Prog. Theor. Exp. Phys. **2021**, 023B01 (28 pages)
DOI: 10.1093/ptep/ptaa186

Renormalizable $SO(10)$ grand unified theory with suppressed dimension-5 proton decays

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Received August 11, 2020; Revised November 24, 2020; Accepted December 17, 2020; Published December 30, 2020

We study a renormalizable supersymmetric (SUSY) $SO(10)$ grand unified theory model where the Yukawa couplings of single **10**, single $\overline{\mathbf{126}}$, and single **120** fields (Y_{10}, Y_{126}, and Y_{120}) account for the quark and lepton Yukawa couplings and the neutrino mass. We pursue the possibility that Y_{10}, Y_{126}, and Y_{120} reproduce the correct quark and lepton masses, Cabibbo–Kobayashi–Maskawa and Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrices and neutrino mass differences, and at the same time suppress dimension-5 proton decays (proton decays via colored Higgsino exchange) through their texture, so that the soft SUSY-breaking scale can be reduced as much as possible without conflicting the current experimental bound on proton decays. We perform a numerical search for such a texture, and investigate implications of that texture on unknown neutrino parameters, the Dirac CP phase of the PMNS matrix, the lightest neutrino mass, and the $(1, 1)$-component of the neutrino mass matrix in the charged lepton basis. Here we concentrate on the case when the active neutrino mass is generated mostly by the Type-2 seesaw mechanism, in which case we can obtain predictions for the neutrino parameters from the condition that dimension-5 proton decays be suppressed as much as possible.

Subject Index B40, B42

1. Introduction

The $SO(10)$ grand unified theory (GUT) [1,2] is a well-motivated scenario beyond the Standard Model (SM), since it unifies the SM gauge groups into an anomaly-free group, it unifies the SM matter fields and the right-handed neutrino of each generation into one **16** representation, and it accommodates the seesaw mechanism for the tiny neutrino mass [3–7]. Renormalizable $SO(10)$ GUT models [8–29], where the electroweak-symmetry-breaking Higgs field originates from **10**, $\overline{\mathbf{126}}$, and **120** fields (or some of them) and the SM Yukawa couplings stem from renormalizable terms $\tilde{Y}_{10}\,\mathbf{16}\,\mathbf{10}\,\mathbf{16} + \tilde{Y}_{126}\,\mathbf{16}\,\overline{\mathbf{126}}\,\mathbf{16} + \tilde{Y}_{120}\,\mathbf{16}\,\mathbf{120}\,\mathbf{16}$ (or part of them) are particularly interesting, because the SM Yukawa couplings and the active neutrino mass are described in a unified manner with fundamental Yukawa couplings $\tilde{Y}_{10}$, $\tilde{Y}_{126}$, and $\tilde{Y}_{120}$. Specifically, the up-type quark, down-type quark, charged lepton, and neutrino Dirac Yukawa matrices are derived as $Y_u = Y_{10}+r_2 Y_{126}+r_3 Y_{120}$, $Y_d = r_1(Y_{10} + Y_{126} + Y_{120})$, $Y_e = r_1(Y_{10} - 3Y_{126} + r_e Y_{120})$, $Y_D = Y_{10} - 3r_2 Y_{126} + r_\nu Y_{120}$, with $Y_{10} \propto \tilde{Y}_{10}$, $Y_{126} \propto \tilde{Y}_{126}$, $Y_{120} \propto \tilde{Y}_{120}$, and $r_1, r_2, r_3, r_e, r_\nu$ being numbers. The Majorana mass for right-handed neutrinos and the Type-2 seesaw [30–32] contribution to the active neutrino mass are both proportional to Y_{126}.

Supersymmetric (SUSY) GUT models are currently severely constrained by the non-observation of proton decay through dimension-5 operators from colored Higgsino exchange [33,34], the most

Funded by SCOAP[3]

stringent bound being on the $p \to K^+\nu$ mode [35]. This constraint is imminent in SUSY renormalizable $SO(10)$ GUT models, because natural unification of the top and bottom quark Yukawa couplings requires $\tan\beta \sim 50$. For such large $\tan\beta$, right-handed dimension-5 operators $E^cU^cU^cD^c$ give a significant contribution to the $p \to K^+\bar{\nu}_\tau$ decay [36], and it is hard to realize a cancellation in the $E^cU^cU^cD^c$ operators' contribution and that of left-handed dimension-5 operators $QQQL$ to the $p \to K^+\bar{\nu}_\tau$ decay *and* a cancellation in the $QQQL$ operators' contributions to the $p \to K^+\bar{\nu}_\mu$ decay. Besides, it is impossible to enhance the colored Higgsino mass well above 2×10^{16} GeV (by some adjustment of the mass spectrum of GUT-scale particles that modifies the unification conditions) because the $SO(10)$ gauge coupling would become non-perturbative immediately above the thresholds of the components of rank-5 $\mathbf{126} + \overline{\mathbf{126}}$ fields.[1] Although one can increase the soft SUSY-breaking scale to suppress dimension-5 proton decays, the higher the SUSY particle masses, the more the naturalness of the electroweak scale is lost. In this situation, it is worth recalling that it is the fundamental Yukawa couplings $\tilde{Y}_{10}$, $\tilde{Y}_{126}$, and $\tilde{Y}_{120}$ that determine the coefficients of the dimension-5 operators. There may be a texture of the fundamental Yukawa couplings that suppresses dimension-5 proton decays and at the same time reproduces the correct quark and lepton Yukawa couplings and neutrino mass matrix. Specifically, as the up quark Yukawa coupling is a specially small Yukawa coupling in the minimal SUSY SM (MSSM) with $\tan\beta \sim 50$, if those components of the Yukawa matrices $\tilde{Y}_{10}$, $\tilde{Y}_{126}$, and $\tilde{Y}_{120}$ responsible for dimension-5 proton decays are related to the up quark Yukawa coupling, then dimension-5 proton decays are maximally suppressed. The above idea has been sought in Refs. [37,38] based on the model that includes single $\mathbf{10}$, single $\overline{\mathbf{126}}$, and single $\mathbf{120}$ fields [17–19].

In this paper we perform a numerical search for such a texture in the model that includes single $\mathbf{10}$, single $\overline{\mathbf{126}}$, and single $\mathbf{120}$ fields with the following steps. First, we spot those components of the Yukawa matrices Y_{10}, Y_{126}, and Y_{120} (proportional to $\tilde{Y}_{10}$, $\tilde{Y}_{126}$, and $\tilde{Y}_{120}$) which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. Next, we numerically fit the experimental data on the quark and lepton masses, Cabibbo–Kobayashi–Maskawa (CKM) and Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrices, and neutrino mass differences in terms of Y_{10}, Y_{126}, and Y_{120}, and meanwhile we minimize the components of Y_{10}, Y_{126}, and Y_{120} spotted above. In this way we numerically discover a texture of the fundamental Yukawa couplings that suppresses dimension-5 proton decays and reproduces the correct fermion data. We further discuss implications of the texture on unknown neutrino parameters, in particular the Dirac CP phase of the PMNS matrix, $\delta_{\rm pmns}$, the lightest neutrino mass, m_1, and the $(1,1)$-component of the neutrino mass matrix in the charged lepton basis, m_{ee}, that regulates the neutrinoless double beta decay.

The present paper focuses on the case when the active neutrino mass is dominated by the Type-2 seesaw contribution coming from the tiny vacuum expectation value (VEV) of the $\overline{\mathbf{126}}$ field, whereas the Type-1 seesaw contribution resulting from integrating out right-handed neutrinos is assumed subdominant. In this case, the neutrino mass matrix is directly proportional to Y_{126} and

[1] We consider that the field-theoretical description breaks down immediately above the GUT-scale, and hence such non-perturbative behavior of the $SO(10)$ gauge theory is not realized in Nature. Additionally, we assume that there are no higher-dimensional operators suppressed by the scale of the breakdown of the field-theoretical description. Therefore, we can neglect the contribution of higher-dimensional operators to the Yukawa coupling unification.

we can derive predictions for the neutrino parameters from the condition that dimension-5 proton decays be suppressed as much as possible.

The paper is organized as follows: In Sect. 2, we review the renormalizable SUSY $SO(10)$ GUT model where the electroweak-symmetry-breaking Higgs field originates from single **10**, single $\overline{\mathbf{126}}$, and single **120** fields. We also rederive the dimension-5 proton decay partial widths, and clarify the relation between the dimension-5 proton decays and the Yukawa couplings Y_{10}, Y_{126}, and Y_{120}. In Sect. 3 we spot those components of the Yukawa matrices Y_{10}, Y_{126}, and Y_{120} which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. In Sect. 4 we perform a numerical search for a texture of Y_{10}, Y_{126}, and Y_{120} that suppresses dimension-5 proton decays and at the same time reproduces the correct fermion data, and discuss a connection between the suppression of dimension-5 proton decays and the neutrino parameters. Section 5 summarizes the paper.

2. Renormalizable SUSY $SO(10)$ GUT

We consider a SUSY $SO(10)$ GUT model that contains fields in **10**, **126**, $\overline{\mathbf{126}}$, and **120** representations, denoted by H, Δ, $\overline{\Delta}$, and Σ, and three matter fields in the **16** representation, denoted by Ψ_i (i is the flavor index). The model also contains fields in **210** and **45** representations, denoted by Φ and A, which are responsible for breaking the $SU(5)$ subgroup of $SO(10)$. The most general renormalizable Yukawa couplings are given by

$$W_{\text{Yukawa}} = (\tilde{Y}_{10})_{ij}\,\Psi_i H \Psi_j + (\tilde{Y}_{126})_{ij}\,\Psi_i \overline{\Delta} \Psi_j + (\tilde{Y}_{120})_{ij}\,\Psi_i \Sigma \Psi_j, \tag{1}$$

where $\tilde{Y}_{10}$ and $\tilde{Y}_{126}$ are 3×3 complex symmetric matrices and $\tilde{Y}_{120}$ is a 3×3 complex antisymmetric matrix, and i, j are the flavor indices that correspond to that of Ψ_i. The electroweak-breaking-Higgs fields of the MSSM, H_u and H_d, are linear combinations of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ components of H, Δ, $\overline{\Delta}$, Σ, and Φ. Accordingly, the Yukawa coupling for up-type quarks, Y_u, for down-type quarks, Y_d, and for charged leptons, Y_e, and the Dirac Yukawa coupling for neutrinos, Y_D, are derived as

$$W_{\text{Yukawa}} \supset (Y_u)_{ij}\, Q_i H_u U_i^c + (Y_d)_{ij}\, Q_i H_d D_i^c + (Y_e)_{ij}\, L_i H_d E_i^c + (Y_D)_{ij}\, L_i H_u N_i^c, \tag{2}$$

where Y_u, Y_d, Y_e, and Y_D are given by

$$Y_u = Y_{10} + r_2\, Y_{126} + r_3\, Y_{120}, \tag{3}$$

$$Y_d = r_1\,(Y_{10} + Y_{126} + Y_{120}), \tag{4}$$

$$Y_e = r_1\,(Y_{10} - 3Y_{126} + r_e\, Y_{120}), \tag{5}$$

$$Y_D = Y_{10} - 3r_2\, Y_{126} + r_D\, Y_{120} \tag{6}$$

at an $SO(10)$ breaking scale. Here, $Y_{10} \propto \tilde{Y}_{10}$, $Y_{126} \propto \tilde{Y}_{126}$, $Y_{120} \propto \tilde{Y}_{120}$, and r_1, r_2, r_3, r_e, r_D are numbers. By a phase redefinition, we take r_1 to be real positive.

Majorana mass for the right-handed neutrinos is obtained as $(Y_{126})_{ij}\,\overline{v}_R\, N_i^c N_j^c$, where $\overline{v}_R$ denotes $\overline{\Delta}$'s VEV. Integrating out N_i^c yields an effective operator $L_i H_u L_j H_u$, which we call the Type-1 seesaw contribution. Additionally, the $(\mathbf{1}, \mathbf{3}, 1)$ component of $\overline{\Delta}$ mixes with that of E after $SO(10)$ breaking. Integrating out the $(\mathbf{1}, \mathbf{3}, 1)$ components yields an effective operator $L_i H_u L_j H_u$, which we call the Type-2 seesaw contribution. This paper centers on the case where the Type-2 seesaw contribution dominates over the Type-1, in which case the Wilson coefficient of the Weinberg operator $(C_\nu)_{ij}L_iH_uL_jH_u$ satisfies

$$(C_\nu)_{ij} \;\propto\; (Y_{126})_{ij} \tag{7}$$

at an $SO(10)$ breaking scale. In Appendix B, we present an example of VEV configurations that realize the dominance of the Type-2 seesaw contribution.

H, Δ, $\bar{\Delta}$, Σ, and Φ contain pairs of $(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$ and $(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$ components, which we call "colored Higgs fields" and denote by H_C^A and $\overline{H}_C^B$ (A, B are labels), respectively. Exchange of $H_C^A, \overline{H}_C^B$ gives rise to dimension-5 operators inducing a proton decay. Those couplings of $H_C^A, \overline{H}_C^B$ which contribute to such operators are

$$W_{\rm Yukawa} \supset \sum_A \left[\frac{1}{2}(Y_L^A)_{ij}\, Q_i H_C^A Q_j + (\overline{Y}_L^A)_{ij}\, Q_i \overline{H}_C^A L_j \right.$$
$$\left. +\; (Y_R^A)_{ij}\, E_i^c H_C^A U_j^c + (\overline{Y}_R^A)_{ij}\, U_i^c \overline{H}_C^A D_j^c \right], \tag{8}$$

where $\overline{Y}_L^A$, Y_R^A, and $\overline{Y}_R^A$ are proportional to Y_{10}, Y_{126}, or Y_{120}, and Y_L^A are proportional to Y_{10} or Y_{126}. After integrating out $H_C^A, \overline{H}_C^B$, we get effective dimension-5 operators contributing to proton decay,

$$-\,W_5 \;=\; \frac{1}{2} C_{5L}^{ijkl}\,(Q_kQ_l)(Q_iL_j) + C_{5R}^{ijkl}\, E_k^c U_l^c U_i^c D_j^c \tag{9}$$

(in the first term, isospin indices are summed in each bracket), where

$$C_{5L}^{ijkl}(\mu = \mu_{H_C}) = \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_L^A)_{kl}(\overline{Y}_L^B)_{ij} - \frac{1}{2}(Y_L^A)_{li}(\overline{Y}_L^B)_{kj} - \frac{1}{2}(Y_L^A)_{ik}(\overline{Y}_L^B)_{lj} \right\}\Bigg|_{\mu=\mu_{H_C}}, \tag{10}$$

$$C_{5R}^{ijkl}(\mu = \mu_{H_C}) = \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{kl}(\overline{Y}_R^B)_{ij} - (Y_R^A)_{ki}(\overline{Y}_R^B)_{lj} \right\}\Bigg|_{\mu=\mu_{H_C}}, \tag{11}$$

$\mathcal{M}_{H_C}$ denotes the mass matrix for $H_C^A, \overline{H}_C^B$, and μ_{H_C} is taken around the eigenvalues of $\mathcal{M}_{H_C}$.

We concentrate on the $(Q_kQ_l)(Q_iL_j)$ operators' contributions to the $p \to K^+\bar{\nu}_\alpha$ ($\alpha = e, \mu, \tau$) and $p \to K^0 e_\beta^+$ ($e_\beta = e, \mu$) decays and the $E_k^c U_l^c U_i^c D_j^c$ operators' contribution to the $p \to K^+\bar{\nu}_\tau$ decay. For other decay modes, the $(Q_kQ_l)(Q_iL_j)$ operators' contributions to the $N \to \pi e_\beta^+$ and $p \to \eta e_\beta^+$ decays are suppressed in the same texture that suppresses the above contributions, as we comment in Sect. 3. The rest of the decay modes are bounded only weakly [39] and so we do not discuss them in this paper.

The contribution of the $C_{5L}^{ijkl}(Q_kQ_l)(Q_iL_j)$ term to the $p \to K^+\bar{\nu}_\alpha$ ($\alpha = e, \mu, \tau$) decays is given by

$$\Gamma(p \to K^+\bar{\nu}_\alpha)|_{{\rm from}\, C_{5L}} = \mathcal{C} \left| \beta_H(\mu_{\rm had}) \frac{1}{f_\pi} \left\{ \left(1 + \frac{D}{3} + F\right) C_{LL}^{s\alpha\, ud}(\mu_{\rm had}) + \frac{2D}{3} C_{LL}^{d\alpha\, us}(\mu_{\rm had}) \right\} \right|^2. \tag{12}$$

Here, $\mathcal{C} = \frac{m_N}{64\pi}\left(1 - \frac{m_K^2}{m_N^2}\right)^2$, with m_N denoting the nucleon mass and m_K the kaon mass, α_H, β_H denote hadronic matrix elements, D, F are parameters of the baryon chiral Lagrangian, and C_{LL} are

Wilson coefficients of the effective Lagrangian, $-\mathcal{L}_6 \supset C_{LL}^{ijkl}(\psi_{u_L k}\psi_{d_L l})(\psi_{d_L i}\psi_{\nu_L j})$, where ψ denotes an SM Weyl spinor and the spinor index is summed in each bracket. The Wilson coefficients C_{LL} satisfy[2]

$$C_{LL}^{s\alpha\, ud}(\mu_{\rm had}) = A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY})\frac{M_{\widetilde{W}}}{m_{\tilde{q}}^2}\,\mathcal{F}\,g_2^2\left(C_{5L}^{s\alpha\, ud} - C_{5L}^{u\alpha\, ds}\right)\Big|_{\mu=\mu_{\rm SUSY}}, \tag{13}$$

$$C_{LL}^{d\alpha\, su}(\mu_{\rm had}) = A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY})\frac{M_{\widetilde{W}}}{m_{\tilde{q}}^2}\,\mathcal{F}\,g_2^2\left(C_{5L}^{d\alpha\, us} - C_{5L}^{u\alpha\, ds}\right)\Big|_{\mu=\mu_{\rm SUSY}}. \tag{14}$$

Here, $\mathcal{F}$ is a loop function factor given by $\mathcal{F} = \frac{1}{x-y}\left(\frac{x}{1-x}\log x - \frac{y}{1-y}\log y\right)/16\pi^2 + \frac{1}{x-1}\left(\frac{x}{1-x}\log x + 1\right)/16\pi^2$ with $x = |M_{\widetilde{W}}|^2/m_{\tilde{q}}^2$ and $y = m_{\tilde{\ell}^\alpha}^2/m_{\tilde{q}}^2$. Also, $M_{\widetilde{W}}$ denotes the Wino mass, $m_{\tilde{q}}$ the first- and second-generation left-handed squark masses (which are usually degenerate), and $m_{\tilde{\ell}^\alpha}$ the mass of the left-handed slepton of flavor α. $A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY})$ accounts for SM renormalization group (RG) corrections in the evolution from soft SUSY-breaking scale $\mu_{\rm SUSY}$ to a hadronic scale where the values of α_H, β_H are reported. Here we neglect SM RG corrections involving the u, d, s-quark and charged lepton Yukawa couplings, and accordingly, quark flavor mixings along the RG evolution are neglected. The Wilson coefficients C_{5L} are related to the colored Higgs Yukawa couplings as

$$\begin{aligned}
&C_{5L}^{s\alpha\, ud}(\mu_{\rm SUSY}) - C_{5L}^{u\alpha\, ds}(\mu_{\rm SUSY}) \\
&\quad = A_L^\alpha(\mu_{\rm SUSY}, \mu_{H_C})\sum_{A,B}(\mathcal{M}_{H_C}^{-1})_{AB}\;\frac{3}{2}\left\{(Y_L^A)_{ud}(\overline{Y}_L^B)_{s\alpha} - (Y_L^A)_{ds}(\overline{Y}_L^B)_{u\alpha}\right\}\Big|_{\mu=\mu_{H_C}},
\end{aligned} \tag{15}$$

$$\begin{aligned}
&C_{5L}^{d\alpha\, us}(\mu_{\rm SUSY}) - C_{5L}^{u\alpha\, ds}(\mu_{\rm SUSY}) \\
&\quad = A_L^\alpha(\mu_{\rm SUSY}, \mu_{H_C})\sum_{A,B}(\mathcal{M}_{H_C}^{-1})_{AB}\;\frac{3}{2}\left\{(Y_L^A)_{us}(\overline{Y}_L^B)_{d\alpha} - (Y_L^A)_{ds}(\overline{Y}_L^B)_{u\alpha}\right\}\Big|_{\mu=\mu_{H_C}},
\end{aligned} \tag{16}$$

where $A_L^\alpha(\mu_{\rm SUSY}, \mu_{H_C})$ accounts for MSSM RG corrections in the evolution from μ_{H_C} to $\mu_{\rm SUSY}$.

The contribution of the $C_{5L}^{ijkl}(Q_kQ_l)(Q_iL_j)$ term to the $p \to K^0 e_\beta^+$ ($e_\beta = e, \mu$) decays is given by

$$\Gamma(p \to K^0 e_\beta^+) \;=\; \mathcal{C}\,\left|\beta_H(\mu_{\rm had})\frac{1}{f_\pi}\,(1-D+F)\,\overline{C}_{LL}^{u\beta\, us}(\mu_{\rm had})\right|^2. \tag{17}$$

Here, $\overline{C}_{LL}$ are Wilson coefficients of the effective Lagrangian,

$$-\mathcal{L}_6 \supset \overline{C}_{LL}^{ijkl}(\psi_{u_L k}\psi_{d_L l})(\psi_{u_L i}\psi_{e_L j}),$$

which satisfy

$$\overline{C}_{LL}^{u\beta\, us}(\mu_{\rm had}) = A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY})\frac{M_{\widetilde{W}}}{m_{\tilde{q}}^2}\,\mathcal{F}\,g_2^2\left(-C_{5L}^{u\beta\, us} + C_{5L}^{s\beta\, uu}\right)\Big|_{\mu=\mu_{\rm SUSY}}, \tag{18}$$

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[2] When writing $C_{5L}^{s\alpha\, du}$, we mean that Q_i is in the flavor basis where the down-type quark Yukawa coupling is diagonal, and that the down-type quark component of Q_i is exactly the s quark (the up-type quark component of Q_i is a mixture of u, c, t). Likewise, Q_k is in the flavor basis where the down-type quark Yukawa coupling is diagonal and its down-type component is exactly the d quark, and Q_l is in the flavor basis where the up-type quark Yukawa coupling is diagonal and its up-type quark component is exactly the u quark. The same rule applies to $C_{5L}^{u\alpha\, ds}$ and others.

where $A_{LL}(\mu_{\rm had},\mu_{\rm SUSY})$ accounts for SM RG corrections. The Wilson coefficients C_{5L} are related to the colored Higgs Yukawa couplings as

$$\begin{aligned} & C_{5L}^{u\beta\, us}(\mu_{\rm SUSY}) - C_{5L}^{s\beta\, uu}(\mu_{\rm SUSY}) \\ & \quad = A_L^{\beta}(\mu_{\rm SUSY},\mu_{H_C}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB}\ \frac{3}{2} \left\{ (Y_L^A)_{us}(\overline{Y}_L^B)_{u\beta} - (Y_L^A)_{uu}(\overline{Y}_L^B)_{s\beta} \right\}\Big|_{\mu=\mu_{H_C}}, \end{aligned} \tag{19}$$

where $A_L^{\beta}(\mu_{\rm SUSY},\mu_{H_C})$ accounts for MSSM RG corrections.

The contribution of the $C_{5R}^{ijkl}\, E_k^c U_l^c U_i^c D_j^c$ term to the $p \to K^+\bar{\nu}_\tau$ decay is given by

$$\Gamma(p \to K^+\bar{\nu}_\tau)|_{{\rm from}\, C_{5R}} = \mathcal{C} \left| \alpha_H(\mu_{\rm had}) \frac{1}{f_\pi} \left\{ \left(1 + \frac{D}{3} + F\right) C_{RL}^{ud\,\tau s}(\mu_{\rm had}) + \frac{2D}{3} C_{RL}^{us\,\tau d}(\mu_{\rm had}) \right\} \right|^2. \tag{20}$$

Here, C_{RL} are Wilson coefficients of the effective Lagrangian,

$$-\mathcal{L}_6 \supset C_{RL}^{ijkl}(\psi_{\nu_L k}\psi_{d_L l})(\psi_{u_R^c i}\psi_{d_R^c j}),$$

which satisfy[3]

$$C_{RL}^{ud\,\tau s}(\mu_{\rm had}) \;=\; A_{RL}(\mu_{\rm had},\mu_{\rm SUSY}) \frac{\mu_H}{m_{\tilde{t}_R}^2}\, \mathcal{F}'\ (V_{ts}^{\rm ckm})^*\, y_t y_\tau\, C_{5R}^{ud\tau t}\big|_{\mu=\mu_{\rm SUSY}}, \tag{21}$$

$$C_{RL}^{us\,\tau d}(\mu_{\rm had}) \;=\; A_{RL}(\mu_{\rm had},\mu_{\rm SUSY}) \frac{\mu_H}{m_{\tilde{t}_R}^2}\, \mathcal{F}'\ (V_{td}^{\rm ckm})^*\, y_t y_\tau\, C_{5R}^{us\tau t}\big|_{\mu=\mu_{\rm SUSY}}, \tag{22}$$

where $V_{ij}^{\rm ckm}$ denotes the (i,j)-component of the CKM matrix. Here, $\mathcal{F}'$ is another loop function factor given by $\mathcal{F}' = \frac{1}{x-y}\big(\frac{x}{1-x}\log x - \frac{y}{1-y}\log y\big)/16\pi^2$, with $x = |\mu_H|^2/m_{\tilde{t}_R}^2$ and $y = m_{\tilde{\tau}_R}^2/m_{\tilde{t}_R}^2$. Also, μ_H denotes the μ-term, $m_{\tilde{t}_R}$ the mass of the right-handed top squark, and $m_{\tilde{\tau}_R}$ the mass of the right-handed tau slepton. $A_{RL}(\mu_{\rm had},\mu_{\rm SUSY})$ accounts for SM RG corrections. The Wilson coefficients C_{5R} are related to the colored Higgs Yukawa couplings as

$$C_{5R}^{ud\tau t}(\mu_{\rm SUSY}) = A_R^{\tau t}(\mu_{\rm SUSY},\mu_{H_C}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{\tau t}(\overline{Y}_R^B)_{ud} - (Y_R^A)_{\tau u}(\overline{Y}_R^B)_{td} \right\}\Big|_{\mu=\mu_{H_C}}, \tag{23}$$

$$C_{5R}^{us\tau t}(\mu_{\rm SUSY}) = A_R^{\tau t}(\mu_{\rm SUSY},\mu_{H_C}) \sum_{A,B} (\mathcal{M}_{H_C}^{-1})_{AB} \left\{ (Y_R^A)_{\tau t}(\overline{Y}_R^B)_{us} - (Y_R^A)_{\tau u}(\overline{Y}_R^B)_{ts} \right\}\Big|_{\mu=\mu_{H_C}}, \tag{24}$$

where $A_R^{\tau t}(\mu_{\rm SUSY},\mu_{H_C})$ accounts for MSSM RG corrections.

[3] y_t and y_τ in Eqs. (21) and (22) are Yukawa couplings of MSSM and so already include the factors of $1/\sin\beta$ and $1/\cos\beta$, respectively.

The flavor-dependent terms in Eqs. (15), (16), (19), (23), and (24) are related to the fundamental Yukawa couplings Y_{10}, Y_{126}, and Y_{120} as follows. Since Y_L^A is defined as $W_{\rm Yukawa} \supset (Y_L^A)_{ij}\, Q_i H_C^A Q_j$, its flavor indices are symmetric and thus Y_L^A is not proportional to Y_{120}. Therefore, we can write, without loss of generality, with $\alpha = e, \mu, \tau$ and $\beta = e, \mu$,

$$\sum_{A,B}(\mathcal{M}_{H_C}^{-1})_{AB}\Big\{(Y_L^A)_{ud}(\overline{Y}_L^B)_{s\alpha} - (Y_L^A)_{ds}(\overline{Y}_L^B)_{u\alpha}\Big\}$$
$$\begin{aligned} = \frac{1}{M_{H_C}}\Big[&a\left\{(Y_{10})_{u_Ld_L}(Y_{10})_{s_L\alpha_L} - (Y_{10})_{d_Ls_L}(Y_{10})_{u_L\alpha_L}\right\} \\ &+ b\left\{(Y_{10})_{u_Ld_L}(Y_{126})_{s_L\alpha_L} - (Y_{10})_{d_Ls_L}(Y_{126})_{u_L\alpha_L}\right\} \\ &+ c\left\{(Y_{10})_{u_Ld_L}(Y_{120})_{s_L\alpha_L} - (Y_{10})_{d_Ls_L}(Y_{120})_{u_L\alpha_L}\right\} \\ &+ d\left\{(Y_{126})_{u_Ld_L}(Y_{10})_{s_L\alpha_L} - (Y_{126})_{d_Ls_L}(Y_{10})_{u_L\alpha_L}\right\} \\ &+ e\left\{(Y_{126})_{u_Ld_L}(Y_{126})_{s_L\alpha_L} - (Y_{126})_{d_Ls_L}(Y_{126})_{u_L\alpha_L}\right\} \\ &+ f\left\{(Y_{126})_{u_Ld_L}(Y_{120})_{s_L\alpha_L} - (Y_{126})_{d_Ls_L}(Y_{120})_{u_L\alpha_L}\right\}\Big], \end{aligned} \tag{25}$$

$$\sum_{A,B}(\mathcal{M}_{H_C}^{-1})_{AB}\left\{(Y_L^A)_{us}(\overline{Y}_L^B)_{d\alpha} - (Y_L^A)_{ds}(\overline{Y}_L^B)_{u\alpha}\right\}$$
$$= (\text{Above expression with the exchange } d_L \leftrightarrow s_L), \tag{26}$$

$$\sum_{A,B}(\mathcal{M}_{H_C}^{-1})_{AB}\Big\{(Y_L^A)_{us}(\overline{Y}_L^B)_{u\beta} - (Y_L^A)_{uu}(\overline{Y}_L^B)_{s\beta}\Big\}$$
$$\begin{aligned} = \frac{1}{M_{H_C}}\Big[&a\left\{(Y_{10})_{u_Ls_L}(Y_{10})_{u_L\beta_L} - (Y_{10})_{u_Lu_L}(Y_{10})_{s_L\beta_L}\right\} \\ &+ b\left\{(Y_{10})_{u_Ls_L}(Y_{126})_{u_L\beta_L} - (Y_{10})_{u_Lu_L}(Y_{126})_{s_L\beta_L}\right\} \\ &+ c\left\{(Y_{10})_{u_Ls_L}(Y_{120})_{u_L\beta_L} - (Y_{10})_{u_Lu_L}(Y_{120})_{s_L\beta_L}\right\} \\ &+ d\left\{(Y_{126})_{u_Ls_L}(Y_{10})_{u_L\beta_L} - (Y_{126})_{u_Lu_L}(Y_{10})_{s_L\beta_L}\right\} \\ &+ e\left\{(Y_{126})_{u_Ls_L}(Y_{126})_{u_L\beta_L} - (Y_{126})_{u_Lu_L}(Y_{126})_{s_L\beta_L}\right\} \\ &+ f\left\{(Y_{126})_{u_Ls_L}(Y_{120})_{u_L\beta_L} - (Y_{126})_{u_Lu_L}(Y_{120})_{s_L\beta_L}\right\}\Big], \end{aligned} \tag{27}$$

where M_{H_C} denotes a typical value of the eigenvalues of $\mathcal{M}_{H_C}$, and $a, b, c, d, e, f, g, h, j$ are numbers determined from the colored Higgs mass matrix [40–45]. Here, $(Y_{10})_{u_Ld_L}$ denotes the $(1,1)$-component of Y_{10} in the term $(Y_{10})_{ij}\, \Psi_i H \Psi_j$ in the flavor basis where the left-handed up-type quark component of Ψ_i has the diagonalized up-type quark Yukawa coupling, and the left-handed down-type quark component of Ψ_j has the diagonalized down-type quark Yukawa coupling. $(Y_{10})_{d_Ls_L}$, $(Y_{126})_{u_Ld_L}$, and others are defined analogously. Since each of $Y_R^A, \overline{Y}_R^A$ is proportional to Y_{10}, Y_{126}, or Y_{120}, we can write

$$\sum_{A,B}(\mathcal{M}_{H_C}^{-1})_{AB}\Big\{(Y_R^A)_{\tau t}(\overline{Y}_R^B)_{ud} - (Y_R^A)_{\tau u}(\overline{Y}_R^B)_{td}\Big\}$$
$$= \frac{1}{M_{H_C}}\Big[a\left\{(Y_{10})_{\tau_Rt_R}(Y_{10})_{u_Rd_R} - (Y_{10})_{\tau_Ru_R}(Y_{10})_{t_Rd_R}\right\}$$

$$
\begin{aligned}
&+ b\left\{(Y_{10})_{\tau_R t_R}(Y_{126})_{u_R d_R} - (Y_{10})_{\tau_R u_R}(Y_{126})_{t_R d_R}\right\} \\
&+ c\left\{(Y_{10})_{\tau_R t_R}(Y_{120})_{u_R d_R} - (Y_{10})_{\tau_R u_R}(Y_{120})_{t_R d_R}\right\} \\
&+ d\left\{(Y_{126})_{\tau_R t_R}(Y_{10})_{u_R d_R} - (Y_{126})_{\tau_R u_R}(Y_{10})_{t_R d_R}\right\} \\
&+ e\left\{(Y_{126})_{\tau_R t_R}(Y_{126})_{u_R d_R} - (Y_{126})_{\tau_R u_R}(Y_{126})_{t_R d_R}\right\} \\
&+ f\left\{(Y_{126})_{\tau_R t_R}(Y_{120})_{u_R d_R} - (Y_{126})_{\tau_R u_R}(Y_{120})_{t_R d_R}\right\} \\
&+ g\left\{(Y_{120})_{\tau_R t_R}(Y_{10})_{u_R d_R} - (Y_{120})_{\tau_R u_R}(Y_{10})_{t_R d_R}\right\} \\
&+ h\left\{(Y_{120})_{\tau_R t_R}(Y_{126})_{u_R d_R} - (Y_{120})_{\tau_R u_R}(Y_{126})_{t_R d_R}\right\} \\
&+ j\left\{(Y_{120})_{\tau_R t_R}(Y_{120})_{u_R d_R} - (Y_{120})_{\tau_R u_R}(Y_{120})_{t_R d_R}\right\}\Big], \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{A,B}(\mathcal{M}_{H_C}^{-1})_{AB}\left\{(Y_R^A)_{\tau t}(\overline{Y}_R^B)_{us} - (Y_R^A)_{\tau u}(\overline{Y}_R^B)_{ts}\right\} \\
&\qquad = \text{(Above expression with the replacement } d_R \to s_R\text{)}, \qquad (29)
\end{aligned}
$$

where a, b, c, d, e, f are the same numbers as in Eqs. (25)–(27).

3. Components of the Yukawa matrices that can be reduced

We spot those components of the Yukawa matrices Y_{10}, Y_{126}, and Y_{120} which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. Specifically, we attempt to reduce the pair products of the components of Y_{10}, Y_{126}, and Y_{120} that appear in Eqs. (25)–(29) (e.g. $(Y_{10})_{\tau_R t_R}(Y_{10})_{u_R d_R}$) to the order of the up quark Yukawa coupling times the top quark Yukawa coupling $O(y_u\, y_t)$. As a matter of fact, some pair products cannot simultaneously be reduced because of the requirement that Y_{10}, Y_{126}, and Y_{120} reproduce the correct quark and lepton Yukawa couplings. In this circumstance we tune the colored Higgs mass matrix such that the coefficients $a, b, c, d, e, f, g, h, j$ in Eqs. (25)–(29) realize cancellations among the problematic pair products. Finally, we present "those components of the Yukawa matrices Y_{10}, Y_{126}, and Y_{120} that can be reduced," as well as an example of the colored Higgs mass matrix that gives coefficients $a, b, c, d, e, f, g, h, j$ that realize the abovementioned cancellations.

First, focus on Eq. (28). We have $(Y_{10})_{\tau_R t_R} + r_2(Y_{126})_{\tau_R t_R} + r_3(Y_{120})_{\tau_R t_R} = y_t\times$ (t_L–τ_R part of the mixing matrix), and since t_L and τ_R are both third-generation components, the t_L–τ_R part of the mixing matrix is almost maximal. The component $(Y_{120})_{\tau_R t_R}$ is suppressed compared to $(Y_{10})_{\tau_R t_R}$, and $(Y_{126})_{\tau_R t_R}$ because Y_{120} is an antisymmetric matrix. Consequently, one or both of $(Y_{10})_{\tau_R t_R}$ and $(Y_{126})_{\tau_R t_R}$ are always on the order of the top quark Yukawa coupling y_t. Hence, in order to reduce the Yukawa coupling pair products in Eq. (28), it is necessary to reduce

$$(Y_{10})_{u_R d_R}, \quad (Y_{126})_{u_R d_R}, \quad \text{and} \quad (Y_{120})_{u_R d_R}. \qquad (30)$$

Note that although Y_{120} is an antisymmetric matrix, $(Y_{120})_{u_R d_R}$ is not necessarily suppressed to $O(y_u)$ or below. This is because $(Y_{120})_{u_R d_R}$ can be on the order of $(Y_{120})_{u_R s_R}$ times the Cabibbo angle $\lambda \simeq 0.22$, and $(Y_{120})_{u_R s_R}$ can be on the order of $0.22 \times \frac{y_t}{y_b} y_s$, as we see later, so that $(Y_{120})_{u_R d_R}$ can be as large as $0.22^2 \times \frac{y_t}{y_b} y_s$, which is much greater than y_u.

Equation (28) also contains terms of the form $(Y_A)_{\tau_R u_R}(Y_B)_{t_R d_R}$ $(A, B = 10, 126, 120)$. They can be estimated to be $\sin^2\theta_{13}^{\rm ckm}\, y_t^2$ ($\theta_{ij}^{\rm ckm}$ denotes the (i, j)-mixing angle of the CKM matrix), which is numerically close to $y_u y_t$. Hence, we do not need to reduce $(Y_A)_{\tau_R u_R}$ or $(Y_A)_{t_R d_R}$ further.

Now focus on Eq. (29). For the same reason as above, we have to reduce

$$(Y_{10})_{u_R s_R}, \quad (Y_{126})_{u_R s_R}, \quad \text{and} \quad (Y_{120})_{u_R s_R}. \tag{31}$$

Equation (29) also contains terms of the form $(Y_A)_{\tau_R u_R}(Y_B)_{t_R s_R}$ $(A, B = 10, 126, 120)$, which are estimated to be $\sin\theta_{13}^{\rm ckm}\sin\theta_{23}^{\rm ckm}\, y_t^2$. They contribute to the $p \to K^+\bar{\nu}_\tau$ decay amplitude by a similar amount to the terms $(Y_A)_{\tau_R u_R}(Y_B)_{t_R d_R}$ in Eq. (28), because these terms enter the decay amplitude in the form $V_{ts}^{\rm ckm}(Y_A)_{\tau_R u_R}(Y_B)_{t_R d_R} + V_{td}^{\rm ckm}(Y_A)_{\tau_R u_R}(Y_B)_{t_R s_R}$ and the CKM matrix satisfies $|V_{ts}^{\rm ckm}| \simeq \sin\theta_{23}^{\rm ckm}$ and $|V_{td}^{\rm ckm}| \sim \sin\theta_{13}^{\rm ckm}$. Therefore, we tolerate the terms $(Y_A)_{\tau_R u_R}(Y_B)_{t_R s_R}$ and do not reduce $(Y_A)_{\tau_R u_R}$ or $(Y_A)_{t_R s_R}$ further.

As a matter of fact, it is impossible to simultaneously reduce $(Y_{10})_{u_R s_R}$, $(Y_{126})_{u_R s_R}$, and $(Y_{120})_{u_R s_R}$ to $O(y_u)$. This is because Eq. (4) gives

$$\begin{aligned}(Y_{10})_{u_R s_R} + (Y_{126})_{u_R s_R} + (Y_{120})_{u_R s_R} &= \frac{1}{r_1}(Y_d)_{u_R s_R} \\ &\simeq \frac{y_t}{y_b} y_s \times (s_L\text{–}u_R \text{ part of the mixing matrix}),\end{aligned} \tag{32}$$

where r_1 is estimated to be y_b/y_t so that the top and bottom quark Yukawa couplings are reproduced. The s_L–u_R part of the mixing matrix is estimated to be the Cabibbo angle $\lambda \simeq 0.22$, and thus we get $(Y_{10})_{u_R s_R} + (Y_{126})_{u_R s_R} + (Y_{120})_{u_R s_R} \simeq 0.22 \times \frac{y_t}{y_b} y_s$, which is much greater than the up quark Yukawa coupling y_u.

A way out is to adjust the colored Higgs mass matrix such that the coefficients c, f in Eqs. (25)–(29) are zero,

$$c = f = 0. \tag{33}$$

Then we are exempted from reducing $(Y_{120})_{u_R s_R}$, because $(Y_{120})_{u_R s_R}$ appears only in the term $(Y_{120})_{\tau_R t_R}(Y_{120})_{u_R s_R}$ and the component $(Y_{120})_{\tau_R t_R}$ is suppressed because Y_{120} is an antisymmetric matrix. As a bonus, it is no longer necessary to reduce $(Y_{120})_{u_R d_R}$.

Next, focus on Eqs. (25) and (26). Since α ranges over all three flavors, it is difficult to reduce $(Y_A)_{s_L \alpha_L}$, $(Y_A)_{d_L \alpha_L}$, and $(Y_A)_{u_L \alpha_L}$ $(A = 10, 126, 120)$ for all α. Hence, we leave these Yukawa couplings untouched and instead reduce $(Y_B)_{u_L d_L}$, $(Y_B)_{u_L s_L}$, and $(Y_B)_{d_L s_L}$ $(B = 10, 126)$ (one side of the Yukawa coupling pair products).

Unfortunately, at least one of $(Y_{10})_{u_L s_L}$, $(Y_{10})_{d_L s_L}$, $(Y_{126})_{u_L s_L}$, and $(Y_{126})_{d_L s_L}$ is on the order of $V_{cd}^{\rm ckm}\frac{y_t}{y_b} y_s$, and consequently some of the Yukawa coupling pair products in Eqs. (25) and (26) cannot be suppressed to $O(y_u y_t)$ for all α. This is seen from the two equalities

$$(Y_{10})_{s_L c_L} + (Y_{126})_{s_L c_L} + (Y_{120})_{s_L c_L} \simeq \frac{y_t}{y_b} y_s \times (c_L\text{–}s_R \text{ part of the mixing matrix}), \tag{34}$$

$$\begin{aligned}&(Y_{10})_{d_L s_L} + (Y_{126})_{d_L s_L} - V_{ud}^{\rm ckm}\left\{(Y_{10})_{u_L s_L} + (Y_{126})_{u_L s_L}\right\} \\ &\qquad = V_{cd}^{\rm ckm}\left\{(Y_{10})_{c_L s_L} + (Y_{126})_{c_L s_L}\right\} + V_{td}^{\rm ckm}\left\{(Y_{10})_{t_L s_L} + (Y_{126})_{t_L s_L}\right\}.\end{aligned} \tag{35}$$

Since c_L and s_R are both second-generation components, the c_L–s_R part of the mixing matrix in Eq. (34) is nearly maximal. Also, $(Y_{120})_{s_L c_L}$ is suppressed compared to $(Y_{10})_{c_L s_L}$ and $(Y_{126})_{c_L s_L}$ because Y_{120} is an antisymmetric matrix. Hence, we have $(Y_{10})_{c_L s_L} + (Y_{126})_{c_L s_L} \simeq \frac{y_t}{y_b} y_s(\mu_{H_C})$, and from Eq. (35) we conclude that at least one of $(Y_{10})_{u_L s_L}$, $(Y_{126})_{u_L s_L}$, $(Y_{10})_{d_L s_L}$, and $(Y_{126})_{d_L s_L}$ is on the order of $V^{\rm ckm}_{cd} \frac{y_t}{y_b} y_s$.[4]

A natural way out is to reduce $(Y_{10})_{u_L s_L}$ and $(Y_{126})_{u_L s_L}$, while tuning coefficients a, b, d, e such that $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$ hold. This choice is because $(Y_{10})_{u_L s_L}$ and $(Y_{126})_{u_L s_L}$ can more easily be related to the tiny up quark Yukawa coupling.

Finally, focus on Eq. (27). Since we leave $(Y_A)_{s_L e_L}$ and $(Y_A)_{s_L \mu_L}$ untouched, we have to reduce $(Y_{10})_{u_L u_L}$ and $(Y_{126})_{u_L u_L}$.

To sum up, in order to suppress dimension-5 proton decays, we have to reduce the following Yukawa couplings:

$$
\begin{aligned}
&(Y_{10})_{u_R d_R},\ (Y_{126})_{u_R d_R},\ (Y_{10})_{u_R s_R},\ (Y_{126})_{u_R s_R} \\
&(Y_{10})_{u_L d_L},\ (Y_{126})_{u_L d_L},\ (Y_{10})_{u_L u_L},\ (Y_{126})_{u_L u_L},\ (Y_{10})_{u_L s_L},\ (Y_{126})_{u_L s_L}.
\end{aligned} \tag{36}
$$

Meanwhile, we have to adjust the colored Higgs mass matrix such that $c = f = 0$, $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$, and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$.

We comment on the $N \to \pi e^+_\beta$ and $p \to \eta e^+_\beta$ decays. Their decay amplitudes contain terms obtained by replacing s with d in Eq. (27). Therefore, by reducing $(Y_{10})_{u_L d_L}$, $(Y_{126})_{u_L d_L}$, $(Y_{10})_{u_L u_L}$, and $(Y_{126})_{u_L u_L}$, these decay modes are also suppressed.

We present an example of the colored Higgs mass matrix that realizes $c = f = 0$ and $a/d = b/e$. The latter is a necessity condition for $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$.

To study the colored Higgs mass matrix, we have to write the superpotential for the H, Δ, $\overline{\Delta}$, Σ, Φ, and A fields, introduce $SO(10)$-breaking VEVs, and specify the colored Higgs components of the fields. To this end, we use the result of Ref. [42]. The notation for fields is common between this paper and Ref. [42], except that the **120** field is written as D in Ref. [42]. We define the couplings, coupling constants, and masses for the fields according to Eqs. (2) and (3) of Ref. [42] (our definition of the coupling constants and masses is reviewed in Appendix A). We employ the same notation for the VEVs of Δ, $\overline{\Delta}$, Φ, and A as Ref. [42], and write the $(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$ and $(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$ components as in Table 3 of Ref. [42].[5]

Now we present the example of the colored Higgs mass matrix. It satisfies

$$
\begin{aligned}
&\lambda_{18} = 0, \quad \lambda_{20} = 0, \quad \frac{\lambda_{21}}{\lambda_{19}} = 3\frac{\lambda_{17}}{\lambda_{16}}, \\
&iA_1 = -\frac{1}{6}\frac{\lambda_{21}}{\lambda_{19}}\Phi_3, \quad iA_2 = -\frac{\sqrt{3}}{6}\frac{\lambda_{21}}{\lambda_{19}}\Phi_2.
\end{aligned} \tag{37}
$$

[4] One might have hoped that the term $V^{\rm ckm}_{td}\left\{(Y_{10})_{t_L s_L} + (Y_{126})_{t_L s_L}\right\}$ cancels the term $V^{\rm ckm}_{cd}\left\{(Y_{10})_{c_L s_L} + (Y_{126})_{c_L s_L}\right\}$, but this is not compatible with the correct quark Yukawa couplings.

[5] We have confirmed that the mass matrix of the $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields given in Eq. (68) of Ref. [42] is correct. However, we argue that in the colored Higgs mass matrix given in Eq. (69) of Ref. [42], the sign of the term $\frac{2}{5}\sqrt{\frac{2}{3}}\lambda_7 A_2$ in $m^{(3,1,-\frac{1}{3})}_{77}$ should be minus. Otherwise, we have confirmed that Eq. (69) of Ref. [42] is correct. We argue that in the superpotential of the VEVs given in Eq. (27) of Ref. [42], the sign of the term $\lambda_6[A_1(-\frac{1}{5}) + A_2(-\frac{3}{5\sqrt{6}})]$ should be flipped. Otherwise, we have confirmed that Eq. (27) of Ref. [42] is correct.

The "texture" of Eq. (37) cannot be derived from any symmetry, and so fine-tunings of superpotential parameters are needed for its realization. These fine-tunings are natural at the quantum level due to the non-renormalization theorem. The VEV configuration in the second line of Eq. (37) can satisfy the F-flatness conditions (displayed in Eq. (28) of Ref. [42]). Given Eq. (37), the colored Higgs mass matrix is given by

$$W \supset \left(H^{(\bar{3},1,\frac{1}{3})} \quad \overline{\Delta}^{(\bar{3},1,\frac{1}{3})}_{(6,1,1)} \quad \Delta^{(\bar{3},1,\frac{1}{3})}_{(6,1,1)} \quad \Delta^{(\bar{3},1,\frac{1}{3})}_{(\overline{10},1,3)} \quad \Phi^{(\bar{3},1,\frac{1}{3})} \quad \Sigma^{(\bar{3},1,\frac{1}{3})}_{(6,1,3)} \quad \Sigma^{(\bar{3},1,\frac{1}{3})}_{(\overline{10},1,1)} \right) \mathcal{M}_{H_C} \begin{pmatrix} H^{(3,1,-\frac{1}{3})} \\ \Delta^{(3,1,-\frac{1}{3})}_{(6,1,1)} \\ \overline{\Delta}^{(3,1,-\frac{1}{3})}_{(6,1,1)} \\ \overline{\Delta}^{(3,1,-\frac{1}{3})}_{(10,1,3)} \\ \Phi^{(3,1,-\frac{1}{3})} \\ \Sigma^{(3,1,-\frac{1}{3})}_{(6,1,3)} \\ \Sigma^{(3,1,-\frac{1}{3})}_{(10,1,1)} \end{pmatrix} \tag{38}$$

where

$$\mathcal{M}_{H_C} =$$
$$\begin{pmatrix}
m_3 & \frac{\lambda_3\Phi_2}{\sqrt{30}} - \frac{\lambda_3\Phi_1}{\sqrt{10}} & -\frac{\lambda_4\Phi_1}{\sqrt{10}} - \frac{\lambda_4\Phi_2}{\sqrt{30}} & -\sqrt{\frac{2}{15}}\lambda_4\Phi_3 & \frac{\lambda_4\bar{v}_R}{\sqrt{5}} & 0 & 0 \\
\frac{\lambda_4\Phi_2}{\sqrt{30}} - \frac{\lambda_4\Phi_1}{\sqrt{10}} & m_2 + i\frac{\lambda_{21}}{\lambda_{19}}\frac{\lambda_6\Phi_2}{30\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\
-\frac{\lambda_3\Phi_1}{\sqrt{10}} - \frac{\lambda_3\Phi_2}{\sqrt{30}} & 0 & m_2 - i\frac{\lambda_{21}}{\lambda_{19}}\frac{\lambda_6\Phi_2}{30\sqrt{2}} & \frac{\lambda_2\Phi_3}{15\sqrt{2}} & -\frac{\lambda_2\bar{v}_R}{10\sqrt{3}} & 0 & 0 \\
-\sqrt{\frac{2}{15}}\lambda_3\Phi_3 & 0 & \frac{\lambda_2\Phi_3}{15\sqrt{2}} & m_{66} & -\frac{\lambda_2\bar{v}_R}{5\sqrt{6}} & 0 & 0 \\
\frac{\lambda_3 v_R}{\sqrt{5}} & 0 & -\frac{\lambda_2 v_R}{10\sqrt{3}} & -\frac{\lambda_2 v_R}{5\sqrt{6}} & m_{77} & 0 & 0 \\
-\lambda_{17}\frac{\Phi_3}{\sqrt{3}} & 0 & \lambda_{21}\frac{\Phi_3}{6\sqrt{5}} & \lambda_{21}\frac{\Phi_2}{3\sqrt{5}} & \frac{\lambda_{21}\bar{v}_R}{2\sqrt{15}} & m_{22} & \frac{2\lambda_{15}\Phi_3}{9} \\
-\sqrt{\frac{2}{3}}\lambda_{17}\Phi_2 & 0 & \lambda_{21}\frac{\Phi_2}{3\sqrt{10}} & \lambda_{21}\frac{\Phi_3}{3\sqrt{10}} & \frac{\lambda_{21}\bar{v}_R}{2\sqrt{15}} & \frac{2\lambda_{15}\Phi_3}{9} & m_{33}
\end{pmatrix}, \tag{39}$$

$$m_{66} = m_2 + \lambda_2\left(\frac{\Phi_1}{10\sqrt{6}} + \frac{\Phi_2}{30\sqrt{2}}\right) - i\frac{\lambda_{21}}{\lambda_{19}}\frac{\lambda_6\Phi_2}{30\sqrt{2}}, \tag{40}$$

$$m_{77} = m_1 + \lambda_1\left(\frac{\Phi_1}{\sqrt{6}} + \frac{\Phi_2}{3\sqrt{2}} + \frac{2\Phi_3}{3}\right) - i\frac{\lambda_{21}}{\lambda_{19}}\frac{\sqrt{2}\lambda_7\Phi_2}{15}, \tag{41}$$

$$m_{22} = m_6 + \frac{1}{3}\sqrt{\frac{2}{3}}\lambda_{15}\Phi_1, \tag{42}$$

$$m_{33} = m_6 + \frac{\sqrt{2}}{9}\lambda_{15}\Phi_2. \tag{43}$$

The Wilson coefficients of the terms $C_{5L}^{ijkl}(Q_kQ_l)(Q_iL_j)$ and $C_{5R}^{ijkl}E_k^cU_l^cU_i^cD_j^c$, which appear after integrating out the colored Higgs fields, are given by

$$C_{5L}^{ijkl}(\mu = \mu_{H_C}) = C_{5R}^{ijkl}(\mu = \mu_{H_C})$$

$$= \begin{pmatrix} (\tilde{Y}_{10})_{kl} & 0 & (\tilde{Y}_{126})_{kl} & (\tilde{Y}_{126})_{kl} & 0 & (\tilde{Y}_{120})_{kl} & (\tilde{Y}_{120})_{kl} \end{pmatrix} \mathcal{M}_{H_C}^{-1} \begin{pmatrix} (\tilde{Y}_{10})_{ij} \\ (\tilde{Y}_{126})_{ij} \\ 0 \\ 0 \\ 0 \\ (\tilde{Y}_{120})_{ij} \\ (\tilde{Y}_{120})_{ij} \end{pmatrix}. \quad (44)$$

First, since the upper-right 5×2 part of $\mathcal{M}_{H_C}$ is zero, the upper-right 5×2 part of the inverse matrix $\mathcal{M}_{H_C}^{-1}$ is also zero. It follows that the terms $(\tilde{Y}_{10})_{kl}(\tilde{Y}_{120})_{ij}$ and $(\tilde{Y}_{126})_{kl}(\tilde{Y}_{120})_{ij}$ do not appear in the Wilson coefficients C_{5L}^{ijkl} and C_{5R}^{ijkl}, and hence $c = f = 0$ in Eqs. (25)–(29). Second, the upper-left 5×5 part of $\mathcal{M}_{H_C}^{-1}$ is exactly the inverse matrix of the same part of $\mathcal{M}_{H_C}$. It is possible to mathematically prove that the components of $\mathcal{M}_{H_C}^{-1}$ satisfy the relation $(\mathcal{M}_{H_C}^{-1})_{11} : (\mathcal{M}_{H_C}^{-1})_{31} : (\mathcal{M}_{H_C}^{-1})_{41} = (\mathcal{M}_{H_C}^{-1})_{12} : (\mathcal{M}_{H_C}^{-1})_{32} : (\mathcal{M}_{H_C}^{-1})_{42}$ when the (3,2)-, (4,2)-, and (5,2)-components of $\mathcal{M}_{H_C}$ are zero as in Eq. (39). Then, since the numbers a, d in Eqs. (28) and (27) are determined by $(\mathcal{M}_{H_C}^{-1})_{11}$, $(\mathcal{M}_{H_C}^{-1})_{31}$, and $(\mathcal{M}_{H_C}^{-1})_{41}$ and the numbers b, e are determined by $(\mathcal{M}_{H_C}^{-1})_{12}$, $(\mathcal{M}_{H_C}^{-1})_{32}$, and $(\mathcal{M}_{H_C}^{-1})_{42}$, we get $a/d = b/e$. We comment that if the model contains a **54**-representation field, its VEV must be 0 to realize the relation $a/d = b/e$.

We are yet to prove that Eq. (37) is compatible with the situation that *all the fields have GUT-scale masses* except for one pair of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields that give the MSSM Higgs fields and a $(\mathbf{1}, \mathbf{3}, 1)$ field that has mass slightly below the GUT scale to realize the Type-2 seesaw mechanism. Also, Eq. (37) must be consistent with the value of a/d that realizes $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$, and with the values of r_1, r_2, r_3, r_e that reproduce the correct fermion data. (Note that common coupling constants enter the colored Higgs mass matrix and the mass matrix of the $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields.) We have numerically checked that under the restriction of Eq. (37) and the condition that the mass matrix of the $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields have one zero eigenvalue, the ratio of the masses of various fields (other than the pair of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields) and the values of a/d, r_1, r_2, r_3, r_e vary in a wide range and there is no correlation among them. It is thus quite likely that the gauge coupling unification is achieved with the help of GUT-scale threshold corrections and the right values of a/d and r_1, r_2, r_3, r_e are obtained even with Eq. (37).

4. Numerical search for the texture of Y_{10}, Y_{126}, and Y_{120}

We search for the texture of the Yukawa couplings Y_{10}, Y_{126}, and Y_{120} discussed in Sect. 3, i.e. the texture which reproduces the correct quark and lepton Yukawa couplings and neutrino mass matrix according to Eqs. (3)–(5) and (7), and in which the components of the Yukawa couplings $(Y_{10})_{u_R d_R}$, $(Y_{126})_{u_R d_R}$, $(Y_{10})_{u_R s_R}$, $(Y_{126})_{u_R s_R}$, $(Y_{10})_{u_L d_L}$, $(Y_{126})_{u_L d_L}$, $(Y_{10})_{u_L u_L}$, $(Y_{126})_{u_L u_L}$, $(Y_{10})_{u_L s_L}$, and $(Y_{126})_{u_L s_L}$ are reduced.

4.1. *Procedures*

First, we numerically calculate the MSSM Yukawa coupling matrices Y_u, Y_d, Y_e at scale $\mu = 2 \times 10^{16}$ GeV in the $\overline{\mathrm{DR}}$ scheme. We also calculate the *flavor-dependent* part of the RG corrections to the coefficient of the $L_i(\mathbf{1}, \mathbf{3}, 1)L_j$ operator ($(\mathbf{1}, \mathbf{3}, 1)$ is a component of $\overline{\Delta}$ and this operator originates from the $\Psi_i\overline{\Delta}\Psi_j$ operator) in the evolution from $\mu = 2 \times 10^{16}$ GeV to $\mu = 10^{13}$ GeV, and to the coefficient of the Weinberg operator in the evolution from $\mu = 10^{13}$ GeV to $\mu = M_Z$, written as R_{ij} and defined as $(C_\nu)_{ij}|_{\mu=M_Z} = c\sum_{k,l} R_{ik}R_{jl}(Y_{126})_{kl}|_{\mu=2\times10^{16}\,\mathrm{GeV}}$, where c is a flavor-independent constant. Here, 10^{13} GeV is the typical scale of the mass of a $(\mathbf{1}, \mathbf{3}, 1)$ particle that is integrated out to obtain the Weinberg operator (see Appendix B).[6] In the calculation of the RG equations, we assume the following spectrum of the pole masses of SUSY particles for concreteness:

$$\begin{aligned} & m_{\tilde{q}} = m_{\tilde{u}^c} = m_{\tilde{d}^c} = m_{\tilde{\ell}} = m_{\tilde{e}^c} = m_{H^0} = m_{H^\pm} = m_A = 20\,\text{TeV}, \\ & M_{\tilde{g}} = M_{\widetilde{W}} = \mu_H = 2\,\text{TeV}, \qquad \tan\beta \; = \; 50. \end{aligned} \tag{45}$$

However, we caution that the values of Y_u, Y_d, Y_e at $\mu = 2 \times 10^{16}$ GeV and R_{ij} only logarithmically depend on the SUSY particle mass spectrum, and so the texture of Y_{10}, Y_{126}, and Y_{120} we seek is not sensitive to the spectrum; for example, multiplying the spectrum by a factor of 10 does not change our results. We adopt the following input values for quark masses and CKM matrix parameters: The isospin-averaged quark mass and strange quark mass in the $\overline{\text{MS}}$ scheme are obtained from lattice calculations in Refs. [46–51] as $\frac{1}{2}(m_u + m_d)(2\,\text{GeV}) = 3.373(80)$ MeV and $m_s(2\,\text{GeV}) = 92.0(2.1)$ MeV. The up and down quark mass ratio is obtained from an estimate in Ref. [52] as $m_u/m_d = 0.46(3)$. The $\overline{\text{MS}}$ charm and bottom quark masses are obtained from quantum chromodynamics (QCD) sum rule calculations in Ref. [53] as $m_c(3\,\text{GeV}) = 0.986 - 9(\alpha_s^{(5)}(M_Z) - 0.1189)/0.002 \pm 0.010$ GeV and $m_b(m_b) = 4.163 + 7(\alpha_s^{(5)}(M_Z) - 0.1189)/0.002 \pm 0.014$ GeV. The top quark pole mass is obtained from $t\bar{t}$+jet events measured by ATLAS [54] as $M_t = 171.1 \pm 1.2$ GeV. The CKM mixing angles and CP phase are calculated from the Wolfenstein parameters in the latest CKM fitter result [55].[7] For the QCD and quantum electrodynamic gauge couplings, we use $\alpha_s^{(5)}(M_Z) = 0.1181$ and $\alpha^{(5)}(M_Z) = 1/127.95$. For the lepton and W, Z, Higgs pole masses, we use the values from the Particle Data Group [39].

The result is given in terms of the singular values of Y_u, Y_d, Y_e and the CKM mixing angles and CP phase at $\mu = 2 \times 10^{16}$ GeV, as well as R_{ij} in the flavor basis where Y_e is diagonal (R_{ij} is also diagonal in this basis); it is tabulated in Table 1. For each singular value of Y_u, Y_d, we present the $1\,\sigma$ error that has propagated from the experimental error of the corresponding input quark mass. For the CKM mixing angles and CP phase, we present $1\,\sigma$ errors that have propagated from experimental errors of the input Wolfenstein parameters.

Next, we fit the MSSM Yukawa couplings Y_u, Y_d, Y_e and the neutrino mixing angles and mass differences with the fundamental Yukawa couplings Y_{10}, Y_{126}, Y_{120} and the numbers r_1, r_2, r_3, r_e according to Eqs. (3)–(5) and (7). Meanwhile, we minimize the following quantity:

$$\sum_{A=10,126} \left\{ \left|(Y_A)_{u_R d_R}\right|^2 + \left|(Y_A)_{u_R s_R}\right|^2 + \left|(Y_A)_{u_L d_L}\right|^2 + \left|(Y_A)_{u_L u_L}\right|^2 + \left|(Y_A)_{u_L s_L}\right|^2 \right\}. \tag{46}$$

To facilitate the analysis, we concentrate on the parameter region where $r_3 = 0$ (which is compatible with any values of r_1, r_2, r_e and Eq. (37)). Then, we obtain $(Y_{10})_{u_R j} + r_2 (Y_{126})_{u_R j} \leq y_u$ and $(Y_{10})_{u_L j} + r_2 (Y_{126})_{u_L j} \leq y_u$ for any flavor index j, and it becomes easier to reduce Eq. (46) to the order of the tiny up quark Yukawa coupling y_u. Given $r_3 = 0$, Eqs. (3)–(5) can be rearranged as follows: We fix the flavor basis such that the left-handed down-type quark components in Ψ_i have diagonal Y_d Yukawa coupling with real positive diagonal components. Y_u, which is still symmetric, is then

[6] In our analysis, we neglect RG corrections involving the coupling of the $L_i(\mathbf{1}, \mathbf{3}, 1)L_j$ operator, since it is much smaller than 1 in all the fitting and minimization results.

[7] Updated results and plots are available at http://ckmfitter.in2p3.fr.

Table 1. The singular values of MSSM Yukawa couplings Y_u, Y_d, and Y_e, and the mixing angles and CP phase of the CKM matrix, at $\mu = 2\times10^{16}$ GeV in the $\overline{\mathrm{DR}}$ scheme. Also shown is the flavor-dependent RG correction R_{ij} for the Weinberg operator C_ν, defined as $(C_\nu)_{ij}|_{\mu=M_Z} = c\sum_{k,l} R_{ik}R_{jl}(Y_{126})_{kl}|_{\mu=2\times10^{16}\,\mathrm{GeV}}$ (c is a flavor-independent constant), in the flavor basis where Y_e is diagonal (R_{ij} is also diagonal in this basis). For each singular value of the quark Yukawa matrices, we present the $1\,\sigma$ error that has propagated from the experimental error of the corresponding input quark mass, and for the CKM parameters, we present $1\,\sigma$ errors that have propagated from experimental errors of the input Wolfenstein parameters.

	Value with Eq. (45)
y_u	$2.69(14)\times10^{-6}$
y_c	0.001384(14)
y_t	0.478(98)
y_d	0.0002908(92)
y_s	0.00579(13)
y_b	0.3552(23)
y_e	0.00012202
y_μ	0.025766
y_τ	0.50441
$\cos\theta_{13}^{\mathrm{ckm}}\sin\theta_{12}^{\mathrm{ckm}}$	0.22474(25)
$\cos\theta_{13}^{\mathrm{ckm}}\sin\theta_{23}^{\mathrm{ckm}}$	0.0398(10)
$\sin\theta_{13}^{\mathrm{ckm}}$	0.00352(21)
δ_{km} (rad)	1.147(33)
R_{ee}	1.00
$R_{\mu\mu}$	1.00
$R_{\tau\tau}$	0.961

written as[8]

$$Y_u = V_{\mathrm{CKM}}^{\mathrm{T}} \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c\, e^{2i\,d_2} & 0 \\ 0 & 0 & y_t\, e^{2i\,d_3} \end{pmatrix} V_{\mathrm{CKM}}, \tag{47}$$

where d_2, d_3 are unknown phases. In the same flavor basis, Y_d becomes

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} V_{dR}, \tag{48}$$

where V_{dR} is an unknown unitary matrix. From Eqs. (4) and (5), and the fact that Y_{10} and Y_{126} are symmetric and Y_{120} is antisymmetric, we get

$$Y_{126} = \frac{1}{1-r_2}\left\{\frac{1}{r_1}\frac{1}{2}\left(Y_d + Y_d^{\mathrm{T}}\right) - Y_u\right\}, \tag{49}$$

$$\frac{1}{r_1}Y_e = Y_u - (3+r_2)Y_{126} + r_e\frac{1}{r_1}\frac{1}{2}\left(Y_d - Y_d^{\mathrm{T}}\right). \tag{50}$$

[8] Note that Y_u in Eq. (2) is the complex conjugate of the SM Y_u defined as $-\mathcal{L} = \bar{q}_L Y_u u_R H$.

We perform the singular value decomposition of Y_e as

$$Y_e = U_{eL} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} U_{eR}^\dagger, \tag{51}$$

and calculate the active neutrino mass matrix in the charged-lepton-diagonal basis as

$$(M_\nu)_{\ell\ell'} \ \propto \ R_{\ell\ell}\, (U_{eL}^{\rm T} Y_{126} U_{eL})_{\ell\ell'}\, R_{\ell'\ell'}, \qquad \ell, \ell' = e, \mu, \tau, \tag{52}$$

where ℓ, ℓ' denote flavor indices for the left-handed charged leptons. Utilizing Eqs. (47)–(52), we perform the fitting as follows. We fix y_u, y_c, y_t and the CKM matrix by the values in Table 1, while we vary y_d/r_1, y_s/r_1, y_b/r_1, the unknown phases d_2, d_3, the unknown unitary matrix V_{dR}, and the complex numbers r_2, r_e. Here we eliminate r_1 by requiring that the central value of the electron Yukawa coupling y_e be reproduced. In this way, we try to reproduce the correct values of y_d, y_s, y_μ, y_τ, $\theta_{12}^{\rm pmns}$, $\theta_{13}^{\rm pmns}$, $\theta_{23}^{\rm pmns}$, and the neutrino mass difference ratio $\Delta m^2_{21}/\Delta m^2_{32}$. Specifically, we require y_d, y_s to fit within their respective $3\,\sigma$ ranges, while we do not constrain y_b because y_b may be subject to sizable GUT-scale threshold corrections. We impose stringent restrictions on the values of neutrino mixing angles and mass differences, because we are primarily interested in the prediction for the neutrino Dirac CP phase from the condition that dimension-5 proton decays be suppressed as much as possible, and so it is essential to suppress variation of the other neutrino parameters. In particular, we require $\sin^2\theta_{12}^{\rm pmns}$, $\sin^2\theta_{13}^{\rm pmns}$, and $\Delta m^2_{21}/\Delta m^2_{32}$ to fit within their respective $1\,\sigma$ ranges as reported by NuFIT 4.1 [56,57]. We assume two narrow benchmark ranges of $\sin^2\theta_{23}^{\rm pmns}$, since the current experimental error of $\sin^2\theta_{23}^{\rm pmns}$ is too large. Only the normal hierarchy of the neutrino mass is considered because no good fitting is obtained with the inverted hierarchy. Finally, since the experimental errors of y_μ, y_τ are tiny, we only require their reproduced values to fit within $\pm 0.1\%$ of their central values. The constraints are summarized in Table 2.

Within the constraints of Table 2, we minimize the quantity in Eq. (46) repeatedly starting from different random values of y_d/r_1, y_s/r_1, y_b/r_1, d_2, d_3, V_{dR}, r_2, and r_e. Each fitting and minimization result is plotted on the planes of the neutrino Dirac CP phase $\delta_{\rm pmns}$, the lightest neutrino mass m_1, and the absolute value of the (1,1)-component of the neutrino mass matrix in the charged-lepton-diagonal basis $|m_{ee}|$, versus the “maximal proton decay amplitude” defined in the next subsection.

4.2. Results

We present plots of the fitting and minimization results obtained by the procedures of Sect. 4.1 on the planes of $\delta_{\rm pmns}$, m_1, $|m_{ee}|$ versus “maximal proton decay amplitude.” The “maximal proton decay amplitude” of the $p \to K^+\nu$ mode, $\tilde{A}(p \to K^+\nu)$, is defined as

$$\begin{aligned}\tilde{A}(p \to K^+\nu)^2 = &\left\{\tilde{A}(p \to K^+\bar{\nu}_\tau)|_{\text{from } C_{5R}} + \tilde{A}(p \to K^+\bar{\nu}_\tau)|_{\text{from } C_{5L}}\right\}^2 \\ &+ \tilde{A}(p \to K^+\bar{\nu}_\mu)^2|_{\text{from } C_{5L}} + \tilde{A}(p \to K^+\bar{\nu}_e)^2|_{\text{from } C_{5L}},\end{aligned} \tag{53}$$

where

$$\begin{aligned}&\tilde{A}(p \to K^+\bar{\nu}_\tau)|_{\text{from } C_{5R}} \\ &\quad = y_t y_\tau \sum_{A,B} \left| \left(1 + \frac{D}{3} + F\right) V_{ts}^{\rm ckm} \left\{(Y_A)_{\tau_R t_R}(Y_B)_{u_R d_R} - (Y_A)_{\tau_R u_R}(Y_B)_{t_R d_R}\right\}\right.\end{aligned}$$

Table 2. Allowed ranges of quantities in the analysis.

	Allowed range
y_u	2.73×10^{-6} (fixed)
y_c	0.001406 (fixed)
y_t	0.4842 (fixed)
y_d	$0.0002953 \pm 0.0000093 \cdot 3$
y_s	$0.00588 \pm 0.00013 \cdot 3$
y_b	unconstrained
y_e	0.00012288 (used to fix r_1)
y_μ	$0.025948 \pm 0.1\%$
y_τ	$0.50625 \pm 0.1\%$
$\cos\theta_{13}^{\rm ckm} \sin\theta_{12}^{\rm ckm}$	0.22474 (fixed)
$\cos\theta_{13}^{\rm ckm} \sin\theta_{23}^{\rm ckm}$	0.0399 (fixed)
$\sin\theta_{13}^{\rm ckm}$	0.00352 (fixed)
$\delta_{\rm km}$ (rad)	1.147 (fixed)
$\sin^2\theta_{12}^{\rm pmns}$	0.310 ± 0.012
$\sin^2\theta_{13}^{\rm pmns}$	0.02237 ± 0.00065
$\sin^2\theta_{23}^{\rm pmns}$	0.45 ± 0.01 or 0.55 ± 0.01
$\Delta m_{21}^2/\Delta m_{32}^2$	0.02923 ± 0.00084
$\delta_{\rm pmns},\ \alpha_2,\ \alpha_3,\ m_1$	unconstrained
$d_2,\ d_3,\ V_{dR}$	unconstrained
r_1	eliminated in favor of y_e
r_3	0 (fixed)
$r_2,\ r_e$	unconstrained

$$+\frac{2D}{3}\,V_{td}^{\rm ckm}\,\{(Y_A)_{\tau_R t_R}(Y_B)_{u_R s_R} - (Y_A)_{\tau_R u_R}(Y_B)_{t_R s_R}\}\Big| \tag{54}$$

(sum over $A, B = (10, 10), (10, 126), (126, 10), (126, 126), (120, 10), (120, 126), (120, 120)$),

$$\tilde{A}(p \to K^+\bar{\nu}_\alpha)|_{\text{from } C_{5L}} = g_2^2 \sum_{A,B=10,126} \frac{3}{2}\left|\left(1+\frac{D}{3}+F\right)(Y_A)_{u_L d_L}(Y_B)_{s_L \alpha_L} + \frac{2D}{3}(Y_A)_{u_L s_L}(Y_B)_{d_L \alpha_L}\right|, \tag{55}$$

with $\alpha = e, \mu, \tau$, and y_t, y_τ, g_2 denoting the top and tau Yukawa couplings and the weak gauge coupling at soft SUSY-breaking scale $\mu_{\rm SUSY}$. The "maximal proton decay amplitudes" of the $p \to K^0 e_\beta^+$ modes, $\tilde{A}(p \to K^0 e_\beta^+)$, are defined as

$$\tilde{A}(p \to K^0 e_\beta^+) = g_2^2 \sum_{A,B=10,126} \frac{3}{2}(1-D+F)\left|(Y_A)_{u_L s_L}(Y_B)_{u_L \beta_L} - (Y_A)_{u_L u_L}(Y_B)_{s_L \beta_L}\right|, \tag{56}$$

with $\beta = e, \mu$. The above $\tilde{A}(p \to K^+\nu)^2$ is related to the $p \to K^+\nu$ decay width when the coefficients a, b, d, e, g, h, j have similar absolute values[9] and the terms with these coefficients interfere maximally constructively under the condition of $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$, and when the contributions from left-handed dimension-5 operators to the $p \to K^+\bar{\nu}_\tau$ mode and

[9] Remember that we are setting $c = f = 0$.

those from right-handed ones interfere maximally constructively. Likewise, $\tilde{A}(p \to K^0 e^+_\beta)^2$ are related to the $p \to K^0 e^+_\beta$ decay widths when the coefficients a, b, d, e have similar absolute values and the terms with these coefficients interfere maximally constructively under the condition of $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$. Therefore, $\tilde{A}(p \to K^+ \nu)$ and $\tilde{A}(p \to K^0 e^+_\beta)$ allow us to estimate how much the texture of the GUT-scale Yukawa couplings Y_{10}, Y_{126}, and Y_{120} contributes to the suppression of dimension-5 proton decays.

As a matter of fact, we have found that $\tilde{A}(p \to K^0 e^+_\beta)$ ($\beta = e, \mu$) are smaller than $\tilde{A}(p \to K^+ \nu)$ in all the fitting and minimization results. Considering that the current experimental bound is more severe for the $p \to K^+ \nu$ decay than for the $p \to K^0 e^+_\beta$ decays, it is phenomenologically more important to study the suppression of $\tilde{A}(p \to K^+ \nu)$ than of $\tilde{A}(p \to K^0 e^+_\beta)$. Therefore, we present the plots of $\delta_{\rm pmns}$, m_1, and $|m_{ee}|$ versus $\tilde{A}(p \to K^+ \nu)$ only, and solely discuss the suppression of $\tilde{A}(p \to K^+ \nu)$.

As a reference, we also present the "minimal proton partial lifetime" $1/\tilde{\Gamma}(p \to K^+ \nu)$ that corresponds to $\tilde{A}(p \to K^+ \nu)$ and is computed for a sample SUSY particle mass spectrum. It is defined as

$$
\begin{aligned}
&\tilde{\Gamma}(p \to K^+ \nu) = \frac{m_N}{64\pi}\left(1 - \frac{m_K^2}{m_N^2}\right)^2 \times \\
&\Bigg[\Bigg\{ \Bigg| \frac{\alpha_H(\mu_{\rm had})}{f_\pi} A_{RL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{\mu_H}{m^2_{\tilde{t}_R}} \mathcal{F}' A_R^{\tau t}(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}(p \to K^+ \bar{\nu}_\tau)|_{\text{from } C_{5R}} \Bigg| \\
&\quad + \Bigg| \frac{\beta_H(\mu_{\rm had})}{f_\pi} A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{M_{\tilde{W}}}{m^2_{\tilde{q}}} \mathcal{F} A_L^{\tau}(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}(p \to K^+ \bar{\nu}_\tau)|_{\text{from } C_{5L}} \Bigg| \Bigg\}^2 \\
&+ \Bigg| \frac{\beta_H(\mu_{\rm had})}{f_\pi} A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{M_{\tilde{W}}}{m^2_{\tilde{q}}} \mathcal{F} A_L(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}(p \to K^+ \bar{\nu}_\mu)|_{\text{from } C_{5L}} \Bigg|^2 \\
&+ \Bigg| \frac{\beta_H(\mu_{\rm had})}{f_\pi} A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{M_{\tilde{W}}}{m^2_{\tilde{q}}} \mathcal{F} A_L(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}(p \to K^+ \bar{\nu}_e)|_{\text{from } C_{5L}} \Bigg|^2 \Bigg], \qquad (57)
\end{aligned}
$$

where the following sample spectrum of the pole masses of SUSY particles is assumed:

$$
\begin{aligned}
&m_{H^0} = m_{H^\pm} = m_A = (\text{sfermion mass}) = 300\,\text{TeV}, \\
&M_{\tilde{g}} = M_{\tilde{W}} = \mu_H = 10\,\text{TeV}, \qquad \tan\beta = 50. \qquad (58)
\end{aligned}
$$

We also assume $M_{H_C} = 2 \times 10^{16}$ GeV. The symbols in Eq. (57) were defined in the paragraphs containing Eqs. (12)–(29),[10] and m_K denotes the kaon mass, m_N the nucleon mass, and f_π the pion decay constant in the chiral limit. The "minimal proton partial lifetime" $1/\tilde{\Gamma}(p \to K^+ \nu)$ is the partial lifetime when the absolute values of the coefficients a, b, d, e, g, h, j are all 1 and the terms with these coefficients interfere maximally constructively under the condition of $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$, and when the contributions from left-handed dimension-5 operators and those from right-handed ones interfere maximally constructively, hence the name. We

[10] We have neglected the muon and electron Yukawa couplings in the calculation of A_L^μ and A_L^e, and rewritten them as A_L.

consider $1/\tilde{\Gamma}(p \to K^+\nu)$ to be a good measure for how much the texture of Y_{10}, Y_{126}, and Y_{120} contributes to the suppression of the $p \to K^+\nu$ decay. We caution that the mass spectrum in Eq. (58) is assumed for reference purposes and has no intrinsic meaning. We also comment that although the mass spectrum in Eq. (58) is different from the one assumed for the RG calculation of the GUT-scale Yukawa couplings, the resulting difference in the GUT-scale Yukawa couplings is less significant than that stemming from experimental uncertainties of the input parameters.

In the evaluation of $\tilde{A}(p \to K^+\nu)$ and $1/\tilde{\Gamma}(p \to K^+\nu)$, we employ the following numerical values and formulas: The baryon chiral Lagrangian parameters are given by $D = 0.804$, $F = 0.463$, and the pion decay constant in the chiral limit is $f_\pi = 0.0868$ GeV [58]. The hadronic form factors for proton decay amplitudes are taken from Ref. [59] as $\alpha_H(\mu_{\rm had}) = -\beta_H(\mu_{\rm had}) = -0.0144\,{\rm GeV}^3$ for $\mu_{\rm had} = 2$ GeV. The RG corrections represented by A_{RL}, A_{LL}, $A_R^{\tau t}$, A_L^τ, and A_L are calculated by using one-loop RG equations [36,60]. We take $\mu_{H_C} = 2 \times 10^{16}$ GeV and $\mu_{\rm SUSY} = 300$ TeV.

Figure 1 displays the results with the higher-octant benchmark where $\sin^2\theta_{23}^{\rm pmns} = 0.55 \pm 0.01$, and Fig. 2 displays those with the lower-octant benchmark where $\sin^2\theta_{23}^{\rm pmns} = 0.45 \pm 0.01$. The left panels show $\delta_{\rm pmns}$, m_1, and $|m_{ee}|$ versus $\tilde{A}(p \to K^+\nu)$, and the right ones show $\delta_{\rm pmns}$, m_1, and $|m_{ee}|$ versus $1/\tilde{\Gamma}(p \to K^+\nu)$. In the plots, each dot corresponds to the result of one fitting and minimization analysis starting from a different random set of values of y_d/r_1, y_s/r_1, y_b/r_1, d_2, d_3, V_{dR}, r_2, and r_e. The horizontal line in each of the right panels corresponds to the current experimental 90% confidence level bound on the $p \to K^+\nu$ partial lifetime, $1/\Gamma(p \to K^+\nu) = 5.9 \times 10^{33}$ yr [35].

From the upper panels of Figs. 1 and 2, we observe that the dimension-5 proton decays are most suppressed for the neutrino Dirac CP phase satisfying $\pi/2 \gtrsim \delta_{\rm pmns} \gtrsim -\pi/2$. From the middle panels, we find that the dimension-5 proton decays are most suppressed for the lightest neutrino mass around $m_1 \simeq 0.003$ eV. From the lower panels, we see that the dimension-5 proton decays are most suppressed when the (1,1)-component of the neutrino mass matrix in the charged lepton basis satisfies $|m_{ee}| \lesssim 0.0002$ eV. The distributions of the fitting and minimization results are qualitatively the same for the higher-octant benchmark with $\sin^2\theta_{23}^{\rm pmns} = 0.55 \pm 0.01$ and the lower-octant benchmark with $\sin^2\theta_{23}^{\rm pmns} = 0.45 \pm 0.01$, which suggests that the results do not depend on the precise value of $\sin^2\theta_{23}^{\rm pmns}$. The predicted range of the Dirac CP phase $\pi/2 \gtrsim \delta_{\rm pmns} \gtrsim -\pi/2$ will be confirmed or falsified in long-baseline neutrino oscillation experiments. At present, NuFit 5.0 [61] reports a slight contradiction between the results of the T2K and the NOvA long-baseline experiments on the Dirac CP phase in the normal mass hierarchy case. Therefore, we cannot currently state that the above predicted range is experimentally favored or disfavored. The predicted values of m_1 and $|m_{ee}|$, with the normal neutrino mass hierarchy, are beyond the reach of ongoing and future cosmological and low-energy experiments.

In the right panels, the dots above the black horizontal line are those fitting and minimization results which always satisfy the current experimental bounds on proton decays for the sample SUSY particle mass spectrum of Eq. (58), and when the colored Higgs mass matrix satisfies $c = f = 0$, $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$, and when the coefficients a, b, d, e, g, h, and j have $O(1)$ absolute value with $M_{H_C} = 2 \times 10^{16}$ GeV. That is to say, the dots above the black horizontal line are *predictions* of the $SO(10)$ GUT with the SUSY particle spectrum of Eq. (58) and the texture of the colored Higgs mass matrix with $c = f = 0$, $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$. The dots slightly below the black horizontal line by an $O(1)$ factor are also viable, because the coefficients a, b, d, e, g, h, and j can vary by $O(1)$. As the dots above or slightly below the black horizontal line satisfy $\pi/2 \gtrsim \delta_{\rm pmns} \gtrsim -\pi/2$, $0.004\,{\rm eV} \gtrsim m_1 \gtrsim 0.002$ eV

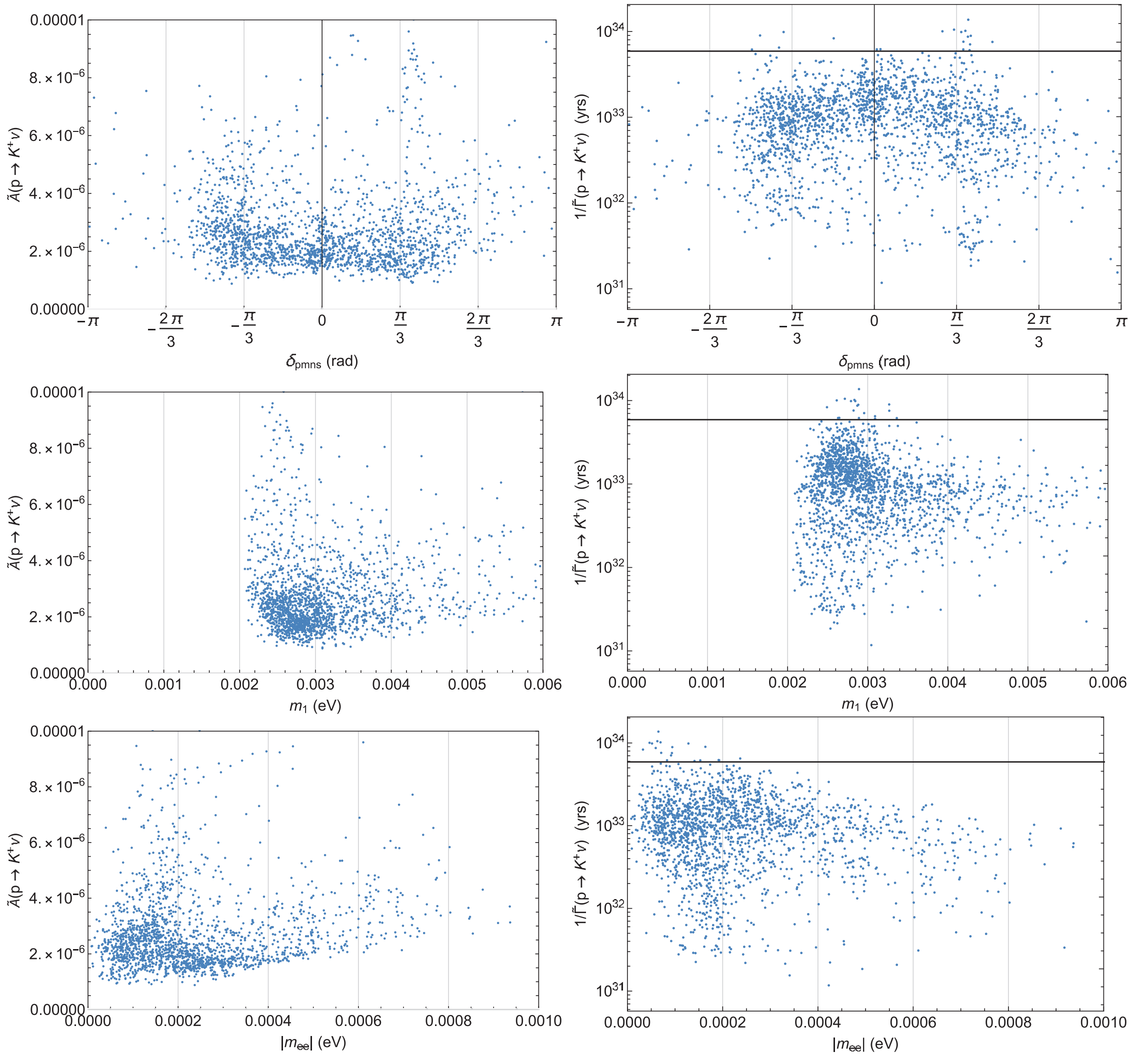

Fig. 1. Results of the fitting and minimization analysis in Sect. 4.1, where the quantity in Eq. (46) is minimized within the constraints of Table 2. Here we choose the higher-octant benchmark where $\sin^2\theta_{23}^{\rm pmns} = 0.55 \pm 0.01$ in Table 2. Each dot corresponds to the result of one analysis starting from a different set of random values of y_d/r_1, y_s/r_1, y_b/r_1, d_2, d_3, V_{dR}, r_2, r_e. The vertical line of the left three panels indicates the "maximal proton decay amplitude" $\tilde{A}(p \to K^+\nu)$ defined in Eq. (53), and that of the right three panels indicates the "minimal proton partial lifetime" $1/\tilde{\Gamma}(p \to K^+\nu)$ defined in Eq. (57). From the upper to the lower panels, the horizontal line indicates the neutrino Dirac CP phase $\delta_{\rm pmns}$, the lightest neutrino mass m_1, and the absolute value of the (1,1)-component of the neutrino mass matrix in the charged-lepton-diagonal basis $|m_{ee}|$. The horizontal line in each of the right panels corresponds to the current experimental 90% CL bound on the $p \to K^+\nu$ partial lifetime, $q1/\Gamma(p \to K^+\nu) = 5.9 \times 10^{33}$ yr.

and $|m_{ee}| < 0.0006$ eV, these values of $\delta_{\rm pmns}$, m_1, and $|m_{ee}|$ are *predictions* of the current model. However, it should be remembered that the SUSY particle spectrum of Eq. (58) is for reference purposes only, and the most important message of the present paper is not that the $SO(10)$ GUT with Eq. (58) predicts the above values of $\delta_{\rm pmns}$, m_1, and $|m_{ee}|$, but that dimension-5 proton decays are most suppressed for these values of $\delta_{\rm pmns}$, m_1, and $|m_{ee}|$ independently of the details of the SUSY particle spectrum.

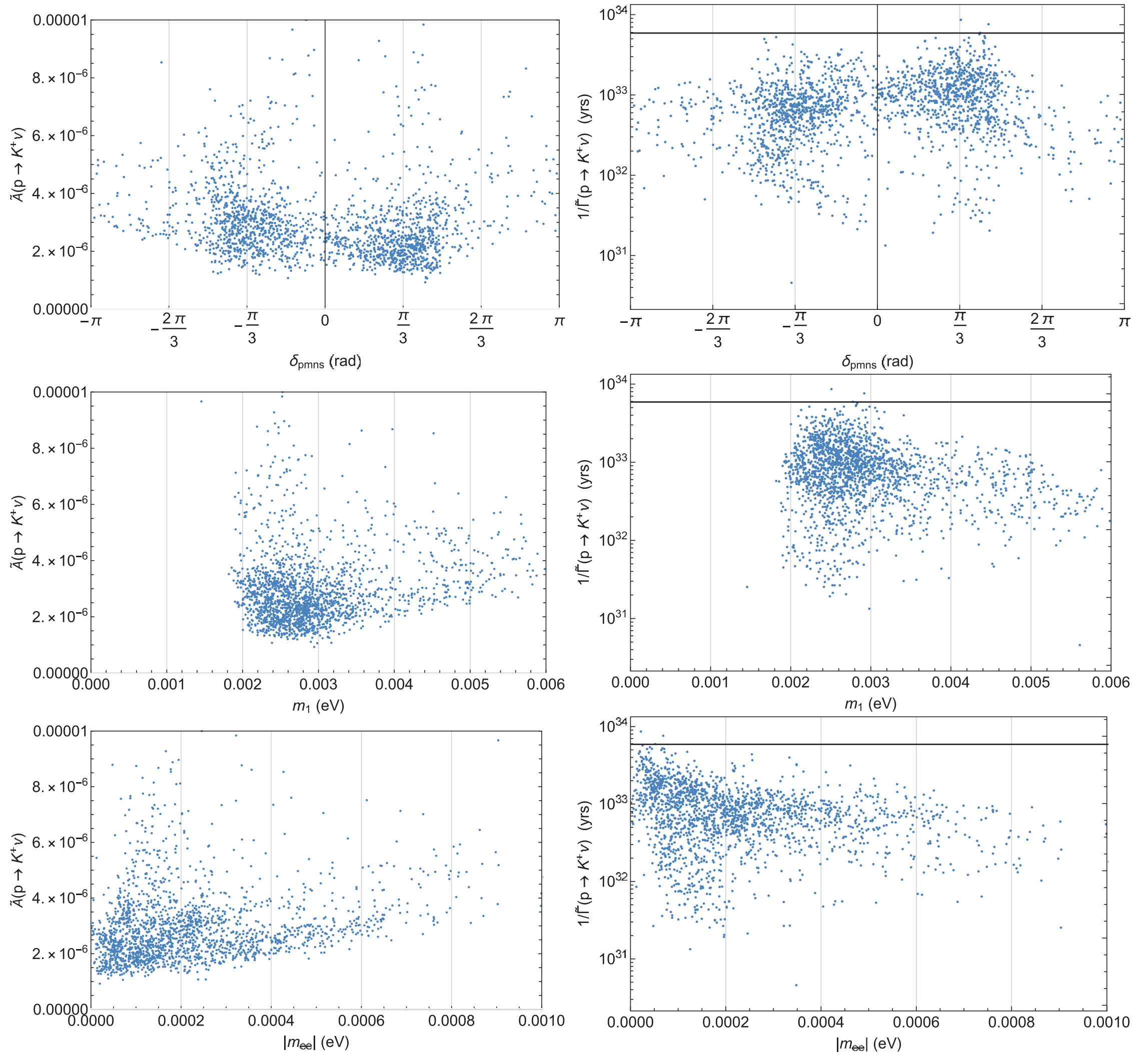

Fig. 2. As Fig. 1 except that we choose the lower-octant benchmark where $\sin^2\theta_{23}^{\rm pmns} = 0.45 \pm 0.01$ in Table 2.

Finally, we discuss the effectiveness of the texture of the colored Higgs mass matrix (texture that gives $c = f = 0, a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$) on the suppression of dimension-5 proton decays. To this end, we compare the "maximal proton decay amplitudes" with and without the colored Higgs mass texture, and the "minimal proton partial lifetimes" with and without that texture, and study how the fitting and minimization results yield different values. We define the "maximal proton decay amplitude without colored Higgs mass texture," $\tilde{A}_{\rm no\,tex}(p \to K^+\nu)$, and the corresponding "minimal proton partial lifetime without colored Higgs mass texture," $1/\tilde{\Gamma}_{\rm no\,tex}(p \to K^+\nu)$, as follows:

$$\tilde{A}_{\rm no\,tex}(p \to K^+\nu)^2 = \left\{\tilde{A}_{\rm no\,tex}(p \to K^+\bar{\nu}_\tau)|_{{\rm from}\ C_{5R}} + \tilde{A}_{\rm no\,tex}(p \to K^+\bar{\nu}_\tau)|_{{\rm from}\ C_{5L}}\right\}^2$$
$$+\,\tilde{A}_{\rm no\,tex}(p \to K^+\bar{\nu}_\mu)^2|_{{\rm from}\ C_{5L}} + \tilde{A}_{\rm no\,tex}(p \to K^+\bar{\nu}_e)^2|_{{\rm from}\ C_{5L}}, \tag{59}$$

where

$$\tilde{A}_{\rm no\,tex}(p \to K^+ \bar{\nu}_\tau)|_{{\rm from}\ C_{5R}}$$
$$= y_t y_\tau \sum_{A,B} \left| \left(1 + \frac{D}{3} + F\right) V_{ts}^{\rm ckm} \{(Y_A)_{\tau_R t_R}(Y_B)_{u_R d_R} - (Y_A)_{\tau_R u_R}(Y_B)_{t_R d_R}\} \right.$$
$$\left. + \frac{2D}{3} V_{td}^{\rm ckm} \{(Y_A)_{\tau_R t_R}(Y_B)_{u_R s_R} - (Y_A)_{\tau_R u_R}(Y_B)_{t_R s_R}\} \right| \tag{60}$$

(sum over $A = 10, 126, 120$ and $B = 10, 26, 120$) and

$$\tilde{A}_{\rm no\,tex}(p \to K^+ \bar{\nu}_\alpha)|_{{\rm from}\ C_{5L}}$$
$$= g_2^2 \sum_{A,B} \frac{3}{2} \left| \left(1 + \frac{D}{3} + F\right) \{(Y_A)_{u_L d_L}(Y_B)_{s_L \alpha_L} - (Y_A)_{d_L s_L}(Y_B)_{u_L \alpha_L}\} \right.$$
$$\left. + \frac{2D}{3} \{(Y_A)_{u_L s_L}(Y_B)_{d_L \alpha_L} - (Y_A)_{d_L s_L}(Y_B)_{u_L \alpha_L}\} \right| \tag{61}$$

(sum over $A = 10, 126$ and $B = 10, 26, 120$), with $\alpha = e, \mu, \tau$, and

$$\tilde{\Gamma}_{\rm no\,tex}(p \to K^+ \nu) =$$
$$\frac{m_N}{64\pi}\left(1 - \frac{m_K^2}{m_N^2}\right)^2$$
$$\times \left[\left\{ \left| \frac{\alpha_H(\mu_{\rm had})}{f_\pi} A_{RL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{\mu_H}{m_{\tilde{t}_R}^2} \mathcal{F}' A_R^{\tau t}(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}_{\rm no\,tex}(p \to K^+ \bar{\nu}_\tau)|_{{\rm from}\ C_{5R}} \right| \right. \right.$$
$$\left. + \left| \frac{\beta_H(\mu_{\rm had})}{f_\pi} A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \mathcal{F} A_L^{\tau}(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}_{\rm no\,tex}(p \to K^+ \bar{\nu}_\tau)|_{{\rm from}\ C_{5L}} \right| \right\}^2$$
$$+ \left| \frac{\beta_H(\mu_{\rm had})}{f_\pi} A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \mathcal{F} A_L(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}_{\rm no\,tex}(p \to K^+ \bar{\nu}_\mu)|_{{\rm from}\ C_{5L}} \right|^2$$
$$\left. + \left| \frac{\beta_H(\mu_{\rm had})}{f_\pi} A_{LL}(\mu_{\rm had}, \mu_{\rm SUSY}) \frac{M_{\tilde{W}}}{m_{\tilde{q}}^2} \mathcal{F} A_L(\mu_{\rm SUSY}, \mu_{H_C}) \frac{1}{M_{H_C}} \tilde{A}_{\rm no\,tex}(p \to K^+ \bar{\nu}_e)|_{{\rm from}\ C_{5L}} \right|^2 \right] \tag{62}$$

with the SUSY particle spectrum of Eq. (58). It is important to note that the terms with coefficient c, f are revived in Eqs. (60) and (61), and the $(Y_A)_{d_L s_L}(Y_B)_{u_L \alpha_L}$ terms are revived in Eq. (61), corresponding to the situation where no texture is assumed for the colored Higgs mass matrix, $|c| = |f| = O(1)$, and all the terms, including those with coefficient c, f, interfere maximally constructively. We plot the fitting and minimization results on the planes of $\delta_{\rm pmns}$, m_1, and $|m_{ee}|$ versus "maximal proton decay amplitude without colored Higgs mass texture," $\tilde{A}_{\rm no\,tex}(p \to K^+\nu)$, or "minimal proton partial lifetime without colored Higgs mass texture," $1/\tilde{\Gamma}_{\rm no\,tex}(p \to K^+\nu)$, in Figs. 3 and 4. Each dot corresponds to one fitting and minimization result that has appeared in Figs. 1 and 2.

Figures 3 and 4 tell us that without the texture of the colored Higgs mass matrix, the fitting and minimization results always give $\tilde{A}(p \to K^+\nu) > 10^{-5}$ and $1/\tilde{\Gamma}(p \to K^+\nu) < 2 \times 10^{31}$ yr (for the SUSY particle spectrum of Eq. (58)), which means that the $p \to K^+\nu$ decay is highly enhanced compared to the case with that texture. We thus conclude that the texture of the colored Higgs mass

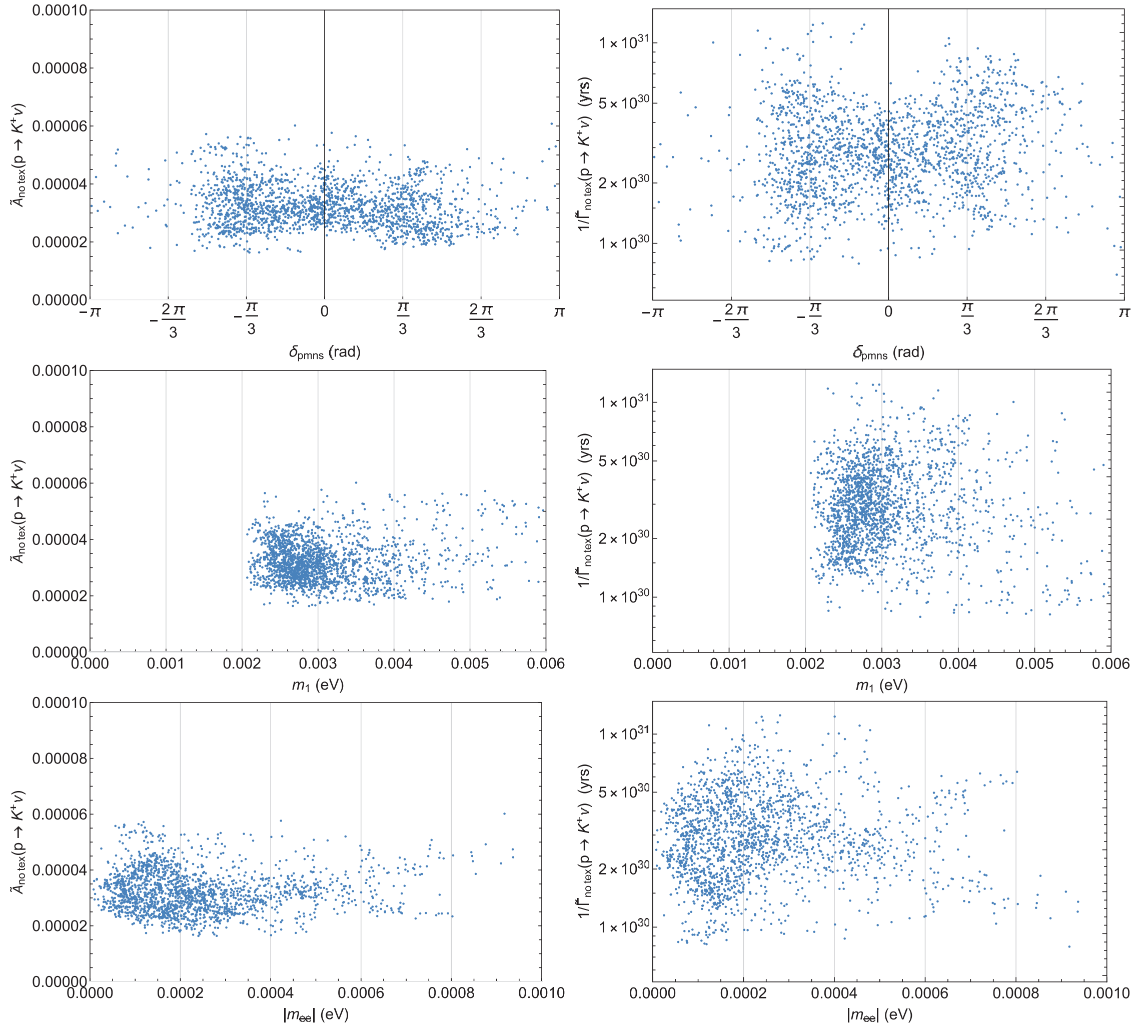

Fig. 3. As Fig. 1 except that we assume no texture for the colored Higgs mass matrix, and accordingly, we assume $|c| = |f| = 1$ and that all the terms, including those with coefficient c, f, interfere maximally constructively.

matrix with $c = f = 0$, $a\,(Y_{10})_{d_L s_L} + d\,(Y_{126})_{d_L s_L} = 0$ and $b\,(Y_{10})_{d_L s_L} + e\,(Y_{126})_{d_L s_L} = 0$, plays an important role in the suppression of dimension-5 proton decays.

5. Summary

In the renormalizable SUSY $SO(10)$ GUT model which includes single **10**, single $\overline{\mathbf{126}}$, and single **120** fields, and where the renormalizable terms $\tilde{Y}_{10}\,\mathbf{16}\,\mathbf{10}\,\mathbf{16} + \tilde{Y}_{126}\,\mathbf{16}\,\overline{\mathbf{126}}\,\mathbf{16} + \tilde{Y}_{120}\,\mathbf{16}\,\mathbf{120}\,\mathbf{16}$ account for the quark and lepton Yukawa couplings and neutrino mass matrix, we have pursued the possibility that a texture of the fundamental Yukawa couplings $\tilde{Y}_{10}$, $\tilde{Y}_{126}$, and $\tilde{Y}_{120}$ suppresses dimension-5 proton decays while reproducing the correct fermion data. Here we have assumed that the active neutrino mass comes mostly from the Type-2 seesaw mechanism. First, we have spotted those components of the Yukawa matrices $Y_{10}(\propto \tilde{Y}_{10})$, $Y_{126}(\propto \tilde{Y}_{126})$, and $Y_{120}(\propto \tilde{Y}_{120})$ which can be reduced to suppress dimension-5 proton decays without conflicting the requirement that they reproduce the correct quark and lepton Yukawa couplings and neutrino mass matrix. Next, we have performed a numerical search for the texture of Y_{10}, Y_{126}, and Y_{120} by fitting the data on the quark and lepton masses,

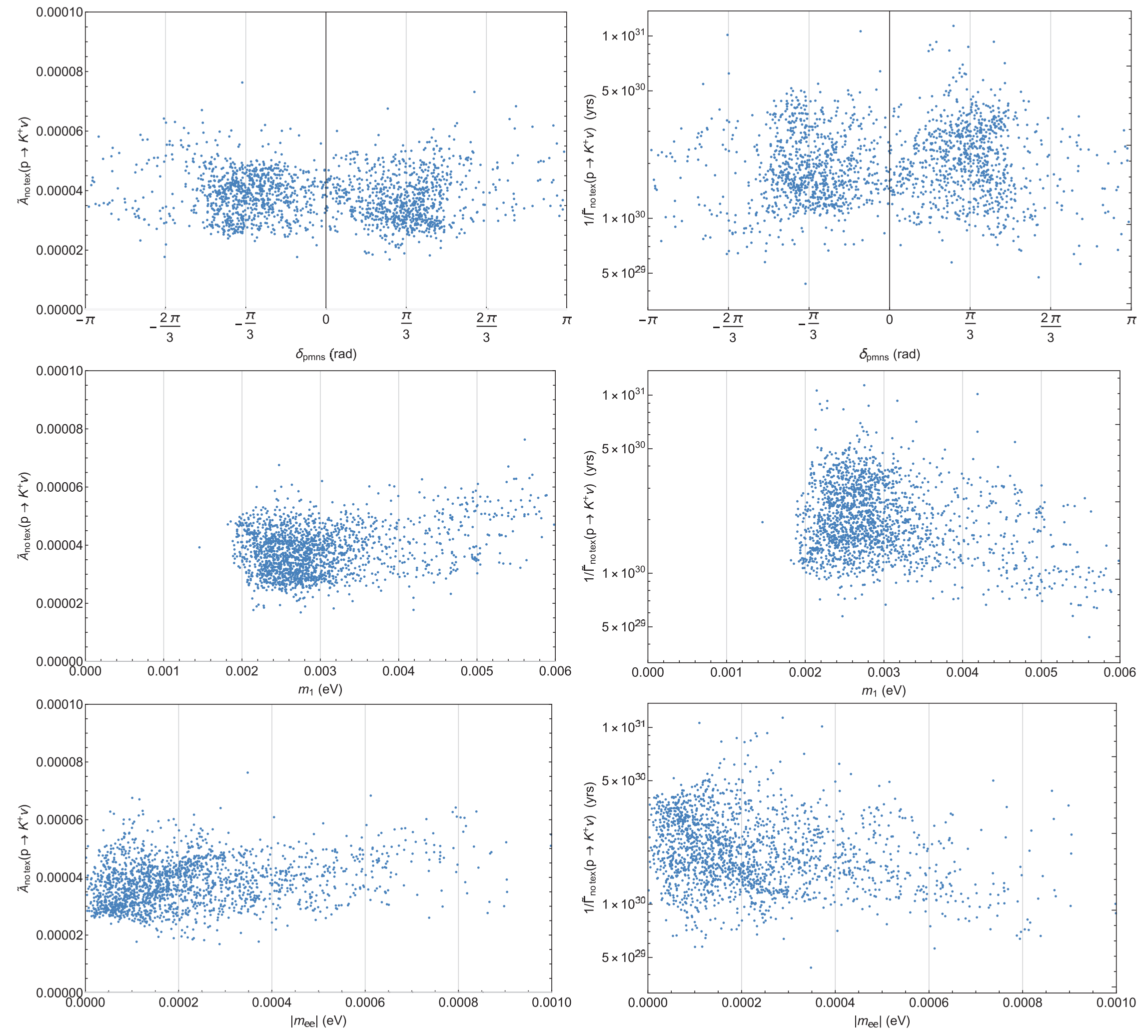

Fig. 4. As Fig. 2 except that we assume no texture for the colored Higgs mass matrix, and accordingly, we assume $|c| = |f| = 1$ and that all the terms, including those with coefficient c, f, interfere maximally constructively.

CKM and PMNS matrices, and neutrino mass differences, at the same time minimizing the above-spotted components of the Yukawa matrices. We have investigated the implications of the texture on unknown neutrino parameters and found that the "maximal proton decay amplitude," $\tilde{A}(p \to K^+\nu)$, which quantifies how much dimension-5 proton decays are suppressed by the Yukawa couplings, is minimized in the region where the neutrino Dirac CP phase satisfies $\pi/2 \gtrsim \delta_{\text{pmns}} \gtrsim -\pi/2$, the lightest neutrino mass is around $m_1 \simeq 0.003$ eV, and the (1,1)-component of the neutrino mass matrix in the charged lepton basis satisfies $|m_{ee}| \lesssim 0.0002$ eV. The above results do not depend on the precise value of the θ_{23} neutrino mixing angle. Additionally, we present the "minimal proton partial lifetime," $1/\tilde{\Gamma}(p \to K^+\nu)$, which corresponds to the "maximal proton decay amplitude" and is computed for a sample SUSY particle mass spectrum.

Acknowledgements

This work is partially supported by Scientific Grants by the Ministry of Education, Culture, Sports, Science and Technology of Japan, Nos. 17K05415, 18H04590, and 19H051061 (NH), and No. 19K147101 (TY).

Funding

Open Access funding: SCOAP[3].

Appendix A. Definitions

We review our definitions of the coupling constants and masses for the H, Δ, $\overline{\Delta}$, Σ, Φ, and A fields in the **10**, **126**, $\overline{\mathbf{126}}$, **120**, **210**, and **45** representations, which follows Eq. (2) of Ref. [42]. The couplings are defined in the same way as Eq. (3) of Ref. [42]. Note that the **120** representation field is written as D in Ref. [42], while we write it as Σ. The coupling constants are defined as

$$\begin{aligned} W = &\frac{1}{2}m_1\Phi^2 + m_2\overline{\Delta}\Delta + \frac{1}{2}m_3H^2 \\ &+ \frac{1}{2}m_4A^2 + \frac{1}{2}m_6\Sigma^2 \\ &+ \lambda_1\Phi^3 + \lambda_2\Phi\overline{\Delta}\Delta + (\lambda_3\Delta + \lambda_4\overline{\Delta})H\Phi \\ &+ \lambda_5A^2\Phi - i\lambda_6A\overline{\Delta}\Delta + \frac{\lambda_7}{120}\varepsilon A\Phi^2 \\ &+ \lambda_{15}\Sigma^2\Phi \\ &+ \Sigma\left\{\lambda_{16}HA + \lambda_{17}H\Phi + (\lambda_{18}\Delta + \lambda_{19}\overline{\Delta})A + (\lambda_{20}\Delta + \lambda_{21}\overline{\Delta})\Phi\right\}, \end{aligned} \tag{A.1}$$

where ε denotes the antisymmetric tensor in $SO(10)$ space.

Appendix B. Example VEV configurations

We present an example of VEV configurations that realize the dominance of the Type-2 seesaw contribution to the active neutrino mass without affecting the gauge coupling unification. Specifically, we elaborate a VEV configuration that renders one $(\mathbf{1}, \mathbf{3}, 1)$ particle, one $(\mathbf{6}, \mathbf{1}, -\frac{2}{3})$ particle, and one $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ particle much lighter than the GUT scale (their masses are shown in Eqs. (B.2)–(B.4)). The $(\mathbf{1}, \mathbf{3}, 1)$ particle comes from the $\Delta, \overline{\Delta}$ fields, and it realizes the Type-2 seesaw mechanism through its coupling with H_uH_u generated by the $\lambda_3\Delta H\Phi$ term and coupling with L_iL_j generated by the $(\tilde{Y}_{126})_{ij}\Psi_i\overline{\Delta}\Psi_j$ term (as shown in Eq. (B.5)). Since the $(\mathbf{1}, \mathbf{3}, 1)$, $(\mathbf{6}, \mathbf{1}, -\frac{2}{3})$, and $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ representations complete the **15** representation of the $SU(5)$ subgroup, that they are lighter than the GUT scale does not affect the gauge coupling unification. Therefore, by making the mass of these particles sufficiently small and by increasing the VEV of $\Delta, \overline{\Delta}$, we can achieve the dominance of the Type-2 seesaw contribution without spoiling the gauge coupling unification.

Our example VEV configuration is given by

$$\begin{aligned} &\left|\lambda_2\Phi_1 - 30\sqrt{6}i\,\frac{\lambda_2\lambda_{19}}{\lambda_6\lambda_{21}}m_2 + 20\sqrt{6}m_2\right| \sim M_{\rm int}, \\ &\left|\lambda_2\Phi_2 - 30\sqrt{2}i\,\frac{\lambda_2\lambda_{19}}{\lambda_6\lambda_{21}}m_2\right| \sim M_{\rm int}, \\ &\left|\lambda_2\Phi_3 + 60i\,\frac{\lambda_2\lambda_{19}}{\lambda_6\lambda_{21}}m_2\right| \sim M_{\rm int}, \\ &M_{\rm int} \ll 2\times10^{16}\,\text{GeV} \sim |\lambda_2\Phi_1| \sim |\lambda_2\Phi_2| \sim |\lambda_2\Phi_3|, \\ &|v_R| > 10^{17}\,\text{GeV}. \end{aligned} \tag{B.1}$$

The first four lines of Eq. (B.1) mean that $\lambda_2\Phi_1 - 30\sqrt{6}i\frac{\lambda_2\lambda_{19}}{\lambda_6\lambda_{21}}m_2 + 20\sqrt{6}m_2$, $\lambda_2\Phi_2 - 30\sqrt{2}i\frac{\lambda_2\lambda_{19}}{\lambda_6\lambda_{21}}m_2$, and $\lambda_2\Phi_3 + 60i\frac{\lambda_2\lambda_{19}}{\lambda_6\lambda_{21}}m_2$ are fine-tuned to nearly 0 as compared to the GUT scale, while $\lambda_2\Phi_1$ and $\lambda_2\Phi_3$ are about the GUT scale. These fine-tunings are not based on any symmetry, but are natural at the quantum level due to the non-renormalization theorem. Equations (B.1) and (37) can simultaneously be consistent with the F-flatness conditions (displayed in Eq. (28) of Ref. [42]) if one tunes m_1, m_4, λ_1, λ_2, and λ_5 appropriately. The relation $|v_R| > 10^{17}$ GeV does not raise the mass of GUT-scale particles much above 2×10^{16} GeV as long as we take λ_2, λ_3, λ_4, λ_6, λ_{18}, λ_{19}, λ_{20}, and λ_{21} sufficiently smaller than 1. We have numerically confirmed that even with the highly restricted VEV configuration and coupling constants of Eqs. (37) and (B.1), the ratio of the masses of various fields (other than the pair of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$ fields with zero mass eigenvalue) and the values of a/d, r_2, r_3, and r_e vary in a wide range and there is no strong correlation among them.

From Eqs. (B.1) and (37), we obtain

$$\left| m_2 - \frac{1}{10\sqrt{6}}\lambda_2\Phi_1 + \frac{1}{10\sqrt{2}}\lambda_2\Phi_2 + \frac{1}{5}\sqrt{\frac{3}{2}}\lambda_6 A_2 \right| \sim M_{\text{int}}, \tag{B.2}$$

$$\left| m_2 + \frac{1}{10\sqrt{6}}\lambda_2\Phi_1 - \frac{1}{30\sqrt{2}}\lambda_2\Phi_2 + \frac{1}{30}\lambda_2\Phi_3 + \frac{1}{5}\lambda_6 A_1 + \frac{1}{5\sqrt{6}}\lambda_6 A_2 \right| \sim M_{\text{int}}, \tag{B.3}$$

$$\left| m_2 + \frac{1}{30\sqrt{2}}\lambda_2\Phi_2 + \frac{1}{60}\lambda_2\Phi_3 + \frac{1}{10}\lambda_6 A_1 + \frac{1}{5}\sqrt{\frac{2}{3}}\lambda_6 A_2 \right| \sim M_{\text{int}}, \tag{B.4}$$

with $M_{\text{int}} \ll 2\times 10^{16}$. The left-hand side of Eq. (B.2) is the mass of the $(\mathbf{1}, \mathbf{3}, 1)$ particle, which comes from $\Delta, \overline{\Delta}$. That of Eq. (B.3) is the mass of the $(\mathbf{6}, \mathbf{1}, -\frac{2}{3})$ particle, which also comes from $\Delta, \overline{\Delta}$. Equation (B.4) and the relation $\lambda_{18} = \lambda_{20} = 0$ guarantee that one eigenvalue of the mass matrix of the $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ fields is about M_{int} (refer to Eq. (65) of Ref. [42]). Since $(\mathbf{1}, \mathbf{3}, 1) + (\mathbf{6}, \mathbf{1}, -\frac{2}{3}) + (\mathbf{3}, \mathbf{2}, \frac{1}{6})$ completes the **15** representation of $SU(5)$, the presence of one $(\mathbf{1}, \mathbf{3}, 1)$, one $(\mathbf{6}, \mathbf{1}, -\frac{2}{3})$, and one $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ particles with mass of order M_{int} does not affect the gauge coupling unification irrespective of the value of M_{int}. Also, the unified gauge coupling is perturbative around the GUT scale for any value of M_{int}. Therefore, we can take M_{int} arbitrarily small.

The $(\mathbf{1}, \mathbf{3}, 1)$ particle, coming from $\Delta, \overline{\Delta}$, generates the Type-2 seesaw contribution to the active neutrino mass through the couplings of $\lambda_3\, \Delta H \Phi$ and $(\tilde{Y}_{126})_{ij}\Psi_i\overline{\Delta}\Psi_j$. The Type-2 seesaw contribution is estimated to be

$$m_\nu^{\text{Type-2}} \sim Y_{126}\,\lambda_3\, U_H U_\Phi\, \frac{v^2}{M_{\text{int}}}, \tag{B.5}$$

where $v = 246$ GeV, and U_H, U_Φ denote the ratio of MSSM Higgs H_u in the H and Φ fields, respectively. We have found numerically that $|U_H|$ is around 1, and that $|U_\Phi|$ varies over a wide range. The Type-1 seesaw contribution is estimated to be

$$m_\nu^{\text{Type-1}} \sim Y_D Y_{126}^{-1} Y_D^{\text{T}}\, \frac{v^2}{|v_R|}, \tag{B.6}$$

where Y_D is given in Eq. (6). To compare the Type-2 and Type-1 seesaw contributions, we evaluate the largest singular value of Y_{126} and that of $Y_D Y_{126}^{-1} Y_D^{\text{T}}$ for each fitting result (corresponding to each dot in Figs. 1 and 2). In fact, since we have not performed a fitting of coefficient r_D in Y_D, we substitute $Y_D' Y_{126}^{-1}(Y_D')^{\text{T}}$ with $Y_D' = Y_{10} - 3r_2 Y_{126}$ for $Y_D Y_{126}^{-1} Y_D^{\text{T}}$. We think that the singular values

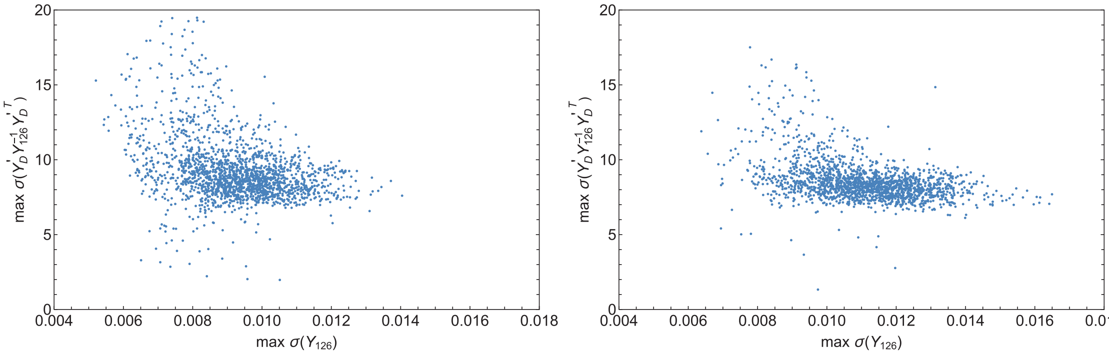

Fig. B.1. The largest singular value of Y_{126} and $\max\sigma(Y_{126})$, and $Y_D' Y_{126}^{-1}(Y_D')^{\mathrm{T}}$ with $Y_D' = Y_{10} - 3r_2 Y_{126}$, $\max\sigma(Y_D' Y_{126}^{-1}(Y_D')^{\mathrm{T}})$. Each dot corresponds to the result of one fitting and minimization analysis, and hence to one dot in Figs. 1 and 2. The left panel shows the results for the higher-octant benchmark where $\sin^2\theta_{23}^{\mathrm{pmns}} = 0.55 \pm 0.01$, and the right panel shows those for the lower-octant benchmark where $\sin^2\theta_{23}^{\mathrm{pmns}} = 0.45 \pm 0.01$.

of $Y_D' Y_{126}^{-1}(Y_D')^{\mathrm{T}}$ approximate well those of $Y_D Y_{126}^{-1} Y_D^{\mathrm{T}}$. Plots of the largest singular values of Y_{126} and $Y_D' Y_{126}^{-1}(Y_D')^{\mathrm{T}}$ are shown in Fig. B.1.

From Fig. B.1, we see that the largest singular value of Y_{126} ranges from 0.004 to 0.018, while that of $Y_D' Y_{126}^{-1}(Y_D')^T$ is below 20, for both benchmarks. Since we have assumed $|v_R| > 10^{17}$ GeV, the Type-1 seesaw contribution is negligible for the active neutrino mass. On the other hand, if we take

$$M_{\mathrm{int}} \sim |\lambda_3 U_H U_\Phi| \times 10^{13}\,\mathrm{GeV}, \tag{B.7}$$

the Type-2 seesaw contribution accounts for the active neutrino mass.

We can also achieve the dominance of the Type-2 seesaw contribution by adding a **54** representation field. Its coupling with Δ^2 gives rise to the mixing of $(\mathbf{1}, \mathbf{3}, 1)$ components of the **54** field and Δ when Δ develops a VEV. Also, the coupling of the **54** field with H^2 and other fields generates the coupling of the $(\mathbf{1}, \mathbf{3}, 1)$ particles with H_u^2. One can decrease the mass of the $(\mathbf{1}, \mathbf{3}, 1)$ particles without affecting the gauge coupling unification by fine-tuning the VEVs and coupling constants as studied in Ref. [63]. In this way, the Type-2 seesaw contribution can be made dominant.

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