



# Parity-violating $\pi NN$ coupling constant from the flavor-conserving effective weak chiral Lagrangian



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## ABSTRACT

We investigate the parity-violating pion–nucleon–nucleon coupling constant  $h_{\pi NN}^1$ , based on the chiral quark–soliton model. We employ an effective weak Hamiltonian that takes into account the next-to-leading order corrections from QCD to the weak interactions at the quark level. Using the gradient expansion, we derive the leading-order effective weak chiral Lagrangian with the low-energy constants determined. The effective weak chiral Lagrangian is incorporated in the chiral quark–soliton model to calculate the parity-violating  $\pi NN$  constant  $h_{\pi NN}^1$ . We obtain a value of about  $10^{-7}$  at the leading order. The corrections from the next-to-leading order reduce the leading order result by about 20%.

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## 1. Introduction

The electroweak interactions have been tested and confirmed mainly by parity-violating lepton scattering, decays of hadrons, and  $\beta$  decays of nuclei. Recently, Parity-violating (PV) hadronic processes play yet another important role of a touchstone to examine the standard model (SM) and physics beyond the standard model (BSM) (see for example recent reviews [1–6]). There are mainly two different ways of describing PV hadronic reactions: One is to consider one-boson exchanges such as  $\pi$ ,  $\rho$ , and  $\omega$  mesons à la the strong nucleon–nucleon ( $NN$ ) potential [7–9]. The other is to employ effective field theory [6,10]. In both methods, the PV pion–nucleon coupling constant is the most essential quantity, since it governs the PV hadronic processes in long range. Desplanques, Donoghue and Holstein (DDH) [9] estimated the value of the PV  $\pi NN$  coupling constant, also known as the so-called “DDH best value”:  $h_{\pi NN}^1 = 4.6 \times 10^{-7}$ . A great deal of experimental and theoretical efforts have been devoted to extracting the precise value of the  $\pi NN$  coupling constant (for recent reviews, see [11,12]), since the contribution of  $\pi$ -exchange is dominant over those of  $\rho$ - and  $\omega$ -meson exchanges in the PV asymmetry in  $\bar{n}p \rightarrow d\gamma$  [13–15], and  $\bar{n}d \rightarrow t\gamma$  [16]. The PV  $\pi NN$  coupling constant has been studied in various different theoretical approaches

such as the Skyrme models [17–19], quark models [20], the chiral-quark soliton model [21,22], QCD sum rule [23], and so on. However, all these values of  $h_{\pi NN}^1$  are far from consensus and are given in the wide range between  $10^{-8}$  [17] and  $\sim 5 \times 10^{-7}$  [19]. A recent analysis of lattice QCD yields  $h_{\pi NN}^{1,\text{con}} = (1.099 \pm 0.505^{+0.058}_{-0.064}) \times 10^{-7}$  for which only the contribution of the connected diagrams to  $h_{\pi NN}^1$  has been considered [24]. On the experimental side, though the accuracy of the measurements has been much improved, an upper bound on the value of  $h_{\pi NN}^1$  [25] is only known. Thus, more systematic and quantitative studies are required in order to obtain the value of the PV  $\pi NN$  coupling constant.

The main dynamical origin of hadronic parity violation comes from the flavor-conserving effective weak Hamiltonian, which was already investigated [8,9,26–30]. In particular, the PV  $\pi NN$  coupling constant can be obtained from the isovector ( $\Delta I = 1$ ) effective weak Hamiltonian, which was first derived in Ref. [30] at the one-loop level with the effects of heavy quarks taken into account. Very recently, Tiburzi [31] investigated systematically the  $\Delta S = 0$  effective weak Hamiltonian with QCD corrections at next-to-leading order (NLO). The effects of the NLO corrections have changed the Wilson coefficients about (10–20)% at the typical scale of light hadrons ( $\mu = 1$  GeV). Considering the fact that the PV  $\pi NN$  coupling constant is very tiny, we expect that the corrections from QCD at NLO may come into play. Thus, it is of great interest to examine the NLO corrections to the PV  $\pi NN$  coupling constant.

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In the present work, we investigate the PV  $\pi NN$  coupling constant,  $h_{\pi NN}^1$ , within the framework of the chiral quark-soliton model ( $\chi$ QSM) together with the effective weak Hamiltonian at NLO [31]. The  $\chi$ QSM is constructed based on the effective chiral action that explains both the pseudoscalar mesons and low-lying baryons on the equal footing [32] (see also a review [33]). The presence of the  $N_c$  valence quarks makes the vacuum polarized. As a result, a baryon is described as a soliton of the classical pion mean field. Since the  $\chi$ QSM is constructed in a relativistic field-theoretic way, it has been very successful in explaining not only usual properties of baryons such as the vector form factors [34–36], axial-vector form factors and semileptonic decay constants [37,38], but also internal structures of the nucleon like the parton distributions [39–41], generalized parton distributions [42, 43], transversities [44–48], etc. Moreover, the  $\chi$ QSM provides a more realistic picture than the Skyrme model, because the effective chiral action on which the model is based contains all orders of the effective chiral Lagrangian including the Gasser–Leutwyler terms [49,50], Wess–Zumino term [51], and so on. The gradient expansion of the effective chiral action yields the corresponding low-energy constants in good agreement with the empirical data [52, 53]. Recently, the present authors computed the PV  $\pi NN$  coupling constant [22] in the same framework, employing the effective weak Hamiltonian from Ref. [9]. We first derived the effective weak chiral Lagrangian, based on the nonlocal chiral-quark model (N $\chi$ QM) from the instanton vacuum associating with the effective weak Hamiltonian [21]. If one performs the gradient expansion for the effective chiral action of the  $\chi$ QSM with the effective weak Hamiltonian, we would obtain exactly the same expressions starting directly from the effective weak chiral Lagrangian. Using this gradient expansion, we were able to obtain the PV  $\pi NN$  coupling constant to be about  $1 \times 10^{-8}$  at  $\mu = 1$  GeV. We also found that the  $h_{\pi NN}^1$  is rather sensitive to the Wilson coefficients. In this respect, it is of great importance to reexamine the PV  $\pi NN$  coupling constant, the  $\Delta I = 1$  effective weak Hamiltonian being employed with the NLO QCD effects. As we will show in this work, the value of  $h_{\pi NN}^1$  indeed turns out to be different from the previous result. Moreover, the effects from the NLO corrections reduce the leading-order result by about 20%.

The paper is organized in the following order: In Section 2, we present briefly the general procedure to obtain the PV  $\pi NN$  coupling constants within the  $\chi$ QSM. We derive the flavor-conserving effective weak chiral Lagrangian, with the nonlocal chiral quark model from the instanton vacuum. In Section 3 we compute the correlation function corresponding to the PV  $\pi NN$  coupling constant, and discuss the result. In Section 4, we conclude the work.

## 2. $\Delta I = 1$ effective weak chiral Lagrangian

We start with the  $\Delta I = 1$  flavor-conserving effective weak Hamiltonian including the NLO corrections [31], which is expressed as

$$\mathcal{H}_W^{\Delta I=1} = \frac{G_F}{\sqrt{2}} \frac{\sin^2 \theta_W}{3} \sum_{i=1}^8 c_i(\mu) O_i(\mu), \quad (1)$$

where  $G_F$  and  $\theta_W$  denote the Fermi constant and the Weinberg angle, respectively. The eight different operators  $O_i$  are defined generically as two-body operators:  $O_i = (\bar{\psi} \gamma_\mu \gamma_5 M_i \psi) (\bar{\psi} \gamma_\mu N_i \psi)$ . The  $c_i$  stands for the Wilson coefficient corresponding to the  $O_i$ , which depends on the renormalization scale  $\mu$ . Applying the following Fiertz identity to  $O_2, O_4, O_6$ , and  $O_8$ ,

$$\delta_{bc} \delta_{ad} = \frac{1}{2} (t^A)_{ab} (t^A)_{cd} \quad (2)$$

where  $t^A$  denote the Gell-Mann matrices in color space, we are able to express the effective weak Hamiltonian in the following form

$$\begin{aligned} \mathcal{H}_W^{\Delta I=1} = & \frac{G_F}{\sqrt{6}} \frac{\sin^2 \theta_W}{3} \left\{ (\bar{\psi} \gamma_\mu \gamma_5 \lambda_3 \psi) \left( \bar{\psi} \gamma_\mu \left[ \lambda_0 \left( \sqrt{2} c_1 + \frac{c_5}{\sqrt{2}} \right) \right. \right. \right. \\ & \left. \left. \left. + \lambda_8 (c_1 - c_5) \right] \psi \right) + \frac{1}{2} (\bar{\psi} \gamma_\mu \gamma_5 \lambda_3 t^A \psi) \left( \bar{\psi} \gamma_\mu \left[ \lambda_0 \left( \sqrt{2} c_2 \right. \right. \right. \right. \\ & \left. \left. \left. + \frac{c_6}{\sqrt{2}} \right) + \lambda_8 (c_2 - c_6) \right] t^A \psi \right) \\ & + (\bar{\psi} \gamma_\mu \lambda_3 \psi) \left( \bar{\psi} \gamma_\mu \gamma_5 \left[ \lambda_0 \left( \sqrt{2} c_3 + \frac{c_7}{\sqrt{2}} \right) \right. \right. \right. \\ & \left. \left. \left. + \lambda_8 (c_3 - c_7) \right] \psi \right) \\ & + \frac{1}{2} (\bar{\psi} \gamma_\mu \lambda_3 t^A \psi) \left( \bar{\psi} \gamma_\mu \gamma_5 \left[ \lambda_0 \left( \sqrt{2} c_4 + \frac{c_8}{\sqrt{2}} \right) \right. \right. \right. \\ & \left. \left. \left. + \lambda_8 (c_4 - c_8) \right] t^A \psi \right) \right\}. \quad (3) \end{aligned}$$

We rewrite the Hamiltonian in terms of the *effective* four-quark operators that contain already the Wilson coefficients  $\mathcal{Q}_i(z; \mu)$

$$\mathcal{H}_W^{\Delta I=1} = \frac{G_F}{\sqrt{6}} \frac{\sin^2 \theta_W}{3} (\mathcal{Q}_1 + \mathcal{Q}_2 + \mathcal{Q}_3 + \mathcal{Q}_4), \quad (4)$$

where the four-quark operators  $\mathcal{Q}_i(z; \mu)$  are defined as

$$\mathcal{Q}_i(z; \mu) = \alpha_i \left( \bar{\psi} \Gamma_1^{(i)} \psi \right) \left( \bar{\psi} \Gamma_2^{(i)} \psi \right), \quad (5)$$

where  $\alpha_i = 1$  for  $i = 1, 3$  and  $\alpha_i = 1/2$  for  $i = 2, 4$ . The  $\Gamma_j^{(i)}$  are defined as

$$\begin{aligned} \Gamma_1^{(1)} &= \gamma_\mu \gamma_5 \lambda_3, & \Gamma_2^{(1)} &= \gamma_\mu \Lambda^{(1)}, \\ \Gamma_1^{(2)} &= \gamma_\mu \gamma_5 \lambda_3 t^A, & \Gamma_2^{(2)} &= \gamma_\mu \Lambda^{(2)} t^A, \\ \Gamma_1^{(3)} &= \gamma_\mu \lambda_3, & \Gamma_2^{(3)} &= \gamma_\mu \gamma_5 \Lambda^{(3)}, \\ \Gamma_1^{(4)} &= \gamma_\mu \lambda_3 t^A, & \Gamma_2^{(4)} &= \gamma_\mu \gamma_5 \Lambda^{(4)} t^A \end{aligned} \quad (6)$$

with flavor matrices defined as

$$\begin{aligned} \Lambda^{(1)} &= \lambda_0 \left( \sqrt{2} c_1 + \frac{c_5}{\sqrt{2}} \right) + \lambda_8 (c_1 - c_5), \\ \Lambda^{(2)} &= \lambda_0 \left( \sqrt{2} c_2 + \frac{c_6}{\sqrt{2}} \right) + \lambda_8 (c_2 - c_6), \\ \Lambda^{(3)} &= \lambda_0 \left( \sqrt{2} c_3 + \frac{c_7}{\sqrt{2}} \right) + \lambda_8 (c_3 - c_7), \\ \Lambda^{(4)} &= \lambda_0 \left( \sqrt{2} c_4 + \frac{c_8}{\sqrt{2}} \right) + \lambda_8 (c_4 - c_8). \end{aligned} \quad (7)$$

In order to compute the  $\Delta I = 1$  flavor-conserving effective weak chiral Lagrangian, we employ the N $\chi$ QM from the instanton vacuum. The effective weak chiral Lagrangian is defined as a vacuum expectation value (VEV) of the effective weak Hamiltonian [54,55]

$$\mathcal{L}_W^{\Delta I=1} = \int D\psi D\psi^\dagger \mathcal{H}_W^{\Delta I=1} \exp \left[ \int d^4z \psi^\dagger(z) \mathcal{D} \psi(z) \right], \quad (8)$$

where  $\mathcal{D}$  represents the nonlocal covariant Dirac operator defined as

$$\mathcal{D}(-i\partial) \equiv i\gamma_\mu \partial_\mu + i\sqrt{M(-i\partial)} U \gamma_5 \sqrt{M(-i\partial)}, \quad (9)$$

and  $U \gamma_5$  represents the chiral field defined as

$$U^{\gamma_5} = \frac{1 + \gamma_5}{2} U + \frac{1 - \gamma_5}{2} U^\dagger \quad (10)$$

with the Goldstone boson field  $U = \exp(i\lambda^a \pi^a / f_\pi)$ . Then, the flavor-conserving effective weak chiral Lagrangian can be expressed in terms of the VEV of the four-quark operator

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{6}} \frac{\sin^2 \theta_W}{3} \sum_{i=1}^4 \langle \mathcal{Q}_i \rangle. \quad (11)$$

We refer to Refs. [21,54,55] for details of how to compute the VEV of  $\mathcal{Q}_i$ .

The flavor-conserving effective weak chiral Lagrangian in the  $\Delta I = 1$  channel is obtained in terms of the low-energy constants  $\mathcal{N}_i$  and  $\mathcal{M}_i$

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\Delta I=1} = & \mathcal{N}_1 \langle (R_\mu - L_\mu) \lambda_3 \rangle \langle (R_\mu + L_\mu) \lambda_0 \rangle \\ & + \mathcal{N}_2 \langle (R_\mu - L_\mu) \lambda_3 \rangle \langle (R_\mu + L_\mu) \lambda_8 \rangle \\ & + \mathcal{N}_3 \langle (R_\mu - L_\mu) \lambda_0 \rangle \langle (R_\mu + L_\mu) \lambda_3 \rangle \\ & + \mathcal{N}_4 \langle (R_\mu - L_\mu) \lambda_8 \rangle \langle (R_\mu + L_\mu) \lambda_3 \rangle \\ & + \mathcal{N}_5 \langle \lambda_3 U \lambda_0 U^\dagger - \lambda_3 U^\dagger \lambda_0 U \rangle + \mathcal{N}_6 \langle \lambda_3 U \lambda_8 U^\dagger - \lambda_3 U^\dagger \lambda_8 U \rangle \\ & + \mathcal{N}_7 \langle L_\mu \lambda_3 L_\mu U^\dagger \lambda_0 U - R_\mu \lambda_3 R_\mu U \lambda_0 U^\dagger \rangle \\ & + \mathcal{N}_8 \langle L_\mu \lambda_3 L_\mu U^\dagger \lambda_8 U - R_\mu \lambda_3 R_\mu U \lambda_8 U^\dagger \rangle \\ & + \mathcal{N}_9 \langle (\lambda_3 R_\mu R_\mu + R_\mu R_\mu \lambda_3) U \lambda_0 U^\dagger \\ & - (\lambda_3 L_\mu L_\mu + L_\mu L_\mu \lambda_3) U^\dagger \lambda_0 U \rangle \\ & + \mathcal{N}_{10} \langle (\lambda_3 R_\mu R_\mu + R_\mu R_\mu \lambda_3) U \lambda_8 U^\dagger \\ & - (\lambda_3 L_\mu L_\mu + L_\mu L_\mu \lambda_3) U^\dagger \lambda_8 U \rangle \\ & + \mathcal{N}_{11} \langle (R_\mu \lambda_3 R_\mu - L_\mu \lambda_3 L_\mu) \lambda_0 \rangle \\ & + \mathcal{N}_{12} \langle (R_\mu \lambda_3 R_\mu - L_\mu \lambda_3 L_\mu) \lambda_8 \rangle, \\ & + \mathcal{M}_1 \langle \lambda_3 U \lambda_0 U^\dagger - \lambda_3 U^\dagger \lambda_0 U \rangle \\ & + \mathcal{M}_2 \langle \lambda_3 U \lambda_8 U^\dagger - \lambda_3 U^\dagger \lambda_8 U \rangle \\ & + \mathcal{M}_3 \langle L_\mu \lambda_3 L_\mu U^\dagger \lambda_0 U - R_\mu \lambda_3 R_\mu U \lambda_0 U^\dagger \rangle \\ & + \mathcal{M}_4 \langle L_\mu \lambda_3 L_\mu U^\dagger \lambda_8 U - R_\mu \lambda_3 R_\mu U \lambda_8 U^\dagger \rangle \\ & + \mathcal{M}_5 \langle (\lambda_3 R_\mu R_\mu + R_\mu R_\mu \lambda_3) U \lambda_0 U^\dagger \\ & - (\lambda_3 L_\mu L_\mu + L_\mu L_\mu \lambda_3) U^\dagger \lambda_0 U \rangle \\ & + \mathcal{M}_6 \langle (\lambda_3 R_\mu R_\mu + R_\mu R_\mu \lambda_3) U \lambda_8 U^\dagger \\ & - (\lambda_3 L_\mu L_\mu + L_\mu L_\mu \lambda_3) U^\dagger \lambda_8 U \rangle \\ & + \mathcal{M}_7 \langle (R_\mu \lambda_3 R_\mu - L_\mu \lambda_3 L_\mu) \lambda_0 \rangle \\ & + \mathcal{M}_8 \langle (R_\mu \lambda_3 R_\mu - L_\mu \lambda_3 L_\mu) \lambda_8 \rangle \end{aligned} \quad (12)$$

where  $\langle \dots \rangle$  means the trace over the flavor. The right and left currents  $R_\mu$  and  $L_\mu$  are defined respectively as

$$R_\mu = iU \partial_\mu U^\dagger, \quad L_\mu = iU^\dagger \partial_\mu U. \quad (13)$$

The weak low-energy constants (WLECs)  $\mathcal{N}_i$  are the leading order in the large  $N_c$  limit whereas  $\mathcal{M}_i$  are of the subleading order. They are expressed as

$$\begin{aligned} \mathcal{N}_1 &= 4N_c^2 J_2^2 \mathcal{C} \left( \sqrt{2}c_1 + \frac{c_5}{\sqrt{2}} \right), \quad \mathcal{N}_2 = 4N_c^2 J_2^2 \mathcal{C} (c_1 - c_5), \\ \mathcal{N}_3 &= 4N_c^2 J_2^2 \mathcal{C} \left( \sqrt{2}c_3 + \frac{c_7}{\sqrt{2}} \right), \quad \mathcal{N}_4 = 4N_c^2 J_2^2 \mathcal{C} (c_3 - c_7), \\ \mathcal{N}_5 &= 8N_c^2 J_1^2 \mathcal{C} \left( \sqrt{2}c_2 - \sqrt{2}c_4 + \frac{c_6}{\sqrt{2}} - \frac{c_8}{\sqrt{2}} \right), \end{aligned}$$

$$\begin{aligned} \mathcal{N}_6 &= 8N_c^2 J_1^2 \mathcal{C} (c_2 - c_4 - c_6 + c_8), \\ \mathcal{N}_7 &= 16N_c^2 J_1 J_3 \mathcal{C} \left( \sqrt{2}c_2 - \sqrt{2}c_4 + \frac{c_6}{\sqrt{2}} - \frac{c_8}{\sqrt{2}} \right), \\ \mathcal{N}_8 &= 16N_c^2 J_1 J_3 \mathcal{C} (c_2 - c_4 - c_6 + c_8), \\ \mathcal{N}_9 &= 8N_c^2 J_1 J_4 \mathcal{C} \left( \sqrt{2}c_2 - \sqrt{2}c_4 + \frac{c_6}{\sqrt{2}} - \frac{c_8}{\sqrt{2}} \right), \\ \mathcal{N}_{10} &= 8N_c^2 J_1 J_4 \mathcal{C} (c_2 - c_4 - c_6 + c_8), \\ \mathcal{N}_{11} &= 4N_c^2 J_2^2 \mathcal{C} \left( \sqrt{2}c_2 + \sqrt{2}c_4 + \frac{c_6}{\sqrt{2}} + \frac{c_8}{\sqrt{2}} \right), \\ \mathcal{N}_{12} &= 4N_c^2 J_2^2 \mathcal{C} (c_2 + c_4 - c_6 - c_8), \\ \mathcal{M}_1 &= 8N_c J_1^2 \mathcal{C} \left( \sqrt{2}c_1 - \sqrt{2}c_3 + \frac{c_5}{\sqrt{2}} - \frac{c_7}{\sqrt{2}} \right), \\ \mathcal{M}_2 &= 8N_c J_1^2 \mathcal{C} (c_1 - c_3 - c_5 + c_7), \\ \mathcal{M}_3 &= 16N_c J_1 J_3 \mathcal{C} \left( \sqrt{2}c_1 - \sqrt{2}c_3 + \frac{c_5}{\sqrt{2}} - \frac{c_7}{\sqrt{2}} \right), \\ \mathcal{M}_4 &= 16N_c J_1 J_3 \mathcal{C} (c_1 - c_3 - c_5 + c_7), \\ \mathcal{M}_5 &= 8N_c J_1 J_4 \mathcal{C} \left( \sqrt{2}c_1 - \sqrt{2}c_3 + \frac{c_5}{\sqrt{2}} - \frac{c_7}{\sqrt{2}} \right), \\ \mathcal{M}_6 &= 8N_c J_1 J_4 \mathcal{C} (c_1 - c_3 - c_5 + c_7), \\ \mathcal{M}_7 &= 4N_c J_2^2 \mathcal{C} \left( \sqrt{2}c_1 + \sqrt{2}c_3 + \frac{c_5}{\sqrt{2}} + \frac{c_7}{\sqrt{2}} \right), \\ \mathcal{M}_8 &= 4N_c J_2^2 \mathcal{C} (c_1 + c_3 - c_5 - c_7). \end{aligned} \quad (14)$$

Definitions of the integrals  $J_1$ ,  $J_2$ ,  $J_3$ , and  $J_4$  are given in Ref. [21], and the constant  $\mathcal{C}$  contains the Fermi constant and the Weinberg angle

$$\mathcal{C} = \frac{G_F}{\sqrt{6}} \frac{\sin^2 \theta_W}{3}. \quad (15)$$

To compute the WLECs in Eq. (14), we use the momentum-dependent quark mass derived from the instanton vacuum [56]

$$M(k) = M_0 F^2(k\rho) \quad (16)$$

with

$$F(k\rho) = 2z \left( I_0(z)K_1(z) - I_1(z)K_0(z) - \frac{1}{z} I_1(z)K_1(z) \right), \quad (17)$$

where  $I_i$  and  $K_i$  are the modified Bessel functions, and  $z = k/2\Lambda$ . The value of the dynamical quark mass at the zero virtuality of the quark is also obtained from the instanton vacuum, i.e.  $M_0 = 350$  MeV, given the average size of the instanton and the inter-distance between instantons  $R \approx 1$  fm [56]. The parameter  $\Lambda$  is determined to reproduce the physical value of  $f_\pi$ .

As discussed already in Ref. [55], the vector and axial-vector currents are not conserved in the presence of the nonlocal interaction arising from the momentum-dependent quark mass, that is, the corresponding gauge symmetries are broken. In order to keep the currents conserved, we need to make the effective chiral action gauge-invariant. In Ref. [57,58], the gauged effective chiral action was derived, based on the instanton vacuum. Had we naively computed  $J_2 = f_\pi^2/4N_c$  without the current conservation being considered, then we would have ended up with the Pagels–Stokar formula for  $f_\pi^2$  [59], which does not satisfy the gauge invariance. The numerical results for the integrals are obtained as

$$\begin{aligned} J_1 &= (-112.31)^3 \text{ MeV}^3, \quad J_2 = (26.673)^2 \text{ MeV}^2, \\ J_3 &= -1.7403 \text{ MeV}, \quad J_4 = -0.601 \text{ MeV}. \end{aligned} \quad (18)$$

**Table 1**  
The Wilson coefficients  $c_i$  taken from Ref. [31].

	LO	NLO (Z)	NLO (Z + W)
$c_1$	0.264	-0.054	-0.055
$c_2$	0.981	0.803	0.810
$c_3$	-0.592	-0.629	-0.627
$c_4$	0	0	0
$c_5$	5.97	4.85	5.09
$c_6$	-2.30	-2.14	-2.55
$c_7$	5.12	4.27	4.51
$c_8$	-3.29	-2.94	-3.36

**Table 2**  
The results for the low-energy constants in units of  $10^{-4}$  MeV<sup>2</sup>. Notations are the same as in Table 1.

WLEC	LO	NLO (Z)	NLO (Z + W)
$\mathcal{N}_1$	0.307	0.224	0.235
$\mathcal{N}_2$	-0.381	-0.328	-0.344
$\mathcal{N}_3$	0.186	0.142	0.154
$\mathcal{N}_4$	-0.382	-0.327	-0.343
$\mathcal{N}_5$	1.106	0.901	0.910
$\mathcal{N}_6$	-0.005	0.002	0
$\mathcal{N}_7$	2.716	2.214	2.236
$\mathcal{N}_8$	-0.012	0.004	0
$\mathcal{N}_9$	0.469	0.382	0.386
$\mathcal{N}_{10}$	-0.002	0.001	0
$\mathcal{N}_{11}$	-0.171	-0.164	-0.203
$\mathcal{N}_{12}$	0.439	0.393	0.449
$\mathcal{M}_1$	0.320	0.216	0.215
$\mathcal{M}_2$	0.001	-0.001	-0.001
$\mathcal{M}_3$	0.786	0.531	0.529
$\mathcal{M}_4$	0.003	-0.002	-0.004
$\mathcal{M}_5$	0.136	0.092	0.091
$\mathcal{M}_6$	0.0004	-0.0004	-0.001
$\mathcal{M}_7$	0.164	0.122	0.130
$\mathcal{M}_8$	-0.254	-0.218	-0.229

Note that the value of  $J_1$  is related to that of the quark condensate  $\langle \bar{\psi}\psi \rangle_M = (-257.13 \text{ MeV})^3$  and that of  $J_2$  corresponds to the value of  $f_\pi = 92.4 \text{ MeV}$ .

In Table 1, the values of the Wilson coefficients taken from Ref. [31] are listed. The first column lists the results for the Wilson coefficients in the LO, and the second and third ones correspond to those from the NLO contributions together with the LO terms. The Z and Z + W in the second and third columns stand respectively for the considerations of Z and Z + W boson exchanges. As already discussed in Ref. [31], there are certain effects from the NLO contributions.

Based on these values of the Wilson coefficients, we list in Table 2 the results for the WLECs given in Eq. (14). Note that the WLECs  $\mathcal{N}_6$ ,  $\mathcal{N}_8$ , and  $\mathcal{N}_{10}$  are null. This is due to the fact that they correspond to the operators

$$O'_2 = -O_2 + O_4 + O_6 - O_8, \quad (19)$$

for which the corresponding Wilson coefficient vanishes because  $O'_2$  is not generated by QCD radiative corrections [31].

Though there are arguments that sizable contributions in  $\Delta I = 1$  channel come from the operators with strangeness [19], we will restrict ourselves to the case of SU(2). The calculation in SU(2) has several merits in particular in the present work. Firstly, the chiral solitonic approach in SU(2) is much simpler and physically clearer than that in SU(3). Secondly, the SU(2) approach allows one to understand better the PV  $\pi NN$  constant based on the effective weak Hamiltonian. A more quantitative work within SU(3) will appear elsewhere. In the case of SU(2), we reduce  $\lambda_0$ ,  $\lambda_3$ , and  $\lambda_8$  to

**Table 3**  
The values of  $\beta_i$  in units of  $10^{-4}$  MeV<sup>2</sup>. Notations are the same as in Table 1.

	WLEC	LO	NLO (Z)	NLO (Z + W)
$\beta_1$		0.031	-0.006	-0.006
$\beta_2$		-0.069	-0.073	-0.073
$\beta_3$		-1.334	-1.092	-1.102
$\beta_4$		-0.434	-0.309	-0.308

$$\lambda_0 \rightarrow \sqrt{\frac{2}{3}}\mathbf{1}, \quad \lambda_3 \rightarrow \tau_3, \quad \lambda_8 \rightarrow \frac{1}{\sqrt{3}}\mathbf{1}. \quad (20)$$

As a result, the  $\Delta I = 1$  effective weak Lagrangian is simplified as

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{SU}(2)} = & \beta_1 \langle (R_\mu - L_\mu)\tau_3 \rangle \langle R_\mu + L_\mu \rangle \\ & + \beta_2 \langle R_\mu - L_\mu \rangle \langle (R_\mu + L_\mu)\tau_3 \rangle \\ & + \beta_3 \langle (R_\mu R_\mu - L_\mu L_\mu)\tau_3 \rangle + \beta_4 \langle (R_\mu R_\mu - L_\mu L_\mu)\tau_3 \rangle, \end{aligned} \quad (21)$$

where  $\beta_i$  are defined in terms of the WLECs

$$\begin{aligned} \beta_1 &= \frac{1}{\sqrt{3}} (\sqrt{2}\mathcal{N}_1 + \mathcal{N}_2), \quad \beta_2 = \frac{1}{\sqrt{3}} (\sqrt{2}\mathcal{N}_3 + \mathcal{N}_4), \\ \beta_3 &= \frac{1}{\sqrt{3}} [\sqrt{2}\mathcal{N}_{11} + \mathcal{N}_{12} + \sqrt{2}(2\mathcal{N}_9 - \mathcal{N}_7) + 2\mathcal{N}_{10} - \mathcal{N}_8], \\ \beta_4 &= \frac{1}{\sqrt{3}} [\sqrt{2}\mathcal{M}_7 + \mathcal{M}_8 + \sqrt{2}(2\mathcal{M}_5 - \mathcal{M}_3) + 2\mathcal{M}_6 - \mathcal{M}_4]. \end{aligned} \quad (22)$$

As will be shown soon,  $\beta_1$  and  $\beta_2$  do not contribute at all to the PV  $\pi NN$  coupling constant. On the other hand,  $\beta_3$  and  $\beta_4$  do come into play, so that we need to examine them in detail. We can explicitly express  $\beta_3$  and  $\beta_4$  in terms of the Wilson coefficients such that we can see which terms contribute dominantly to the PV  $\pi NN$  coupling constant.  $\beta_3$  and  $\beta_4$  are rewritten as

$$\begin{aligned} \beta_3 &= \frac{G_F \sin^2 \theta_W}{12\sqrt{2}} [(c_2 + c_4)f_\pi^4 - 16N_c(c_2 - c_4)\langle \bar{\psi}\psi \rangle_M (J_3 - J_4)], \\ \beta_4 &= \frac{G_F \sin^2 \theta_W}{12\sqrt{2}N_c} [(c_1 + c_3)f_\pi^4 - 16N_c(c_1 - c_3)\langle \bar{\psi}\psi \rangle_M (J_3 - J_4)], \end{aligned} \quad (23)$$

which clearly shows that  $\beta_4$  is the subleading order in the large  $N_c$  limit with respect to  $\beta_3$ . Note that the structure of the  $\beta_4$  is the same as that of  $\beta_3$  except for the Wilson coefficients and the  $1/N_c$  factor. The magnitudes of the second terms in Eq. (23) are much larger than those of the first ones, so we can ignore approximately the first terms. That is,  $\beta_3$  and  $\beta_4$  can be expressed as

$$\beta_3 \approx \frac{4G_F \sin^2 \theta_W N_c}{3\sqrt{2}} c_2 \langle \bar{\psi}\psi \rangle_M (J_4 - J_3), \quad \beta_4 \approx -\frac{c_3}{c_2 N_c} \beta_3, \quad (24)$$

which indicates that  $\beta_3$  is larger than  $\beta_4$  approximately by (70–75)%.

In Table 3, we list the results for the  $\beta_i$ . Note that  $\beta_3$  and  $\beta_4$  have the same sign because  $c_2$  and  $c_3$  have different relative signs as shown in Table 1. The magnitude of  $\beta_3$  indeed turns out to be about 75% larger than the  $\beta_4$ , as expected from Eq. (24).

### 3. Parity-violating $\pi NN$ coupling constant

We are now in a position to determine the PV  $\pi NN$  coupling constant. Starting from Eq. (21), we are able to derive the PV  $\pi NN$  coupling constant. We already have shown explicitly how one can obtain the PV  $\pi NN$  coupling constant, based on the  $\chi$ QSM [22].

Thus, we want to briefly explain the procedure of computing the  $h_{\pi NN}^1$  within the model. The PV  $\pi NN$  coupling constant can be derived by solving the following matrix element:

$$\begin{aligned} \langle N | \mathcal{H}_W^{\Delta I=1} | \pi^a N \rangle &= \frac{G_F}{\sqrt{6}} \frac{\sin^2 \theta_W}{3} \sum_{i=1}^4 \langle N | \mathcal{Q}_i(z; \mu) | \pi^a N \rangle \\ &= \frac{G_F}{\sqrt{6}} \frac{\sin^2 \theta_W}{3} \sum_{i=1}^4 \int d^4 \xi (k^2 + m_\pi^2) \\ &\quad \times e^{ik \cdot \xi} \langle N | \mathcal{T} [\mathcal{Q}_i(z; \mu) \pi^a(\xi)] | N \rangle, \end{aligned} \quad (25)$$

where the nucleon states can be constructed by using the Ioffe-type current in Euclidean space ( $x_0 = -ix_4$ ) [32,33]:

$$\begin{aligned} |N(p_1)\rangle &= \lim_{y_4 \rightarrow -\infty} e^{p_4 y_4} \mathcal{N}^*(p_1) \int d^3 y e^{i\mathbf{p}_1 \cdot \mathbf{y}} J_N^\dagger(y) |0\rangle, \\ \langle N(p_2)| &= \lim_{x_4 \rightarrow +\infty} e^{-p_0 x_4} \mathcal{N}(p_2) \int d^3 x e^{-i\mathbf{p}_2 \cdot \mathbf{x}} \langle 0 | J_N(x). \end{aligned} \quad (26)$$

The  $J_N^\dagger$  ( $J_N$ ) constitutes  $N_c$  quarks

$$J_N(x) = \frac{1}{N_c!} \epsilon^{c_1 c_2 \dots c_{N_c}} \Gamma_{(TT_3)(JJ_3)}^{s_1 s_2 \dots s_{N_c}} \psi_{s_1 c_1}(x) \dots \psi_{s_{N_c} c_{N_c}}(x), \quad (27)$$

where  $s_1 \dots s_{N_c}$  and  $c_1 \dots c_{N_c}$  stand for spin-isospin and color indices, respectively. The  $\Gamma_{(TT_3)(JJ_3)}^{(s)}$  provides the quantum numbers  $(TT_3)(JJ_3)$  for the nucleon:  $T = 1/2$ ,  $Y = 1$  and  $J = 1/2$ . The nucleon creation operator  $J_N^\dagger$  can be obtained by taking the Hermitian conjugate of  $J_N$ . The matrix element in Eq. (25) is just the four-point correlation function given as

$$\lim_{\substack{y_0 \rightarrow -\infty \\ x_0 \rightarrow +\infty}} \sum_{i=1}^4 \langle 0 | \mathcal{T} [J_N(x) \mathcal{Q}^i(z; \mu) \partial_\mu A_\mu^a(\xi) J_N^\dagger(y)] | 0 \rangle = \lim_{\substack{y_0 \rightarrow -\infty \\ x_0 \rightarrow +\infty}} \mathcal{K}, \quad (28)$$

where  $A_\mu^a$  stands for the axial-vector current. Note that we have used the partial conservation of the axial-vector current (PCAC). In principle, the four-point correlation function  $\mathcal{K}$  can be computed by solving the following functional integral

$$\begin{aligned} \mathcal{K} &= \frac{1}{\mathcal{Z}} \int D\psi D\psi^\dagger DU J_N(x) \mathcal{Q}^i(z; \mu) \partial_\mu A_\mu^a(\xi) J_N^\dagger(y) \\ &\quad \times \exp \left[ \int d^4 x \psi^\dagger \left( i\hat{\not{p}} + i\sqrt{M(-\partial^2)} U \gamma_5 \sqrt{M(-\partial^2)} \right) \psi \right]. \end{aligned} \quad (29)$$

As was already mentioned in the previous work [22], it is extremely complicated to deal with Eq. (29) technically, since the PV  $\pi NN$  coupling constant arises from both the two-body quark operators  $\mathcal{Q}^i$  and the axial-vector one, which causes laborious triple sums over quark levels already at the leading order in the large  $N_c$  expansion. Thus, we employ the gradient expansion method as in Ref. [22]. In the gradient expansion,  $(\beta U/M) \ll 1$  is used as an expansion method [32] to expand the quark propagator in the pion background field, with the pion momentum assumed to be small. This approximation of the gradient expansion is known to describe static properties of the nucleon qualitatively well [32]. Equivalently, we can directly start from the effective weak chiral Lagrangian in Eqs. (12), (21) already derived in the previous Section.

The classical soliton is assumed to have a hedgehog symmetry, so that it can be parametrized in terms of the soliton profile function  $P(r)$

$$U_0 = \exp(i\boldsymbol{\tau} \cdot \hat{\mathbf{r}} P(r)). \quad (30)$$

In principle,  $P(r)$  can be found by solving the equations of motion self-consistently [33]. However, we will employ a parametrized form of  $P(r)$  which is very close to the self-consistent one. The classical soliton field can be fluctuated such that the pion field can be coupled to a  $\Delta I = 1$  two-body quark operator

$$U = \exp\left(i \frac{\boldsymbol{\tau} \cdot \boldsymbol{\pi}}{2f_\pi}\right) U_0 \exp\left(i \frac{\boldsymbol{\tau} \cdot \boldsymbol{\pi}}{2f_\pi}\right). \quad (31)$$

Since the traces of the left and right currents over flavor space vanish, i.e.

$$\langle R_\mu + L_\mu \rangle = 0, \quad \langle R_\mu - L_\mu \rangle = 0, \quad (32)$$

the terms with  $\mathcal{N}_1$ ,  $\mathcal{N}_2$ ,  $\mathcal{N}_3$ , and  $\mathcal{N}_4$  do not contribute to  $h_{\pi NN}^1$  as shown in our previous analysis [22] with the DDH effective Hamiltonian [9]. Considering the fact that  $\mathcal{N}_1$  and  $\mathcal{N}_2$  contain the Wilson coefficient  $c_5$ , which is the most dominant one, and  $\mathcal{N}_3$  and  $\mathcal{N}_4$  have  $c_7$  that is the second largest one, one can explain a part of the reason why  $h_{\pi NN}^1$  turns out to be rather small in the present approach.

When it comes to all other terms, we can approximately rewrite  $L_\mu^2$  and  $R_\mu^2$  as

$$\begin{aligned} L_\mu L_\mu &\simeq \frac{i}{2f_\pi} \left( L_\mu^0 L_\mu^0 \boldsymbol{\tau} \cdot \boldsymbol{\pi} - \boldsymbol{\tau} \cdot \boldsymbol{\pi} L_\mu^0 L_\mu^0 \right), \\ R_\mu R_\mu &\simeq \frac{i}{2f_\pi} \left( \boldsymbol{\tau} \cdot \boldsymbol{\pi} R_\mu^0 R_\mu^0 - R_\mu^0 R_\mu^0 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \right) \end{aligned} \quad (33)$$

with  $L_\mu^0 = iU_0^\dagger \partial_\mu U_0$  and  $R_\mu^0 = iU_0 \partial_\mu U_0^\dagger$ , so that we get

$$\langle (R_\mu R_\mu - L_\mu L_\mu) \tau_3 \rangle = i \frac{\sqrt{2}}{f_\pi} \langle (R_\mu^0 R_\mu^0 + L_\mu^0 L_\mu^0) (\tau^- \pi^+ - \tau^+ \pi^-) \rangle, \quad (34)$$

where  $\tau^\pm$  and  $\pi^\pm$  are defined in the spherical basis as

$$\tau^\pm = \frac{1}{2} (\tau^1 \pm i\tau^2), \quad \pi^\pm = \frac{1}{\sqrt{2}} (\pi^1 \pm i\pi^2). \quad (35)$$

The relevant effective Lagrangian is then expressed as

$$\mathcal{L}_{\text{eff}}^{\text{SU}(2)} = (\beta_3 + \beta_4) \frac{i\sqrt{2}}{f_\pi} \langle (R_\mu^0 R_\mu^0 + L_\mu^0 L_\mu^0) (\tau^+ \pi^- - \tau^- \pi^+) \rangle, \quad (36)$$

where  $\beta_3$  and  $\beta_4$  are defined already in Eq. (22).

Since we have already explained how the quantization of the soliton is performed in the context of the PV  $\pi NN$  coupling constant in Ref. [22], we proceed to compute the  $h_{\pi NN}^1$  within this framework. For simplicity, let us consider the PV process  $n + \pi^+ \rightarrow p$ . Then, we need to compute the following trace

$$\langle (R_\mu^0 R_\mu^0 + L_\mu^0 L_\mu^0) \tau^+ \rangle. \quad (37)$$

Defining isovector fields  $r_\mu^i$  and  $l_\mu^i$  as

$$R_\mu^0 = -r_\mu^i \tau^i, \quad L_\mu^0 = -l_\mu^i \tau^i \quad (38)$$

and using an identity  $\langle \tau^i \tau^j \tau^k \rangle = 2i\epsilon^{ijk}$ , we obtain

$$\begin{aligned} \langle R_\mu^0 R_\mu^0 \tau^+ \rangle &= (r_\mu^1 + ir_\mu^2) r_\mu^3 - r_\mu^3 (r_\mu^1 + ir_\mu^2), \\ \langle L_\mu^0 L_\mu^0 \tau^+ \rangle &= (l_\mu^1 + il_\mu^2) l_\mu^3 - l_\mu^3 (l_\mu^1 + il_\mu^2). \end{aligned} \quad (39)$$

Thus, we arrive at the final form of the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{SU}(2)} &= (\beta_3 + \beta_4) \frac{i\sqrt{2}}{f_\pi} \\ &\quad \times \left[ (r_\mu^1 + ir_\mu^2) r_\mu^3 - r_\mu^3 (r_\mu^1 + ir_\mu^2) + (l \rightarrow l) \right] \pi^-, \end{aligned} \quad (40)$$

from which we can derive the PV  $\pi NN$  coupling constant. Using the collective quantization discussed in Ref. [22], we get

$$\begin{aligned} & \int d^3x \langle p \uparrow | r_\mu^3 (r_\mu^1 + i r_\mu^2) | n \uparrow \rangle = - \int d^3x \langle p \uparrow | (r_\mu^1 + i r_\mu^2) r_\mu^3 | n \uparrow \rangle \\ & = \frac{2\pi}{3I^2} \int dr r^2 \sin^2 P(r) \left[ \sin^2 P(r) - 3 \cos^2 P(r) \right], \end{aligned} \quad (41)$$

where  $I$  denotes the moment of inertia [32] expressed as

$$\begin{aligned} I & = \frac{N_c}{12} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \text{Tr} \left( \tau^i \frac{1}{\omega + iH} \tau^i \frac{1}{\omega + iH} \right) \\ & \approx \frac{8}{3} \pi f_\pi^2 \int_0^\infty dr r^2 \sin^2 P(r). \end{aligned} \quad (42)$$

Here,  $\omega$  is the energy frequencies of the quark levels and Tr stands for the functional trace over coordinate space, isospin and Dirac spin space. The second term was derived approximately by the gradient expansion. Similarly, we obtain the same result for  $l_\mu$ . Having carried out the calculation of the matrix element for the collective operators, we finally derive the PV  $\pi NN$  coupling constant  $h_{\pi NN}^1$  as

$$\begin{aligned} h_{\pi NN}^1 & = i \langle p \uparrow | \mathcal{L}_{\text{eff}}^{\text{SU}(2)} | n \uparrow, \pi^+ \rangle \\ & = \frac{8\sqrt{2}\pi}{3f_\pi I^2} (\beta_3 + \beta_4) \\ & \quad \times \int dr r^2 \sin^2 P(r) \left[ \sin^2 P(r) - 3 \cos^2 P(r) \right]. \end{aligned} \quad (43)$$

It is interesting to see that Eq. (43) is exactly the same as the expression obtained in Ref. [22] except for the coefficient  $\beta_3 + \beta_4$ .

In order to compute the PV  $\pi NN$  coupling constant, we employ the following numerical values of the constants involved in the present work: the Fermi constant  $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$ , the Weinberg angle  $\sin^2 \theta_W = 0.23116$ , and the pion decay constant  $f_\pi = 0.0924 \text{ GeV}$ . Concerning the profile function, we have already examined the dependence of  $h_{\pi NN}^1$  on types of the profile functions [22]. In the present work, we employ the physical profile function expressed as

$$P(r) = \begin{cases} 2 \arctan \left( \frac{r_0}{r} \right)^2, & (r \leq r_x), \\ P_0 e^{-m_\pi r} (1 + m_\pi r) / r^2, & (r > r_x), \end{cases} \quad (44)$$

where  $r_0$  is defined as  $r_0 = \sqrt{\frac{3g_A}{16\pi f_\pi^2}}$  with the axial-vector constant  $g_A = 1.26$ .  $P_0$  and  $r_x$  are given as  $P_0 = 2r_0^2$ , and  $r_x = 0.752 \text{ fm}$ , respectively. The profile function in Eq. (44) satisfies a correct behavior of the Yukawa tail. Then, the moment of inertia is obtained to be  $I = 3.32 \text{ GeV}^{-1}$ .

Numerical results for  $h_{\pi NN}^1$  are summarized in Table 4. In Ref. [31], it was shown that NLO contributions alter the values of

**Table 4**  
 $h_{\pi NN}^1$  in units of  $10^{-8}$ . Notations are the same as in Table 1.

	LO	NLO (Z)	NLO (Z + W)
$h_{\pi NN}^1$	10.96	8.69	8.74

**Table 5**  
 $h_{\pi NN}^1$  in units of  $10^{-8}$  with various theoretical works.

DDH [9]	DZ [20]	KS [19]	QCD sum rules [23]	Skyrme Model [18]	Lattice QCD [24]	Present work
45.6	11.4	60	2	8.0 – 13	$10.99 \pm 5.05_{-0.64}^{+0.58}$	8.74

the Wilson coefficients at  $\mu = 1 \text{ GeV}$  by about (10–20)%, which actually lessens the value of the  $h_{\pi NN}^1$  by about 25% as shown in Table 4. As already examined in Eqs. (23), (24),  $\beta_3$  plays a dominant role in determining  $h_{\pi NN}^1$ . Thus, the most important operator in the  $\Delta I = 1$  effective weak Hamiltonian is  $O_2$ , which contains the Wilson coefficient  $c_2$ . As clearly shown in Table 4, the NLO QCD radiative corrections suppress the PV  $\pi NN$  coupling constant. In fact, we have shown already in the previous work [22], the QCD radiative corrections strongly diminish the value of  $h_{\pi NN}^1$ . This behavior contrasts with the case of  $K$  nonleptonic decays, where the penguin diagrams enhance the contribution to the  $\Delta I = 1/2$  channel. Concerning the dependence on the profile functions, the linear and arctangent functions give the NLO (Z + W) results  $6.44 \times 10^{-8}$  and  $7.53 \times 10^{-8}$ , respectively. Therefore dependence on the profile function does not change the essential conclusion of this work.

In Table 5, we compare the present result with those of various theoretical works. The present result turns out to be about 5 times smaller than the DDH “best value”. We find that the result from the QCD sum rules predicts the smallest value of  $h_{\pi NN}^1$  whereas Ref. [19] yields the largest result, in which the importance of the strangeness contribution was emphasized. Compared to the value of  $h_{\pi NN}^1$  from lattice QCD with connected diagrams considered only, the present result is in good agreement with it. On the experiment side, forbidden  $\gamma$ -ray transition of  $^{18}\text{F}$  reported an upper limit  $h_{\pi NN}^1 < 1.3 \times 10^{-6}$  [60,61]. Recent analyses of the PV analyzing power in the  $pp$  scattering give wide ranges,  $h_{\pi NN}^1 = (1.1 \pm 2) \times 10^{-6}$  [62], and  $-1 \times 10^{-6} < h_{\pi NN}^1 < 2 \times 10^{-6}$  [63]. Our result is within the range of these experiment analyses, but the large magnitude of the boundary values stimulates extension of our investigation to the SU(3) flavor space.

#### 4. Summary and outlook

In the present work, we investigated the parity-violating pion-nucleon coupling constant. Starting from the  $\Delta I = 1$  effective weak Hamiltonian [31] that considered the next-to-leading order QCD radiative corrections, we derived the effective weak chiral Lagrangian with the weak low-energy constants determined in the  $\Delta I = 1$  and  $\Delta S = 0$  channel. In order to calculate the parity-violating pion-nucleon coupling constant  $h_{\pi NN}^1$ , we employed the chiral quark-soliton model. Using the gradient expansion, which is equivalent to using the effective weak chiral Lagrangian directly, we were able to compute the values of  $h_{\pi NN}^1$ . We found that the first four terms of the Lagrangian did not contribute at all to  $h_{\pi NN}^1$ , which partially explains why the value of  $h_{\pi NN}^1$  should be small. It was also found that the main contribution to  $h_{\pi NN}^1$  arose from the operator  $O_2$  in the effective weak Hamiltonian. We also noted that the next-to-leading order QCD radiative corrections further suppress the value of  $h_{\pi NN}^1$  and as a result we obtained  $h_{\pi NN}^1 = 8.74 \times 10^{-8}$ . We compared this result with those from various theoretical models including the recent result from lattice QCD. The present result was shown to be in agreement with that from lattice QCD.

The present work can be extended to the SU(3) case in which the strange quark comes into play. Another merit of the chiral quark-soliton model is that the explicit breaking of flavor SU(3) symmetry can be treated systematically, the strange quark mass being considered as a perturbation. Thus, it is interesting to examine the contribution of the strange quark and its current quark

mass to the parity-violating pion–nucleon coupling constant. Other coupling constants such as  $h_{\rho NN}$  and  $h_{\omega NN}$  can be studied within the same framework. The related works are under way.

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