A note on trans-Planckian tail effects

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Abstract. We study the proposal by Mersini et al. [1] that the observed dark energy might be explained by the back-reaction of the set of *tail modes* in a theory with a dispersion relation in which the mode frequency decays exponentially in the trans-Planckian regime. The matter tail modes are frozen out, however they induce metric fluctuations. The energy-momentum tensor with which the tail modes effect the background geometry obtains contributions from both metric and matter fluctuations. We calculate the equation of state induced by the tail modes taking into account the gravitational contribution. We find that, in contrast to the case of frozen super-Hubble cosmological fluctuations, in this case the matter perturbations dominate, and they yield an equation of state which to leading order takes the form of a positive cosmological constant.

Contents

1 Introduction

In an interesting paper, Mersini et al. [1] have put forwards the suggestion that the energy density stored in trans-Planckian modes provides a candidate for the observed dark energy of the universe. The authors of [1] assumed that the dispersion relation for the fluctuation modes of some matter field φ is dramatically modified for wave numbers larger than some critical value k_c. If we think in terms of waves propagating in an inhomogeneous medium, it is reasonable to assume that the dispersion relation for the mode propagation will get modified when the mode start probing the underlying structure of the background, in our case, the trans-Planckian regime [2]. Instead of the usual linear dispersion relation for the frequency ω_k, namely $\omega_k \sim k$, a decaying function of Epstein class [3] of the form

$$\omega_k^2 = k^2 \left(\frac{\epsilon_1}{1+e^x} + \frac{\epsilon_3 e^x}{(1+e^x)^2} \right) , \tag{1.1}$$

with

$$x \equiv \left(\frac{k}{k_c} \right) \tag{1.2}$$

was considered. Here, ϵ_1 and ϵ_3 are constants. The dispersion function above encapsulates the T-duality behaviour. To obtain the linear dispersion relation for $k \ll k_c$, the values ϵ_1 and ϵ_3 must satisfy the constraint

$$\frac{\epsilon_1}{2} + \frac{\epsilon_3}{4} = 1. \tag{1.3}$$

The value of k_c is given by the high energy scale of the new physics (e.g. string physics) which leads to the change in the dispersion relation.

The family of dispersion models chosen attenuates to zero in the trans-Planckian regime, thereby producing ultralow frequencies at very short distances. The total energy contribution of the modes produced is then finite, eliminating the need for renormalization.

The modes with very high momenta but ultralow frequencies $w(k)$ are frozen for as long as the Hubble expansion rate of the universe dominates over their frequencies. These modes correspond to the ones with $k > k_H$ where k_H is determined via

$$\omega^2(k_H) = H^2, \tag{1.4}$$

H being the Hubble expansion rate. In [1, 4] these are called the *tail modes*. It then follows that the tail modes have not decayed and redshifted away but are still frozen today.

The energy density in the tail modes can be computed in the standard way [1, 5]

$$\langle \rho_{\text{tail}} \rangle = \frac{1}{2\pi^2} \int_{k_H}^{\infty} k dk \int \omega_k d\omega |\beta_k|^2, \tag{1.5}$$

where β_k are the Bogoliubov coefficients which give the excitation level of the mode. These coefficients were calculated in [1] assuming the modes begin in the state which minimizes the Hamiltonian (the same prescription as the one used in [6]). Not unexpectedly, it was found that these coefficients decay exponentially as $k \to \infty$. Rather surprisingly, it was found then that $\langle \rho_{\text{tail}} \rangle$ is comparable in amplitude to the observed dark energy density, without any fine-tunning and with the Planck mass m_{pl} being the fundamental scale k_c of the theory.

In this note we take a closer look at this suggestion. It is not sufficient to simply calculate the energy density stored in the tail modes, but we also have to verify that their equation of state is really that of dark energy. In addition, we must consider the effects of the gravitational fluctuations which are induced by the matter fluctuations. As studied in [7, 8], the effect of the matter and metric fluctuations on the background cosmology can be described in terms of an *effective energy momentum tensor* $\tau_{\mu\nu}$. Here, we study the equation of state of $\tau_{\mu\nu}$ for the tail modes and we find that, given certain assumptions, it indeed has the right form to be a candidate for dark energy.

In the following, we first review the effective energy momentum tensor $\tau_{\mu\nu}$ of cosmological fluctuations. In section 3 we then study the equation of state of the tail modes contribution to $\tau_{\mu\nu}$. We end with a discussion of our results. Throughout this paper we will use natural units in which the speed of light and Planck's constant are set to 1. Greek letters are used for space-time indices and latin indices run over spatial dimensions only.

2 Effective energy momentum tensor for cosmological fluctuations

As is well known, cosmological fluctuations are observed to be small today on large scales (scales which we see in the cosmic microwave background), and hence a linear analysis of the fluctuations on top of the evolving background cosmology is usually a good starting point. Linear fluctuations can be analyzed in Fourier space where each Fourier mode evolves independently. The Einstein field equations, however, are highly nonlinear and even at the classical level fluctuations at second order influence the background. So, in this paper we will be concerned with the leading effects which the fluctuations have on the background beyond the linear treatment. At quadratic order there is a coupling between the Fourier modes. In particular, a Fourier mode with wave vector **k** can combine with a mode with wave vector $-\mathbf{k}$ to yield a correction to the background, which is the $\mathbf{k} = \mathbf{0}$ mode. This effect is called *back-reaction*.

On the other hand, matter fluctuations induce, via the Einstein constraint equations, metric fluctuations. These metric fluctuations will also back-react on the homogeneous background metric. The evolution of the full metric is governed by the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2.1}$$

where $G_{\mu\nu}$ is the Einstein tensor of the metric $g_{\mu\nu}$, $T_{\mu\nu}$ is the energy-momentum tensor of matter, and G is Newton's gravitational constant.

At linear order in the amplitude of cosmological fluctuations, the metric and matter can be written as

$$ds^2 = (1 + 2\phi(\mathbf{x}, t))dt^2 - a(t)^2 \left[(1 - 2\psi(\mathbf{x}, t))\gamma_{ij} dx^i dx^j \right], \tag{2.2}$$

and

$$\varphi(\mathbf{x}, t) = \varphi_o(t) + \delta\varphi(\mathbf{x}, t) \, . \tag{2.3}$$

We have chosen a particular coordinate system (longitudinal gauge) in order to write the metric in the form (2.2)[1] In the longitudinal gauge the metric fluctuations ϕ and ψ coincide with Bardeen's gauge invariant variables Φ and Ψ. In the absence of anisotropic stress ϕ and ψ are equal. In (2.2), γ_{ij} is the background metric of the constant time hypersurfaces. For vanishing spatial curvature (the case we will discuss) we simply have $\gamma_{ij} = \delta_{ij}$. The background metric is given by the scale factor $a(t)$, the background matter by $\varphi_0(t)$. We will only consider scalar metric fluctuations. Vector perturbations decay in an expanding universe, and since they are not seeded at linear order by matter fluctuations we can neglect them. In the next section we will also comment on the possible role of gravitational waves.

To find out how the linear fluctuations effect the background (an effect which is quadratic in the amplitude of the inhomogeneities) we insert (2.2) and (2.3) into the Einstein equations (2.1) and expand to second order. The zero'th order terms obey the background equations, the linear terms are assumed to satisfy the linear perturbation equations. The terms on the left hand side of (2.1) which are quadratic can be moved to the right hand side of the equation, where they combine with the quadratic terms in $T_{\mu\nu}$ to form an *effective energy-momentum tensor*

$$\tau_{\mu\nu}(\mathbf{x}, t) \equiv T_{\mu\nu}^{(2)} - \frac{1}{8\pi G} G_{\mu\nu}^{(2)} \, , \tag{2.4}$$

where the superscript (2) indicates the order of the terms.

In the presence of fluctuations, the background metric is modified. The corrected background is

$$g_{\mu\nu}^{(bg)}(t) \equiv g_{\mu\nu}^{(0)}(t) + \delta g^{(bg,2)}(t) \, , \tag{2.5}$$

where the first term is the background metric and the second one indicates the quadratic corrections to the background. Following the method proposed in [7, 8] and reviewed in [11], we can extract the corrections to the background by taking the spatial average of (2.1) expanded to second order. The equation then takes the form

$$G_{\mu\nu}(g_{\alpha\beta}^{(bg)}) = 8\pi G \tau_{\mu\nu}(t) \, , \tag{2.6}$$

where $\tau_{\mu\nu}(t)$ is the spatial average of (2.4).

The form of the effective energy-momentum tensor $\tau_{\mu\nu}$ was derived in [7, 8] with the result

$$\tau_{00} = \frac{1}{8\pi G}[+12H\langle\phi\dot{\phi}\rangle - 3\langle(\dot{\phi})^2\rangle + 9a^{-2}\langle(\nabla\phi)^2\rangle] + \frac{1}{2}\langle(\delta\dot{\varphi})^2\rangle + \frac{1}{2}a^{-2}\langle(\nabla\delta\varphi)^2\rangle \tag{2.7}$$

$$+ \frac{1}{2}V''(\varphi_0)\langle\delta\varphi^2\rangle + 2V'(\varphi_0)\langle\phi\delta\varphi\rangle$$

and

$$\tau_{ij} = a^2\delta_{ij}\left\{ \frac{1}{8\pi G}\left[(24H^2 + 16\dot{H})\langle\phi^2\rangle + 24H\langle\dot{\phi}\phi\rangle + \langle(\dot{\phi})^2\rangle + 4\langle\phi\ddot{\phi}\rangle - \frac{4}{3}a^{-2}\langle(\nabla\phi)^2\rangle \right] \right.$$

$$\left. + 4\dot{\varphi}_0^2\langle\phi^2\rangle + \frac{1}{2}\langle(\delta\dot{\varphi})^2\rangle - \frac{1}{6}a^{-2}\langle(\nabla\delta\varphi)^2\rangle - 4\dot{\varphi}_0\langle\delta\dot{\varphi}\phi\rangle - \frac{1}{2}V''(\varphi_0)\langle\delta\varphi^2\rangle + 2V'(\varphi_0)\langle\phi\delta\varphi\rangle \right\} . \tag{2.8}$$

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[1]See [9] for an in-depth review of the theory of cosmological fluctuations and [10] for an introductory overview.

Inserting these expressions into (2.6) allows the determination of the effect of cosmological perturbations on the background metric.

In [7, 8, 12, 13] the backreaction of long wavelength modes was studied. In [7, 8] it was found that the contribution of super-Hubble modes to $\tau_{\mu\nu}$ acts like a negative cosmological constant, and hence can potentially lead to a dynamical relaxation of the cosmological constant, as discussed in [11]. The effect is easy to understand heuristically: for super-Hubble modes the contribution of spatial gradients is negligible. Since the dominant mode of ϕ is constant in time on super-Hubble scales, if the equation of state of matter is constant then time derivative terms are also negligible. Thus, the equation of state of $\tau_{\mu\nu}$ has to be that of a cosmological constant. Matter fluctuations carry positive energy, but lead to metric potential wells which have negative gravitational energy. On super-Hubble scales the negativity of the gravitational energy overcomes the positivity of the matter energy, hence explaining the sign of the effect.

An important issue first raised in [14] is whether the resulting correction to the background metric is physical, or whether it is equivalent to a second order time reparametrization. In fact, in the case of long wavelength (super-Hubble) adiabatic fluctuations it can be shown [15, 16] that the effect is indeed not physically measurable. The local expansion rate of space computed at a fixed value of the only clock field in the problem, the matter field φ, is independent of whether there are fluctuations ϕ or not. However, in our universe there are several matter fields. In particular, we measure time in terms of the temperature of the CMB, i.e. in terms of a clock field which has a negligible effect on the expansion of space. Since we are interested in effects in the current universe, we can assume that time is measured from the CMB and thus back-reaction effects computed via (2.6) are physical [17]. This is similar to the case discussed in [18] where it was shown that the backreaction of long wavelengths fluctuations is physical and leads in fact to a decrease in the cosmological constant.

3   Tail mode contribution

We now want to study the contribution of tail modes to $\tau_{\mu\nu}$ in the model of [1], where the dispersion relation is dramatically modified in the ultraviolet. The modification of the dispersion relation can be taken into account by replacing the ∇ operator in Fourier space by $\omega_k \nabla$ in the relations (2.7) and (2.8).

Whereas for the standard dispersion relation the contribution of short wavelength modes to $\tau_{\mu\nu}$ is divergent, it is convergent in our case. Since the dynamics of the tail modes is frozen (no oscillations on the time scale k^{-1}) we might expect that, in analogy to the case of the frozen super-Hubble modes, the contribution of the tail modes might look like a cosmological constant, and might hence contribute to dark energy. Here we study this question.

Matter and metric fluctuations $\delta\varphi$ and ϕ are not independent. They are related through the $0i$ component of the Einstein equations,

$$\dot{\phi} + H\phi \,=\, 4\pi G\dot{\varphi}_0\delta\varphi. \tag{3.1}$$

Note that if we were to use this equation to determine $\delta\varphi$ from ϕ, there would be a singularity when $\dot{\varphi}_0 = 0$. This corresponds to the breakdown of longitudinal gauge. However, we will be using this equation to determine the metric fluctuations from the matter inhomogeneities, and thus no problem in applying (3.1) arises.

The above equation has a homogeneous solution, which is constant modulo Hubble damping, and an inhomogeneous solution determined by the matter fluctuations, which to

leading order in H obeys the equation

$$\dot{\phi} \approx 4\pi G\dot{\varphi}_0\delta\varphi\,. \tag{3.2}$$

We study back-reaction in the matter-dominated phase of Standard Big Bang cosmology. We will use the scalar field φ to model pressureless matter. A massive scalar field with potential $V(\varphi) = m^2\varphi^2/2$ has an equation of state whose time average can yield pressureless matter. The mass m sets the time scale on which the equation of state oscillates about $p = 0$, where p denotes the pressure. This time scale should be microscopic in order not to lead to cosmologically relevant effects. Hence, we require $m \gg H$.

The oscillations of the matter fluctuations corresponding to the tail modes, which we are considering, are prevented due to the modified dispersion relation chosen. Hence, from equation (3.1) we see that the oscillations of ϕ are actually driven by the oscillating background. Therefore, the fast oscillating quantities are $\varphi_0(t)$, $\dot{\varphi}_0(t)$, and ϕ. If we are interested in the time-averaged equation of state of $\tau_{\mu\nu}$, we can drop all terms which are linear in rapidly oscillating functions.

Hence, if $\varphi_0(t)$ is oscillating as

$$\varphi_0(t) = \mathcal{A}\sin(mt), \tag{3.3}$$

then ϕ will oscillate as

$$\phi = \frac{1}{m}\tilde{\mathcal{A}}\sin(mt), \tag{3.4}$$

with space-dependent amplitude $\tilde{\mathcal{A}}$ determined by the space-dependent matter fluctuation via

$$\tilde{\mathcal{A}} = 4\pi Gm\mathcal{A}\delta\varphi\,. \tag{3.5}$$

In order for φ to dominate the energy density we have (from the Friedmann equation)

$$m^2\mathcal{A}^2 \approx m_{\rm pl}^2 H^2\,, \tag{3.6}$$

where $m_{\rm pl}$ is the reduced Planck mass and $\mathcal{A}$ is the amplitude of the scalar field. Hence

$$\mathcal{A} \approx \frac{H}{m}m_{\rm pl} \ll m_{\rm pl}\,. \tag{3.7}$$

In the following we will use these results to estimate the magnitude of the tail mode contribution to $\tau_{\mu\nu}$.

We first compare the magnitude of the gravitational terms with those of the matter terms. The matter terms are

$$\tau_{00}^m = \frac{1}{2}\left[\langle(\delta\dot{\varphi})^2\rangle + a^{-2}\langle(\nabla\delta\varphi)^2\rangle + V''(\varphi_0)\langle\delta\varphi^2\rangle\right] \tag{3.8}$$

and

$$\tau_{ii}^m = \frac{1}{2}\langle(\delta\dot{\varphi})^2\rangle - \frac{a^{-2}}{6}\langle(\nabla\delta\varphi)^2\rangle - \frac{V''(\varphi_0)}{2}\langle\delta\varphi^2\rangle, \tag{3.9}$$

where we recall that the ∇ operator is the modified one. Since the tail modes are frozen and the ∇ operator is the modified one, the first two terms on the right hand side of each equation are suppressed, and the third term dominates. Hence, the induced equation of state of these matter terms is

$$p \simeq -\rho\,, \tag{3.10}$$

where ρ is the energy density. The correction terms coming from the first two terms on the right hand side of each equation leads to

$$w \equiv \frac{p}{\rho} > -1. \tag{3.11}$$

Now let us turn to the terms which are quadratic in the gravitational potential ϕ. Since the rate of change of ϕ has magnitude $m\phi$, we can neglect the terms proportional to H and $\dot{H}$ (and consequently also $\dot{\varphi}_0^2$) in (2.7) and (2.8). So we obtain

$$\tau_{00}^g \approx -3m_{\rm pl}^2 m^2 \langle \phi^2 \rangle + 9m_{\rm pl}^2 a^{-2} \langle (\nabla\phi)^2 \rangle \tag{3.12}$$

and

$$\tau_{ii}^g \approx -3m_{\rm pl}^2 m^2 \langle \phi^2 \rangle - \frac{4}{3} m_{\rm pl}^2 a^{-2} \langle (\nabla\phi)^2 \rangle. \tag{3.13}$$

Now let us estimate the magnitude of the matter term $\langle (\delta\varphi)^2 \rangle$ in order to compare the matter and the gravitational contributions. From eq. (3.2) we have that $m_{\rm pl}^2 \phi \approx \varphi_0 \delta\varphi$, since both ϕ and φ_0 are fast oscillating quantities. Then we have

$$\delta\varphi^2 \approx \frac{m_{\rm pl}^4 \phi^2}{\mathcal{A}^2}. \tag{3.14}$$

Substituting $\mathcal{A}$ from eq. (3.7) and considering that $\tau_{00}^m \approx m^2 \langle \delta\varphi^2 \rangle$, we obtain

$$\tau_{00}^m \approx \frac{m^4 m_{\rm pl}^2 \langle \phi^2 \rangle}{H^2} = m^2 m_{\rm pl}^2 \langle \phi^2 \rangle \left(\frac{m^2}{H^2} \right). \tag{3.15}$$

Comparing the above equation with τ_{00}^g, we can see that $\tau_{00}^m \approx (m^2/H^2)\tau_{00}^g$. Since $m \gg H$, we conclude that the matter contribution to the effective energy-momentum tensor dominates over the gravitational contribution.

The time average of the cross terms (the terms involving one factor of ϕ and one factor of $\delta\varphi$) vanishes and hence we can neglect these terms. Thus, in summary, we find that the matter terms in the effective energy-momentum tensor $\tau_{\mu\nu}$ dominate. They lead to an equation of state $w = -1$ with positive effective energy density. Therefore, it appears that, even when taking gravitational effects into account, the tail modes can provide a candidate for dark energy.

For completeness, we are now going to estimate the effective energy-momentum tensor of the gravitational waves for this class of dispersion relation.

In the case of gravitational waves the metric assumes the form

$$ds^2 = dt^2 - a^2(t)(\delta_{ik} + h_{ik})dx^i dx^k, \tag{3.16}$$

where h_{ik} is defined as the transverse traceless part of the metric perturbations.

In [7, 8] it was shown that, neglecting the coupling of the gravitational waves to matter, the effective energy momentum tensor of gravitational waves is given by

$$8\pi G \tau_0^0 = \frac{\dot{a}}{a} \langle \dot{h}_{kl} h_{kl} \rangle + \frac{1}{8} \left(\langle \dot{h}_{kl} \dot{h}_{kl} \rangle + \frac{1}{a^2} \langle h_{kl,m} h_{kl,m} \rangle \right) \tag{3.17}$$

and

$$-\frac{8\pi G}{3} \tau_i^i = \frac{7}{24a^2} \langle h_{kl,m} h_{kl,m} \rangle - \frac{5}{24} \langle \dot{h}_{kl} \dot{h}_{kl} \rangle, \tag{3.18}$$

where we assumed that the gravitational wave field is isotropic.

In the tensorial case, the quantity that can be interpreted as the pressure is [7, 8]

$$p_{gw} = -\frac{1}{3}\tau_i^i - \frac{1}{6H}\langle \dot{h}^{ij}h_{ij}\rangle(\rho_0 + p_0),\qquad(3.19)$$

while the energy density is still given by $\rho_{gw} = \tau_0^0$.

Since we are interested in the tail modes, which are frozen, we can neglect all terms with either space or time derivatives. Thus, all terms are negligible, and we conclude that the tail modes of the spectrum of gravitational waves only have a negligible effect on the effective energy-momentum tensor of back-reaction.

4 Discussion

We have studied the equation of state of the effective energy-momentum tensor with which tail modes back react on the background space-time. The context of our study is a proposal by Mersini et al. [1] that the dispersion relation of fluctuations might be highly distorted in the trans-Planckian regime, such that for very large values of k the effective frequency ω_k becomes smaller than the Hubble expansion rate H (*tail modes*). This class of dispersion relation reflects the T-duality symmetry of string theory and leads naturally to a finite vacuum energy. It was found in [1] that the energy density stored in the tail modes coincides with the energy density of the observed dark energy. So it was suggested that, with the correct equation of state, this contribution could account for the observed dark energy without any fine-tuning.

In order to verify if this effect could really represent dark energy, we have explicitly calculated the equation of state induced by the tail modes, including not only the contribution of the matter fluctuations but also that of the metric fluctuations. We have shown that (in contrast to what happens for super-Hubble modes in the case of a standard dispersion relation) the effective energy-momentum tensor is dominated by the contributions from the matter terms which, to first approximation, leads to an equation of state of dark energy with positive energy density. We also analysed the contribution of gravitational waves in this regime and found that it is negligible.

In conclusion, our results support the suggestion that the backreaction of the trans-Planckian modes might account for the observed dark energy.

Acknowledgments

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