions of order $1/Q$. At the QED and QCD leading order approximation, the virtual photon produces a quark and an antiquark pair $e^+e^- \to \gamma^* \to q + \bar{q}$, which then hadronize and produce two back-to-back jets. Further, we assume that we are in the "leading hadron approximation", where a significant fraction of the energy in each jet is carried by the observed hadrons, that is $P_h \cdot P_\Lambda \sim Q^2$. We can use these two large momenta to define light-like directions to conveniently decompose the hadronic tensor, as done earlier for other reactions [22, 24]. For this, we choose a coordinate system where the transverse components of the light-cone momenta[1] of P_h, P_Λ vanish, $\boldsymbol{P}_{hT} = 0$ and $\boldsymbol{P}_{\Lambda T} = 0$. Introducing two light-like vectors n_+ and n_-, such that $n_+^2 = n_-^2 = 0$ and $n_+n_- = 1$, we can decompose the relevant large momenta as

$$P_h^\mu = \frac{M_h^2}{z_h Q\sqrt{2}} n_-^\mu + \frac{z_h Q}{\sqrt{2}} n_+^\mu \approx \frac{z_h Q}{\sqrt{2}} n_+^\mu, \tag{2.3}$$

$$P_\Lambda^\mu = \frac{z_\Lambda Q}{\sqrt{2}} n_-^\mu + \frac{M_\Lambda^2}{z_\Lambda Q\sqrt{2}} n_+^\mu \approx \frac{z_\Lambda Q}{\sqrt{2}} n_-^\mu, \tag{2.4}$$

$$q^\mu = \frac{Q}{\sqrt{2}} n_-^\mu + \frac{Q}{\sqrt{2}} n_+^\mu + q_T^\mu, \tag{2.5}$$

[1]The light-cone components of a 4-vector a are defined as $a = (a^+, a^-, \boldsymbol{a}_T)$, where $a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$.

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where M_Λ is the mass of Λ, and the momentum fractions are defined as

$$z_h = \frac{2P_h \cdot q}{Q^2} \approx \frac{P_h^+}{q^+} \equiv z, \tag{2.6}$$

$$z_\Lambda = \frac{2P_\Lambda \cdot q}{Q^2} \approx \frac{P_\Lambda^-}{q^-} \equiv \bar{z}, \tag{2.7}$$

and

$$-q_T^2 = Q_T^2 \ll Q^2. \tag{2.8}$$

In this system, for any 4-vector, we can obtain the components orthogonal to $n_\pm$ using

$$g_T^{\mu\nu} = g^{\mu\nu} - n_+^\mu n_-^\nu - n_+^\nu n_-^\mu, \tag{2.9}$$

$$\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} n_{+\rho} n_{-\sigma}, \tag{2.10}$$

where $g^{\mu\nu}$ is the metric tensor and we use the convention $\epsilon^{0123} = +1$.

The leptonic tensor, on the other hand, is usually evaluated in the coordinate system depicted in figure 1, where $\boldsymbol{P}_\Lambda = \boldsymbol{q}_\perp = 0$. In this system, we can again define normalized time-like and space like vectors along the dominant momenta

$$\hat{t} = \frac{q}{Q}, \tag{2.11}$$

$$\hat{v} = 2\frac{P_\Lambda}{\bar{z}Q} - \hat{t}\,. \tag{2.12}$$

The projections of the 4-vectors onto the two orthogonal $\perp$ directions can be obtained using the tensors

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \hat{t}^\mu \hat{t}^\nu + \hat{v}^\nu \hat{v}^\mu, \tag{2.13}$$

$$\epsilon_\perp^{\mu\nu} = -\epsilon^{\mu\nu\rho\sigma} \hat{t}_\rho \hat{v}_\sigma. \tag{2.14}$$

Interestingly, the two perpendicular projection tensors differ only by a factor of order Q_T/Q, since

$$g_\perp^{\mu\nu} = g_T^{\mu\nu} - \frac{n_+^\mu q_T^\nu + n_+^\nu q_T^\mu}{Q}\,. \tag{2.15}$$

Thus we can neglect the differences between the $_T$ and $_\perp$ components of the vectors in this work.

2.2 Cross section

The cross section for this process is given by the convolution of the leptonic and hadronic tensors

$$\frac{2P_1^0 2P_2^0 2P_\Lambda^0 d\sigma}{d^3\boldsymbol{P}_1 d^3\boldsymbol{P}_2 d^3\boldsymbol{P}_\Lambda} = \frac{\alpha_{em}^2}{Q^6} L_{\mu\nu} W_{(3h)}^{\mu\nu}, \tag{2.16}$$

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where $L_{\mu\nu}$ is the leptonic tensor, $W^{\mu\nu}_{(3h)}$ is the hadronic tensor, and α_{em} is the fine structure constant. The leptonic tensor can be evaluated using standard methods (e.g. [24]), yielding

$$L_{\mu\nu} = Q^2\Big[-2A(y)g^{\mu\nu}_{\perp} + 4B(y)\hat{v}^{\mu}\hat{v}^{\nu} - 4B(y)\Big(\hat{l}^{\mu}_{\perp}\hat{l}^{\nu}_{\perp} + \frac{1}{2}g^{\mu\nu}_{\perp}\Big) - 2C(y)B^{1/2}(y)\Big(\hat{v}^{\mu}\hat{l}^{\nu}_{\perp} + \hat{v}^{\nu}\hat{l}^{\mu}_{\perp}\Big)\Big], \tag{2.17}$$

where the normalized perpendicular part of the leptons's 4-momentum is defined as

$$\hat{l}^{\mu}_{\perp} = \frac{l^{\mu}_{\perp}}{|\boldsymbol{l}_{\perp}|}. \tag{2.18}$$

The usual coefficient functions are given by

$$A(y) = \frac{1}{2} - y + y^2, \tag{2.19}$$

$$B(y) = y(1-y), \tag{2.20}$$

$$C(y) = 1 - 2y, \tag{2.21}$$

where the lepton momentum fractions are defined as

$$y = \frac{P_h \cdot l}{P_h \cdot q} \approx \frac{l^-}{q^-} = \frac{1+\cos\theta_2}{2}. \tag{2.22}$$

Here θ_2 is the angle between the 3-momentum of the electron l and the $\hat{\boldsymbol{z}}$ axis in the center-of-mass frame,

The phase space factor can be then written in the conventional notation

$$d^9V \equiv \frac{d^3\boldsymbol{P}_1 d^3\boldsymbol{P}_2 d^3\boldsymbol{P}_{\Lambda}}{2P_1^0 2P_2^0 2P_{\Lambda}^0} = \frac{d^2\boldsymbol{P}_{h\perp}dz\ d\varphi_R dM_h^2 d\xi}{8z}\frac{Q^2\bar{z}d\bar{z}d\Omega_{\Lambda}}{8}, \tag{2.23}$$

where we used $\boldsymbol{P}_{\Lambda\perp} = 0$ and the leading order relations in the hard scale Q

$$P_h^- \ll P_h^+, \to E_h \sim \frac{1}{\sqrt{2}}P_h^+, \tag{2.24}$$

$$R^- \ll R^+, \to E_R \sim \frac{1}{\sqrt{2}}R^+, \tag{2.25}$$

$$\frac{R^+}{P_h^+} = \xi - 1/2 \sim \frac{E_R}{E_h}, \tag{2.26}$$

and we replaced $|\boldsymbol{R}_T|$ with the invariant mass of the hadron pair M_h using the relation

$$\boldsymbol{R}_T^2 = \xi(1-\xi)M_h^2 - M_1^2(1-\xi) - M_2^2\xi. \tag{2.27}$$

The spherical angle of the Λ hadron can be expressed in terms of y and the azimuthal angle of $\boldsymbol{l}_{\perp}$ as

$$d\Omega_{\Lambda} = 2dyd\varphi_L, \tag{2.28}$$

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yielding

$$d^9V = \frac{Q^2}{32}\frac{\bar{z}}{z} d^2\boldsymbol{P}_{h\perp} dz\; d\varphi_R\; dM_h^2\; d\xi\; d\bar{z}\; dy\; d\varphi_L \tag{2.29}$$

$$= z\bar{z}\frac{Q^2}{32} d^2\boldsymbol{q}_T\; dz\; d\varphi_R\; dM_h^2\; d\xi\; d\bar{z}\; dy\; d\varphi_L\,, \tag{2.30}$$

where in the last expression we substituted $\boldsymbol{P}_{h\perp}$ with the transverse component of the intermediate virtual photon's 3-momentum in the light-cone frame $\boldsymbol{q}_T = -\boldsymbol{P}_{h\perp}/z$.

The hadronic tensor is defined as

$$W^{\mu\nu}_{(3h)}(q; P_h, R, P_\Lambda) = \frac{1}{(2\pi)^7}\sum_X \int \frac{d^3\boldsymbol{P}_X}{(2\pi)^3 2P_X^0}\; (2\pi)^4\; \delta(q - P_X - P_h - P_\Lambda) \tag{2.31}$$

$$\times \langle 0|J^\mu(0)|P_X; P_h, R, P_\Lambda\rangle\langle P_X; P_h, R, P_\Lambda|J^\nu(0)|0\rangle\,.$$

Using the parton picture, we can decompose the hadronic tensor in terms of the quark-quark correlators Δ and $\bar{\Delta}$ for the production of a hadron pair and a hyperon in the fragmentation of the quark and the antiquark with momenta k and $\bar{k}$, respectively

$$W^{\mu\nu}_{(3h)} \approx \frac{3}{(2\pi)^3}\sum_a e_a^2 \int d^4k\; d^4\bar{k}\; \delta^4(q - k - \bar{k})\mathrm{Tr}\Big[\bar{\Delta}(\bar{k}, P_\Lambda)\gamma^\mu\Delta(k, P_h, R)\gamma^\mu\Big], \tag{2.32}$$

where a and e_a denote the flavor and the fractional electric charge of the fragmenting quark, while the pre factor is the number of quark colors $N_c = 3$. We can then integrate the k^+ and $\bar{k}^-$ components using the δ function and rewrite the hadronic tensor as a convolution of the conventional integrated quark-quark correlators

$$W^{\mu\nu}_{(3h)} \approx \frac{3(32z)(4\bar{z})}{(2\pi)^3}\sum_a e_a^2 \int d^2\boldsymbol{k}_T d^2\bar{\boldsymbol{k}}_T\; \delta^2(\boldsymbol{q}_T - \boldsymbol{k}_T - \bar{\boldsymbol{k}}_T) \tag{2.33}$$

$$\times \mathrm{Tr}\Big[\bar{\Delta}(\bar{z}, \bar{\boldsymbol{k}}_T)_{\bar{k}^- = P_\Lambda^-/\bar{z}}\gamma^\mu\Delta(z, \xi, \boldsymbol{k}_T, \boldsymbol{R}_T)_{k^+ = P_h^+/z}\gamma^\mu\Big],$$

where the light-cone momentum fractions for the dihadron correlators are defined in terms of those for the individual hadrons,

$$z = z_1 + z_2, \tag{2.34}$$

$$\xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}\,, \tag{2.35}$$

where $z_i = P_i^+/k^+$ are the light-cone momentum fractions of each hadron in the pair.

The two-hadron fragmentation of a quark is described by a quark-quark correlator [22, 26–28]

$$\Delta_{ij}(k; P_h, R) = \sum_X \int d^4\zeta e^{ik\cdot\zeta}\langle 0|\psi_i(\zeta)|P_h R, X\rangle\langle P_h R, X|\bar{\psi}_j(0)|0\rangle, \tag{2.36}$$

which, for the case of an unpolarized hadron pair and at the leading twist approximation, is parametrized via four DiFFs

$$\frac{1}{32z}\int dk^-\Delta(k, P_h, R)|_{k^+ = P_h^+/z} \equiv \Delta(z, \xi, \boldsymbol{k}_T, \boldsymbol{R}_T) \tag{2.37}$$

$$= \frac{1}{4\pi}\frac{1}{4}\left\{D_1\not{n}_+ - G_1^\perp\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_T^\rho R_T^\sigma}{M_h^2}\gamma_5 + H_1^\sphericalangle\frac{\sigma_{\mu\nu}R_T^\mu n_+^\nu}{M_h} + H_1^\perp\frac{\sigma_{\mu\nu}k_T^\mu n_+^\nu}{M_h}\right\}.$$

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Here D_1 is the unpolarized DiFF, while the helicity-dependent DiFF, $G_1^\perp$, describes the correlation of the fragmenting quark's longitudinal polarization with the vector product of the transverse momenta of the two hadrons. The two remaining DiFFs, $H_1^\sphericalangle$ (IFF) and $H_1^\perp$, describe the correlations between the quark transverse polarization with the relative and total transverse momenta of the pair, respectively. The DiFFs here are defined in a frame where their total momentum 3-momentum is along the $\hat{z}$ axis, and we can choose as two transverse vectors $\boldsymbol{R}_T$ and $\boldsymbol{k}_T$, which are the transverse components of R and k in this system. We can relate $\boldsymbol{k}_T = -\boldsymbol{P}_\perp/z$ to the transverse component of P in a frame where k has no transverse component. In general, DiFFs depend on $z, \xi, \boldsymbol{k}_T, \boldsymbol{R}_T$. It is worth noting, that only $\boldsymbol{k}_T \cdot \boldsymbol{R}_T$ correlations are possible for our scalar functions and so the azimuthal dependence of the DiFFs can be expanded in a Fourier cosine series

$$D_1(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \cos(\varphi_{KR})) = \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\cos(n \cdot \varphi_{KR})}{1 + \delta^{0,n}} \; D_1^{[n]}(z, \xi, |\boldsymbol{k}_T|, |\boldsymbol{R}_T|)\,, \tag{2.38}$$

where $\varphi_{KR} \equiv \varphi_k - \varphi_R$ is the relative azimuthal angle between $\boldsymbol{k}_T$ and $\boldsymbol{R}_T$.

For single polarized hadron production the quark-quark correlator now depends on the momentum of the fragmenting quark k, and the momentum P_Λ and polarization $\boldsymbol{S}_\Lambda$ of the produced hadron. Once again we can integrate over the large component of the light-cone momentum

$$\frac{1}{4z} \int dk^+ \Delta(k, P_\Lambda, \boldsymbol{S}_\Lambda)_{k^- = P_\Lambda^-/z} \equiv \Delta(z, \boldsymbol{k}_T, \boldsymbol{S}_\Lambda)\,, \tag{2.39}$$

where $\boldsymbol{k}_T$ is the transverse component of the fragmenting quark's momentum in a system with the $\hat{z}$ axis pointed along the 3-momentum $\boldsymbol{P}_\Lambda$. Following the notation in ref. [29], we can expand the correlator in terms of the relevant leading-twist transverse momentum dependent (TMD) FFs

$$\begin{aligned}\Delta(z, \boldsymbol{k}_T, \boldsymbol{S}_\Lambda) = \frac{1}{4}\Bigg\{ & D_1 \not{n}_+ + D_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^\mu n_+^\nu k_T^\rho S_T^\sigma}{M_h} \\ & - \left(\lambda_\Lambda G_{1L} + G_{1T} \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{M_h}\right) \not{n}_+ \gamma_5 - H_{1T}\, i\sigma_{\mu\nu} S_T^\mu n_+^\nu \gamma_5 \\ & - \left(\lambda_\Lambda H_{1L}^\perp + H_{1T}^\perp \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{M_h}\right) i\sigma_{\mu\nu} k_T^\mu n_+^\nu \gamma_5 + H_1^\perp \frac{\sigma_{\mu\nu} k_T^\mu n_+^\nu}{M_h} \Bigg\},\end{aligned} \tag{2.40}$$

where λ_Λ and $\boldsymbol{S}_T$ are the helicity and the transverse polarization vectors of the produced Λ hadron. The FFs are functions of z and $\boldsymbol{P}_{\Lambda\perp}^2$. Here $\boldsymbol{P}_{\Lambda\perp} = -z\boldsymbol{k}_T$ is the transverse momentum of the produced hadron in a system where the 3-momentum of the quark $\boldsymbol{k}$ has no transverse component.

Finally, we can calculate the cross section

$$\begin{aligned}\frac{d\sigma\Big(e^+e^- \to (h_1h_2) + \Lambda + X\Big)}{d^2\boldsymbol{q}_T\; dz\; d\varphi_R\; dM_h^2\; d\xi\; d\bar{z}\; dy} = & \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} z^2 \bar{z}^2 \sum_a e_a^2 \\ & \times \Bigg\{ A(y)\; \mathcal{F}\Big[D_1^{a\to h_1h_2} D_1^{\bar{a}\to\Lambda}\Big] \\ & - S_T A(y)\; \mathcal{F}\left[\frac{\bar{k}_T}{M_\Lambda} \sin(\varphi_{\bar{k}} - \varphi_S)\; D_1^{a\to h_1h_2} D_{1T}^{\perp,\bar{a}\to\Lambda}\right]\end{aligned} \tag{2.41}$$

$$
\begin{aligned}
&+ \lambda_\Lambda A(y)\ \mathcal{F}\left[\frac{k_T R_T}{M_h^2}\sin(\varphi_k-\varphi_R)\ G_1^{\perp,a\to h_1h_2}G_{1L}^{\bar a\to\Lambda}\right]\\
&+ S_T A(y)\ \mathcal{F}\left[\frac{k_T R_T}{M_h^2}\sin(\varphi_k-\varphi_R)\frac{\bar k_T}{M_\Lambda}\cos(\varphi_{\bar k}-\varphi_S)\ G_1^{\perp,a\to h_1h_2}G_{1T}^{\bar a\to\Lambda}\right]\\
&+ S_T B(y)\ \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\sin(\varphi_k+\varphi_S)H_1^{\perp,a\to h_1h_2}\\
&\qquad\qquad + \frac{R_T}{M_h}\sin(\varphi_R+\varphi_S)H_1^{\sphericalangle,a\to h_1h_2}\Bigg)\ H_{1T}^{\bar a\to\Lambda}\Bigg]\\
&+ \lambda_\Lambda B(y)\ \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\sin(\varphi_k+\varphi_{\bar k})H_1^{\perp,a\to h_1h_2}\\
&\qquad\qquad + \frac{R_T}{M_h}\sin(\varphi_R+\varphi_{\bar k})H_1^{\sphericalangle,a\to h_1h_2}\Bigg)\ \frac{\bar k_T}{M_\Lambda}H_{1L}^{\perp,\bar a\to\Lambda}\Bigg]\\
&+ S_T B(y)\ \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\sin(\varphi_k+\varphi_{\bar k})H_1^{\perp,a\to h_1h_2}\\
&\qquad\qquad + \frac{R_T}{M_h}\sin(\varphi_R+\varphi_{\bar k})H_1^{\sphericalangle,a\to h_1h_2}\Bigg)\ \frac{\bar k_T^2}{M_\Lambda^2}\cos(\varphi_{\bar k}-\varphi_S)H_{1T}^{\perp,\bar a\to\Lambda}\Bigg]\\
&+ B(y)\ \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\cos(\varphi_k+\varphi_{\bar k})H_1^{\perp,a\to h_1h_2}\\
&\qquad\qquad + \frac{R_T}{M_h}\cos(\varphi_R+\varphi_{\bar k})H_1^{\sphericalangle,a\to h_1h_2}\Big)\ \frac{\bar k_T}{M_\Lambda}H_1^{\perp,\bar a\to\Lambda}\Bigg]\Bigg\},
\end{aligned}
$$

where φ_S is the azimuthal angle of $\boldsymbol{S}_T$ and we have integrated over the trivial dependence on the azimuthal angle of the lepton plane φ_L. The transverse momentum convolution, $\mathcal{F}$, is defined as

$$
\begin{aligned}
&\mathcal{F}[wD^a\bar D^{\bar a}] \qquad (2.42)\\
&\quad=\int d^2\boldsymbol{k}_T d^2\bar{\boldsymbol{k}}_T\delta^2(\boldsymbol{k}_T+\bar{\boldsymbol{k}}_T-\boldsymbol{q}_T)w(\boldsymbol{k}_T,\bar{\boldsymbol{k}}_T,\boldsymbol{R}_T,\bar{\boldsymbol{R}}_T)D^a(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)D^{\bar a}(\bar z,\bar{\boldsymbol{k}}_T^2)\,.
\end{aligned}
$$

Integrating the cross section over $\boldsymbol{q}_T$, φ_R and ξ yields the unpolarized "collinear" cross section

$$
\frac{d\sigma\Big(e^+e^-\to(h_1h_2)+\Lambda+X\Big)}{dz\ dM_h^2\ d\bar z\ dy}=\frac{3\alpha_{em}^2}{(2\pi)^2Q^2}A(y)\sum_a e_a^2\ D_1^{a\to h_1h_2}(z,M_h^2)\ \bar D_1^{\bar a\to\Lambda}(\bar z), \quad (2.43)
$$

where

$$
\begin{aligned}
D_1^a(z,M_h^2)&=z^2\int d^2\boldsymbol{k}_T\int d\xi\int d\varphi_R\, D_1^a(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \qquad (2.44)\\
&=z^2\int d^2\boldsymbol{k}_T\int d\xi\, D_1^{a,[0]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|),
\end{aligned}
$$

and

$$
D_1^{\bar a}(z)=\bar z^2\int d^2\bar{\boldsymbol{k}}_T D_1^{\bar a}(\bar z,\bar{\boldsymbol{k}}_T^2). \qquad (2.45)
$$

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Further integrating the result in eq. (2.43) over M_h^2 we recover the same expression as for two back-to-back hadron production in eq. (77) of ref. [24]. This is natural, as the integrated DIFF

$$D_1^{a\to h_1h_2}(z) \equiv \int dM_h^2\; D_1^{a\to h_1h_2}(z, M_h^2), \tag{2.46}$$

can be thought of describing the collinear fragmentation into a "parent" particle that carries the total light-cone momentum fraction z of the pair h_1h_2.

The cross sections of the measurements of a hadron pair in the same hemisphere, such as those recently performed by the BELLE collaboration [5, 30], involve a sum over all the fragmenting quark flavors. Thus, leveraging the relatively well-determined single hadron FFs in a combined analysis of the single hadron pair cross section with that in eq. (2.43) would enable the flavor separation of the unpolarized DiFFs.

For example, for DiFF into $\pi^+\pi^-$ pairs, the isospin and charge symmetries entail

$$D_1^{u\to\pi^+\pi^-} = D_1^{\bar{u}\to\pi^+\pi^-} \approx D_1^{d\to\pi^+\pi^-} = D_1^{\bar{d}\to\pi^+\pi^-}, \tag{2.47}$$

$$D_1^{s\to\pi^+\pi^-} = D_1^{\bar{s}\to\pi^+\pi^-}. \tag{2.48}$$

For the one pair inclusive production, omitting the contributions of all the heavier flavors, we can approximate

$$d\sigma(e^+e^-\to(h_1h_2)+X) \sim \sum_q e_q^2\; D_1^{q\to\pi^+\pi^-} \approx \frac{5}{9}D_1^{u\to\pi^+\pi^-}(z) + \frac{1}{9}D_1^{s\to\pi^+\pi^-}(z)\,. \tag{2.49}$$

Hence the separation of the light and strange quark DiFF contributions is not possible without additional input. For the associated hadron production in the opposite hemisphere, the cross section can be approximated as

$$d\sigma(e^+e^-\to(h_1h_2)+\pi^++X) \sim \frac{5}{9}D_1^{u\to\pi^+\pi^-}(z)D_1^{u^+\to\pi^+}(\bar{z}) + \frac{1}{9}D_1^{s\to\pi^+\pi^-}(z)D_1^{s^+\to\pi^+}(\bar{z}), \tag{2.50}$$

where

$$D_1^{q^+\to h}(\bar{z}) \equiv D_1^{q\to h}(\bar{z}) + D_1^{\bar{q}\to h}(\bar{z}). \tag{2.51}$$

The unpolarized FFs to charged pions have been phenomenologically extracted from experimental measurement by a number of different groups [31–35]. Thus the measurement of the cross section in eq. (2.50) would allow us to extract the flavor dependence of the DiFF $D_1^{q\to\pi^+\pi^-}$. Similar arguments can be used to explore other hadron channels and combinations. For example, choosing a π^+K^+ pair on one side and a K^- hadron on the other side would significantly enhance the "double favored" channel ($\bar{s}\to\pi^+K^+, s\to K^-$) for moderate to large values of z_1, z_2, and $\bar{z}$, as can also be seen in hadronization models [36–39]. The contributions of all the other quark flavors would include at least one "unfavored" fragmentation, which will be suppressed for our chosen region of light-cone momentum fractions. Thus, such a measurement would allow us to extract a DiFF for a specific quark flavor, $\bar{s}\to\pi^+K^+$. In principle, a global fit, using all possible measurements of the single and dihadron fragmentations would yield the best estimates of these functions, with the most realistic uncertainties.

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3 The azimuthal asymmetries

In this section we will discuss several asymmetries that give access to various combinations of polarization-dependent FFs and DiFFs, that can be measured at BELLE and the upcoming BELLE II experiments, similar to the measurements of the asymmetries for back-to-back hadrons [28, 40] and back-to-back hadron pairs [22, 23].

We define the integral of the cross section in eq. (2.41), weighted with an arbitrary function $\mathcal{I}$, as

$$\langle \mathcal{I} \rangle \equiv \int d\xi \int d\varphi_R \int d^2\boldsymbol{q}_T \, \mathcal{I} \, \frac{d\sigma\Big(e^+e^- \to (h_1h_2) + \Lambda + X\Big)}{d^2\boldsymbol{q}_T \, dz \, d\varphi_R \, dM_h^2 \, d\xi \, d\bar{z} \, dy}, \tag{3.1}$$

where the unpolarized integrated cross section in eq. (2.50) is simply

$$\frac{d\sigma\Big(e^+e^- \to (h_1h_2) + \Lambda + X\Big)}{dz \, dM_h^2 \, d\bar{z} \, dy} = \langle 1 \rangle. \tag{3.2}$$

3.1 The asymmetry induced by the Collins effect for unpolarized Λ

For an unpolarized Λ, only the first and the last terms in eq. (2.41) contribute. In addition to the unpolarized term, the correlation of the transverse polarizations of the fragmenting quark and antiquark pair is described by the convolution of the Collins function $H_1^{\perp,\bar{a}\to\Lambda}$ for a Λ with the two dihadron analogues $H_1^{\perp,a\to h_1h_2}, H_1^{\sphericalangle,a\to h_1h_2}$ for the hadron pair h_1h_2. Thus, we can access this term by considering the following weighted average

$$\begin{aligned}\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} \frac{B(y)}{M_\Lambda^2 M_h} \\ \times \sum_a e_a^2 \int d\xi \int d\varphi_R \int d^2\boldsymbol{q}_T \int d^2\boldsymbol{k}_T \int d^2\bar{\boldsymbol{k}}_T \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) q_T \cos(\varphi_q + \varphi_R) \\ \times \Big[\Big(k_T \bar{k}_T \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp,a\to h_1h_2} + R_T \bar{k}_T \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle,a\to h_1h_2} \Big) H_1^{\perp,\bar{a}\to\Lambda} \Big],\end{aligned} \tag{3.3}$$

where we can use the δ function to write

$$\int d^2\boldsymbol{q}_T \; \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) \; q_T \cos(\varphi_q + \varphi_R) = (k_T \cos(\varphi_k + \varphi_R) + \bar{k}_T \cos(\varphi_{\bar{k}} + \varphi_R)). \tag{3.4}$$

We further use the Fourier cosine decomposition for the DiFFs in eq. (2.38) to conclude

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle,a\to h_1h_2}(z, M_h^2) \; H_1^{\perp\bar{a},[1]}(\bar{z}), \tag{3.5}$$

noting that

$$\begin{aligned}H_1^{\sphericalangle,a\to h_1h_2}(z, M_h^2) \\ \equiv z^2 \int d^2\boldsymbol{k}_T \int d\xi \left[\frac{|\boldsymbol{R}_T|}{M_h} H_1^{\sphericalangle,[0]}(z, \xi, |\boldsymbol{k}_T|, |\boldsymbol{R}_T|) + \frac{|\boldsymbol{k}_T|}{M_h} H_1^{\perp,[1]}(z, \xi, |\boldsymbol{k}_T|, |\boldsymbol{R}_T|) \right],\end{aligned} \tag{3.6}$$

is the same integrated IFF that enters in SIDIS [17] and in e^+e^- two hadron pair [23] asymmetries, while

$$H_1^{\perp\bar{a},[1]}(\bar{z}) \equiv \bar{z}^2 \int d^2\bar{\boldsymbol{k}}_T \, \frac{\bar{\boldsymbol{k}}_T^2}{2M_\Lambda^2} \, H_1^{\perp,\bar{a}\to\Lambda}(z,\bar{\boldsymbol{k}}_T^2), \tag{3.7}$$

is the first moment of the Collins function.

The corresponding weighted azimuthal asymmetry then takes a simple form

$$A^{Coll} = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \, H_1^{\sphericalangle,a\to h_1h_2}(z,M_h^2) \, H_1^{\perp\bar{a},[1]}(\bar{z})}{\sum_a e_a^2 \, D_1^{a\to h_1h_2}(z,M_h^2) \, \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}. \tag{3.8}$$

3.2 The asymmetries for a longitudinally polarized Λ

To devise the measurements involving the polarization of the produced Λ, it is important to underline that the cross section expression in eq. (2.41) is a conditional probability of the process for the given polarization $\boldsymbol{S}_\Lambda$. We have no a priory knowledge of this polarization, and it cannot be measured directly on an event-by-event basis. Thus, we need to extract the polarization $\boldsymbol{s}_\Lambda$ that Λ acquires in the back-to-back fragmentation process, which can be measured in experiment by considering the average angular distributions of the final state particles. Employing the spin density matrix formalism, we can infer from general considerations [29, 41, 42], that the cross section of this process can be expressed as

$$\frac{d\sigma}{dV} = \alpha + \boldsymbol{\beta}\cdot\boldsymbol{S}_\Lambda \sim 1 + \boldsymbol{S}_\Lambda\cdot\boldsymbol{s}_\Lambda. \tag{3.9}$$

Thus, the acquired polarization vector is simply given by

$$\boldsymbol{s}_\Lambda = \frac{\boldsymbol{\beta}}{\alpha}, \tag{3.10}$$

where α and $\boldsymbol{\beta}$ are only functions of the momentum of Λ and the polarization of the fragmenting quark. The latter in our work is expressed using the correlations with the momenta of h_1, h_2, so that the acquired polarization $\boldsymbol{s}_\Lambda$ only depends on the momenta of the observed particles. Further, it is clear from eq. (3.9), that the average of the acquired polarization over the kinematic variables is given by

$$\langle\boldsymbol{s}_\Lambda\rangle = \frac{\langle\boldsymbol{\beta}\rangle}{\langle\alpha\rangle}, \tag{3.11}$$

as the probability of the process itself is given by α. The coefficients α and $\boldsymbol{\beta}$ can be easily read off directly from eq. (2.41). For example, α corresponds to the first and the last terms on the right hand side of eq. (2.41), while the longitudinal part β_L of the coefficient $\boldsymbol{\beta}$ is given by the terms multiplied by λ_Λ. In section 4 we present an example of a measurement for the longitudinal polarization of Λ^0 baryon.

Let us consider the correlations for the longitudinally polarized Λ that are manifest in two terms in eq. (2.41). The term that involves $G_1^{\perp,a\to h_1h_2}$ and $G_{1L}^{\bar{a}\to\Lambda}$ describes the

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correlations between the longitudinal polarizations of the fragmenting quark and anti-quark. We can access the helicity of Λ by considering the weighted average

$$\langle \beta_L \rangle_{G_1^\perp G_{1L}} = \left\langle \frac{q_T}{M_h} \sin(\varphi_q - \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} A(y) \sum_a e_a^2 \; G_1^{\perp,a\to h_1h_2}(z, M_h^2) \; G_{1L}^{\bar{a}\to\Lambda}(\bar{z}), \tag{3.12}$$

where

$$\begin{aligned} &G_1^{\perp,a\to h_1h_2}(z, M_h^2) \\ &\quad \equiv z^2 \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{2M_h^2} \int d\xi \frac{R_T}{M_h} \Big(G_1^{\perp,[0]}(z, \xi, |\boldsymbol{k}_T|, |\boldsymbol{R}_T|) - G_1^{\perp,[2]}(z, \xi, |\boldsymbol{k}_T|, |\boldsymbol{R}_T|) \Big), \end{aligned} \tag{3.13}$$

is the same integrated helicity-dependent DiFF as in refs. [20, 23] and

$$G_{1L}^{\bar{a}\to\Lambda}(\bar{z}) \equiv \bar{z}^2 \int d^2\bar{\boldsymbol{k}}_T \; G_{1L}^{\bar{a}\to\Lambda}(\bar{z}, \bar{\boldsymbol{k}}_T^2), \tag{3.14}$$

is the integrated helicity FF. Thus, the acquired longitudinal polarization s_L of Λ is

$$\langle s_L \rangle^{\sin(\varphi_q - \varphi_R)} (z, M_h^2, \bar{z}, y) = \frac{\sum_a e_a^2 \; G_1^{\perp,a\to h_1h_2}(z, M_h^2) \; G_{1L}^{\bar{a}\to\Lambda}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}, \tag{3.15}$$

A second contribution is acquired by the correlation of the transverse polarizations of the quark and the anti quark, which couples the two analogues of Collin DiFFs on one side with the "Kotzinian-Mulders" type FF. The relevant weighting for this term is

$$\langle \beta_L \rangle_{H_1^\sphericalangle H_{1L}^\perp} = \left\langle \frac{q_T}{M_\Lambda} \sin(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle,a\to h_1h_2}(z, M_h^2) \; H_{1L}^{\perp\bar{a},[1]}(\bar{z}), \tag{3.16}$$

where

$$H_{1L}^{\perp\bar{a},[1]}(\bar{z}) \equiv \bar{z}^2 \int d^2\bar{\boldsymbol{k}}_T \; \frac{\bar{\boldsymbol{k}}_T^2}{2M_\Lambda^2} \; H_{1L}^{\perp,\bar{a}\to\Lambda}(z, \bar{\boldsymbol{k}}_T^2). \tag{3.17}$$

The corresponding acquired helicity is

$$\langle s_L \rangle^{\sin(\varphi_q + \varphi_R)} (z, M_h^2, \bar{z}, y) = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \; H_1^{\sphericalangle,a\to h_1h_2}(z, M_h^2) \; H_{1L}^{\perp\bar{a},[1]}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}. \tag{3.18}$$

3.3 The asymmetries for a transversely polarized Λ

Finally, we extract the correlations for the transverse polarization of Λ. The are four relevant terms contributing to the transverse part of $\boldsymbol{\beta}_T$ in eq. (2.41). Here we can proceed by considering the $\hat{x}$ and $\hat{y}$ components of this vector, labelled β_x and β_y, respectively.

The transverse polarization acquired by the "polarizing" FF, $D_{1T}^\perp$ mixes with the contribution involving the transversity- and pretzelocity-like FFs of Λ.

$$\begin{aligned} \langle \beta_x \rangle^{\sin(\varphi_q)} &= \langle q_T \sin(\varphi_q) \rangle \\ &= \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} \sum_a e_a^2 \Big\{ - A(y) M_\Lambda D_1^{a\to h_1h_2}(z, M_h^2) \; D_{1T}^{\perp\bar{a},[1]}(\bar{z}) \\ &\qquad + B(y) M_h H_1^{\perp,a\to h_1h_2}(z, M_h^2) \; H_1^{\bar{a}\to\Lambda}(\bar{z}) \Big\}, \end{aligned} \tag{3.19}$$

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$$\langle\beta_y\rangle^{\cos(\varphi_q)} = \langle q_T\cos(\varphi_q)\rangle$$
$$= \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2}\sum_a e_a^2\Big\{A(y)M_\Lambda D_1^{a\to h_1h_2}(z,M_h^2)\ D_{1T}^{\perp\bar{a},[1]}(\bar{z}) + B(y)M_h H_1^{\perp,a\to h_1h_2}(z,M_h^2)\ H_1^{\bar{a}\to\Lambda}(\bar{z})\Big\}, \quad (3.20)$$

where

$$H_1^{\perp,a\to h_1h_2}(z,M_h^2) \quad (3.21)$$
$$\equiv z^2\int d^2\boldsymbol{k}_T\int d\xi\left[\frac{|\boldsymbol{k}_T||\boldsymbol{R}_T|}{2M_h^2}H_1^{\sphericalangle,[1]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)+\frac{|\boldsymbol{k}_T|^2}{2M_h^2}H_1^{\perp,[0]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)\right],$$

and

$$D_{1T}^{\perp\bar{a},[1]}(\bar{z}) \equiv \bar{z}^2\int d^2\bar{\boldsymbol{k}}_T\ \frac{\bar{\boldsymbol{k}}_T^2}{2M_\Lambda^2}D_{1T}^{\perp\bar{a},[1]}(\bar{z},\bar{\boldsymbol{k}}_T^2), \quad (3.22)$$
$$H_1^{\bar{a}}(\bar{z}) \equiv \bar{z}^2\int d^2\bar{\boldsymbol{k}}_T\ \left(H_{1T}^{\bar{a}}(\bar{z},\bar{\boldsymbol{k}}_T^2)+\frac{\bar{\boldsymbol{k}}_T^2}{2M_\Lambda^2}H_{1T}^{\perp\bar{a}}(\bar{z},\bar{\boldsymbol{k}}_T^2)\right). \quad (3.23)$$

This is analogous to the associated Λ production asymmetry proposed in ref. [19] and the additional q_T weighting in our case allows us to disentangle the convolution of Λ FFs and DiFFs into "collinear" products of the corresponding moments. Here, the contributions involving the chiral-odd FFs, ignored in [19], enter with different relative sign to those containing the "polarizing" FF. Thus, we can form linear combinations of the two terms to access the contributions from the individual structure functions

$$\langle\beta_y\rangle^{\cos(\varphi_q)} - \langle\beta_x\rangle^{\sin(\varphi_q)} = M_\Lambda\frac{3\alpha_{em}^2}{2\pi^2Q^2}A(y)\sum_a e_a^2 D_1^{a\to h_1h_2}(z,M_h^2)\ D_{1T}^{\perp\bar{a},[1]}(\bar{z}), \quad (3.24)$$

$$\langle\beta_y\rangle^{\cos(\varphi_q)} + \langle\beta_x\rangle^{\sin(\varphi_q)} = M_h\frac{3\alpha_{em}^2}{2\pi^2Q^2}B(y)\sum_a e_a^2 H_1^{\perp,a\to h_1h_2}(z,M_h^2)\ H_1^{\bar{a}\to\Lambda}(\bar{z}), \quad (3.25)$$

and the corresponding polarizations

$$\frac{\langle s_y\rangle^{\cos(\varphi_q)}(z,M_h^2,\bar{z},y) - \langle s_x\rangle^{\sin(\varphi_q)}(z,M_h^2,\bar{z},y)}{M_\Lambda} = 2\frac{\sum_a e_a^2 D_1^{a\to h_1h_2}(z,M_h^2)\ D_{1T}^{\perp\bar{a},[1]}(\bar{z})}{\sum_a e_a^2 D_1^{a\to h_1h_2}(z,M_h^2)\ \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}, \quad (3.26)$$

$$\frac{\langle s_y\rangle^{\cos(\varphi_q)}(z,M_h^2,\bar{z},y) + \langle s_x\rangle^{\sin(\varphi_q)}(z,M_h^2,\bar{z},y)}{M_h} = \frac{2B(y)}{A(y)}\frac{\sum_a e_a^2 H_1^{\perp,a\to h_1h_2}(z,M_h^2)\ H_1^{\bar{a}\to\Lambda}(\bar{z})}{\sum_a e_a^2 D_1^{a\to h_1h_2}(z,M_h^2)\ \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}. \quad (3.27)$$

It is important to stress, that here we are operating with the measured polarizations along the $\hat{x}$ and $\hat{y}$.

The contribution from the term involving $G_1^{\perp,a\to h_1h_2}$ and another "worm-gear" type FF, $G_{1T}^{\bar{a}}$, which describes the longitudinal quark antiquark polarization correlations, also

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admixes with the contributions from several other terms when calculating the weighted moments, for example with a weight $\langle q_T^2 \sin(\varphi_q - \varphi_R)\cos(\varphi_q)\rangle$. Here, we omit the resulting long and convoluted expression (even by the standards of this manuscript) for brevity.

The last contribution to the transverse polarization can be obtained using weights that only involve the azimuthal angle φ_R and no additional factors of q_T

$$\langle \boldsymbol{\beta}_x \rangle_{H_1^\sphericalangle H_1}^{\sin(\varphi_R)} = \langle \boldsymbol{\beta}_y \rangle_{H_1^\sphericalangle H_1}^{\cos(\varphi_R)} = \frac{3\alpha_{em}^2}{8\pi^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\bar{a}\to\Lambda}(\bar{z}). \tag{3.28}$$

The corresponding acquired transverse polarization is

$$\begin{aligned} \langle \boldsymbol{s}_T \rangle_x^{\sin(\varphi_R)}(z, M_h^2, \bar{z}, y) &= \langle \boldsymbol{s}_T \rangle_y^{\cos(\varphi_R)}(z, M_h^2, \bar{z}, y) \\ &= \frac{1}{2}\frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\bar{a}}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}. \end{aligned} \tag{3.29}$$

This is similar to the IFF asymmetry in SIDIS, where the IFF is multiplied by the integrated transversity PDF. Here, similar to the integrated transversity PDF, the polarized FF H_1 is a combination of the TMD "transversity" FF H_{1T} and TMD "pretzelocity" $H_{1T}^\perp$, see eq. (3.23).

4 Treatment of the polarized Λ^0

In this section we will elaborate the treatment of the measurements involving polarized Λ^0 production, where either the longitudinal or the transverse polarization is leveraged to access the relevant structure functions in the cross section. The determination of the final state polarization is possible for this hyperon, as it undergoes "self-analyzing" weak decays into a baryon-meson pair. The two hadronic decay channels into $p+\pi^-$ and $n+\pi^0$ have relative branching ratios of 64% and 36%, respectively. An important aspect of such decays is the correlation between the polarization vector of the decaying hyperon and the produced hadron's momentum. In particular, taking the $p+\pi^-$ channel as an example, the decay rate is described as

$$\frac{dN}{N d\cos\theta} \sim 1 + \alpha_\Lambda S_\Lambda \cos(\theta), \tag{4.1}$$

where S_Λ is the modulus of the polarization of the Λ^0, and θ is the angle between the decay proton momentum and the Λ^0 polarization vector in its rest frame. For the longitudinal polarization of the Λ^0 this angle is calculated along the direction of the $\hat{z}$ axis defined in section I, figure 1, as we can simply boost along $\hat{z}$ to the Λ^0 rest frame. The decay parameter α_Λ has been measured in a number of experiments in the 60's and 70's [43], with the Particle Data Group average value of $\alpha_\Lambda = 0.642$ [44], which describes the admixture of $s-$ and $p-$ partial waves of the p, π^- system [45].

The idea of using Λ^0's as a quark longitudinal spin polarimeter in SIDIS and $e^+e^- \to \Lambda^0 + X$ annihilation reactions was originally proposed in refs. [46–48] and for a quark transverse spin polarimeter in SIDIS in ref. [49]. Since then a number of experimental and

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phenomenological studies of hyperon polarization in these reactions have been performed, see for examples refs. [50–56]. It is worth mentioning that substantial longitudinal polarization of Λ^0 ($\lambda_\Lambda = -0.320$ for $z > 0.3$) was observed by ALEPH ref. [50], indicating the large analyzing power of this measurement. Similarly, the BELLE collaboration has recently measured a transverse polarization of the Λ^0 of order 0.1 via the so-called "polarizing" fragmentation function ref. [57, 58], observing significant analyzing power for the transverse polarization as well.

The important aspect of our proposed measurements is that the polarization of the Λ^0 couples with the azimuthal angles of the dihadron pair on the other side. On the other hand, the hyperon itself is not directly detected in the experiment, but rather is seen through its decay products. In such a decay, the polarization of the Λ^0 induces modulations of the angular distributions of the decay products. For example, for the helicity-dependent modulations involving the helicity λ_Λ, the polar angle of the decay product p will be modulated by $1 + \lambda_\Lambda \cos(\theta_p)$, where θ_p is the polar angle of the proton in the rest frame of Λ^0.

Thus, we need to consider the full observed final state in our process $e^+e^- \to (h_1h_2) + \Lambda^0 + X \to (h_1h_2) + (p + \pi^-) + X$. For extraction of the asymmetry in eq. (3.15), we should consider a weighted integral of the complete final state

$$\left\langle \cos(\theta_p) \frac{q_T}{M_h} \sin(\varphi_q - \varphi_R) \right\rangle \sim \alpha_\Lambda \; G_1^{\perp, a \to h_1 h_2} \; G_{1L}^{\bar{a} \to \Lambda}, \tag{4.2}$$

where the measurement of the weighted asymmetries with the detected final state hadrons now involves also the integration over the phase space of the proton. Similar expressions can also be derived for the components of the transverse polarization of Λ^0.

Such measurements should be possible with the much improved apparatus of the BELLE II Collaboration in the near future.

5 Conclusions

The study of the quark hadronization in e^+e^- experiments is a crucial part of the international efforts to map the 3-dimensional structure of the nucleon and to understand the spin-orbit correlations in the strong interactions. The measurements of the Collins effect and the two-hadron interference effect at BELLE [2–5] and BaBar [6] have been used to extract the nucleon transversity PDF.

The theoretical underpinning for such measurements of the Collins effect in two back-to-back hadron production was given by Boer and collaborators in refs. [24, 25, 28], where the relevant asymmetry involves convolution of two Collins FFs for the produced hadrons on each side. A measurement with a polarized hadron on one side and an unpolarized one on the other side was proposed in ref. [19] to test the universality of the T-odd "polarizing" FF entering the corresponding cross section. Recently, the work by [59] developed an approach for calculations of the polarized back-to-back hadron pair production in e^+e^- in an arbitrary frame and to arbitrary precision. The measurement of the spin-dependent DiFF was first proposed in ref. [22] and recently revised and corrected in ref. [23]. Here,

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the cross section for producing two back-to-back hadron pairs includes convolutions of two DiFFs, one for each pair involved. Note, that the unpolarized FFs can be measured in single inclusive hadron production, while the unpolarized DiFFs are measured by detecting two inclusive hadrons in the same jet [5].

In this work, we proposed a new inclusive measurement, where an unpolarized hadron pair is detected back-to-back with a single hadron, that may or may not be polarized. We derived the expression for the corresponding cross section in eq. (2.41), which involves convolutions of the DiFFs for the hadron pair with the FFs for the hadron on the other side. A number of new exciting measurements were then discussed. For example, the measurement of the unpolarized cross section will enable us to determine the flavor dependence of the DiFF using our knowledge of the ordinary unpolarized FFs, which is not possible with the other e^+e^- measurements. Moreover, we can measure a product of the Collins function of a meson with the IFF of the hadron pair eq. (3.8), that would provide additional information to constrain both functions. This is crucial in improving our knowledge of the transversity PDF from SIDIS measurements involving one and two hadron inclusive final states.

The measurement of the hadron polarization dependent asymmetries gives access to a wide variety of combinations of polarized spin 1/2 baryon FFs and polarized DiFFs, as discussed in section 3. Here we have eight TMD FFs for the baryon convoluted with four DiFFs for the unpolarized hadron pair, whereas in the case of two back-to-back dihadron pairs there are only the four DiFFs involved from each side [23]. Nonetheless, further developing the weighted asymmetry method we used in accessing the helicity-dependent DiFFs in ref. [20], we were able to access the individual combination of TMD FFs with DiFFs, disentangling their transverse momentum convolutions.

In section 3.2 we discussed a critical point for the measurements involving a polarized final state hadron. This polarization is acquired during the hadronization process, in correlation with the momenta of the observed final state particles. Using the spin density matrix formalism, we expressed the polarization vector of Λ by utilizing the eq. (2.41). Then the measurements of different components of this polarization when analyzed using a particular weighted azimuthal modulations allows us to access specific combinations of FFs with DiFFs. For example, the weighted average in eq. (3.15) extracts a product of the helicity-dependent DiFF $G_1^\perp$ with the "collinear" helicity FF G_{1L} when measuring the longitudinal polarization of the Λ. Such a measurement will complement the previously proposed measurements of $G_1^\perp$ in e^+e^- annihilation and SIDIS [20, 60], aimed at testing the universality of $G_1^\perp$. The transverse polarization-dependent modulation in eq. (3.29) yields a product of the collinear IFF with the collinear "transversity" FF. The last one is strikingly similar to SIDIS, where a similar modulation yields a product of the IFF with the transversity PDF [17].

Another interesting example is the measurement of the transverse polarization components of the hadron for the weighted azimuthal modulation in eq. (3.19), involving the components of the transverse momentum $\boldsymbol{q}_T$, which yields an admixture of two terms. One is the product of the unpolarized DiFF with the T-odd "polarizing" FF of Λ, while the second is the product of the Collins-like DiFF with transversity-like FF of Λ. This is an analogue of the associated production measurement proposed in ref. [19], where the contri-

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butions of the chiral-odd functions (such as Collins function and the transversity-like FF) were neglected. Note, that only the convolution of the chiral-odd unpolarized DiFF with the chiral-odd "polarizing" FF contribute to the perpendicular components of polarization to $\boldsymbol{P}_{h\perp}$, which was considered in Ref. [19]. Our results demonstrate that we can separate the chiral-odd and chiral-even contributions by forming linear combinations of the $x-$ and $y-$components of the transverse polarization.

The measurement of the baryon polarization was discussed in section 4. Such measurements for the Λ^0 hyperon through the "self-analyzing" weak decays have been long employed as polarimeters. Here, we discussed the relevant weighted asymmetries for the full detected final state particles to measure the relevant structure functions within the "narrow width approximation" for the treatment if the intermediate Λ^0 baryon, as described for the longitudinal polarization case in eq. (4.2).

This new process completes the set of all possible inclusive measurements with up to two produced hadrons on each side. It presents an exciting new opportunity for exploring the hadronization process using data already collected by the BELLE experiment, as well as that to be taken in the upcoming BELLE II experiment. A combined global analysis of these measurements along with the other inclusive hadron production reactions in electron-positron annihilation will allow us to acquire a good understanding of both single hadron and dihadron fragmentations.

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