

# Gluon emission in interaction of two reggeons

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**Abstract** The vertex is constructed for gluon production in the interaction of two reggeons coupled to projectiles and two reggeons coupled to targets. The vertex can be used to build cross sections for collisions of two pairs of nucleons in AA scattering. The transversality of the constructed vertex is demonstrated as well as its good behaviour at large longitudinal momenta necessary for applications. Poles at zero values of longitudinal momenta are discussed and it is found that they remain in the amplitudes unlike in the case of a single projectile.

## 1 Introduction

In the QCD hadronic interactions at high energies in the Regge kinematics, when the transferred transverse momenta are much smaller than the energies, can be described by the interaction of normal gluons with reggeised ones (“reggeons”). The latter combine into pomerons coupled to participant colourless projectiles and targets. The simplest case is the collision of the highly virtual gluon with a hadron or a nucleus. It has been studied long ago and solved by the formulation of the BFKL [1, 2], Balitski [3] and Kovchegov [4] equations for the non-integrated gluon densities, which allow one to calculate the relevant cross sections. With some ingenuity this approach can be generalised to pp or pA scattering.

However, nucleus–nucleus collisions present a more difficult problem. In the case of heavy nuclei the total cross sections can be treated within the effective pomeron interaction formalism [5]. For the inclusive gluon production a general formalism was developed in [6–8] in the framework of the colour glass condensate (CGC) approach. The inclusive cross sections were expressed via averages of the gluon potentials in the field of the colliding nuclei, developed in rapidity according to the so-called JIMWLK functional equations.

These averages can only be found by numerical methods. Several attempts to find analytic expressions for the inclusive gluon production have lead to only approximate [9] or partial and inconclusive results [10, 11].

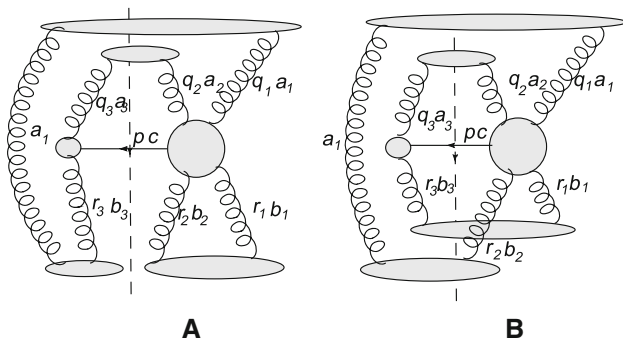
The BFKL approach presents an alternative (in all probability equivalent, in principle) way to study this problem. It may produce analytic formulae for the cross sections and also allow one to study the case of light nuclei, when the leading contributions appear to be of the subleading order in  $N_c$  [12] and a direct application of the CGC approach does not seem to be possible. In the BFKL approach the problem requires knowledge of amplitudes for gluon production in interaction of many reggeons coupled to the projectile with many reggeons coupled to the target.

The simplest non-trivial case is production of gluons in interaction of two reggeons coupled to the projectile with two reggeons coupled to the target. For heavy nuclei, in the lowest order, this means interaction of two nucleons from each of the nuclei, as illustrated in Fig. 1. One observes that apart from the well-known production amplitude in collision between two colourless objects there enters a more complicated vertex when the gluon is produced by a pair of reggeons coupled to projectile and target nucleons (the  $RR \rightarrow RRP$  vertex where P stands for “particle”, that is, the gluon).

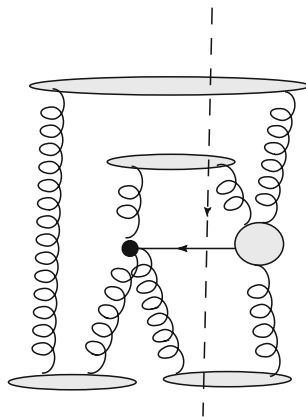
In this paper we study this  $RR \rightarrow RRP$  vertex both with a real (on-mass-shell) and virtual (off-mass-shell) gluon. The latter is needed if one wants to construct the vertex for transition of three reggeons into three reggeons ( $RRR \rightarrow RRR$ ), which enters the kernel of the equation for the odderon or the higher pomeron made of the three reggeons. Note that for the odderon the  $RRR \rightarrow RRR$  vertex, integrated over the longitudinal momenta, was derived in [13].

Note that for collisions of heavy nuclei the contribution shown in Fig. 1 is only a particular term corresponding to the interaction of only two pairs of nucleons. By itself it corresponds only gluon production in collision of two deuterons. But even in this simple case one should additionally consider contributions from cutting the vertex itself, as shown

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**Fig. 1** Gluon production by pair of nucleons of the projectile in collision with a pair of nucleons from the target. *Solid lines* correspond to particles, wavy ones to reggeons



**Fig. 2** Gluon production by pair of nucleons of the projectile in collision with a pair of nucleons from the target from cutting the  $RR \rightarrow RRP$  vertex. The *left vertex* is induced

in Fig. 2. As seen from Fig. 1, the immediate application of the vertex  $RR \rightarrow RRP$  is to give a non-trivial contribution to the diffractive gluon production in deuteron–proton collisions (Fig. 1a).

In this paper we do not attempt to calculate this contribution, which requires a lot more of analytical and numerical efforts. Our aim is to just derive the vertex itself and demonstrate its basic properties important to its subsequent applications: transversality, vanishing at large longitudinal momenta and presence or absence of poles at zero values of the latter.

The main tool for the calculation of the vertex is the Lipatov effective action [14], which gives the rules for the reggeon–particle interaction at a given rapidity and introduces the so-called induced vertices for this interaction. Some of the full and induced vertices derived from this action have already been found in [15]. The induced vertex  $RP \rightarrow RP$  is new and will be derived here.

The paper is organised as follows. In Sect. 2 we derive the vertex  $RR \rightarrow RRP$  off the mass shell. In Sect. 3 we demonstrate its transversality. Section 4 is devoted to the study of the vertex at high longitudinal momenta. In Sect. 5 we derive

the vertex on the mass shell. In Sect. 6 we investigate the pole singularities at zero values of the longitudinal momenta. In Sect. 7 we draw some conclusions.

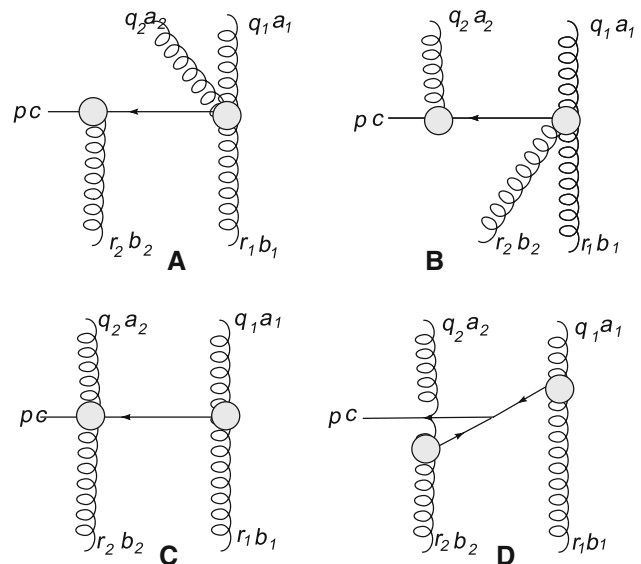
## 2 Vertex $RR \rightarrow RRP$ with a virtual emitted gluon

In the framework of the effective action the vertex  $RR \rightarrow RRP$  is constructed as a sum of four diagrams, shown in Fig. 3a–d with subsequent symmetrisation in the reggeons attached to the projectile (upper in Fig. 3) and those attached to the target (lower in Fig. 3). The blobs in Fig. 3 denote full (basic plus induced) vertices. Solid lines denote gluons, wavy ones correspond to reggeons. We denote momenta and colours of upper reggeons from right to left as  $q_1, a_1$  and  $q_2, a_2$  and those for lower reggeons as  $r_1, b_1$  and  $r_2, b_2$ . The emitted gluon has momentum, polarisation and colour  $p, \mu, c$ . The reggeons carry polarisation vectors  $n^\pm$  with  $n_+^+ = n_-^+ = n_-^- = n_+^- = 0, n_+^- = n_-^+ = 1$ .

### 2.1 Figure 3a

As is clear from the figure we already know all the building blocks for the construction of the vertex. The vertex on the right  $RR \rightarrow RP$  can be found from the vertex  $R \rightarrow RRP$ , calculated in [15] after changing the direction of reggeon propagation and notations of the momenta. From the same publication one can extract the vertex on the left  $P \rightarrow RP$ .

In this way we find the vertex  $RR \rightarrow RP$  on the right in the form



**Fig. 3** Different contributions to the vertex  $RR \rightarrow RRP$ . *Solid lines* correspond to particles, wavy ones to reggeons

$$\bar{V}_\nu = i \frac{f^{b_1 a_1 e} f^{e a_2 d}}{t_1^2 + i0} \left( \bar{A} t_\nu - \bar{B} q_{1\nu} - \bar{C} q_{2\nu} + \bar{D} n_\nu^+ + \bar{E} n_\nu^- \right), \tag{1}$$

where  $t = p + r_2 = q_1 + q_2 - r_1, t_1 = q_1 - r_1$  and

$$\begin{aligned} \bar{A} &= 3t_- + \frac{r_1^2}{q_{1+}}, \quad \bar{B} = 4t_-, \quad \bar{C} = 4t_- + 2\frac{r_1^2}{q_{1+}}, \\ \bar{D} &= -\frac{r_1^2(r_1 - q_1)^2}{t_+ q_{1+}} - 2t_- \frac{r_1^2}{q_{1+}} - 4t_-^2, \\ \bar{E} &= -\left( -(r_1 + q_1)(t + q_2) + q_2^2 - q_1^2 + (r_1 - q_1)^2 \right. \\ &\quad \left. + 2r_{1-q_1+} \right) + \left( -2r_{1-} + \frac{r_1^2}{q_{1+}} \right) \left( t_+ + q_{2+} - \frac{q_2^2}{r_{1-}} \right). \end{aligned} \tag{2}$$

Note that this vertex is transversal ( $t\bar{V} = 0$ ).

On the left of the diagram Fig. 3a there stands the vertex  $R \rightarrow RP$ , given by the tensor [15]

$$\begin{aligned} X_{\mu\nu} &= -g f^{db_2c} \left( (p+t)_+ g_{\mu\nu} + (p-2t)_\mu n_\nu^+ \right. \\ &\quad \left. + (t-2p)_\nu n_\mu^+ - n_\mu^+ n_\nu^+ \frac{r_2^2}{p_+} \right). \end{aligned} \tag{3}$$

This vertex is not orthogonal to  $p$  or  $t$  separately. However, the product  $(pXt) = 0$ . The term  $t_\nu n_\mu^+$  does not contribute due to the transversality of  $\bar{V}_\nu$  and will be dropped.

Multiplying  $X$  by  $\bar{V}$  on the right we obtain the contribution to the vertex  $RR \rightarrow RRP$  from the diagram in Fig. 3a:

$$\begin{aligned} \mathcal{A}_{1\mu} &= -g^3 C_1 \frac{1}{(t_1^2 + i0)(t^2 + i0)} \\ &\quad \times \left( a_\mu \bar{A} - b_\mu \bar{B} - c_\mu \bar{C} + d_\mu \bar{D} + e_\mu \bar{E} \right), \end{aligned} \tag{4}$$

where vectors  $a, \dots, e$  are

$$\begin{aligned} a_\mu &= p_\mu p_+ - n_\mu^+ (t^2 + p^2), \\ b_\mu &= 2p_+ q_{1\mu} + (p-2t)_\mu q_{1+} - n_\mu^+ \left( 2(pq_1) + r_2^2 \frac{q_{1+}}{p_+} \right), \\ c_\mu &= 2p_+ q_{2\mu} + (p-2t)_\mu q_{2+} - n_\mu^+ \left( 2(pq_2) + r_2^2 \frac{q_{2+}}{p_+} \right), \\ d_\mu &= 0, \\ e_\mu &= 2p_+ n_\mu^- + (p-2t)_\mu - n_\mu^+ \left( 2p_- + \frac{r_2^2}{p_+} \right) \end{aligned}$$

and the colour coefficient  $C_1$  is

$$C_1 = f^{db_2c} f^{b_1 a_1 e} f^{e a_2 d}. \tag{5}$$

### 2.2 Figure 3b

In this case the  $R \rightarrow RRP$  vertex on the right can be taken directly from [15] duly changing the notations. We get for it

$$\begin{aligned} V_\nu &= i \frac{f^{db_2e} f^{eb_1 a_1}}{t_1^2 + i0} \left\{ q_{1+}(4r_1 + \bar{t})_\nu - \left[ (q_1 + r_1)(\bar{t} - r_2) \right. \right. \\ &\quad \left. \left. + r_2^2 - r_1^2 + (q_1 - r_1)^2 + 2q_{1+r_1-} \right] n_\nu^+ \right. \\ &\quad \left. + \frac{q_1^2 (q_1 - r_1)^2}{\bar{t}_- r_{1-}} n_\nu^- + \left( 2q_{1+} - \frac{q_1^2}{r_{1-}} \right) \right. \\ &\quad \left. \times \left[ -2q_{1+n_\nu^-} + (\bar{t} + 2r_2)_\nu + \left( \bar{t}_- - r_{2-} + \frac{r_2^2}{q_{1+}} \right) n_\nu^+ \right] \right\}, \end{aligned}$$

where  $\bar{t} = q_1 - r_1 - r_2$ . We rewrite it as

$$\begin{aligned} V_\nu &= i \frac{f^{db_2e} f^{eb_1 a_1}}{(q_1 - r_1)^2 + i0} \left\{ A \bar{t}_\nu + B r_{1\nu} + C r_{2\nu} + D n_\nu^- + E n_\nu^+ \right\}, \end{aligned}$$

where

$$\begin{aligned} A &= 3q_{1+} - \frac{q_1^2}{r_{1-}}, \quad B = 4q_{1+}, \quad C = 2 \left( 2q_{1+} - \frac{q_1^2}{r_{1-}} \right), \\ D &= \frac{q_1^2 (q_1 - r_1)^2}{\bar{t}_- r_{1-}} - 2q_{1+} \left( 2q_{1+} - \frac{q_1^2}{r_{1-}} \right), \\ E &= -\left( (q_1 + r_1)(\bar{t} - r_2) + r_2^2 - r_1^2 + (q_1 - r_1)^2 + 2q_{1+r_1-} \right) \\ &\quad + \left( 2q_{1+} - \frac{q_1^2}{r_{1-}} \right) \left( \bar{t}_- - r_{2-} + \frac{r_2^2}{q_{1+}} \right). \end{aligned} \tag{6}$$

Vertex  $\bar{X}$  in the diagram on the left is obtained from (3) by inversion of reggeons with  $p$  and  $\bar{t}$  preserved:

$$\begin{aligned} \bar{X}_{\mu\nu} &= -g f^{da_2c} \\ &\quad \times \left( 2p_- g_{\mu\nu} + (p-2\bar{t})_\mu n_\nu^- + n_\mu^- (\bar{t} - 2p)_\nu - n_\mu^- n_\nu^- \frac{q_2^2}{p_-} \right). \end{aligned} \tag{7}$$

Multiplying it by  $V$  on the right we get the contribution to the vertex  $RR \rightarrow RRP$  from the diagram in Fig. 3b:

$$\begin{aligned} \mathcal{A}_{2\mu} &= -g^3 C_2 \frac{1}{(t_1^2 + i0)(\bar{t}^2 + i0)} \\ &\quad \times \left( \bar{a}_\mu A + \bar{b}_\mu B + \bar{c}_\mu C + \bar{d}_\mu D + \bar{e}_\mu E \right), \end{aligned} \tag{8}$$

where the vectors  $\bar{a}, \dots, \bar{e}$  are

$$\begin{aligned} \bar{a}_\mu &= p_\mu p_- - n_\mu^- (\bar{t}^2 + p^2), \\ \bar{b}_\mu &= 2p_- r_{1\mu} + (p - 2\bar{t})_\mu r_{1-} - n_\mu^- \left( 2(pr_1) + q_2^2 \frac{r_{1-}}{p_-} \right), \\ \bar{c}_\mu &= 2p_- r_{2\mu} + (p - 2\bar{t})_\mu r_{2-} - n_\mu^- \left( 2(pr_2) + q_2^2 \frac{r_{2-}}{p_-} \right), \\ \bar{d}_\mu &= 0, \\ \bar{e}_\mu &= 2p_- n_\mu^+ + (p - 2\bar{t})_\mu - n_\mu^- \left( 2p_+ + \frac{q_2^2}{p_-} \right) \end{aligned}$$

and the colour factor is

$$C_2 = f^{da_2c} f^{db_2e} f^{eb_1a_1}. \tag{9}$$

### 2.3 Figure 3c

Here on the right we have the well-known Lipatov vertex  $f^{a_1b_1d} L_{1\nu}$ , where the momentum part is

$$L_{1\nu} = a_{1\nu} + b_1 n_\nu^+ + c_1 n_\nu^-. \tag{10}$$

Here

$$a_1 = q_1 + r_1, \quad b_1 = \frac{r_1^2}{q_{1+}} - 2r_{1-}, \quad c_1 = \frac{q_1^2}{r_{1-}} - 2q_{1+}.$$

On the left, however, we have a new vertex  $RP \rightarrow RP$ . Using the effective action we find that it consists of two terms, the term coming from the standard 4-gluon interaction  $Z_1$  and the induced term  $Z_2$ . Calculations give

$$\begin{aligned} Z_{\mu\nu}^{(1)} &= i g^2 \left[ f^{a_2ce} f^{b_2de} (2n_\mu^+ n_\nu^- - n_\mu^- n_\nu^+ - g_{\mu\nu}) \right. \\ &\quad \left. + f^{a_2de} f^{b_2ce} (2n_\mu^- n_\nu^+ - n_\mu^+ n_\nu^- - g_{\mu\nu}) \right] \end{aligned} \tag{11}$$

and

$$\begin{aligned} Z_{\mu\nu}^{(2)} &= i g^2 q_{2\perp}^2 \left( \frac{f^{a_2ce} f^{b_2de}}{-p_- r_{2-}} + \frac{f^{a_2de} f^{b_2ce}}{t_{1-} r_{2-}} \right) n_\mu^- n_\nu^- + i g^2 r_{2\perp}^2 \\ &\quad \times \left( \frac{f^{a_2ce} f^{b_2de}}{-t_{1+} q_{2+}} + \frac{f^{a_2de} f^{b_2ce}}{p_+ q_{2+}} \right) n_\mu^+ n_\nu^+. \end{aligned} \tag{12}$$

Multiplying these terms by the Lipatov vertex from the right we respectively get the two terms

$$\begin{aligned} \mathcal{B}_\mu^{(1)} &= g^3 f^{a_1b_1d} \frac{1}{t_1^2 + i0} \left\{ f^{a_2ce} f^{b_2de} \left[ 2a_{1-} n_\mu^+ - a_{1+} n_\mu^- - a_{1\mu} \right. \right. \\ &\quad \left. \left. + n_\mu^+ \left( \frac{r_1^2}{q_{1+}} - 2r_{1-} \right) - 2n_\mu^- \left( \frac{q_1^2}{r_{1-}} - 2q_{1+} \right) \right] \right. \\ &\quad \left. + f^{a_2de} f^{b_2ce} \left[ 2a_{1+} n_\mu^- - a_{1-} n_\mu^+ - a_{1\mu} - 2n_\mu^+ \right. \right. \\ &\quad \left. \left. + \left( \frac{r_1^2}{q_{1+}} - 2r_{1-} \right) n_\mu^- \left( \frac{q_1^2}{r_{1-}} - 2q_{1+} \right) \right] \right\} \end{aligned}$$

and

$$\begin{aligned} \mathcal{B}_\mu^{(2)} &= g^3 f^{a_1b_1d} \frac{1}{t^2 + i0} \left[ n_\mu^- z_1 \left( \frac{r_1^2}{q_{1+}} + a_{1-} - 2r_{1-} \right) \right. \\ &\quad \left. + n_\mu^+ z_2 \left( \frac{q_1^2}{r_{1-}} + a_{1+} - 2q_{1+} \right) \right], \end{aligned}$$

where we denoted

$$z_1 \equiv q_{2\perp}^2 \left( \frac{f^{a_2ce} f^{b_2de}}{-p_- r_{2-}} + \frac{f^{a_2de} f^{b_2ce}}{t_{1-} r_{2-}} \right)$$

and

$$z_2 \equiv r_{2\perp}^2 \left( \frac{f^{a_2ce} f^{b_2de}}{-t_{1+} q_{2+}} + \frac{f^{a_2de} f^{b_2ce}}{p_+ q_{2+}} \right).$$

In practice it is convenient to split the total contribution into two parts having different colour factors. So the contribution from Fig. 3c consists of two different amplitudes,

$$\begin{aligned} \mathcal{A}_{3\mu} &= g^3 C_3 \frac{1}{t_1^2 + i0} \\ &\quad \times \left[ 2a_{1-} n_\mu^+ - a_{1+} n_\mu^- - a_{1\mu} + n_\mu^+ \left( \frac{r_1^2}{q_{1+}} - 2r_{1-} \right) \right. \\ &\quad \left. - 2n_\mu^- \left( \frac{q_1^2}{r_{1-}} - 2q_{1+} \right) \right. \\ &\quad \left. + n_\mu^- \frac{q_2^2}{-p_- r_{2-}} \left( \frac{r_1^2}{q_{1+}} + a_{1-} - 2r_{1-} \right) \right. \\ &\quad \left. + n_\mu^+ \frac{r_1^2}{-t_{1+} q_{2+}} \left( \frac{q_1^2}{r_{1-}} + a_{1+} - 2q_{1+} \right) \right] \end{aligned} \tag{13}$$

and

$$\begin{aligned} \mathcal{A}_{4\mu} &= g^3 C_4 \frac{1}{t_1^2 + i0} \\ &\quad \times \left[ 2a_{1+} n_\mu^- - a_{1-} n_\mu^+ - a_{1\mu} \right. \\ &\quad \left. - 2n_\mu^+ \left( \frac{r_1^2}{q_{1+}} - 2r_{1-} \right) + n_\mu^- \left( \frac{q_1^2}{r_{1-}} - 2q_{1+} \right) \right. \\ &\quad \left. + n_\mu^- \frac{q_2^2}{t_{1-} r_{2-}} \left( \frac{r_1^2}{q_{1+}} + a_{1-} - 2r_{1-} \right) \right. \\ &\quad \left. + n_\mu^+ \frac{r_1^2}{p_+ q_{2+}} \left( \frac{q_1^2}{r_{1-}} + a_{1+} - 2q_{1+} \right) \right], \end{aligned} \tag{14}$$

where the colour factors are

$$C_3 = f^{a_1b_1d} f^{a_2ce} f^{b_2de} = -C_2 \tag{15}$$

and

$$C_4 = f^{a_1b_1d} f^{a_2de} f^{b_2ce} = C_1. \tag{16}$$

### 2.4 Figure 3d

The contribution from this diagram comes from two Lipatov vertices  $L_{1\nu_1}$  and  $L_{2\nu_2}$  coupled to the triple gluon vertex

$$\Gamma_{\nu_1\mu,\nu_2}(t_1, p, t_2) = -gf^{d_1cd_2} \left( (p+t_2)_{\nu_1} g_{\mu\nu_2} + (t_1-t_2)_\mu g_{\nu_1\nu_2} + (-t_1-p)_{\nu_2} g_{\mu\nu_1} \right),$$

where  $t_1 = q_1 - r_1$ ,  $t_2 = q_2 - r_2$ . The two Lipatov vertices are transversal  $(t_1 L_1) = (t_2 L_2) = 0$ . So in  $\Gamma_{\nu_1\mu\nu_2}$  we can drop terms with  $t_{1\nu_1}$  and  $t_{2\nu_2}$  and take

$$\Gamma_{\nu_1\mu\nu_2} = -f^{d_1cd_2} \left( 2t_{2\nu_1} g_{\mu\nu_2} + (t_1-t_2)_\mu g_{\nu_1\nu_2} - 2t_{1\nu_2} g_{\mu\nu_1} \right). \tag{17}$$

As a result we find a compact expression for the contribution from Fig. 3d

$$\mathcal{A}_{5\mu} = g^3 C_5 \frac{1}{(t_1^2 + i0)(t_2^2 + i0)} \times (2(t_2 L_1) L_{2\mu} - 2(t_1 L_2) L_{1\mu} + (L_1 L_2)(t_1 - t_2)_\mu). \tag{18}$$

Here the colour factor is

$$C_5 = f^{a_1 d_1 b_1} f^{d_1 c d_2} f^{a_2 d_2 b_2} = C_1 + C_2, \tag{19}$$

where we have used the Jacoby identity.

Note that for some calculations a more explicit form may be preferable, which is given in the appendix.

### 2.5 Symmetrisation and particular configurations

Contributions calculated above refer to a fixed order of upper and lower reggeons,

$$\mathcal{A}(q_2, a_2; q_1, a_1 | r_2, b_2; r_1, b_1) \equiv \mathcal{A}(2, 1|2, 1).$$

The total amplitude is obtained after we sum it with the contributions with interchange of upper and lower gluons. For  $\mathcal{A}_i$  with  $i = 1, 2, 3, 4$  each interchange gives a new diagram so that the total amplitude is

$$\mathcal{A}_i^{\text{tot}} = \mathcal{A}_i(2, 1|2, 1) + \mathcal{A}_i(2, 1|1, 2) + \mathcal{A}_i(1, 2|2, 1) + \mathcal{A}_i(1, 2|1, 2), \quad i = 1, 2, 3, 4.$$

For  $i = 5$  a simultaneous interchange of upper and lower reggeons does not give a new contribution. So in this case

$$\mathcal{A}_5^{\text{tot}} = \mathcal{A}_5(2, 1|2, 1) + \mathcal{A}_5(2, 1|1, 2).$$

Each interchange combines the interchanges of momenta and colours. In the general case this introduces a multitude of different colour factors. To simplify we restrict ourselves to colour configurations actually present in the applications. Inspecting Fig. 1 we see that the  $RR \rightarrow RRP$  vertex may

appear in two different colour configurations. One of them, Fig. 1a, is diffractive with respect to the targets but non-diffractive with respect to the projectiles, the D–ND configuration (of course there exists a similar configuration with projectiles and targets reversed). The other configuration, that of Fig. 1b, is non-diffractive with respect to both projectiles and targets, the ND–ND configuration. In the D–ND configuration the general colour coefficient  $C(a_2, a_1 | b_2, b_1)$  is to be convoluted with  $\delta_{b_1, b_2}$ . Then we obtain

$$C(a_2, a_1 | b_2, b_1) \delta_{b_1 b_2} = N f^{a_2 a_1 c} \kappa^{\text{D-ND}}, \tag{20}$$

where  $\kappa$  is just a number, different for different diagrams. In the ND–ND configuration the general colour coefficient is to be convoluted with  $\delta_{a_1 b_2}$  and we obtain

$$C(a_2, a_1 | b_2, b_1) \delta_{a_1 b_2} = N f^{a_2 b_1 c} \kappa^{\text{ND-ND}}, \tag{21}$$

where again the number  $\kappa$  is different for different diagrams. The choice of convolution colours  $a_1$  and  $b_2$  is of course arbitrary due to the symmetry in both upper and lower reggeons.

Using these considerations for both configurations the total amplitudes can be written via their momentum parts as follows. In the D–ND configuration

$$\mathcal{A}_i^{\text{tot}} = N f^{a_2 a_1 c} = \kappa_i^{(1)} \mathcal{A}_i(q_2, q_1 | r_2, r_1) + \kappa_i^{(2)} \mathcal{A}_i(q_2, q_1 | r_1, r_2) + \kappa_i^{(3)} \mathcal{A}_i(q_1, q_2 | r_2, r_1) + \kappa_i^{(4)} \mathcal{A}_i(q_1, q_2 | r_1, r_2). \tag{22}$$

In the ND–ND configuration we have the same formula with  $f^{a_2 a_1 c} \rightarrow f^{a_2 b_1 c}$ .

Simple calculations give for the D–ND configuration

$$\kappa_i^{(1)} = \kappa_i^{(2)} = -\kappa_i^{(3)} = -\kappa_i^{(4)}$$

and

$$\kappa_1^{(1)} = \kappa_4^{(1)} = -\frac{1}{2}, \quad \kappa_2^{(1)} = -\kappa_3^{(1)} = 1, \quad \kappa_5^{(1)} = \frac{1}{2}.$$

We recall that for  $i = 5$  in (22) only the first two terms are to be taken into account.

For the ND–ND configuration, the coefficients  $\kappa_i^{(k)}$  are given in Table 1.

The other coefficients are defined through  $\kappa_1$  and  $\kappa_2$  according to the relations

$$\kappa_3 = -\kappa_2, \quad \kappa_4 = \kappa_1, \quad \kappa_5 = \kappa_1 + \kappa_2.$$

**Table 1**  $\kappa_i^{(k)}$  for ND–ND configuration

$k$	1	2	3	4
$\kappa_1$	1/2	0	1	1
$\kappa_2$	-1	0	-1	-1/2

### 3 Transversality

#### 3.1 Amplitudes $\mathcal{A}_i, i = 1, \dots, 5$

The obtained expressions for the RR  $\rightarrow$  RRP amplitudes are rather cumbersome. A simple method to check transversality is just to calculate the product  $(p\mathcal{A})$  numerically. The corresponding calculations by a FORTRAN program in both D-ND and ND-ND configurations show that the constructed vertex is indeed transversal. Nevertheless it is instructive to see this fact analytically.

We consider subsequently the five amplitudes  $\mathcal{A}_i, i = 1, \dots, 5$ , corresponding to contributions (4), (8), (13), (14) and (18), respectively.

##### 1. $\mathcal{A}_1$

Using the orthogonality of  $\bar{V}$  we find

$$\begin{aligned} X_1 &= (p\mathcal{A}_1) = g^3 C_1 \frac{1}{t_1^2} (\bar{A}t_+ - \bar{B}q_{1+} - \bar{C}q_{2+} + \bar{E}) \\ &= g^3 C_1 \frac{1}{t_1^2} Z_1, \end{aligned} \tag{23}$$

where  $t_1 = q_1 - r_1$  and  $\bar{A}, \dots, \bar{E}$  are given by (2). We find

$$\begin{aligned} Z_1 &= -3r_{1-}(q_{1+} + q_{2+}) + (a_1, t + q_2) + q_1^2 + q_2^2 \\ &\quad + 2r_1^2 - t_1^2 + r_1^2 \frac{q_{2+}}{q_{1+}} - \frac{q_2^2 r_1^2}{q_{1+r_{1-}}}, \end{aligned} \tag{24}$$

where  $a_1 = q_1 + r_1$ .

##### 2. $\mathcal{A}_2$

Similarly to (23) we write

$$\begin{aligned} X_2 &= (p\mathcal{A}^{(2)}) = g^3 C_2 \frac{1}{t_1^2} (At_- + Br_{1-} + Cr_{2-} + E) \\ &= g^3 C_2 \frac{1}{t_1^2} Z_2. \end{aligned} \tag{25}$$

The explicit expressions for  $A, \dots, E$  are given in (6) We find

$$\begin{aligned} Z_2 &= -3q_{1+}(r_{1-} + r_{2-}) - (a_1, \bar{t} - r_2) + r_1^2 + r_2^2 \\ &\quad + 2q_1^2 - t_1^2 + q_1^2 \frac{r_{2-}}{r_{1-}} - \frac{q_1^2 r_2^2}{q_{1+r_{1-}}}. \end{aligned} \tag{26}$$

##### 3. $\mathcal{A}_3$

We have

$$\begin{aligned} p^\mu Z_{\mu\nu}^{(3)} &= i f^{a_2 c e} f^{b_2 d e} \times \left[ n_\nu^+ \left( 2p_+ - \frac{q^2}{r_{2-}} \right) \right. \\ &\quad \left. - n_\nu^- \left( p_- + \frac{r_2^2 p_+}{t_{1+q_{2+}}} \right) - p_\nu \right]. \end{aligned}$$

This has to be multiplied by  $L_{1\nu}$  given by (10). We get

$$X_3 = (p\mathcal{A}^{(3)}) = g^3 C_3 \frac{1}{t_1^2} Z_3.$$

Calculations give

$$\begin{aligned} Z_3 &= 3q_{1+}p_- - (a_1 p) + r_1^2 + r_2^2 + 2q_1^2 + 2q_1^2 \frac{r_{2-}}{r_{1-}} \\ &\quad + q_2^2 \frac{r_{1-}}{r_{2-}} + r_1^2 \frac{q_{2+}}{q_{1+}} + r_2^2 \frac{q_{1+}}{q_{2+}} - \frac{q_2^2 r_1^2}{q_{1+r_{2-}}} - \frac{q_1^2 r_2^2}{q_{1+r_{1-}}} \\ &\quad - \frac{q_1^2 r_2^2}{q_{2+r_{1-}}}. \end{aligned} \tag{27}$$

##### 4. $\mathcal{A}_4$

We have

$$\begin{aligned} p^\mu Z_{\mu\nu}^{(4)} &= i f^{a_2 d e} f^{b_2 d e} \times \left[ n_\nu^+ \left( 2p_- + \frac{r^2}{q_{2+}} \right) \right. \\ &\quad \left. - n_\nu^- \left( p_+ - \frac{q_2^2 p_-}{t_{1-r_{2-}}} \right) - p_\nu \right]. \end{aligned} \tag{28}$$

This again has to be multiplied by (10). We get

$$X_4 = (p\mathcal{A}^{(4)}) = g^3 C_4 \frac{1}{t_1^2} Z_4,$$

where after simple calculations

$$\begin{aligned} Z_4 &= 3r_{1-}p_+ - (a_1 p) - q_1^2 - q_2^2 - 2r_1^2 \\ &\quad - 2r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} - q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} \\ &\quad + \frac{q_1^2 r_2^2}{q_{2+r_{1-}}} + \frac{q_2^2 r_1^2}{q_{1+r_{1-}}} + \frac{q_2^2 r_1^2}{q_{1+r_{2-}}}. \end{aligned} \tag{29}$$

##### 5. $\mathcal{A}_5$

We again write

$$X_5 = (p\mathcal{A}^{(4)}) = -C_5 \frac{1}{t_1^2} Z_5 \tag{30}$$

and find

$$\begin{aligned} Z_5 &= (L_1 L_2) = (a_1 + b_1 n^+ + c_1 n^-, a_2 + b_2 n^+ + c_2 n^-) \\ &= (a_1 a_2) + b_1 a_{2+} + b_2 a_{1+} + c_1 a_{2-} + c_2 a_{1-} \\ &\quad + b_1 c_2 + b_2 c_1. \end{aligned}$$

Note that this is not the total contribution from  $\mathcal{A}^{(5)}$  but only half of it containing  $1/t_1^2$ . The other half contains  $-1/t_2^2$  and so has a different structure from the other amplitudes. It has to be taken into account in amplitudes with  $1 \leftrightarrow 2$ . Also note the sign “-” in (30). Calculations give

$$Z_5 = (a_1 a_2) - q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} - r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} + \frac{q_1^2 r_2^2}{q_{2+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}}. \tag{31}$$

6.  $\mathcal{A}^{\text{tot}}$

Summing our contributions we find

$$X^{\text{tot}} = (p \mathcal{A}^{\text{tot}}) = g^3 \frac{1}{t_1^2} \left( \sum_{i=1}^4 C_i Z_i - C_5 Z_5 \right) = g^3 \frac{1}{t_1^2} \left[ C_1 (Z_1 + Z_4 - Z_5) + C_2 (Z_2 - Z_3 - Z_5) \right]. \tag{32}$$

3.2  $Z_1 + Z_4 - Z_5$

Since our expressions are long enough we separately consider terms with double poles at  $q_{i+} = 0$  and  $r_{i-} = 0, i = 1, 2$ , simple poles and non-singular terms.

Suppressing the common factor  $g^3/t_1^2$ , the double pole contribution is

$$Z_{dp}^{(1+4-5)} = \left( -\frac{q_2^2 r_1^2}{q_{1+} r_{1-}} \right) + \left( \frac{q_2^2 r_1^2}{q_{1+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}} + \frac{q_1^2 r_2^2}{q_{1+} r_{2-}} \right) - \left( \frac{q_1^2 r_2^2}{q_{2+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}} \right) = 0,$$

where the three terms in the brackets correspond to contributions from  $Z_1, Z_4$  and  $Z_5$ . Obviously they cancel.

The single pole contribution is

$$Z_p^{(1+4-5)} = \left( r_1^2 \frac{q_{2+}}{q_{1+}} \right) + \left( -2r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} - q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} \right) - \left( -q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} - r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} \right) = 0.$$

Again the three terms correspond to contributions from  $Z_1, Z_4, Z_5$ .

The three terms with the non-singular contributions are

$$Z_{ns}^{(1+4-5)} = (-3r_{1-}(q_{1+} + q_{2+}) + (a_1, q_1 + 2q_2 - r_1) + q_1^2 + q_2^2 + 2r_1^2 - t_1^2) + (3r_{1-}(q_{1+} + q_{2+}) - (a_1, q_1 + q_2 - r_1 - r_2) - q_1^2 - q_2^2 - 2r_1^2) - (a_1 a_2).$$

We find for the vectors, multiplying  $a_1$  in the first two terms,

$$q_1 + 2q_2 - r_1 - q_1 - q_2 + r_1 + r_2 = a_2.$$

So in the sum all terms cancel except if containing  $t_1^2$ :  $Z_{ns}^{(1+4-5)} = -t_1^2$ . So restoring the suppressed factor we have  $X^{(1+4-5)} = -g^3 C_1$ . This means that the sum of all diagrams considered above is not transversal by itself. Violation of transversality comes from the contribution  $\mathcal{A}_1$ .

3.3  $Z_2 - Z_3 - Z_5$

In a similar fashion here we find the double pole contribution

$$Z_{dp}^{(2+3-5)} = - \left( \frac{q_1^2 r_2^2}{q_{1+} r_{1-}} \right) + \left( \frac{q_2^2 r_1^2}{q_{1+} r_{2-}} + \frac{q_1^2 r_2^2}{q_{1+} r_{1-}} + \frac{q_1^2 r_2^2}{q_{2+} r_{1-}} \right) - \left( \frac{q_1^2 r_2^2}{q_{2+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}} \right) = 0.$$

The three terms in the brackets correspond to contributions from  $Z_2, Z_3$  and  $Z_5$ . They cancel in the sum.

The single pole contribution is

$$Z_p^{(2-3-5)} = + \left( q_1^2 \frac{r_{2-}}{r_{1-}} \right) - \left( 2q_1^2 \frac{r_{2-}}{r_{1-}} + q_2^2 \frac{r_{1-}}{r_{2-}} + r_1^2 \frac{q_{2+}}{q_{1+}} + r_2^2 \frac{q_{1+}}{q_{2+}} \right) - \left( -q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} - r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} \right).$$

They also give zero in the sum.

Finally, the non-singular contribution is

$$Z_{ns}^{(2-3-5)} = +(-3q_{1+}(r_{1-} + r_{2-}) - (a_1, q_1 - r_1 - 2r_2) + r_1^2 + r_2^2 + 2q_1^2 - t_1^2) - (-3q_{1+}(r_{1-} + r_{2-}) - (a_1, q_1 + q_2 - r_1 - r_2) + r_1^2 + r_2^2 + 2q_1^2) - (a_1 a_2).$$

We find for the vectors, multiplying  $a_1$  in the first two terms,

$$-(q_1 - r_1 - 2r_2 - q_1 - q_2 + r_1 + r_2) = a_2.$$

In the sum all terms cancel again except if containing  $t_1^2$ :  $Z_{ns}^{(2-3-5)} = -t_1^2$ . So restoring the factor in front we have  $X^{(2-3-5)} = -g^3 C_2$ . Again the sum of all diagrams considered above is not transversal by itself. In this part violation of transversality comes from contributions  $\mathcal{A}_2$ .

For the sum of all diagrams with fixed reggeon momenta we find

$$X^{\text{tot}} = -g^3 (C_1 + C_2) = -g^3 C_5. \tag{33}$$

This expression is antisymmetric under the interchange  $(a_2, a_1|b_2, b_1) \leftrightarrow (a_1, a_2|b_1, b_2)$  and does not depend on the momenta of the four reggeons. So it will vanish after symmetrisation in the sum  $\mathcal{A}(2, 1|2, 1) + \mathcal{A}(1, 2|1, 2)$ . Thus after symmetrisation we find the transversality of the constructed  $RR \rightarrow RRP$  vertex.

### 4 On-mass-shell amplitudes

On the mass shell, at  $p^2 = 0$ , the physical amplitudes can be written via the physical polarisation vector  $\epsilon_\mu$ , which we choose with the properties

$$(p\epsilon) = (l\epsilon) = 0, \quad \epsilon_+ = 0, \quad \epsilon_- = -\frac{(p\epsilon)_\perp}{p_+}. \tag{34}$$

So the product with any vector  $v$  is

$$(v\epsilon) = (v\epsilon)_\perp - \frac{v_+}{p_+}(p\epsilon)_\perp. \tag{35}$$

The amplitudes  $\mathcal{A}_i, i = 1, \dots, 5$  take the following form on the mass shell multiplied by the polarisation vector  $\epsilon$ :

1.  $\mathcal{A}_1$

Obviously it is sufficient to transform our coefficients  $a, b, \dots, e$ . We have

$$a_\epsilon \equiv (a\epsilon) = 0, \\ b_\epsilon \equiv (b\epsilon) = 2p_+(q_1\epsilon) + q_{1+}(p - 2t, \epsilon).$$

Since  $p - 2t = -p - 2r_2, (p - 2t, \epsilon) = -2(r_2\epsilon)$ , which gives

$$b_\epsilon = 2p_+ \left( (q_1\epsilon)_\perp - \frac{q_{1+}}{p_+}(p\epsilon)_\perp \right) - 2q_{1+}(r_2\epsilon)_\perp \\ = 2p_+(q_1\epsilon)_\perp - 2q_{1+}(p + r_2, \epsilon)_\perp.$$

Similarly,

$$c_\epsilon = 2p_+ \left( (q_2\epsilon)_\perp - \frac{q_{2+}}{p_+}(p\epsilon)_\perp \right) - 2q_{2+}(r_2\epsilon)_\perp \\ = 2p_+(q_2\epsilon)_\perp - 2q_{2+}(p + r_2, \epsilon)_\perp$$

and finally

$$e_\epsilon = -2(r_2\epsilon)_\perp - 2(p\epsilon)_\perp.$$

2.  $\mathcal{A}_2$

Again we transform coefficients  $\bar{a}, \dots, \bar{e}$ . We have

$$\bar{a}_\epsilon = (p\epsilon)_\perp \frac{t^2}{p_+}, \\ \bar{b}_\epsilon = 2p_-(r_1\epsilon) + r_{1-}(p - 2t, \epsilon) + \frac{(p\epsilon)_\perp}{p_+} \\ \times \left( 2(pr_1) + q_2^2 \frac{r_{1-}}{p_-} \right),$$

where  $t = p - q_2$ , so that  $(p - 2t, \epsilon) = 2(q_2\epsilon)$ . We find

$$\bar{b}_\epsilon = 2p_-(r_1\epsilon)_\perp + 2r_{1-} \left( (q_2\epsilon)_\perp - (p\epsilon)_\perp \frac{q_{2+}}{p_+} \right) \\ + 2(p\epsilon)_\perp \left( r_{1-} - r_{1-} \frac{q_2^2}{p_\perp^2} + \frac{(pr_1)_\perp}{p_+} \right).$$

Similarly

$$\bar{c}_\epsilon = 2p_-(r_2\epsilon)_\perp + 2r_{2-} \left( (q_2\epsilon)_\perp - (p\epsilon)_\perp \frac{q_{2+}}{p_+} \right) \\ + 2(p\epsilon)_\perp \left( r_{2-} - r_{2-} \frac{q_2^2}{p_\perp^2} + \frac{(pr_2)_\perp}{p_+} \right).$$

Finally

$$\bar{e}_\epsilon = 2(q_2\epsilon)_\perp + 2(p\epsilon)_\perp \left( 1 - \frac{q_{2+}}{p_+} - \frac{q_2^2}{p_\perp^2} \right).$$

3.  $\mathcal{A}_3$

We write

$$\mathcal{A}_{3\epsilon} = g^3 C_3 \frac{1}{t_1^2} B_3, \tag{36}$$

where

$$B_3 = 2(p\epsilon)_\perp \frac{a_{1+}}{p_+} - (a_1\epsilon)_\perp + 2 \frac{(p\epsilon)_\perp}{p_+} \left( \frac{q_1^2}{r_{1-}} - 2q_{1+} \right) \\ - 2 \frac{(p\epsilon)_\perp q_2^2}{p_\perp^2} \left( \frac{r_1^2}{q_{1+}} - r_{1-} \right).$$

Upon transforming we find

$$B_3 = -(a_1\epsilon)_\perp + 2(p\epsilon)_\perp \\ \times \left[ -\frac{q_{1+}}{p_+} + \frac{q_1^2}{p_+ r_{1-}} - \frac{q_2^2}{p_\perp^2 r_{2-}} \left( \frac{r_1^2}{q_{1+}} - r_{1-} \right) \right],$$

where  $a_1 = q_1 + r_1$

4.  $\mathcal{A}_4$

We write

$$\mathcal{A}_{4\epsilon} = g^3 C_4 \frac{1}{t_1^2} B_4, \tag{37}$$

where



$$B_4 = -(p\epsilon)_\perp \frac{q_{1+}}{p_+} - (a_1\epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+} \left( \frac{q_1^2}{r_{1-}} - 2q_{1+} \right) + \frac{(p\epsilon)_\perp q_2^2}{p_+ r_{1-} r_{2-}} \left( \frac{r_1^2}{q_{1+}} - r_{1-} \right).$$

Again transforming we obtain final expression

$$B_4 = -(a_1\epsilon)_\perp + \frac{(p\epsilon)_\perp}{p_+} \left( q_{1+} - \frac{q_1^2}{r_{1-}} - \frac{q_2^2}{r_{2-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{1-} r_{2-}} \right).$$

5.  $\mathcal{A}_5$

Multiplying by  $\epsilon$  we find

$$L_{1\epsilon} = (L_1\epsilon) = (a_1\epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+} \left( \frac{q_1^2}{r_{1-}} - q_{1+} \right),$$

$$L_{2\epsilon} = (L_2\epsilon) = (a_2\epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+} \left( \frac{q_2^2}{r_{2-}} - q_{2+} \right)$$

and finally

$$(t_1 - t_2)_\epsilon = (t_1 - t_2, \epsilon) = (t_1 - t_2, \epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+} (q_{1+} - q_{2+}).$$

The product  $(\mathcal{A}_5\epsilon)$  will be given by Eq. (18) with vectors substituted by their products with  $\epsilon$  given above.

5 Asymptotics for large  $q_{1+}$  or  $r_{1-}$  with fixed  $p$

For applications the behaviour of the vertex at large values of longitudinal momenta has utter importance, since one has to integrate over them when the vertex is inserted into the amplitude. The necessary condition for the possibility of this integration is that the amplitude should vanish at high values of longitudinal momenta. Note that in the inclusive cross section momentum  $p$  of the observed gluon is fixed. This means that the sums  $q_{1+} + q_{2+}$  and  $r_{1-} + r_{2-}$  remain finite when one of the longitudinal momenta tends to infinity. Having in mind that in the D-ND configuration upper and lower reggeons enter in a different manner we have to study separately the cases of  $q_{1+} \rightarrow \infty$  and  $r_{1-} \rightarrow \infty$ .

5.1  $q_{1+} \rightarrow \infty, p$  fixed

We consider subsequently our amplitudes  $\mathcal{A}_i, i = 1, \dots, 5$ .

1.  $\mathcal{A}_1$

Here  $t = q_1 + q_2 - r_1$ . Since  $q_2 = p - q_1, t$  is finite. Of the two denominators one is finite; the other grows as  $q_{1+}$ . So non-vanishing terms come from the ones in the numerator which grow as  $q_{1+}$  or faster. Inspecting the coefficients  $a, b, c, e$  we conclude:  $a$  and  $e$  are finite,

$$b = 2p_+ q_1 + q_{1+}(p - 2t) - q_{1+} n^+ \left( 2p_- + \frac{r_2^2}{p_+} \right),$$

$$c = 2p_+ q_2 + q_{2+}(p - 2t) - q_{2+} n^+ \left( 2p_- + \frac{r_2^2}{p_+} \right).$$

Turning to  $\bar{A}, \dots, \bar{E}$  we find that  $\bar{A}, \bar{B}$  are finite with  $\bar{B} = \bar{C}$ . So the contribution  $a\bar{A} - b\bar{B} - c\bar{C}$  is finite. We are left with only  $e\bar{E}$ .

At large  $q_{1+}, \bar{E} = \bar{E}_0 - 2q_{2+} r_{1-}$ , where

$$\bar{E}_0 = (r_1 + q_1, t + q_2) + q_2^2 - q_1^2 + (q_1 - r_1)^2 + 2r_{1-} q_{1+} = r_{1-} q_{2+} - r_{1-} q_{1+}.$$

We find  $\bar{E} = -r_{1-}(q_{1+} + q_{2+})$  and it is finite. So there are no growing terms in the numerator and, in the limit  $q_{1+} \rightarrow \infty, \mathcal{A}_1 = 0$ .

2.  $\mathcal{A}_2$

Here  $\bar{t} = q_1 - r_1 - r_2$  and it grows as  $q_{1+}$ . The two denominators both grow as  $q_{1+}$ . Possible terms non-vanishing at  $q_{1+} \rightarrow \infty$  may come from the ones in the numerator growing as  $q_{1+}^2$  or faster.

The coefficients  $\bar{a}, \dots, \bar{e}$  are in this limit

$$\bar{a} = n^- \bar{t}^2, \quad \bar{b} = -2\bar{t} r_{1-}, \quad \bar{c} = -3\bar{t} r_{2-}, \quad \bar{e} = -2\bar{t}.$$

The terms  $A, \dots, E$  are

$$A = 3q_{1+}, \quad B = C = 4q_{1+}, \quad E = E_0 + 2q_{1+}(\bar{t} - r_{2-}),$$

where

$$E_0 = -(q_1 + r_1, \bar{t} - r_2) + r_2^2 - r_1^2 + (q_1 - r_1)^2 + 2q_{1+} r_{1-} = -q_{1+}(\bar{t} - r_2) - q_{1+} r_{1-}.$$

So

$$E = q_{1+}(\bar{t} - r_{2-} - r_{1-}) = -2q_{1+}(r_{1-} + r_{2-}).$$

Thus

$$\bar{a}A + \bar{b}B + \bar{c}C + \bar{e}E = 6q_{1+}^2(r_{1-} + r_{2-}) - 2q_{1+}^2 r_{1-} - 2q_{1+}^2 r_{2-} + 4q_{1+}^2(r_{1-} + r_{2-}) = 2q_{1+}(r_{1-} + r_{2-}).$$

The denominator is  $\bar{r}^2 t_1^2 = 4q_{1+}^2 r_{1-}(r_{1-} + r_{2-})$ . Thus, in the limit  $q_{1+} \rightarrow \infty$ ,

$$A_{2+} = -g^3 C_2 \frac{1}{2r_{1-}}. \tag{38}$$

3.  $A_3$

In the limit  $q_{1+} \rightarrow \infty$  the expression in the square brackets in (13) is  $-a_+ n^- - a + 4n^- q_{1+}$ . So the growing “+” component is just  $2q_{1+}$  and, in the limit,

$$A_{3+} = -g^3 C_3 \frac{1}{r_{1-}}. \tag{39}$$

4.  $A_4$

In the limit  $q_{1+} \rightarrow \infty$  the expression in the square brackets in (14) is  $2q_{1+} n^- - a + n^- q_{1+}$ . So the growing “+” component is just  $-q_{1+}$ , and in this limit

$$A_{4+} = g^3 C_4 \frac{1}{2r_{1-}}. \tag{40}$$

5.  $A_5$

The two denominators grow as  $q_{1+}$  each. So we have to search for terms in the numerator which grow as  $q_{1+}^2$  or faster. We have

$$\begin{aligned} (t_2 L_1) &= (q_2 - r_2, q_1 + r_1) + b_1 q_{2+} - c_1 r_{2-} \\ &= q_{2+} r_{1-} - q_{1+} r_{2-} - 2q_{2+} r_{1-} + 2q_{1+} r_{2-} \\ &= q_{1+} r_{2-} - q_{2+} r_{1-}. \end{aligned}$$

We also have, in the limit  $q_{1+} \rightarrow \infty$ ,  $L^{(2)} = a_2 - 2n^- q_{1+}$ , so that the growing “+” component is  $L_{2+} = -q_{1+}$ . As a result

$$\begin{aligned} L_{2+}(t_2 L_1) - (1 \leftrightarrow 2) &= q_{1+}(q_{2+} r_{1-} - q_{1+} r_{2-}) - q_{2+}(q_{1+} r_{2-} - q_{2+} r_{1-}) \\ &= q_{1+}(q_{2+} r_{1-} - q_{1+} r_{2-} + q_{1+} r_{2-} - q_{2+} r_{1-}) = 0. \end{aligned}$$

So the terms growing with  $q_{1+}$  come from the third term in (18). We find, in the limit  $q_{1+} \rightarrow \infty$ ,

$$\begin{aligned} (L_1 L_2) &= (a_1 a_2) + b_1 a_{2+} + b_2 a_{1+} + c_1 a_{2-} + c_2 a_{1-} \\ &\quad + b_1 c_2 + b_2 c_1 \\ &= q_{1+} r_{2-} + q_{2+} r_{1-} - 2q_{1+} r_{2-} - 2q_{2+} r_{1-} \\ &\quad - 2q_{1+} r_{2-} - 2q_{2+} r_{1-} + 4q_{1+} r_{2-} + 4q_{2+} r_{1-} \\ &= q_{1+} r_{2-} + q_{2+} r_{1-}. \end{aligned}$$

So we find the “+” component,

$$\begin{aligned} (t_{1+} - t_{2+})(L^{(1)} L^{(2)}) &= 2q_{1+}(q_{1+} r_{2-} + q_{2+} r_{1-}) \\ &= 2q_{1+}^2 (r_{2-} - r_{1-}), \end{aligned}$$

and thus

$$A_{5+} = g^3 C_5 \left( \frac{1}{2r_{1-}} - \frac{1}{2r_{2-}} \right). \tag{41}$$

6. After addition of the symmetrised contributions we find for the “+” components suppressing the common factor  $g^3 f^{a_2 a_1 c}$  or  $g^3 f^{a_2 b_1 c}$  for the D–ND and ND–ND configurations, respectively:

$$\begin{aligned} A_2^{\text{tot}} &= -(\kappa_2^{(1)} - \kappa_2^{(3)}) \frac{1}{2r_{1-}} - (\kappa_2^{(2)} - \kappa_2^{(4)}) \frac{1}{2r_{2-}}, \\ A_3^{\text{tot}} &= -(\kappa_3^{(1)} - \kappa_3^{(3)}) \frac{1}{r_{1-}} - (\kappa_3^{(2)} - \kappa_3^{(4)}) \frac{1}{r_{2-}}, \\ A_4^{\text{tot}} &= (\kappa_4^{(1)} - \kappa_4^{(3)}) \frac{1}{2r_{1-}} + (\kappa_4^{(2)} - \kappa_4^{(4)}) \frac{1}{2r_{2-}}, \\ A_5^{\text{tot}} &= \frac{1}{2} (\kappa_5^{(1)} - \kappa_5^{(3)}) \left( \frac{1}{r_{1-}} - \frac{1}{r_{2-}} \right). \end{aligned}$$

For the D–ND configuration we have shown

$$\begin{aligned} \kappa_2^{(1)} = \kappa_2^{(3)} = -\kappa_2^{(2)} = -\kappa_2^{(4)} &= 1, \\ \kappa_3^{(1)} = \kappa_3^{(3)} = -\kappa_3^{(2)} = -\kappa_3^{(4)} &= -1, \\ \kappa_4^{(1)} = \kappa_4^{(3)} = -\kappa_4^{(2)} = -\kappa_4^{(4)} &= -\frac{1}{2}, \\ \kappa_5^{(1)} = \kappa_5^{(3)} = -\kappa_5^{(2)} = -\kappa_5^{(4)} &= \frac{1}{2}. \end{aligned}$$

So all contributions are zero. This means that each of the diagrams studied above separately behaves as  $1/q_{1+}$  as  $q_{1+} \rightarrow \infty$ .

For the ND–ND configuration, using  $\kappa_i^{(k)}$  from Table 1 we find

$$\begin{aligned} A_2^{\text{tot}} &= \left( \frac{1}{2} + \frac{1}{2} \right) \frac{1}{r_{1-}} + \frac{1}{4} \frac{1}{r_{2-}}, \\ &\quad \times A_3^{\text{tot}} = (-1 - 1) \frac{1}{r_{1-}} - \frac{1}{2} \frac{1}{r_{2-}}, \\ A_4^{\text{tot}} &= \left( \frac{1}{4} + \frac{1}{2} \right) \frac{1}{r_{1-}} + \frac{1}{2} \frac{1}{r_{2-}}, \quad A_5^{\text{tot}} = \frac{1}{4} \frac{1}{r_{1-}} - \frac{1}{4} \frac{1}{r_{2-}}. \end{aligned}$$

Subsequent terms on the right-hand side correspond to contributions from  $i = 1, 3$  and 4. We observe that in this case separate contributions do not vanish, in the limit  $q_{1+} \rightarrow \infty$ . However, the sum of them does vanish in this limit.

5.2  $r_{1-} \rightarrow \infty, p$  fixed

Again we subsequently study contributions  $\mathcal{A}_i, i = 1, \dots, 5$ .

1.  $\mathcal{A}_1$

In this case  $t = q_1 + q_2 - r_1$  grows and the two denominators each grow as  $r_{1-}$ . So we have to separate terms in the numerator growing as  $r_{1-}^2$ .

Coefficients  $a, \dots, e$  are at large  $t$

$$a = -n^+ t, \quad b = -2tq_{1+}, \quad c = -2tq_{2+}, \quad e = -2t.$$

The terms  $\bar{A}, \dots, \bar{E}$  are

$$\bar{A} = 3t_-, \quad \bar{B} = \bar{C} = 4t_-, \quad \bar{E} = \bar{E}_0 - 2r_{1-}(t_+ + q_{2+}),$$

where

$$\begin{aligned} \bar{E}_0 &= (r_1 + q_1, t + q_2) - (r_1 - q_1)^2 - 2r_{1-}q_{1+} \\ &= r_{1-}(t_+ + q_{2+}) - r_{1-}q_{1+}. \end{aligned}$$

As a result

$$\bar{E} = -r_{1-}(t_+ + q_{2+}) - r_{1-}q_{1+} = -2r_{1-}(q_{1+} + q_{2+}).$$

Thus the growing “-” component is

$$\begin{aligned} a_- \bar{A} - b_- \bar{B} - c_- \bar{C} + e_- \bar{E} \\ &= -3t_-^2 + 8t_-^2 q_{1+} + 8t_-^2 q_{2+} - 4t_-^2 (q_{1+} + q_{2+}) \\ &= -2t_-^2 (q_{1+} + q_{2+}). \end{aligned}$$

The denominator is  $4t_-^2 q_{1+}(q_{1+} + q_{2+})$ , so that finally

$$\mathcal{A}_{1-} = g^3 C_1 \frac{1}{2q_{1+}}. \tag{42}$$

2.  $\mathcal{A}_2$

Here  $\bar{t} = q_1 - r_1 - r_2$  and is finite. As a result the coefficients  $\bar{a}$  and  $\bar{e}$  are finite. For the rest we have

$$\begin{aligned} \bar{b} &= 2p_- r_1 + r_{1-}(p - 2\bar{t}) - n^- r_{1-} \left( 2p_+ + \frac{q_2^2}{p_-} \right), \\ \bar{c} &= 2p_- r_2 + r_{2-}(p - 2\bar{t}) - n^- r_{2-} \left( 2p_+ + \frac{q_2^2}{p_-} \right). \end{aligned}$$

The terms  $A, \dots, E$  are

$$A = 3q_{1+}, \quad B = C = 4q_{1+}, \quad E = E_0 + 2q_{1+}(\bar{t} - r_{2-}),$$

where

$$\begin{aligned} E_0 &= -(q_1 + r_1)(\bar{t} - r_2) - (q_1 - r_1)^2 - 2q_{1+}r_{1-} \\ &= -q_{1+}(\bar{t} - r_{2-}) - q_{1+}r_{1-}. \end{aligned}$$

So

$$E = q_{1+}(\bar{t} - r_{2-}) - q_{1+}r_{1-} = 2q_{1+}p_-$$

and is finite. We observe that all growing terms cancel and in the limit  $r_{1-} \rightarrow \infty \mathcal{A}_2 = 0$ .

3.  $\mathcal{A}_3$

In the limit  $r_{1-} \rightarrow \infty$  the expression in the square brackets in (13) is  $2r_{1-}n^+ - r_1 - 2n^+r_{1-}$ . So the growing “-” component is  $-r_{1-}$ , and in this limit

$$\mathcal{A}_{3-} = g^3 C_3 \frac{1}{2q_{1+}}. \tag{43}$$

4.  $\mathcal{A}_4$

In the limit  $r_{1-} \rightarrow \infty$  the expression in square brackets in (13) is  $-r_{1-}n^+ - a + 4n^+r_{1-}$ . So the growing “-” component is  $2r_{1-}$ , and in this limit

$$\mathcal{A}_{4-} = -g^3 C_4 \frac{1}{q_{1+}}. \tag{44}$$

5.  $\mathcal{A}_5$

The two denominators grow as  $r_{1-}$  each. So we have to search for terms in the numerators which grow as  $r_{1-}^2$  or faster. We have seen that

$$\begin{aligned} (t_2 L_1) &= (q_2 - r_2, q_1 + r_1) + b_1 q_{2+} - c_1 r_{2-} \\ &= q_{2+r_{1-}} - q_{1+r_{2-}} - 2q_{2+r_{1-}} + 2q_{1+r_{2-}} \\ &= q_{1+r_{2-}} - q_{2+r_{1-}}. \end{aligned}$$

We also have, in the limit  $r_{1-} \rightarrow \infty, L_2 = r_2 - 2n^+r_{1-}$ , so that the growing “-” component is  $L_+^{(2)} = -r_{1-}$ . As a result

$$\begin{aligned} L_{2-}(t_2 L_1) - (1 \leftrightarrow 2) \\ &= r_{1-}(q_{2+r_{1-}} - q_{1+r_{2-}}) - r_{2-}(q_{1+r_{2-}} - q_{2+r_{1-}}) \\ &= r_{1-}(q_{2+r_{1-}} - q_{1+r_{2-}} + q_{1+r_{2-}} - q_{2+r_{1-}}) = 0. \end{aligned}$$

So the terms growing with  $r_{1-}$  come again from the third term in (18). We find, in the limit  $r_{1-} \rightarrow \infty$ ,

$$\begin{aligned} (L_1 L_2) &= (a_1 a_2) + b_1 a_{2+} + b_2 a_{1+} + c_1 a_{2-} + c_2 a_{1-} \\ &\quad + b_1 c_2 + b_2 c_1 \\ &= q_{1+r_{2-}} + q_{2+r_{1-}} - 2q_{1+r_{2-}} - 2q_{2+r_{1-}} \\ &\quad - 2q_{1+r_{2-}} - 2q_{2+r_{1-}} + 4q_{1+r_{2-}} + 4q_{2+r_{1-}} \\ &= q_{1+r_{2-}} + q_{2+r_{1-}}. \end{aligned}$$

So we find the “-” component,

$$(t_{1-} - t_{2-})(L^{(1)}L^{(2)}) = -2r_{1-}(q_{1+}r_{2-} + q_{2+}r_{1-}) = 2r_{1-}^2(q_{1+} - q_{2+}),$$

and as a result

$$\mathcal{A}_-^{(5)} = g^3 C_5 \left( \frac{1}{2q_{1+}} - \frac{1}{8q_{2+}} \right). \tag{45}$$

6. After addition of the symmetrised contributions and suppressing the common factors as before we find for the “-” components

$$\begin{aligned} \mathcal{A}_1^{\text{tot}} &= (\kappa_1^{(1)} + \kappa_1^{(2)}) \frac{1}{2q_{1+}} + (\kappa_1^{(3)} + \kappa_1^{(4)}) \frac{1}{2q_{2+}}, \\ \mathcal{A}_3^{\text{tot}} &= (\kappa_3^{(1)} + \kappa_3^{(2)}) \frac{1}{2q_{1+}} + (\kappa_3^{(3)} + \kappa_3^{(4)}) \frac{1}{2q_{2+}}, \\ \mathcal{A}_4^{\text{tot}} &= - \left( (\kappa_4^{(1)} + \kappa_4^{(2)}) \frac{1}{q_{1+}} - (\kappa_4^{(3)} + \kappa_4^{(4)}) \frac{1}{q_{2+}} \right), \\ \mathcal{A}_5^{\text{tot}} &= \frac{1}{2} (\kappa_5^{(1)} + \kappa_5^{(2)}) \left( \frac{1}{q_{1+}} - \frac{1}{q_{2+}} \right). \end{aligned}$$

For the D-ND configuration

$$\begin{aligned} \kappa_1^{(1)} + \kappa_1^{(2)} &= -1, \kappa_3^{(1)} + \kappa_3^{(2)} = -2, \\ \kappa_4^{(1)} + \kappa_4^{(2)} &= -1, \kappa_5^{(1)} + \kappa_5^{(2)} = 1, \end{aligned}$$

and for all  $i$

$$\kappa_i^{(3)} + \kappa_i^{(4)} = -\kappa_i^{(1)} - \kappa_i^{(2)}.$$

So we get

$$\begin{aligned} \mathcal{A}_1^{\text{tot}} &= -\frac{1}{2} \left( \frac{1}{q_{1+}} - \frac{1}{q_{2+}} \right), \quad \mathcal{A}_3^{\text{tot}} = -\left( \frac{1}{q_{1+}} - \frac{1}{q_{2+}} \right), \\ \mathcal{A}_4^{\text{tot}} &= \left( \frac{1}{q_{1+}} - \frac{1}{q_{2+}} \right), \quad \mathcal{A}_5^{\text{tot}} = \frac{1}{2} \left( \frac{1}{q_{1+}} - \frac{1}{q_{2+}} \right). \end{aligned}$$

So unlike the limit  $q_{1+} \rightarrow \infty$ , in this configuration the individual contributions do not vanish, in the limit  $r_{1-} \rightarrow \infty$ . However, their sum vanishes.

For the ND-ND configuration we find using Table 1

$$\begin{aligned} \mathcal{A}_1^{\text{tot}} &= \frac{1}{2} \left( \frac{1}{2} \frac{1}{q_{1+}} + 2 \frac{1}{q_{2+}} \right), \\ &\times \mathcal{A}_3^{\text{tot}} = \frac{1}{2} = \left( \frac{1}{q_{1+}} + \frac{3}{2} \frac{1}{q_{2+}} \right), \\ \mathcal{A}_4^{\text{tot}} &= -\left( \frac{1}{2} \frac{1}{q_{1+}} + 2 \frac{1}{q_{2+}} \right), \\ &\times \mathcal{A}_5^{\text{tot}} = -\frac{1}{4} \left( \frac{1}{q_{1+}} - \frac{1}{q_{2+}} \right). \end{aligned}$$

The individual contributions do not vanish again. Their sum is

$$\begin{aligned} \mathcal{A}^{\text{tot}} &= \sum_{i=1}^5 \mathcal{A}_i^{\text{tot}} = \frac{1}{q_{1+}} \left( \frac{1}{4} + \frac{1}{2} - \frac{1}{2} - \frac{1}{4} \right) \\ &+ \frac{1}{q_{2+}} \left( 1 + \frac{3}{4} - 2 + \frac{1}{4} \right) = 0. \end{aligned}$$

So it vanishes at  $r_{1-} \rightarrow \infty$ .

Note that at  $r_{1-} \rightarrow \infty$  the leading contribution comes from the “-” component of the vertex. So one may expect a still better convergence for the amplitude on the mass shell multiplied by the polarisation vector  $\epsilon$  with a zero “+” component. Numerical calculations show that this is indeed so in the D-ND configuration due to cancellations in the sum with interchanged  $r_{1-}$  and  $r_{2-}$ . In this case, the vertex behaves as  $1/r_{1-}^2$  at  $r_{1-} \rightarrow \infty$ . However, the same numerical calculations show that this result does not hold for the ND-ND configuration, in which the amplitude behaves as  $1/r_{1-}$ .

### 6 Poles in longitudinal momenta

Here we present contributions with poles at  $q_{1+}, q_{2+}, r_{1-}, r_{2-} = 0$  coming from the induced vertices in the effective action formalism. In the case of gluon production in the collision of a single projectile on several targets, these poles cancel with the singularities coming from rescattering contributions [16–18]. In our case there is no rescattering and one might think that these poles cancel in the total amplitude after taking into account all permutations of the interacting reggeons. This possibility was advocated in [13] for the second order odderon kernel. However, we shall see that, in our case, pole singularities do not cancel and remain in the total production amplitude. For applications this means that one has to fix somehow the way to do longitudinal integrations in the presence of these poles. The requirement of the hermiticity of the effective action and the structure of the simple reggeon exchange prompt using integrations in the principal value sense.

Due to the complicated form of the production amplitudes the simplest way to see the existence of pole singularities at  $q_{1+}, q_{2+}, r_{1-}r_{2-} = 0$  is by a numerical check. It indeed shows that in both the N-ND and the ND-ND configurations the production amplitude contains pole singularities at each  $q_{1+}, q_{2+}, r_{1-}$  and  $r_{2-}$  equal to zero and also double pole singularities at, say,  $q_{1+} = r_{1-} = 0$ .

However, it is instructive to extract the pole contributions in an analytical form to see their character and understand why they cannot cancel. We shall subsequently consider pole contributions from the amplitudes  $\mathcal{A}_i$   $i = 1, \dots, 5$ .

1.  $\mathcal{A}_1$

The coefficients  $a, \dots, e$  are not singular. Singular terms in  $\bar{A}, \dots, \bar{E}$  are

$$\bar{A} = \frac{r_1^2}{q_{1+}}, \quad \bar{B} = 0, \quad \bar{C} = 2\frac{r_1^2}{q_{1+}},$$

$$\bar{E} = 2\frac{r_1^2 q_{2+}}{q_{1+}} - \frac{q_2^2 r_1^2}{q_{1+r_1-}}.$$

Here  $t = q_1 + q_2 - r_1$ . Writing the pole part of  $\mathcal{A}_1$  as

$$\mathcal{A}_1 = -g^3 C_1 \frac{1}{t^2 t_1^2} X_1, \tag{46}$$

we find after trivial calculations

$$X_{1\mu} = \frac{r_1^2}{q_{1+}} \left[ p_+(p_\mu - 4q_{2\mu}) + n_\mu^+(4(pq_2) - 4p - q_{2+} - t^2 - p^2) + 4p_+ q_{2+} n_\mu^- \right]$$

$$- \frac{q_2^2 r_1^2}{q_{1+r_1-}} \left[ (p - 2t)_\mu - n_\mu^+ \left( 2p_- + \frac{r_2^2}{p_+} \right) + 2p_+ n_\mu^- \right]. \tag{47}$$

On the mass shell, multiplied by the polarisation vector, it gives

$$X_{1\epsilon} = -4(q_2\epsilon)_\perp \frac{r_1^2 q_{2+}}{q_{1+}} + 2(p + r_2, \epsilon)_\perp \frac{q_2^2 r_1^2}{q_{1+r_1-}}. \tag{48}$$

2.  $\mathcal{A}_2$

The coefficients  $\bar{a}, \dots, \bar{e}$  are not singular. The singular terms in  $A, \dots, E$  are

$$A = -\frac{q_1^2}{r_{1-}}, \quad B = 0, \quad C = -2\frac{q_1^2}{r_{1-}},$$

$$E = 2r_{2-} \frac{q_1^2}{r_{1-}} - \frac{q_1^2 r_2^2}{q_{1+r_1-}}.$$

Here  $\bar{t} = q_1 - r_1 - r_2$ . We write

$$\mathcal{A}_2 = -g^3 C_2 \frac{1}{\bar{t}^2 t_1^2} X_2. \tag{49}$$

Calculations give

$$X_{2\mu} = \frac{q_1^2}{r_{1-}} \left[ -p_-(p_\mu + r_{2\mu}) + 4p_- r_{2-} n_\mu^+ + n_\mu^- (\bar{t}^2 + p^2 + 4(pr_2) - 4p_+ r_{2-}) - \frac{q_1^2 r_2^2}{q_{1+r_1-}} \right]$$

$$\times \left[ (p - 2\bar{t})_\mu + 2p_- n_\mu^+ - n_\mu^- \left( 2p_+ + \frac{q_2^2}{p_-} \right) \right]. \tag{50}$$

On the mass shell, multiplied by the polarisation vector,

$$X_{2\epsilon} = -\frac{q_1^2}{r_{1-}} \left[ (p\epsilon)_\perp \left( 2r_{2-} + \frac{2(p, 2r_2 - q_2)_\perp + q_2^2}{p_+} \right) + 4p_- (r_2\epsilon)_\perp \right]$$

$$- 2\frac{q_1^2 r_2^2}{q_{1+r_1-}} \left[ (q_2\epsilon)_\perp + (p\epsilon)_\perp \left( \frac{q_{1+}}{p_+} - \frac{q_2^2}{p_-^2} \right) \right]. \tag{51}$$

3.  $\mathcal{A}_3$

As before we write

$$\mathcal{A}_3 = g^3 C_3 \frac{1}{t_1^2} X_3. \tag{52}$$

We find

$$X_{3\mu} = n_\mu^+ \left( \frac{r_1^2}{q_{1+}} + \frac{r_1^2}{q_{2+}} - \frac{q_1^2 r_1^2}{q_{1+q_2+r_1-}} \right) + n_\mu^- \left( \frac{q_2^2 r_{1-}}{r_{2-} p_-} - 2\frac{q_1^2}{r_{1-}} - \frac{q_2^2 r_1^2}{q_{1+r_2-p_-}} \right). \tag{53}$$

On the mass shell, multiplied by the polarisation vector,

$$X_{3\epsilon} = 2(p\epsilon)_\perp \left( \frac{q_1^2}{p_+ r_{1-}} + \frac{q_2^2 r_{1-}}{r_{2-} p_-^2} - \frac{q_2^2 r_1^2}{q_{1+r_2-p_-^2}} \right). \tag{54}$$

4.  $\mathcal{A}_4$

Again we write

$$\mathcal{A}_4 = g^3 C_4 \frac{1}{t_1^2} X_4. \tag{55}$$

We find

$$X_{4\mu} = -n_{\mu}^{+} \left( 2 \frac{r_1^2}{q_{1+}} + \frac{r_1^2 q_{1+}}{p+q_{2+}} - \frac{q_1^2 r_1^2}{p+q_{2+}r_{1-}} \right) + n_{\mu}^{-} \left( \frac{q_1^2}{r_{1-}} + \frac{q_2^2}{r_{2-}} - \frac{q_2^2 r_1^2}{q_{1+r_{1-}r_{2-}} \right). \tag{56}$$

On the mass shell, multiplied by the polarisation vector,

$$X_{4\epsilon} = -\frac{(p\epsilon)_{\perp}}{p_{+}} \left( \frac{q_1^2}{r_{1-}} + \frac{q_2^2}{r_{2-}} - \frac{q_2^2 r_1^2}{q_{1+r_{1-}r_{2-}} \right). \tag{57}$$

5.  $\mathcal{A}_5$

Writing

$$\mathcal{A}_5 = g^3 C_5 \frac{1}{t_1^2 t_2^2} X_5, \tag{58}$$

we use Eq. (59). To simplify quite cumbersome expressions we from the start restrict ourselves to the mass-shell case and multiply  $\mathcal{A}_5$  by the polarisation vector. Singular terms are contained in  $A^{(i)}$ ,  $i = 1, 2, 3, 5$  (the term with  $A^{(4)}$  drops out in our gauge). We find

$$A^{(1)} = 2 \frac{q_2^2 r_{1-}}{r_{2-}} - 2 \frac{r_2^2 q_{1+}}{q_{2+}}, \quad A^{(2)} = -2 \frac{q_1^2 r_{2-}}{r_{1-}} + 2 \frac{r_1^2 q_{2+}}{q_{1+}},$$

$$A^{(3)} = -\frac{q_1^2 r_{2-}}{r_{1-}} - \frac{r_1^2 q_{2+}}{q_{1+}} - \frac{q_2^2 r_{1-}}{r_{2-}} - \frac{r_2^2 q_{1+}}{q_{2+}} + \frac{q_1^2 r_2^2}{q_{2+r_{1-}}} + \frac{q_2^2 r_1^2}{q_{1+r_{2-}}}$$

and the most complicated term

$$A^{(5)} = \frac{q_2^2}{r_{2-}} (a^{(1)}, p + t_2) + \frac{q_1^2 q_2^2}{r_{2-}} + \frac{q_2^2 r_1^2}{r_{2-}} + 4 \frac{q_1^2 q_{1+r_{2-}}}{r_{1-}} + 4 \frac{r_2^2 q_{1+}^2}{q_{2+}} + 4 \frac{q_1^2 q_{2+r_{2-}}}{r_{1-}} + 2 \frac{q_2^2 r_1^2 q_{2+}}{q_{1+r_{2-}}} - (1 \leftrightarrow 2),$$

where  $a^{(1)} = q_1 + r_1$  and  $a^{(2)} = q_2 + r_2$ . The coefficients are

$$(a^{(1)}\epsilon) = (a^{(1)}, \epsilon)_{\perp} - (p\epsilon)_{\perp} \frac{q_{1+}}{p_{+}},$$

$$(a^{(2)}\epsilon) = (a^{(2)}, \epsilon)_{\perp} - (p\epsilon)_{\perp} \frac{q_{2+}}{p_{+}},$$

$$(\tau^{(3)}\epsilon) = (q_1 - r_1 - q_2 + r_2, \epsilon)_{\perp} - (p\epsilon)_{\perp} \frac{q_{1+} - q_{2+}}{p_{+}}.$$

Collecting all terms we find

$$X_{5\epsilon} = (a^{(1)}\epsilon)_{\perp} A^{(1)} + (a^{(2)}\epsilon)_{\perp} A^{(2)} + (t_1 - t_2, \epsilon)_{\perp} A^{(3)} - \frac{(p\epsilon)_{\perp}}{p_{+}} A,$$

where

$$A = \frac{q_2^2}{r_{2-}} (a^{(1)}, p + t_2) + \frac{q_1^2 q_2^2}{r_{2-}} + 2 \frac{q_1^2 q_2^2}{r_{2-}} + 3 \frac{q_1^2 q_{1+r_{2-}}}{r_{1-}} + \frac{r_2^2 q_{1+}^2}{q_{2+}} + 3 \frac{q_1^2 q_{2+r_{2-}}}{r_{1-}} + \frac{q_2^2 r_1^2 q_{2+}}{q_{1+r_{2-}}} - (1 \leftrightarrow 2).$$

As we see each of the amplitudes  $\mathcal{A}_i$ ,  $i = 1, \dots, 5$ , contains both single poles in the longitudinal momenta and double poles in the longitudinal momenta of in-coming and out-going reggeons. Can they cancel in their sum together with terms obtained by the permutation of reggeons? The answer is negative, since different amplitudes contain different denominators, which, moreover, change with the permutation of the reggeons. These denominators depend on the transverse momenta and so have different values. Therefore, at least at fixed transverse momenta the pole singularities contain different (and varying) coefficients in the amplitudes  $\mathcal{A}_i$ ,  $i = 1, \dots, 5$ , and the amplitudes obtained from them by the permutations of reggeons. So unlike the case of a single projectile, in production amplitudes with two projectiles and targets the poles in the longitudinal momenta remain uncanceled, which requires a formulation of the way to do the longitudinal integrations. Integration in the principal value sense is an obvious choice.

7 Conclusions

We have derived the expression for the vertex  $RR \rightarrow RRP$  describing gluon production in interaction of two in-coming and two out-going reggeons. The vertex can be used for calculations of inclusive cross sections for gluon jet production in collision of a pair of projectile nucleons with a pair of target nucleons and also of the diffractive gluon jet production in deuteron–proton collisions. The vertex turns out to be quite complicated but amenable to further analytic and numerical calculations, which we postpone for future publications.

A few important properties of the obtained vertex have been demonstrated. The vertex is transversal in accordance with the gauge invariance. It vanishes when one of the longitudinal momenta goes to infinity, which allows one to subsequently do integrations over the longitudinal momenta in applications.

The vertex contains pole singularities at zero values of the longitudinal momenta inherited from intermediate induced vertices in the framework of effective action. In the spirit of this framework one should consider these poles in the prin-

cial value sense. Note that in contrast to gluon production on several centers by a single projectile, where rescattering effects cancel these poles, in the amplitudes containing the vertex  $RR \rightarrow RRP$ , like in Fig. 1, there are no additional rescattering contributions, so that the mentioned pole singularities are preserved in the amplitudes and should be taken into account in the longitudinal integrations.

Finally, again in contrast to the case of a single projectile [16–18], we find that the structure of the on-mass-shell vertex remains quite complicated and cannot be restored from the purely transverse picture, which is obtained by taking multiple cuts of the amplitude [19]. We believe that this is due to the fact that the amplitude possesses additional singularities, apart from the standard ones corresponding to physical intermediate gluons.

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### Appendix: An alternative form of the amplitude $\mathcal{A}_5$

Here we present a more explicit form for the amplitude  $\mathcal{A}_5$  in terms of coefficients in the two Lipatov vertices  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$ .

We rewrite the triple gluon vertex as

$$\Gamma_{\nu_1\mu,\nu_2}(t_1, p, t_2) = g f^{d_1c,d_2} \left( \tau_{\nu_2}^{(1)} g_{\mu\nu_1} + \tau_{\nu_1}^{(2)} g_{\mu\nu_2} + \tau_{\mu}^{(3)} g_{\nu_1\nu_2} \right),$$

where  $\tau^{(1)} = -t_1 - p, \tau^{(2)} = p + t_2, \tau^{(3)} = t_1 - t_2$ . This allows one to write the final vertex as

$$\mathcal{A}_{5\mu} = g^3 C_5 \frac{1}{4(t_1^2 + i0)(t_2^2 + i0)} \times \left( a_{1\mu} A^{(1)} + a_{2\mu} A^{(2)} + \tau_{\mu}^{(3)} A^{(3)} + n_{\mu}^+ A^{(4)} + n_{\mu}^- A^{(5)} \right), \tag{59}$$

where

$$\begin{aligned} A^{(1)} &= (a_2 \tau^{(1)} + b_2 \tau_+^{(1)} + c_2 \tau_-^{(1)}) \\ A^{(2)} &= (a_1 \tau^{(2)} + b_1 \tau_+^{(2)} + c_1 \tau_-^{(2)}) \\ A^{(3)} &= (a_2 a_1 + b_1 a_{2+} + c_1 a_{2-} + b_2 a_{1+} \\ &\quad + c_2 a_{1-} + b_2 c_1 + b_1 c_2) \\ A^{(4)} &= b_1 (a_2 \tau^{(1)} + b_2 (a_1 \tau^{(2)} + b_1 b_2 (\tau^{(1)} + \tau^{(2)})_+ \\ &\quad + c_2 b_1 \tau_-^{(1)} + b_2 c_1 \tau_-^{(2)}) \\ A^{(5)} &= c_1 (a_2 \tau^{(1)} + c_2 (a_1 \tau^{(2)} + c_1 c_2 (\tau^{(1)} + \tau^{(2)})_- \\ &\quad + c_2 b_1 \tau_+^{(2)} + b_2 c_1 \tau_+^{(1)}). \end{aligned}$$

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