Research Article

R^2 Corrections to the Jet Quenching Parameter

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A calculation of R^2 corrections to the jet quenching parameter from AdS/CFT correspondence is presented. It is shown that these corrections will increase or decrease the jet quenching parameter depending on the coefficients of the high curvature terms.

1. Introduction

The experiments of ultrarelativistic nucleus-nucleus collisions at RHIC and LHC have produced a strongly coupled quark-gluon plasma (sQGP). One of the interesting properties of sQGP is jet quenching phenomenon: due to the interaction with the medium, high energy partons traversing the medium are strongly quenched. This phenomenon is usually characterized by the jet quenching parameter (or transport coefficient) $\hat{q}$ which describes the average transverse momentum square transferred from the traversing parton, per unit mean free path [1, 2].

AdS/CFT duality can explore the strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) plasma through the correspondence between type IIB superstring theory formulated on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM in four dimensions [3–5]. Therefore some quantities, for instance, the jet quenching parameter, can be studied.

By using the AdS/CFT correspondences, Liu et al. [6] have calculated the jet quenching parameter for $N = 4$ SYM theory firstly. Interestingly the magnitude of $\hat{q}_{\text{SYM}}$ turns out to be closer to the value extracted from RHIC data [7, 8] than pQCD result for the typical value of the 't Hooft coupling, $\lambda \simeq 6\pi$, of QCD. This proposal has attracted lots of interest. Thus, after [6] there are many attempts to addressing jet quenching parameter [9–12]. However, in [6] the jet quenching parameter was studied only in infinite-coupling case. Therefore, it is very interesting to investigate the effect of finite-coupling corrections. The purpose of the present work is to study R^2 corrections to the jet quenching parameter.

The organization of this paper is as follows. In the next section, the result of jet quenching parameter from dual gauge theory is briefly reviewed [6]. In Section 3, we consider R^2 corrections to the jet quenching parameter. The last part is devoted to conclusion and discussion.

2. $\hat{q}$ from Dual Gauge Theory

The eikonal approximation relates the jet quenching parameter with the expectation value of an adjoint Wilson loop $W^A[\mathscr{C}]$ with $\mathscr{C}$ a rectangular contour of size $L \times L_-$, where the sides with length L_- run along the light-cone and the limit $L_- \rightarrow \infty$ is taken in the end. Under the dipole approximation, which is valid for small transverse separation L, the jet quenching parameter $\hat{q}$ defined in [1] is extracted from the asymptotic expression for $TL \ll 1$:

$$\left\langle W^A [\mathscr{C}] \right\rangle \approx \exp \left[-\frac{1}{4\sqrt{2}} \hat{q} L_- L^2 \right], \quad (1)$$

where $\langle W^A[\mathscr{C}] \rangle \approx \langle W^F[\mathscr{C}] \rangle^2$ with $\langle W^F[\mathscr{C}] \rangle$ the thermal expectation value in the fundamental representation.

Using AdS/CFT correspondence, we can calculate $\langle W^F[\mathscr{C}] \rangle$ according to

$$\left\langle W^F [\mathscr{C}] \right\rangle \approx \exp \left[-S_I \right], \quad (2)$$

where S_I is the regulated finite on-shell string worldsheet action whose boundary corresponds to the null-like rectangular loop $\mathscr{C}$.

By using such a strategy, they find that [6]

$$\hat{q}_{\text{SYM}} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)}\sqrt{\lambda}T^3 \approx 7.53\sqrt{\lambda}T^3, \quad (3)$$

where T is the temperature of $\mathcal{N} = 4$ SYM plasma.

3. R^2 Corrections to $\hat{q}_{\text{SYM}}$

We now consider the R^2 corrections to $\hat{q}_{\text{SYM}}$. Restricting to the gravity sector in AdS_5, the leading order higher derivative corrections can be written as [13]

$$\begin{aligned} S = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\,\big[R - 2\Lambda \\ + l^2\left(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\big], \end{aligned} \quad (4)$$

where α_i are arbitrary small coefficients and the negative cosmological constant Λ is related to the AdS space radius l by

$$\Lambda = -\frac{6}{l^2}. \quad (5)$$

The black brane solution of AdS_5 space with curvature-squared corrections is given by

$$ds^2 = -f(r)\,dt^2 + \frac{r^2}{l^2}d\vec{x}^2 + \frac{1}{f(r)}dr^2, \quad (6)$$

where

$$f(r) = \frac{r^2}{l^2}\left(1 - \frac{r_0^4}{r^4} + \alpha + \beta\frac{r_0^8}{r^8}\right), \quad (7)$$

with

$$\begin{aligned} \alpha &= \frac{2}{3}\left(10\alpha_1 + 2\alpha_2 + \alpha_3\right), \\ \beta &= 2\alpha_3, \end{aligned} \quad (8)$$

where r denotes the radial coordinate of the black brane geometry, $r = r_0$ is the horizon, and l relates the string tension $1/2\pi\alpha'$ to the 't Hooft coupling constant by $l^2/\alpha' = \sqrt{\lambda}$.

The temperature of $\mathcal{N} = 4$ SYM plasma with R^2 corrections is given by

$$T_1 = \frac{r_0}{\pi l^2}\left(1 + \frac{1}{4}\alpha - \frac{5}{4}\beta\right) = T\left(1 + \frac{1}{4}\alpha - \frac{5}{4}\beta\right). \quad (9)$$

In terms of the light-cone coordinate $x^\mu = (r, x^\pm, x_2, x_3)$, metric (6) becomes

$$\begin{aligned} ds^2 = &-\left[\frac{r^2}{l^2} + f(r)\right]dx^+dx^- + \frac{r^2}{l^2}\left(dx_2^2 + dx_3^2\right) \\ &+ \frac{1}{2}\left[\frac{r^2}{l^2} - f(r)\right]\left[\left(dx^+\right)^2 + \left(dx^-\right)^2\right] + \frac{dr^2}{f(r)}. \end{aligned} \quad (10)$$

The worldsheet in the bulk can be parameterized by τ and σ; then the Nambu-Goto action for the worldsheet becomes

$$S = \frac{1}{2\pi\alpha'}\int d\tau\, d\sigma\sqrt{\det g_{\alpha\beta}}. \quad (11)$$

We set the pair of quarks at $x_2 = \pm L/2$ and choose $x^- = \tau$, $x_2 = \sigma$. In this setup, we can ignore the effect of x^- dependence of the worldsheet, implying $x_3 = \text{const}$, $x^+ = \text{const}$; then the string action is given by

$$S = \frac{\sqrt{2}r_0^2 L_-}{2\pi\alpha' l^2}\int_0^{L/2} d\sigma\sqrt{1 + \frac{r'^2 l^2}{f r^2}}, \quad (12)$$

with $r' = \partial_\sigma r$.

The equation of motion for $r(\sigma)$ reads

$$r'^2 = \gamma^2\frac{r^2 f}{l^2}, \quad (13)$$

where γ is an integration constant. From (13), it is clear that the turning point occurs at $f = 0$, implying $r' = 0$ at the horizon $r = r_0$. Knowing that $r = r_0$ at $\sigma = 0$, then (13) can be integrated as

$$\begin{aligned} \frac{L}{2} &= \int_{r_0}^{\infty}\frac{l\,dr}{\gamma\sqrt{f}r} = \frac{l^2}{\gamma r_0}\int_1^{\infty}\frac{dt}{\sqrt{t^4 - 1 + \alpha t^4 + \beta/t^4}} \\ &= \frac{a l^2}{\gamma r_0}, \end{aligned} \quad (14)$$

where

$$a = \int_1^{\infty}\frac{dt}{\sqrt{t^4 - 1 + \alpha t^4 + \beta/t^4}}. \quad (15)$$

Then we can find the action for the heavy quark pair

$$S = \frac{\pi\sqrt{\lambda}L_- L T_1^2}{2\sqrt{2}}\sqrt{1 + \frac{4a^2}{\pi^2 T_1^2 L^2}}, \quad (16)$$

where we have used the relations $r_0 = \pi l^2 T$ and $l^2/\alpha' = \sqrt{\lambda}$.

This action needs to be subtracted by the self-energy of the two quarks, given by the parallel flat string worldsheets; that is,

$$S_0 = \frac{2L_-}{2\pi\alpha'}\int_{r_0}^{\infty} dr\sqrt{g_{--}g_{rr}} = \frac{a\sqrt{\lambda}L_- T_1}{\sqrt{2}}. \quad (17)$$

Therefore, the net on-shell action is given by

$$S_I = S - S_0 \approx \frac{\pi^2}{8\sqrt{2}a}\sqrt{\lambda}T_1^3 L_- L^2, \quad (18)$$

where we have used $T_1 L \ll 1$.

Note that $\hat{q}$ is related to $\langle W^A[\mathscr{C}]\rangle$ according to

$$\left\langle W^A[\mathscr{C}]\right\rangle \approx \exp\left[-\frac{1}{4\sqrt{2}}\hat{q}L_- L^2\right] \approx \exp\left[-2S_I\right]. \quad (19)$$

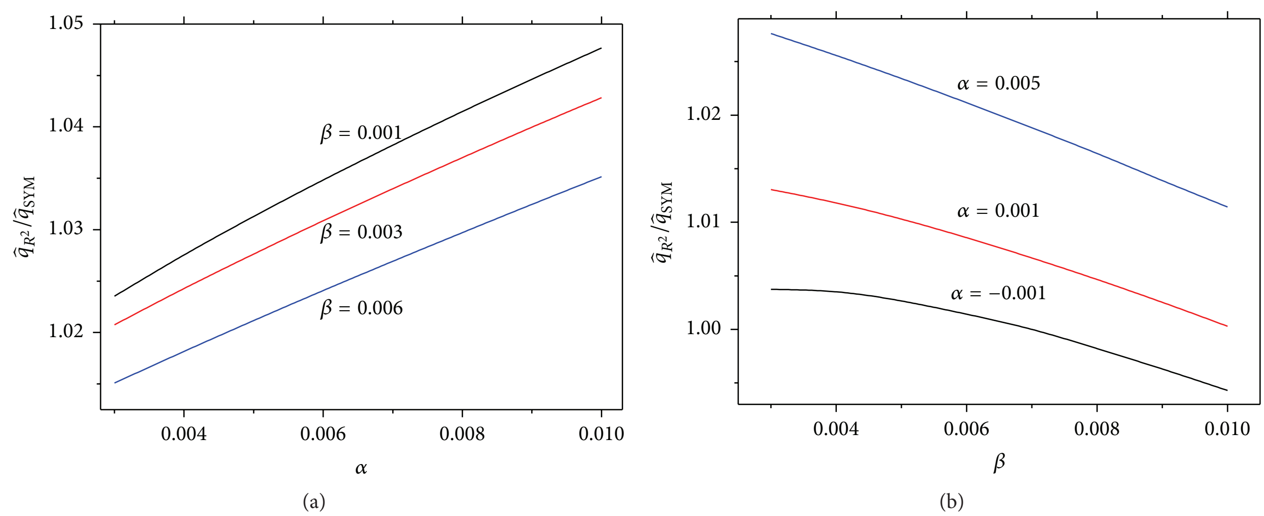

FIGURE 1: (a) $\hat{q}_{R^2}/\hat{q}_{SYM}$ versus α. From top to down $\beta = 0.001, 0.003, 0.006$. (b) $\hat{q}_{R^2}/\hat{q}_{SYM}$ versus β. From top to down $\alpha = 0.005, 0.001, -0.001$.

Plugging (18) into (19), we end up with jet quenching parameter under R^2 corrections:

$$\hat{q}_{R^2} = \frac{\pi^2}{a}\sqrt{\lambda}T_1^3 = \frac{(1+(1/4)\alpha-(5/4)\beta)^3\pi^2}{\int_1^\infty\left(dt/\sqrt{t^4-1+\alpha t^4+\beta/t^4}\right)}\sqrt{\lambda}T^3. \quad (20)$$

We now discuss our result. It is clear that the jet quenching parameter in (3) can be derived from (20) if we neglect the effect of curvature-squared corrections by plugging $\alpha = \beta = 0$ in (20). Furthermore, we compare the jet quenching parameter under R^2 corrections with its counterpart in the case of the dual gauge theory as follows:

$$\frac{\hat{q}_{R^2}}{\hat{q}_{SYM}} = \frac{\Gamma(5/4)\sqrt{\pi}}{\Gamma(3/4)}\frac{(1+(1/4)\alpha-(5/4)\beta)^3}{\int_1^\infty\left(dt/\sqrt{t^4-1+\alpha t^4+\beta/t^4}\right)}. \quad (21)$$

Notice that the result of (21) is related to α and β, or, in other words, it is dependent on α_i. However, the exact interval of α_i has not been determined; we only know that $\alpha_i \sim \alpha'/l^2 \ll 1$ [14]. In order to choose the values of α and β properly, we employ the Gauss-Bonnet gravity [15] which has nice properties that are absent for theories with more general ratios of the α_i's. The Gauss-Bonnet gravity is defined by the following action [16]:

$$S = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left[R - 2\Lambda + \frac{\lambda_{GB}}{2}l^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]. \quad (22)$$

Comparing (4) with (22), we find

$$\alpha_1 = \frac{\lambda_{GB}}{2}, \quad \alpha_2 = -2\lambda_{GB}, \quad \alpha_3 = \frac{\lambda_{GB}}{2}. \quad (23)$$

Plugging (23) into (8), we have

$$\alpha = \lambda_{GB}, \quad \beta = \lambda_{GB}, \quad (24)$$

where λ_{GB} can be constrained by causality and positive-definiteness of the boundary energy density [11]

$$-\frac{7}{36} < \lambda_{GB} \le \frac{9}{100}. \quad (25)$$

Moreover, there is an obstacle to the calculation of (21): we must require that the square root in the denominator be everywhere positive and the temperature in (9) be also positive. Under the above conditions, we find

$$-\frac{7}{36} < \alpha \le \frac{9}{100}, \quad 0 \le \beta \le \frac{9}{100}, \quad \alpha + \beta \ge 0, \quad (26)$$

and then we can discuss the results of (21) for different values of α and β in such a range.

As a concrete example, we plot $\hat{q}_{R^2}/\hat{q}_{SYM}$ versus α with different β and $\hat{q}_{R^2}/\hat{q}_{SYM}$ versus β with different α in Figure 1. In (a) from top to down, $\beta = 0.001, 0.003, 0.006$; $\hat{q}_{R^2}/\hat{q}_{SYM}$ increases at each β as α increases. While in (b) from top to down $\alpha = 0.005, 0.001, -0.001$, at each α; $\widehat{q}_{R^2}/\widehat{q}_{\mathrm{SYM}}$ is a decreasing function of β.

From the figure, it is clear that R^2 corrections can affect the jet quenching parameter. With some chosen values of α and β in this paper, the jet quenching parameter can be larger than or smaller than its counterpart in infinite-coupling case.

4. Conclusion and Discussion

In this paper, we have investigated R^2 corrections to the jet quenching parameter. These corrections are related to curvature-squared corrections in the corresponding gravity dual. It is shown that the corrections will increase or decrease the jet quenching parameter depending on the coefficients of the high curvature terms. Interestingly, under R^2 corrections, the drag force is also larger than or smaller than that in the infinite-coupling case [17]. However, we should admit that we cannot predict a result for $\mathcal{N} = 4$ SYM because the first higher derivative correction in weakly curved type II B backgrounds enters at order R^4 and not R^2.

Actually, there are some other important finite-coupling corrections to the jet quenching parameter. For example, the subleading order corrections to the jet quenching parameter (3) due to worldsheet fluctuations can be found in [10]:

$$\widehat{q}'_{\mathrm{SYM}} = \widehat{q}_{\mathrm{SYM}}\left(1 - 1.97\lambda^{-1/2}\right). \tag{27}$$

Other corrections of $O(1/N_c)$ and higher orders in $1/\sqrt{\lambda}$ are also discussed in [11, 12]. However, $1/\lambda$ corrections on jet quenching parameter are as yet undetermined; we hope to report our progress in this regard in future.

Competing Interests

The authors declare that they have no competing interests.

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