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# Heavy baryon spectroscopy with relativistic kinematics

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ABSTRACT

We present a comparative Faddeev study of heavy baryon spectroscopy with nonrelativistic and relativistic kinematics. We show results for different standard hyperfine interactions with both kinematics in an attempt to learn about the light quark dynamics. We highlight the properties of particular states accessible in nowadays laboratories that would help in discriminating between different dynamical models. The advance in the knowledge of light quark dynamics is a key tool for the understanding of the existence of exotic hadrons.



## 1. Introduction

Recently, there have been exciting developments in heavy-baryon physics, both theoretically and experimentally. Results from LHC and future charm-bottom factories are expected to add to the excitement in this field in the near future. A major focus of activity with the current LHCb detector is the study of the properties of beauty and charm hadrons [1,2]. Baryonic states containing two heavy quarks should be visible with the current detector, as the $\Xi_{cc}$ isodoublet. However, current set of data will certainly be insufficient for angular analyses aimed at confirming the quark model predictions for the spin–parity of these states. These studies will require the statistics and improved triggering of the LHCb upgrade [3]. In the next few years, more results are expected to appear in currently running experiments, e.g. Belle [4], BES-III [5] and the future PANDA experiment at the FAIR facility [6]. Tremendous efforts are also being done in lattice QCD simulations to minimize the systematic errors in the prediction of ground and excited heavy baryon masses [7–14].

Baryons containing heavy quarks provide an interesting laboratory for studying QCD. They combine two different regimes: the slow relative motion of the heavy quark with the relativistic motion of the light quarks. While the mass of heavy baryons is measured as part of the discovery process, no spin or parity quantum numbers of a given state have been measured experimentally, but they are assigned based on quark model expectations. Such properties can only be extracted by studying angular distributions of the particle decays, that are available only for the lightest and most abundant species. For excited heavy baryons the data sets are typically one order of magnitude smaller than for heavy mesons and therefore the knowledge of radially and orbitally excited states is very much limited. Therefore, guidelines for assigning quantum numbers to new states or to indicate new states to look for are required by experiment. Likewise, forthcoming experimental studies will also help to constrain theoretical models and to advance in the understanding of QCD realizations at low energies. We do understand ground state heavy quark baryons, both in the quark model and in the lattice QCD. The main issue is therefore to determine quantum numbers of excited states. Here, a coherent theoretical and experimental effort is required.

On the other hand, the advent of the $XYZ$ puzzle, new particles that hint at four-quark matter [15], has made manifest our poor knowledge about the light quark dynamics and the urgent need of a better understanding before any explanation about states already observed or predictions of other resonances could be thoughtfully made. Particular dynamics proposed predicted a handful of exotics that had not been observed and therefore could be used as a hint of the inadequacy of the dynamical model. In this respect, it has been suggested that we can learn more on light quark dynamics from singly heavy baryons, $Qqq$, than from the light baryon sector, $qqq$ [16].

Heavy baryons have been the matter of study during the last two decades [17–39]. After the discovery of the first charmed baryons, several theoretical works [17–21] based on potential models developed for the light baryon or meson spectra started analyzing properties of the observed and expected states. Later on, Capstick and Isgur [22] studied heavy baryon systems in a relativized quark potential model applying a variational approach to obtain the mass eigenvalues and bound state wave functions by using a harmonic oscillator basis. Roncaglia et al. [23] predicted

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the masses of baryons containing one or two heavy quarks using the Feynman–Hellmann theorem and semiempirical mass formulas. Silvestre-Brac [24] studied ground state charmed and bottom baryons using Faddeev equations in configuration space. Excited states were studied by diagonalization in a harmonic oscillator basis up to eight quanta. Jenkins [25] studied heavy baryon masses in a combined expansion in $1/m_Q$, $1/N_c$, and SU(3) flavor symmetry breaking. Bowler et al. [26] made an exploratory study using lattice techniques to predict charmed and bottom baryons. Mathur et al. [27] gave a more precise prediction of the masses of charmed and bottom baryons from quenched lattice QCD. Ebert et al. [28] calculated the masses of ground state heavy baryons with the relativistic quark–diquark approximation. QCD sum rules have been also applied to study heavy baryon masses [29,30]. Stimulated by the experimental progress, there had been several theoretical papers on the heavy baryon spectra using a perturbative treatment of the hyperfine interaction in the quark model [31], heavy quark effective field theory [32], an exact solution of the Schrödinger equation by the Faddeev method in momentum space [33,34], a variational calculation in a harmonic oscillator expansion [35], and a relativistic quark–diquark approximation [36]. Dynamically generated baryon resonances in the charm-sector have also been recently studied by a unitary baryon–meson coupled-channel model [37] and also incorporating heavy-quark spin symmetry [38,39]. Comprehensive reviews discussing the recent experimental progress and an overview of theoretical approaches can be found in Refs. [40,41]

Our purpose in this letter is to scrutinize particular features of heavy hadron spectroscopy that may help in distinguishing between the different dynamical quark models. As discussed above, in the literature there are plenty of studies of heavy baryons with a great variety of interactions, different kinematics and calculation techniques. We pretend to learn about the possible realizations of QCD at low energies by comparing the predictions of the different dynamical models used in the light baryon sector together with different kinematics when applied to the heavy baryon sector. We will try to minimize the changes in an attempt to find predictions that may allow us in discriminating between the different dynamical models. If one finds outstanding differences, the advent of new experimental data will be an important test that will tell us about the adequate realization of QCD al low energies. For this objective heavy hadrons containing a single heavy quark are particularly interesting because they merge the relativistic light quark dynamics and the dynamics of a heavy and light quark in such a way that they can be easily isolated, obtaining a composition of two two-body problems.

The paper is organized as follows. In Section 2 we review Faddeev solution of the three-body bound-state problem in momentum space. In Section 3 we will describe the relevant features of the heavy baryon spectra from the quark-model point of view and we will define the method and interacting potentials we will use. In Section 4 we show the results for the models defined in Section 3 and we analyze the main differences in their predictions. Finally, in Section 5 we will summarize our most important findings.

## 2. Formalism

Let us briefly revise the most important aspects of the solution of the Schrödinger equation with relativistic kinematics for the bound-state problem. The three-body Schrödinger equation can be written as,

$$|\psi\rangle = G_0(W_0)[V_1+V_2+V_3]|\psi\rangle, \tag{1}$$

where if one assumes that the three particles are in the c.m. system, i.e., $\vec{k}_1+\vec{k}_2+\vec{k}_3=0$, then $W_0$ is the invariant mass of the system and

$$G_0(W_0) = \frac{1}{W_0-\omega_1(k_1)-\omega_2(k_2)-\omega_3(k_3)}, \tag{2}$$

with $\omega_i(k_i)=\sqrt{m_i^2+k_i^2}$. Making the Faddeev decomposition

$$|\psi\rangle = |\phi_1\rangle+|\phi_2\rangle+|\phi_3\rangle, \tag{3}$$

one obtains the Faddeev equations

$$|\phi_i\rangle = G_0(W_0)t_i(W_0)\big[|\phi_j\rangle+|\phi_k\rangle\big], \tag{4}$$

with

$$t_i(W_0) = V_i + V_i G_0(W_0) t_i(W_0). \tag{5}$$

Our basis states are $|\vec{p}_i\vec{q}_i\rangle$ where $\vec{p}_i$ is the relative momentum of the pair $jk$ measured in the c.m. frame of the pair (that is, the frame in which particle $j$ has momentum $\vec{p}_i$ and particle $k$ has momentum $-\vec{p}_i$) and $\vec{q}_i=-\vec{k}_i$ is the relative momentum between the pair $jk$ and particle $i$ measured in the three-body c.m. frame (that is, the frame in which the pair $jk$ has total momentum $\vec{q}_i$ and particle $i$ has momentum $-\vec{q}_i$). In terms of these relative momenta the propagator of three free particles (2) takes the form

$$G_0(W_0;p_i,q_i) = \frac{1}{W_0-W_i(p_i,q_i)-\omega_i(q_i)}, \tag{6}$$

where

$$W_i(p_iq_i) = \sqrt{\omega^2(p_i)+q_i^2}, \tag{7}$$

and

$$\omega(p_i) = \sqrt{m_j^2+p_i^2}+\sqrt{m_k^2+p_i^2} \equiv \omega_j(p_i)+\omega_k(p_i). \tag{8}$$

Thus, if the single-particle states are normalized invariantly on the mass shell, i.e., $\langle\vec{k}_i|\vec{k}_i'\rangle = 2\omega_i(k_i)\delta(\vec{k}_i-\vec{k}_i')$, then the invariant volume element for three particles in the three-body c.m. frame can be written in terms of the corresponding volume element for the relative momenta as

$$\frac{d\vec{k}_1}{2\omega_1(k_1)}\frac{d\vec{k}_2}{2\omega_2(k_2)}\frac{d\vec{k}_3}{2\omega_3(k_3)}\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3) = \frac{\omega(p_i)}{8W_i(p_iq_i)\omega_i(q_i)\omega_j(p_i)\omega_k(p_i)}d\vec{p}_i d\vec{q}_i. \tag{9}$$

The basis states $|\vec{p}_i\vec{q}_i\rangle$ are normalized as

$$\langle\vec{p}_i\vec{q}_i|\vec{p}_i{}'\vec{q}_i{}'\rangle = \frac{8W_i(p_iq_i)\omega_i(q_i)\omega_j(p_i)\omega_k(p_i)}{\omega(p_i)}\delta(\vec{p}_i-\vec{p}_i{}')\delta(\vec{q}_i-\vec{q}_i{}'), \tag{10}$$

and satisfy the completeness relation

$$1 = \int\frac{\omega(p_i)}{8W_i(p_iq_i)\omega_i(q_i)\omega_j(p_i)\omega_k(p_i)}d\vec{p}_i d\vec{q}_i\,|\vec{p}_i\vec{q}_i\rangle\langle\vec{p}_i\vec{q}_i|. \tag{11}$$

The matrix elements of the two-body interaction in the three-body c.m. frame are connected to the corresponding matrix elements in the two-body c.m. frame by [42]

$$\langle\vec{p}_i\vec{q}_i|V_i|\vec{p}_i{}'\vec{q}_i{}'\rangle = 8\omega_i(q_i)\left[\frac{W_i(p_iq_i)\omega_j(p_i)\omega_k(p_i)W_i(p_i'q_i)\omega_j(p_i')\omega_k(p_i')}{\omega(p_i)\omega(p_i')}\right]^{1/2} \times\delta(\vec{q}_i-\vec{q}_i{}')V_i(\vec{p}_i,\vec{p}_i{}'), \tag{12}$$

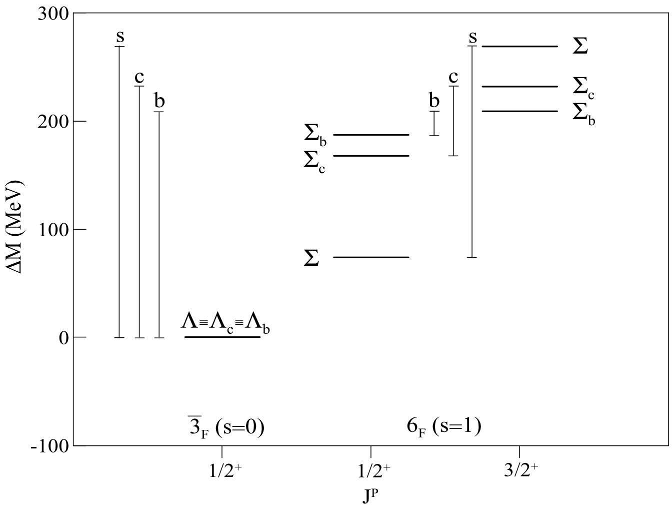


**Fig. 1.** Experimental mass difference [43] among the $\Sigma_Q(3/2^+)$, the $\Sigma_Q(1/2^+)$ and the $\Lambda_Q(1/2^+)$ baryons for $Q = s, c,$ and $b$.

with $V_i(\vec{p}_i, \vec{p}_i')$ the usual Fourier transform of the potential.

If we introduce a complete set of basis states into Eq. (5) and use Eq. (12), we obtain that $\langle \vec{p}_i\vec{q}_i|t_i|\vec{p}_i'\vec{q}_i'\rangle$ is given by an expression similar to (12) with $V_i(\vec{p}_i, \vec{p}_i')$ replaced by $t_i(W_0, q_i; \vec{p}_i, \vec{p}_i')$, where $t_i(W_0, q_i; \vec{p}_i, \vec{p}_i')$ satisfies the Lippmann–Schwinger equation

$$t_i\big(W_0, \vec{q}_i; \vec{p}_i, \vec{p}_i'\big) = V_i\big(\vec{p}_i, \vec{p}_i'\big) + \int d\vec{p}_i''V_i\big(\vec{p}_i, \vec{p}_i''\big)G_0\big(W_0; p_i''q_i\big)t_i\big(W_0, \vec{q}_i; \vec{p}_i'', \vec{p}_i'\big). \quad (13)$$

Following the same procedure in Eqs. (4) one finds that, in the absence of noncentral forces, the Faddeev equations for the bound-state problem of the three-quark system can be written as

$$\begin{aligned}&\big\langle p_iq_i; \ell_i\lambda_iS_iT_i\big|\phi_i^{LST}\big\rangle \\ &= G_0(W_0; p_i, q_i)\sum_{j\neq i}\sum_{\ell_j\lambda_jS_jT_j}\frac{1}{2}\int_{-1}^{1}d\cos\theta\int_0^\infty q_j^2dq_j\frac{1}{\omega_j(q_j)\omega_k(q_k)} \\ &\times\left[\frac{W_i(p_i'q_i)\omega_j(p_i')\omega_k(p_i')W_j(p_jq_j)\omega_k(p_j)\omega_i(p_j)}{\omega(p_i')\omega(p_j)}\right]^{1/2} \\ &\times t_i^{\ell_iS_iT_i}\big(W_0, q_i; p_i, p_i'\big)A_L^{\ell_i\lambda_i\ell_j\lambda_j}\big(p_i'q_ip_jq_j\big) \\ &\times\langle S_iT_i|S_jT_j\rangle_{ST}\langle p_jq_j; \ell_j\lambda_jS_jT_j|\phi_j^{LST}\rangle,\end{aligned} \quad (14)$$

with $\omega_k(q_k) \equiv \sqrt{m_k^2 + q_i^2 + q_j^2 + 2q_iq_j\cos\theta}$. $S_i$ and $T_i$ are the spin and isospin of the pair $jk$ while $S$ and $T$ are the total spin and isospin. $\ell_i$ ($\vec{p}_i$) is the orbital angular momentum (momentum) of the pair $jk$, $\lambda_i$ ($\vec{q}_i$) is the orbital angular momentum (momentum) of particle $i$ with respect to the pair $jk$, and $L$ is the total orbital angular momentum. The spin–isospin and orbital angular momentum recoupling coefficients $\langle S_iT_i|S_jT_j\rangle_{ST}$ and $A_L^{\ell_i\lambda_i\ell_j\lambda_j}(p_i'q_ip_jq_j)$ are given by Eqs. (24)–(27) and (33) of Ref. [42]. In Ref. [42] it is also demonstrated that in the nonrelativisitc limit these relativistic Faddeev equations go into the nonrelativisitc ones.

## 3. Discussion

Let us start by discussing Fig. 1. In this figure we have plotted the experimental mass difference [43] among the $\Sigma_Q(3/2^+)$, the $\Sigma_Q(1/2^+)$ and the $\Lambda_Q(1/2^+)$ baryons for $Q = s, c,$ and $b$. When the heavy quark mass $m_Q \to \infty$, the angular momentum of the light degrees of freedom is a good quantum number. Thus, heavy quark baryons belong to either flavor SU(3) antisymmetric $\bar{\mathbf{3}}_\mathbf{F}$ or symmetric $\mathbf{6}_\mathbf{F}$ representations. The spin of the light diquark is 0 for $\bar{\mathbf{3}}_\mathbf{F}$, while it is 1 for $\mathbf{6}_\mathbf{F}$. Thus, the spin of the ground state baryons is 1/2 for $\bar{\mathbf{3}}_\mathbf{F}$, representing among others the $\Lambda_Q(1/2^+)$ baryon, while it can be both 1/2 or 3/2 for $\mathbf{6}_\mathbf{F}$, allocating among others the $\Sigma_Q(1/2^+)$ and the $\Sigma_Q(3/2^+)$. Therefore heavy hadrons form doublets. As can be seen in Fig. 1, $\Sigma_Q(1/2^+)$ and $\Sigma_Q(3/2^+)$ will be degenerate in the heavy quark limit, their mass splitting being caused by the spin–spin interaction at the order $1/m_Q$. As also seeing in this figure, the mass difference between states belonging to the flavor $\bar{\mathbf{3}}_\mathbf{F}$ and $\mathbf{6}_\mathbf{F}$ representations tends to a constant when the heavy quark mass $m_Q \to \infty$, due to the dynamics of the light diquark subsystem. Thus:

$$\begin{aligned}&\mathrm{M}\big[\Sigma_Q\big(3/2^+\big)\big] - \mathrm{M}\big[\Sigma_Q\big(1/2^+\big)\big] \Rightarrow \Delta\mathrm{M}\big([\mathbf{6}_\mathbf{F}] - [\mathbf{6}_\mathbf{F}]\big) \equiv V_{qQ} \\ &\mathrm{M}\big[\Sigma_Q\big(3/2^+\big)\big] - \mathrm{M}\big[\Lambda_Q\big(1/2^+\big)\big] \Rightarrow \Delta\mathrm{M}\big([\mathbf{6}_\mathbf{F}] - [\bar{\mathbf{3}}_\mathbf{F}]\big) \equiv V_{qq}.\end{aligned} \quad (15)$$

Let us note that in $\Lambda_Q(1/2^+)$ there is an attractive $ud$ diquark ("good" diquark) with color $\bar{\mathbf{3}}$, spin 0 and isospin 0,[1] whereas in $\Sigma_Q(1/2^+)$ and $\Sigma_Q(3/2^+)$ there is a repulsive $ud$ diquark ("bad" diquark) with color $\bar{\mathbf{3}}$, but spin 1 and isospin 1. Thus, single heavy baryons are ideal laboratories for testing the dynamics of the different two-quark subsystems: heavy–light and light–light, that may drive important differences in the final spectrum. Effects like the degeneracy of the members of the flavor $\mathbf{6}_\mathbf{F}$ representation when $m_Q \to \infty$ can be, for example, taken into account systematically in the framework of heavy quark effective field theory (HQET). However, the mass difference between states belonging to the flavor $\bar{\mathbf{3}}_\mathbf{F}$ and $\mathbf{6}_\mathbf{F}$ representations, mainly due to the dynamics of the light diquark, is hard to accommodate in any heavy quark mass expansion. Therefore, exact solutions of the three-body problem for heavy hadrons are theoretically desirable because they will serve to test the reliability of the predictions of approximate techniques, that would only be exact in the infinite heavy-quark mass limit, as could be heavy quark mass expansions, variational calculations, or quark–diquark approximations.

Most of the potentials used in the literature to study heavy baryons differ essentially on the treatment of the hyperfine interaction and the value of the parameters used. Among them, the model proposed by Bhaduri et al. [19] works quite well both in the meson and baryon sectors. For the case of baryons, it reads,

$$V_{\mathrm{Bha}}(r) = \frac{1}{2}\left[-\frac{\kappa}{r} + \lambda r - C + \frac{\kappa}{m_im_j}\frac{e^{(-r/r_0)}}{rr_0^2}\vec{\sigma}_i\cdot\vec{\sigma}_j\right], \quad (16)$$

with $\kappa = 0.52$, $\lambda = 0.186$ GeV$^2$, $C = 0.9135$ GeV, $r_0 = 2.305$ GeV$^{-1}$ ($r_0 = 0.45$ fm), $m_u = m_d = 0.337$ GeV, $m_c = 1.870$ GeV. The parameters were essentially fitted in the charmonium system.

An alternative to the Bhaduri model based just on a one-gluon exchange (OGE) interaction emerged with models aiming a coherent understanding of the hadron spectra and the hadron–hadron interactions [46,47]. These hybrid models combine the hyperfine effect of the OGE potential and a pseudoscalar interaction between the light quarks consequence of the spontaneous breaking of chiral symmetry. The simplest way of considering the pseudoscalar potential is to supplement the interaction of Eq. (16) by a standard one-pion exchange potential,

[1] The attractive $ud$ diquark is a source to form the color superconductivity in quark matter at high density [44,45]

**Table 1**
Different models used for the study of heavy baryons.

| Model | Kinematics | | Interacting potential | |
|---|---|---|---|---|
| | No Rel | Rel | Bha | Bha + Ps |
| A | X | | X | |
| B | | X | X | |
| C | X | | | X |
| D | | X | | X |

$$V_{\rm Ps}(r) = \frac{1}{3}\alpha_{\rm ch}\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} m_\pi \left[ Y(m_\pi r) - \frac{\Lambda_\pi^3}{m_\pi^3} Y(\Lambda_\pi r)\right] \times \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j , \quad (17)$$

where $m_\pi = 0.7$ fm$^{-1}$ is the pion mass, $\alpha_{ch} = 0.029$ is the chiral coupling constant, $\Lambda_\pi = 4.2$ fm$^{-1}$ is a cutoff parameter, and $Y(x)$ is the standard Yukawa function $Y(x) = e^{-x}/x$.

As has been discussed in the introduction, there are plenty of studies of heavy baryons [17–39] with a great variety of interactions, different kinematics and calculation techniques. Our aim in this work is to learn about the possible realizations of QCD at low energies by comparing the predictions of different dynamical models used in the light baryon sector together with different kinematics when applied to the heavy baryon sector. The particular dynamics of these three-body systems combining two different regimes: the slow relative motion of the heavy quark with the relativistic motion of the light quarks, provides with an interesting laboratory for studying QCD. We will try to minimize the changes in an attempt to find predictions that may allow us in discriminating between the different dynamical models. If one finds outstanding differences, the advent of new experimental data will be an important test for the predictions of the different models that will tell us about the adequate realizations of QCD al low energies. As it has also been discussed in the introduction this could be relevant for the study of exotic hadrons [15,48–51].

The models we will use are summarized in Table 1. Model A corresponds to a simple Bhaduri potential, Eq. (16), with nonrelativistic kinematics. Model B represents the Bhaduri interaction but with relativistic kinematics. Model C stands for a Bhaduri potential supplemented by a pseudoscalar source of hyperfine interaction, Eq. (17), with nonrelativistic kinematics. Finally, Model D will be the last interacting potential but with relativistic kinematics. We will try to highlight the most important differences among the patterns for the heavy baryon sector predicted by the different models as well as to point to those states whose experimental measurement or lattice QCD simulation will help in the assignment of quantum numbers to new states.

## 4. Results

Let us start by showing the comparison with results presented in the literature as well as the convergence of the Faddeev method. Using Model A in the Feshbach–Rubinow variational approach [52], Bhaduri et al. [19] obtained for the masses of the $\Lambda_c(1/2^+)$, the $\Sigma_c(1/2^+)$, and the $\Sigma_c(3/2^+)$ the results shown in the second column of Table 2. In order to compare with these results, we have calculated the masses of these states considering only the three-body configurations where all orbital angular momenta $\ell_i, \lambda_i \le 1$. As can be seen, our results for the nonrelativistic case, third column of Table 2, are in good agreement with those of Ref. [19], since the accuracy of the Feshbach–Rubinow method is about 20 MeV. We have also studied the convergence with respect to the number of Faddeev amplitudes considered, the results are shown in Table 3. As one can see in the second and third columns of this table, the largest effect of higher order Faddeev amplitudes is obtained for states with a spin 0 diquark, the $\Lambda_c(1/2^+)$, the nonrelativistic results being almost fully converged when considering all Faddeev amplitudes in $S$ and $P$ waves.

**Table 2**
Masses of $\Lambda_c(1/2^+)$, $\Sigma_c(1/2^+)$, and $\Sigma_c(3/2^+)$ (in MeV) obtained with the potential of Eq. (16) for both the nonrelativistic (Model A) and the relativistic (Model B) Faddeev equations including three-body configurations with $\ell_i, \lambda_i \le 1$ compared to the results of Ref. [19].

| | Ref. [19] | Model A ($r_0 = 0.45$ fm) | Model B ($r_0 = 0.45$ fm) |
|---|---|---|---|
| $\Lambda_c(1/2^+)$ | 2334 | 2331 | 364 |
| $\Sigma_c(1/2^+)$ | 2511 | 2498 | 2249 |
| $\Sigma_c(3/2^+)$ | 2585 | 2570 | 2374 |

**Table 3**
Masses of $\Lambda_c(1/2^+)$ and $\Sigma_c(1/2^+)$ (in MeV) obtained with the potential of Eq. (16) for the nonrelativistic (Model A) and the relativistic (Model B) Faddeev equations including the three-body configurations with the indicated $\ell_i$ and $\lambda_i$.

| $\ell_i, \lambda_i$ | Model A ($r_0 = 0.45$ fm) | | Model B ($r_0 = 0.75$ fm) | |
|---|---|---|---|---|
| | $\Lambda_c(1/2^+)$ | $\Sigma_c(1/2^+)$ | $\Lambda_c(1/2^+)$ | $\Sigma_c(1/2^+)$ |
| $\le 1$ | 2331 | 2498 | 2304 | 2460 |
| $\le 2$ | 2323 | 2495 | 2294 | 2454 |
| $\le 3$ | 2320 | 2495 | 2288 | 2452 |
| $\le 4$ | 2319 | 2494 | 2286 | 2451 |

In the fourth column of Table 2 we also show the results for Model B, the relativistic Bhaduri potential, using the same set of parameters as in Model A. As can be seen the relativistic effects are very large, particularly for the $\Lambda_c(1/2^+)$, the state with a good *ud* diquark, and they tend to lower the masses. The $\Lambda_c(1/2^+)$ mass is lowered by 1964 MeV, while the $\Sigma_c$ mass is lowered by 249 MeV for the $1/2^+$ state and 196 MeV for the $3/2^+$ state, so that the mass splitting $\Sigma_c$–$\Lambda_c$, which is 167 MeV for the nonrelativistic case, increases to 1885 MeV for the relativistic one, i.e., to about eleven times the experimental value. From the results of Table 2, one may think that a relativistic description of the heavy baryon spectrum based on the potential of Eq. (16) would not be possible. This, however, is not the case as it will be shown below. The large sensitivity to relativistic effects shown in Table 2, as we will see next, is due to a particular feature of the interaction at short distances. The last term of the interaction of Eq. (16) contains a smeared-out $\delta$ function with smearing parameter $r_0$. The $\delta$ function must be smeared-out otherwise the mass of the nucleon would collapse to $-\infty$ when $r_0 \to 0$ [53]. We show this effect in Fig. 2, where we plot the eigenvalue of the Schrödinger equation[2] for $\Lambda_c(1/2)^+$, $\Sigma_c(1/2^+)$, and $\Sigma_c(3/2^+)$ as a function of the parameter $r_0$ using the relativistic Faddeev equations, Model B.

As one can see from Fig. 2, the relativistic kinematics produces two main effects: first of all, as already noticed, it lowers the masses of the baryons and second, one needs of larger values of the parameter $r_0$ to avoid that the mass of the $\Lambda_c$ (or any other baryon with a "good" diquark) would collapse to negative values. Thus, while $r_0 = 0.45$ fm is a reasonable value for nonrelativistic kinematics, it is not so in the case of the relativistic one where instead the value of $r_0$ should be larger, $r_0 = 0.75$ fm, in order to get the correct excitation energy (one also has to redefine the constant $C$ in Eq. (16) that for the relativistic Model B with $r_0 = 0.75$ fm should be 782 MeV). Another aspect that deserves attention when fitting the parameters of the smeared $\delta$ function is the number of Faddeev amplitudes considered. In the last two columns of Table 3 we show the masses of $\Lambda_c(1/2^+)$

[2] The mass is given by $M = m_1 + m_2 + m_3 - \frac{3}{2}C + E$, where $E$ is the eigenvalue of the Schrödinger equation.

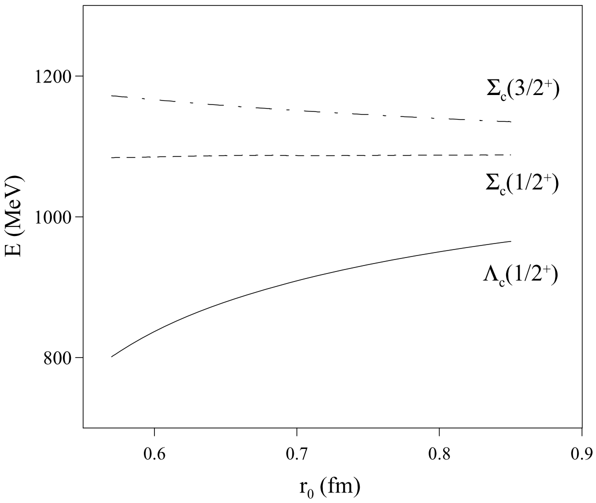


**Fig. 2.** Variation of the masses of $\Sigma_c(3/2^+)$, $\Sigma_c(1/2^+)$ and $\Lambda_c(1/2^+)$ with the regularization parameter of the spin–spin interaction $r_0$ for Model B. See text for discussion.

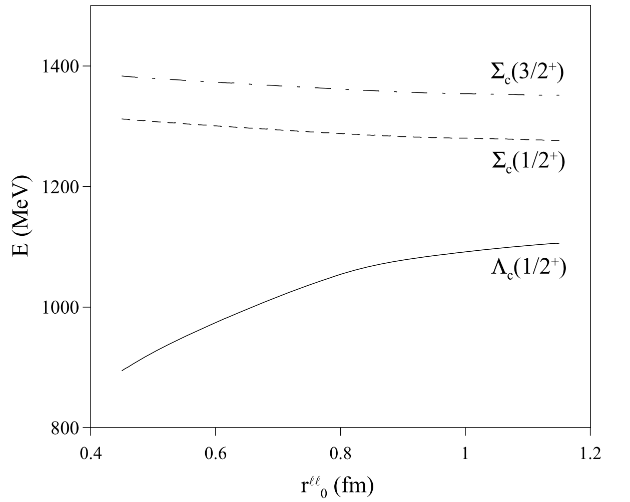


**Fig. 3.** Variation of the masses of $\Sigma_c(3/2^+)$, $\Sigma_c(1/2^+)$ and $\Lambda_c(1/2^+)$ with the regularization parameter of the spin–spin interaction between the light quarks $r_0^{\ell\ell}$ for Model C.

**Table 4**
Excitation spectra of $\Lambda_c$ and $\Sigma_c$ baryons, $M[J^P]-M[\Lambda_c(1/2^+)]$, for the nonrelativistic (Model A) and relativistic (Model B) Bhaduri potential. The star stands for radial excitations. Masses are in MeV. Experimental data are from Ref. [43].

| State | $J^P$ | Model A ($r_0 = 0.45$ fm) | Model B ($r_0 = 0.75$ fm) | Exp. |
|---|---|---|---|---|
| $\Lambda_c$ | $1/2^{+*}$ | 610 | 417 | – |
| | $3/2^+$ | 697 | 666 | – |
| | $3/2^{+*}$ | 925 | 817 | – |
| | $1/2^-$ | 371 | 372 | 324 |
| $\Sigma_c$ | $1/2^+$ | 175 | 165 | 167 |
| | $1/2^{+*}$ | 772 | 830 | – |
| | $3/2^+$ | 248 | 226 | 232 |
| | $1/2^-$ | 540 | 518 | – |

and $\Sigma_c(1/2^+)$ obtained from the potential of Eq. (16) for the relativistic Faddeev equations including the three-body configurations with the indicated $\ell_i$ and $\lambda_i$. As one can see the problem is fully converged when considering all three-body amplitudes with $\ell_i$ and $\lambda_i \le 4$ (what means that considering another orbital angular momentum induces a change in the energy eigenvalue smaller than 1 MeV). Model B with $r_0$ =0.75 fm allows to correctly reproduce the experimental data, supporting a description of the heavy baryon spectra based on the Bhaduri potential with relativistic kinematics. It gives $M[\Sigma_c(1/2^+)]-M[\Lambda_c(1/2^+)] = 165$ MeV, to be compared with the experimental result of 167 MeV [43], and $M[\Sigma_c(3/2^+)] - M[\Sigma_c(1/2^+)] = 61$ MeV, to be compared with the experimental result of 65 MeV [43].

It is interesting to note that in Ref. [42] we used almost the same regularization for the $\delta$ function, $r_0 = 0.74$ fm, to get the correct description of the light baryon masses in the relativistic Bhaduri model. As we can see here with the same value we get a description of the same quality as in the nonrelativistic case for the lowest known excitations of the charmed baryon spectrum. The behavior of the "good" diquark baryons $(N, \Lambda, \Lambda_c, \ldots)$ masses in the relativistic theory is a consequence of being more sensitive to the form of the interaction at short distances, due to the nonrelativistic propagator falling down as $1/k^2$ while the relativistic one does it as $1/k$. That is the reason why the pathological effects produced by the $\delta$ function appear in the case of the relativistic theory much sooner, i.e., at larger values of $r_0$ than in the case of the nonrelativistic one.

It seems therefore that a correct description of the low lying states of the charmed baryon spectrum[3] can be obtained by means of a nonrelativistic and a relativistic treatment of the kinetic energy operator. In the following we will try to see if this modification has any important effect in other states of the spectra. Thus, we show in Table 4 the excitation spectra of $\Lambda_c$ and $\Sigma_c$ baryons for the nonrelativistic (Model A) and relativistic (Model B) Bhaduri potential. As can be seen the mass difference between the members of the flavor $\mathbf{6_F}$ representation, controlled by the $Qq$ interaction, as well as the mass difference between the members of the flavor $\mathbf{\bar{3}_F}$ and $\mathbf{6_F}$ representations, controlled by the $qq$ interaction, can be perfectly described in both cases. One could have fitted the strength of confinement to get a better description of the first negative parity state, but it is not the purpose of the present work to get the better $\chi^2$ fit to the experimental data but to highlight the mechanisms controlling the relevant aspects of the heavy baryon spectrum.[4] To this respect we find the first relevant difference between the relativistic and the nonrelativistic treatment of the kinetic energy operator in the heavy baryon spectroscopy that resembles what has been observed in the case of the light baryon spectroscopy [42,54–56]. Being the spin–spin part fixed, the most important difference comes from the radial excitation of the ground state baryon with a "good" diquark, the $\Lambda_c(1/2^+)$. Giving the same excitation for the negative parity state, the nonrelativistic case predicts a $1/2^+$ excited state that it is 200 MeV higher than the relativistic state. The relativistic description gives an almost degenerate positive and negative $\Lambda_c$ excited states. As mentioned above, the tendency to the level inversion between the negative and positive parity excited states of "good" diquark baryons takes place also in the light baryon spectra, and it is thus a remnant of the relativistic light quarks dynamics as we will observe in the following. A second difference comes from the radial excitation of the $\Lambda_c(3/2^+)$. It corresponds to the configuration with total orbital momentum and total spin $(L, S) = (0, 3/2)$, being the ground state $(L, S) = (2, 1/2)$. The configuration in $S$ wave is lowered by the relativistic kinetic energy operator as compared to the $D$ wave, giving 100 MeV difference. The other states of the spectra are predicted by both models with approximately the same mass.

[3] Note that the present reasoning would remain valid for bottom baryons.
[4] Reducing the strength of confinement would lower correspondingly all states in the spectrum.

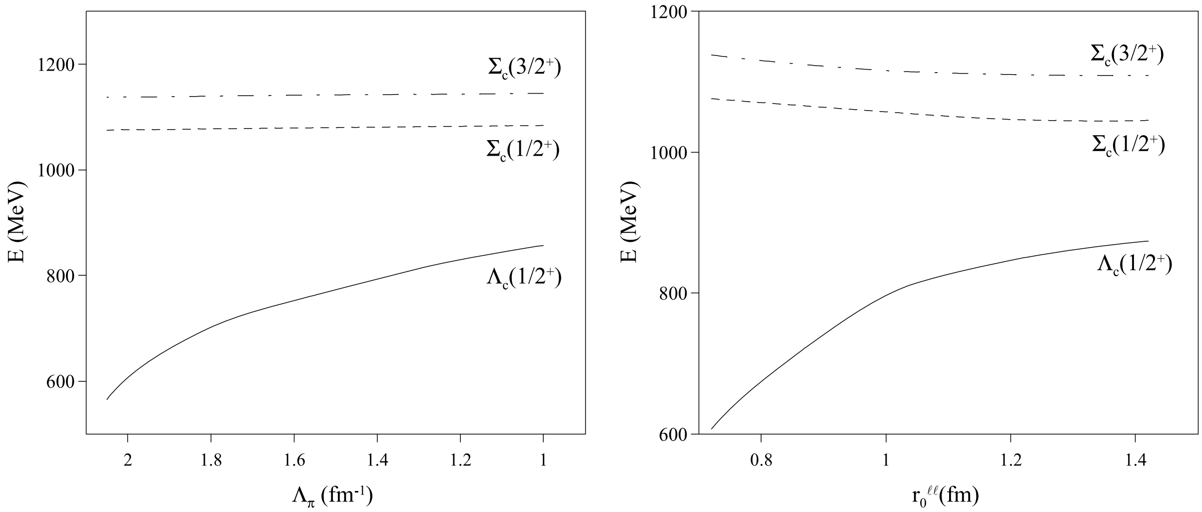


**Fig. 4.** Variation of the masses of $\Sigma_c(3/2^+)$, $\Sigma_c(1/2^+)$ and $\Lambda_c(1/2^+)$ with (a) the regularization parameter of the pseudoscalar interaction $\Lambda_\pi$, and (b) the regularization parameter of the OGE spin–spin interaction $r_0^{\ell\ell}$, for Model D.

**Table 5**
Excitation spectra of $\Lambda_c$ and $\Sigma_c$ baryons, $M[J^P] - M[\Lambda_c(1/2^+)]$, for the nonrelativistic (Model C) and relativistic (Model D) Bhaduri plus pseudoscalar potential. The star stands for radial excitations. Masses are in MeV. Experimental data are from Ref. [43].

| State | $J^P$ | Model C ($\Lambda_\pi = 4.2$ fm$^{-1}$) | Model D ($\Lambda_\pi = 1.9$ fm$^{-1}$) | Exp. |
|---|---|---|---|---|
| $\Lambda_c$ | $1/2^{+*}$ | 623 | 464 | – |
| | $3/2^+$ | 726 | 688 | – |
| | $3/2^{+*}$ | 966 | 857 | – |
| | $1/2^-$ | 387 | 383 | 324 |
| $\Sigma_c$ | $1/2^+$ | 169 | 171 | 167 |
| | $1/2^{+*}$ | 779 | 824 | – |
| | $3/2^+$ | 245 | 235 | 232 |
| | $1/2^-$ | 544 | 538 | – |

Let us now analyze the second alternative for the description of the hyperfine splitting in the meson and baryon spectra coming from the hybrid models mentioned in Section 3, where the Bhaduri potential is supplemented by a pseudoscalar exchange between the light quarks, Models C and D in Table 1. When the pseudoscalar exchange is added to the standard Bhaduri potential, the strength of the hyperfine interaction between the light quarks is increased. As explained above, this enlarges the mass difference between the $\Sigma$'s and the $\Lambda$'s (members of the flavor $\mathbf{6_F}$ and $\bar{\mathbf{3}}_\mathbf{F}$ representations, respectively). Thus, one has to reduce the strength of the OGE as noticed in the description of the NN interaction [46]. In Fig. 3 we have represented the variation of the masses of $\Sigma_c(3/2^+)$, $\Sigma_c(1/2^+)$ and $\Lambda_c(1/2^+)$ with the regularization parameter of the spin–spin interaction between the light quarks $r_0^{\ell\ell}$ for Model C, containing OGE and a pseudoscalar interaction with nonrelativistic kinematics. A correct description of the low-lying spectrum can be obtained with the standard pseudoscalar interaction if one takes $r_0^{\ell\ell} = 1.40$ fm. The results are shown in Table 5.

When the relativistic kinematics is considered for this model we note the same effect observed previously. As can be seen in Eq. (17) the pseudoscalar potential consists of a Yukawa interaction with a range equal to the inverse of the pion mass plus a smeared-out delta function with smearing parameter $1/\Lambda_\pi$. Therefore, also in this case the mass of the baryon with a good diquark, the $\Lambda_c$, will collapse if $1/\Lambda_\pi \le 0.1$ fm in the case of the nonrelativistic kinematics and if $1/\Lambda_\pi \le 0.45$ fm in the case of the relativistic one. Thus, the cutoff parameter $\Lambda_\pi$ must be smaller than $\sim$10 fm$^{-1}$ in the case of the nonrelativistic kinematics and $\sim$2.2 fm$^{-1}$ in the case of the relativistic one. As explained above, this is due to the nonrelativistic propagator falling down as $1/k^2$ while the relativistic one doing it as $1/k$. In Fig. 4(a) we have represented the variation of the masses of $\Sigma_c(3/2^+)$, $\Sigma_c(1/2^+)$ and $\Lambda_c(1/2^+)$ with the regularization parameter of the pseudoscalar interaction, $\Lambda_\pi$, for Model D. Stable results compatible with the experimental data are obtained with a similar regularization of the pseudoscalar interaction we have used in Ref. [54] for the study of the light baryon spectrum, $\Lambda_\pi = 1.9$ fm$^{-1}$. Once we have fixed the regularization of the pseudoscalar interaction, as in the nonrelativistic case, one has to refit the strength of the OGE between the light quarks. In Fig. 4(b) we have represented the variation of the masses of $\Sigma_c(3/2^+)$, $\Sigma_c(1/2^+)$ and $\Lambda_c(1/2^+)$ with the regularization parameter of the spin–spin interaction between the light quarks $r_0^{\ell\ell}$ for Model D. Once again a nice description of the splittings between the members of the flavor $\mathbf{6_F}$ and $\bar{\mathbf{3}}_\mathbf{F}$ representations can be obtained using the same regularization as in the light baryon spectra [54].

We show in Table 5 the excitation spectra of $\Lambda_c$ and $\Sigma_c$ baryons for the nonrelativistic (Model C) and relativistic (Model D) Bhaduri plus pseudoscalar potential. As can be seen the mass difference between the members of the flavor $\mathbf{6_F}$ representation, controlled by the $Qq$ interaction, as well as the mass difference between the members of the flavor $\bar{\mathbf{3}}_\mathbf{F}$ and $\mathbf{6_F}$ representations, controlled by the $qq$ interaction, is again perfectly described in both cases. As said in the case of the Bhaduri model, one could fit the strength of confinement to get a correct description of the first negative parity state. We confirm the relevant differences between the relativistic and the nonrelativistic description noted in the case of the Bhaduri model. Being the spin–spin part fixed, we observed again how the radial excitation of the ground state baryon with a "good" diquark, the $\Lambda_c(1/2^+)$, has a larger excitation energy than the negative parity state in the nonrelativistic case, being both almost degenerate in the relativistic one, a remnant of the relativistic light quarks dynamics. The second difference observed with the Bhaduri model is also preserved here. The $S$ wave $\Lambda_c(3/2^+)$, $(L,S) = (0,3/2)$, is lowered by the relativistic theory as compared to the ground state $D$ wave, $(L,S) = (2,1/2)$, giving 100 MeV difference. The other states of the spectra are predicted by both models with approximately the same mass.

At difference with the light baryon spectrum, the combined effect of the pseudoscalar exchange together with relativistic kinematics is not translated into a more effective level ordering

**Table 6**
Excitation spectra of double charmed baryons, $M[J^P] - M[\Xi_{cc}(1/2^+)]$, as compared to recent lattice calculations. The star stands for radial excitations. Masses are in MeV.

| State | $J^P$ | Model A | Model B | Ref. [7] | Ref. [8] | Ref. [10] |
|---|---|---|---|---|---|---|
| $\Xi_{cc}$ | $1/2^{+*}$ | 462 | 437 | – | – | – |
| | $3/2^+$ | 80 | 53 | 53±64 | 103±35 | 109±49 |
| | $1/2^-$ | 332 | 324 | – | – | – |

effect [54]. This was already noticed when studying the strange baryon spectrum [57]. In the case of $\Lambda$ baryons the two light quarks are in a flavor antisymmetric spin 0 state, the pseudoscalar and the one-gluon exchange forces being both attractive. For $\Sigma$ baryons they are in a flavor symmetric spin 1 state. The pseudoscalar force, being still attractive, is suppressed by one order of magnitude due to the expectation value of the $(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\lambda}_i \cdot \vec{\lambda}_j)$ operator [57] and the one-gluon exchange between the two light quarks becomes repulsive. Therefore, the attraction is provided by the interaction between the light diquark and the heavy quark, which for heavy quarks $c$ or $b$ is given only by the one-gluon exchange potential. The simpler wave function of the charmed baryons, containing only good light diquarks in the members of the flavor $\bar{\mathbf{3}}_\mathbf{F}$ representation, the $\Lambda_c$'s, and only bad diquarks in the members of the flavor $\mathbf{6}_\mathbf{F}$ representation, the $\Sigma_c$'s, makes the $(\vec{\sigma} \cdot \vec{\sigma})(\vec{\tau} \cdot \vec{\tau})$ structure less effective when giving attraction for symmetric spin–isospin pairs and repulsion for antisymmetric ones.

Finally, all these models could be applied to double heavy baryons where relativistic effects are clearly perturbative. As the kinematics determined the parameters of the model in the single heavy baryon sector, the different models will give rise to different predictions. We show in Table 6 the predictions of Models A and B compared to recent lattice QCD calculations. As can be seen the relativistic theory would give rise to a smaller spin splitting. However we can see how the difference between the negative and the positive parity states is minimized regarding the kinematics chosen, due to the nonexistence of baryons with good diquarks as a consequence of the restrictions imposed by the Pauli principle. In this system all states belong to the flavor $\mathbf{6}_\mathbf{F}$ representation, and the spin splitting $M[\Xi_{cc}(3/2^+)] - M[\Xi_{cc}(1/2^+)]$ comes determined exactly by the same interaction responsible for the $M[\Sigma_c(3/2^+)] - M[\Sigma_c(1/2^+)]$ splitting, the dynamics of the $qQ$ subsystem (see Eq. (15)). As we can see in Table 6 the predictions of lattice QCD calculation are still uncertain, but the progress in this field as well as the measurement of double charmed baryons in current factories will help in determining the correct dynamics at the level of quarks. Among the baryons with two heavy quarks the first question to be settled is where do exactly these states lie. In any case, the excited spectra of double charmed baryons do not depend much on the mass of the heavy quarks, and therefore the predicted excited spectra should serve as a guideline for potential future experiments looking for such states.

From the point of view of lattice QCD, determining resonance energies is no trivial matter. Currently, there is a clear yet challenging way for obtaining energies of resonances that decay into two-particle states. Unfortunately, there is currently no formalism that would allow the determination of resonances that have large overlap with three-particle states. It will take a few more years for the community to have these issues well under control to be able to study such resonances [58]. Thus, the $\Lambda_c(3/2^+)$, is currently out of the scope of what is accessible via lattice QCD. It has a predicted mass in the order of 3 GeV, which is of the order of 800 MeV above the lowest $\Lambda_c$. Therefore, this state is most likely a hadronic resonance with potential large overlap with $\Lambda_c(1/2^+)\pi^+\pi^-$, among other few particle states, just like $\Lambda_c(5/2^+)$. Consequently one cannot apply all lattice techniques used for the ground state calculation. However, the first radial and orbital excitations of the good diquark baryon, $\Lambda_c(1/2^+)$ would be accessible in the near future.

## 5. Conclusions

We have presented a comparative Faddeev study of baryons with one and two heavy quarks with nonrelativistic and relativistic kinematics. We have used different standard interacting potentials appearing in the literature that differ in the description of the hyperfine splitting between the light quarks as well as in the parameters trying to minimize the changes to get as much information as possible from the dynamics. We have analyzed in detail the performance of the relativistic and nonrelativistic Faddeev equations for the different interacting potentials used. We have identified particular states of singly heavy baryons whose masses will be different depending on the particular dynamics governing the interaction between the two light quarks. In particular, the measurement and identification of the radial excitation of the $\Lambda_c(1/2^+)$ will provide enough information to clarify the relativistic or nonrelativistic kinematics of the light quarks inside heavy baryons. The consequent measurement of the spin splitting in the double heavy baryon sector will provide definitive constraints about the light quark dynamics. The radial excitation of the $\Lambda_c(3/2^+)$ will also shed hints about the dynamical model used, but its large excitation energy and significant overlapping with three-body decay channels makes it more adequate as a theoretical test for future lattice QCD simulations.

We have clearly illustrated the uncertain situation when using just the quark model energy for assigning quantum numbers to new observed heavy baryon resonances. We have also illustrated the correlation between the predictions for the single and double heavy baryon systems. Angular analysis based on larger statistics and improved triggering in forthcoming experiments will help us in the coherent theoretical and experimental effort needed to learn about quarks dynamics. We have analyzed the interplay between the kinematics and the spin splitting, already observed for the light baryons, that translates into the relative position between the radial and orbital excitations of baryons with good diquarks, that may also constitute a basic ingredient for the description of heavy baryons. Our findings would be equally applicable to the bottom sector.

Heavy baryons constitute an extremely interesting problem joining the dynamics of light–light and heavy–light subsystems in an amazing manner. While the mass difference between members of the same SU(3) configuration, either $\bar{\mathbf{3}}_\mathbf{F}$ or $\mathbf{6}_\mathbf{F}$, is determined by the interaction in the light–heavy quark subsystem, the mass difference between members of different representations comes mainly determined by the dynamics of the light diquark, and should therefore be determined in consistency with the light baryon spectra.

The study of heavy baryons may provide us with critical insights into the nature of QCD. There are still important questions to be resolved on the nature of the short-distance interactions of the quarks. The parametrization of the true degrees of freedom of any theory becomes a challenge that will allow to advance in the understanding of QCD. The advent of experimental data for singly and double heavy baryons could be the appropriate laboratory to understand the dynamics of the light quark subsystem. Heavy baryon spectroscopy may play a pivotal role in answering these questions. Meanwhile, the encouraging results of this letter suggest a complete study of the heavy baryon spectroscopy to have at our disposal the complete pattern of models based on relativistic kinematics [59].

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