Corrigendum


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By mistake, Eqs. (7)–(10) of Ref. [1] have been taken from an outdated manuscript version of Ref. [2] and not from the cited original publication. These four equations must be replaced by

\[ R(\alpha) \approx R_n \left| \cos\left(\alpha - \alpha_n^f\right)/2\right| \]  
(7)

with

\[ R_n = \text{Max}[R(\alpha), 2\pi n \leq \alpha \leq 2\pi(n + 1)]. \]
(8)

and

\[ R(\alpha) \approx R_{n_f} \left| \cos\left(\alpha - \alpha_{n_f}^f\right)/2(n_f + 1 - n_0)\right| \]  
(9)

with

\[ R_{n_f} = \text{Max}[R(\alpha), 2\pi n_0 \leq \alpha \leq 2\pi(n_f + 1)]. \]
(10)

The quantities \( \alpha_n^f \) and \( \alpha_{n_f}^f \) are the positions where \( R(\alpha) \) in Eqs. (7) and (9) have their maxima, respectively. In Fig. 1(a) the approximation based on these equations is compared with exact results obtained on the basis of Ref. [3]. The position of the spikes are now exactly reproduced. However, for small \( R \)'s there remain some deviations. In particular, for the lowest orbit the expression \( (\alpha_n^f)^{(1/2)} \) of [2, Eq. (13)] gets imaginary for \( R/r_0 < 0.246 \) and causes a little kink, see Fig. 1(a). This fact prompted Dubbers [2] replacing for \( R \leq 0.34 \) the quantity \( \alpha_n^f(R) \) by the approximation which reads corrected [4] \( \alpha_0[1 + (R/r_0)^2/(8\sin^2\alpha_0/2)]. \)

In addition, the statement in Ref. [2] that normalization is preserved could not be confirmed, even not with the corrected formulas. Numerical integration with an estimated accuracy in the order of \( 10^{-6} \) gave for the first three orbits deviations of +17.7\% (with the above approximation, +12.6\% without it), 
-2.07\%, -0.32\%,
-0.10\%,
-0.04\% from the associated solid angles 0.2042, 0.2653, 0.1326, 0.0796, 0.0531. For outer orbits the approximation of Dubbers is extremely good.

According to Dubbers [4], the approximations (7) and (9) can still be improved by adapting the width of the cosine functions appropriately, separately for the rising and falling branches, such that their zeros lie exactly at \( n_0, n_f + 1, n_f + 2, \ldots \) where they belong to. In this way, the above mentioned kink disappears completely.

Furthermore, in equations (13), (14) and (15) of Ref. [1] the expression \( \sqrt{1 - (R/R_0)^2} \) must be replaced by \( \sqrt{1 - (R/R_0)^2}. \)

The last paragraph of the 2. section [1] reads now: The result after summation over all \( n \), including \( n_f \),

\[ \frac{dP}{d(R/r_0)} = \sum_n dP_n/R(r_0) + dP_{n_f}/d(R/r_0) \]

(18)

is shown in Fig. 1(b). Still deviations from the exact results exist which are clearly visible for \( R/r_0 \lesssim 0.5 \). Therefore, a warning may be appropriate to employ even this improved mathematical approximation which preserves normalization when striving for high precision.

It should finally be mentioned that with Eq. (11) of Ref. [1], i.e.,

\[ \frac{dP_n(R)}{dR} = \frac{dP(\cos \theta_{ln})/d\cos \theta}{|R(\cos \theta_{ln})|} + \frac{dP(\cos \theta_{ln+1})/d\cos \theta}{|R(\cos \theta_{ln+1})|} \]

(1)

and Eq. (18) the exact mono-energetic point spread function (PSF) can be obtained which coincides with the black curves in Fig. 1. To
achieve this, the real zeros \( \cos \theta \vert_k \) and derivatives must directly be derived from the exact formula of Dubbers [2, Eq. (11)], i.e.,

\[
R(\alpha) = 2r_0 \sqrt{1 - \frac{a_0^2}{\alpha^2}} \left| \sin(1/2 \alpha) \right|
\]

(2)

with \( \alpha = z_0/(r_0 \cos \theta) \). The required derivatives in Eq. (1) read

\[
R'(\cos \theta \vert_k) = -\frac{z_0 (1 - \cos^2 \theta) \cos(\alpha/2) + 2r_0 \cos^3 \theta \sin(\alpha/2)}{\cos^2 \theta \sqrt{1 - \cos^2 \theta}} \bigg|_{\cos \theta = \cos \theta \vert_k}.
\]

(3)

The quantities \( \cos \theta \vert_k \) denote an infinite manifold of solutions of Eq. (2) for every preselected \( R \). With the described approach to find the invertible function of Eq. (2), approximations like Eqs. (7) and (9), or even more sophisticated ones, are not required. As a final remark, the treatment presented in Ref. [3] is completely equivalent to such an approach which follows Ref. [5, pp. 95–96].

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**References**


