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The three-body decays $B_{(s)} \to \rho f_0(X) \to \rho\pi^-\pi^+$ in pQCD factorization approach

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Abstract

In this paper, we study three-body decays $B_{(s)} \to \rho f_0(X) \to \rho\pi^-\pi^+$ in the perturbative QCD (pQCD) factorization approach. By using two-pion distribution amplitudes (DAs), these decays can proceed mainly via quasi-two-body channels. The Flatté model for the $f_0(980)$ resonance and the Breit–Wigner formula for the $f_0(500)$, $f_0(1500)$, and $f_0(1790)$ resonances are adopted to parameterize the time-like scalar form factors. We also use Bugg's model for the wide $f_0(500)$ as a comparison. We evaluate S-wave resonance contributions, and show the contributions to the $B_{(s)} \to \rho\pi^+\pi^-$ decay spectrums with respect to the two-pion invariant mass $M(\pi^+\pi^-)$. The pQCD prediction of $B \to \rho f_0(500)$ is $\mathcal{B}(B^0 \to \rho^0 f_0(500)) = 4.43 \times 10^{-8}$, $\mathcal{B}(B^+ \to \rho^+ f_0(500)) = 3.84 \times 10^{-8}$ for Breit–Wigner model, and $\mathcal{B}(B^0 \to \rho^0 f_0(500)) = 4.91 \times 10^{-8}$, $\mathcal{B}(B^+ \to \rho^+ f_0(500)) = 4.07 \times 10^{-8}$ for Bugg's model. We also predict other resonance contributions $\mathcal{B}(B_s \to \rho^0 f_0(980)) = 8.91 \times 10^{-8}$, $\mathcal{B}(B_s \to \rho^0 f_0(1500)) = 1.43 \times 10^{-8}$ and $\mathcal{B}(B_s \to \rho^0 f_0(1790)) = 2.78 \times 10^{-9}$. This study in pQCD approach can provide a ready reference to the running and forthcoming experiments. Further improvements in theories and experiments are expected so that we can understand the inner structure of the scalar mesons f_0.

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1. Introduction

Recently, the first experimental achievements for the three-body hadronic decays are reported in the LHCb Collaboration. They give the branching fractions and sizable direct CP asymmetries for $B \to KKK$, $B \to K\pi\pi$ and $B \to \pi\pi\pi$ decays [1,2]. These new experimental measurements evoke much more theoretical studies on three-body decays. Understanding of such decays is becoming more and more important, especially because it provides a serious background when we want to extract information from the resonance decays. Thus, it's very important to develop theoretical studies in this direction for hadronic B decays.

Based on the k_T factorization theorem, pQCD approach for two-body B meson decays has been developed by many researcher over many years, and this formalism which is shown to be infrared-finite, gauge-invariant, and consistent with the factorization assumption in the heavy-quark limit [3–7] has been successful. Sachrajda et al. have investigated the reliability of Sudakov effects, and have expressed an opposite opinion on the effect of Sudakov suppression in [8]. The conclusion is that Sudakov suppression are so weak that they can not be applied in B decays. However, their conclusion is drawn based on a very sharp B meson wave function, which is not favored by experimental data.

Recently, this method has been applied to the three-body B meson decays [9–13] in which the two-hadron distribution amplitudes were introduced [14–16] to make it possible to calculate the three-body final states. For these decays, the end-point singularities are smeared by the two-pion invariant mass [9], and collinear factorization holds. It's not practical to compute the hard b-quark decay kernels, which involve two virtual gluons at leading order, due to a large number of diagrams. Besides, the region which contain the two hard gluons, is power-suppressed and can be neglected.

A factorization formula for a $B \to h_1h_2h_3$ decay amplitude is written as

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1h_2} \otimes \Phi_{h_3}, \tag{1}$$

where the LO hard kernel H contains a single hard gluon exchange in three-body hadronic decays in a similar way as the one for the two-body decays. The functions Φ are the wave functions for the B and the final state mesons, which absorbs the non-perturbative dynamics characterized by the soft scale $\bar{\Lambda}$ in the process. In this region all final-state mesons carry momenta of $O(M_B)$, and all three pairs have invariant masses of $O(M_B^2)$, where M_B is the B meson mass. The dominant contribution comes from this region, where at least one pair of light mesons has the invariant mass below $O(\bar{\Lambda}M_B)$ [9], $\bar{\Lambda} = M_B - m_b$ being the B meson and b quark mass difference. This configuration contains two energetic mesons almost collimating to each other, in which both resonance contributions and nonresonance contributions through two-body decays can be included by means of an appropriate parametrization of two-meson DA $\Phi_{h_1h_2}$ [14–16].

This paper is organized as follows. After the introduction, we give a factorization formula, describe all the wave functions in Sec. 2. We give the pQCD analytic formulae for the amplitudes in Sec. 3, and in Sec. 4, we calculate and present the numerical results for resonance decays. A brief summary is given in the final section.

2. Theoretical framework

We define the kinematics for the $B \to M_{2\pi} M_\rho$ decay, in which the B meson transit into a pair of pions. In the light-cone coordinates, we assume that the two pions and ρ are moving along the direction of $n = (1, 0, 0_T)$ and $v = (0, 1, 0_T)$, respectively. In the rest frame of the B meson, the

π^+ and π^- mesons carry the momenta P_1 and P_2, respectively. The B meson momentum P_B, the total momentum of the two pions, $P = P_1 + P_2$, and the ρ meson momentum P_3 are taken as

$$P_B = \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad P = \frac{M_B}{\sqrt{2}}(1-r^2, \eta, \mathbf{0}_T), \quad P_3 = \frac{M_B}{\sqrt{2}}(r^2, 1-\eta, \mathbf{0}_T), \tag{2}$$

where $\omega^2 = P^2$ is the invariant mass of the two-pion system, $r = m_\rho/M_B$, the variable $\eta = \omega^2/[(1-r^2)M_B^2]$.

The pQCD results depend on the inputs for the nonperturbative parameters, such as distribution amplitudes, decay constants, and chiral scales for pseudoscalar mesons. For the B meson, the DA has been widely adopted as

$$\phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{xM_B}{\omega_b}\right)^2 - \frac{\omega_b^2 b^2}{2}\right], \tag{3}$$

with the Gaussian form being motivated by the oscillator model in [19]. The normalization constant N_B is related to the decay constant f_B through

$$\int_0^1 dx\, \phi_B(x, b=0) = \frac{f_B}{2\sqrt{2N_c}}, \tag{4}$$

with the number of color's degree of freedom $N_c = 3$, the shape parameter ω_b and the momentum fraction x of the spectator quark in the B meson. b is the conjugate variable to the spectator transverse momentum k_T.

The wave functions for the B and ρ mesons are given as

$$\Phi_B(x,b) = \frac{1}{\sqrt{2N_c}}(\not{P}_B + M_B)\gamma_5\phi_B(x,b), \tag{5}$$

$$\Phi_\rho(P_3, x) = \frac{1}{\sqrt{2N_c}}[\not{\epsilon} M_\rho\phi_\rho(x) + \not{\epsilon}\not{P}_3\phi_\rho^t(x) + M_\rho\phi_\rho^s(x)] \tag{6}$$

where the distribution amplitudes ϕ_ρ, ϕ_ρ^t, ϕ_ρ^s are collected into [20]

$$\phi_\rho(x) = \frac{3f_\rho}{\sqrt{6}}x(1-x)[1 + a_{2\rho}^{\parallel}C_2^{3/2}(t)], \tag{7}$$

$$\phi_\rho^t(x) = \frac{f_\rho^T}{2\sqrt{6}}[3t^2 + 0.3t^2(5t^2-3) + 0.21(3 - 30t^2 + 35t^4)],$$

$$\phi_\rho^s(x) = \frac{3f_\rho^T}{2\sqrt{6}}(1-2x)[1 + 0.76(10x^2 - 10x + 1)].$$

For the two pions, we write DAs as [21]

$$\Phi_{2\pi}(z,\zeta,\omega^2) = \frac{1}{\sqrt{2N_c}}\left[\not{P}\phi_v(z,\zeta,\omega^2) + \omega\phi_s(z,\zeta,\omega^2) + \omega(\not{n}\not{v} - 1)\phi_t(z,\zeta,\omega^2)\right], \tag{8}$$

where n and v are the unit vectors pointing to the plus and minus directions in the light-cone coordinates. We define $\zeta = P_1^+/P^+$, as the π^+ meson momentum fraction. We give the leading term in the complete Gegenbauer expansion of $\phi_i(z,\zeta,\omega^2)$ [14–18,22]:

$$\phi_v = \frac{9F_s(\omega^2)}{\sqrt{2N_c}}a_2^{I=0}z(1-z)(1-2z), \quad \phi_s = \frac{F_s(\omega^2)}{\sqrt{2N_c}}, \quad \phi_t = \frac{F_s(\omega^2)}{\sqrt{2N_c}}(1-2z), \tag{9}$$

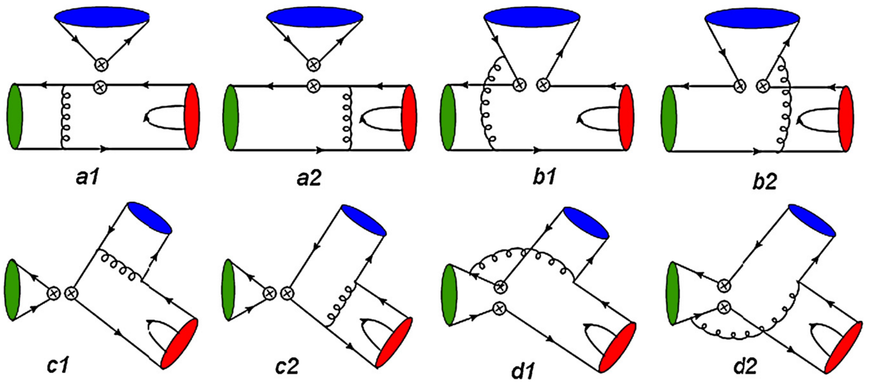

Fig. 1. Diagrams contributing to the $B \to \rho\pi\pi$ decays.

where the two-meson DA $\phi_v(z,\zeta,\omega^2)$ is the twist-2 component, and $\phi_s(z,\zeta,\omega^2)$, $\phi_t(z,\zeta,\omega^2)$ are the twist-3 components. $F_s(\omega^2)$ on the r.h.s. of the equation are time-like scalar form factors.

3. Perturbative calculations in pQCD approach

In this section, the hard part H(t) will be calculated. This part involves the four quark operators and the necessary hard gluon connecting the spectator quark and the four quark operators. The amplitudes for each diagram including wave functions are illustrated by Fig. 1. The red bubble denotes the two-pion distribution amplitude, the green one denotes the B distribution amplitude.

We simplify the distribution amplitudes, $\phi_B = \phi_B(x,b)$, $\phi_\rho^{(t,s)} = \phi_\rho^{(t,s)}(y)$, $\phi_{v,s,t} = \phi_{v,s,t}(z,\zeta,\omega^2)$, and denote $1-x$ by $\bar{x}$ below. We first calculate the usual factorizable emission diagrams (a1) and (a2), the total contribution of (V−A)(V−A) and (V−A)(V+A) currents is

$$
\begin{aligned}
F_e^{LL}(\zeta,\eta) + F_e^{LR}(\zeta,\eta) = {} & 8\pi f_\rho r M_B^3 C_F \int_0^1 dx dz \int_0^\infty b_1 db_1 b_2 db_2 \phi_B \\
& \times \Big\{ -\Big[\sqrt{\eta \bar{r}^2}\phi_s(\bar{\eta}(1-2z\bar{r}^2) + r^2(2\eta z - 1)) \\
& + \sqrt{\eta \bar{r}^2}\phi_t(\bar{\eta} - 2z\bar{\eta}\bar{r}^2 - 2\eta r^2 z + r^2) \\
& + \phi_v(\bar{\eta}\bar{r}^2 + z\bar{\eta}\bar{r}^{2^2} - \eta r^2(\eta z + 1))\Big] D_1(t_1)E_1(t_1)h(m_{\alpha 1}, m_{\beta 1}, b_2, b_1)S_t(z) \\
& + \Big[2\sqrt{\eta \bar{r}^2}\phi_s(r^2(\eta - x) - \bar{\eta}\bar{r}^2) + \bar{r}^2\phi_v(\eta\bar{\eta} + r^2(x-\eta))\Big] \\
& \cdot D_1(t_2)E_1(t_2)h(m_{\alpha 2}, m_{\beta 2}, b_1, b_2)S_t(x)\Big\},
\end{aligned} \tag{10}
$$

the hard functions can be found in Refs. [23,24],

$$
\begin{aligned}
& h(m_\alpha, m_\beta, b_1, b_2) \\
& = [\theta(b_1 - b_2)I_0(m_\alpha b_2)K_0(m_\alpha b_1) + \theta(b_2 - b_1)I_0(m_\alpha b_1)K_0(m_\alpha b_2)]K_0(m_\beta b_2)
\end{aligned} \tag{11}
$$

where K_0, I_0 denote modified Bessel functions of order zero, $m_{\alpha 1} = M_B\sqrt{z\bar{r^2}}$, $m_{\beta 1} = M_B\sqrt{zx\bar{r^2}}$, $m_{\alpha 2} = M_B\sqrt{(x-\eta)\bar{r^2}}$, $m_{\beta 2} = m_{\beta 1}$, and the hard scale $t_i = \max[\sqrt{|m_{\alpha i}^2|}, \sqrt{|m_{\beta i}^2|}, b_j, b_k]$. In the hard kernels, the S_t re-sums the threshold logarithms $\ln^2 z$ to all orders and it has been parameterized as [25]

$$S_t(z) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[z(1-z)]^c, \tag{12}$$

where $c = 0.3$. The evolution factors $E_1(t) = \alpha_s(t)S_B(t)S_{2\pi}(t)$. The Sudakov exponents associated with all mesons of the original and final state are written as

$$S_B(t) = \exp\left[-s(xP_B^+, b_1) - \frac{5}{3}\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right], \tag{13}$$

$$S_\rho(t) = \exp\left[-s(yP_3^-, b_3) - s((1-y)P_3^-, b_3) - 2\int_{1/b_3}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right], \tag{14}$$

$$S_{2\pi}(t) = \exp\left[-s(zP^+, b_2) - s((1-z)P^+, b_2) - 2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right], \tag{15}$$

where the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The explicit expression of the exponent s is shown in [26–28]. The coefficient $D_1 = \xi_u d_2 - \xi_t\,(d_4 + d_{10})$ is for B^+ decays, $D_1 = [\xi_u d_1 - \xi_t\,(3d_7 + 3d_9)\,/2]/\sqrt{2}$ is for B_s^0 decays associated with the $s\bar{s}$ component, and $D_1 = [\xi_u d_1 - \xi_t\,(3d_7 + 3d_9)\,/2 + \xi_t(d_4 - d_{10}/2)]/\sqrt{2}$ is for B^0 decays with the $n\bar{n}$ ($u\bar{u}$ or $d\bar{d}$) component. In this paper, $\xi_u = V_{ub}^* V_{ud}$, $\xi_t = V_{tb}^* V_{td}$. The coefficients d_i are the combinations of the ordinary Wilson coefficients C_i as in Ref. [13].

For the unfactorizable diagrams (b1) and (b2), (V−A)(V−A) and (V−A)(V+A) currents contributions are given as

$$\begin{aligned}
M_e^{LL}(\zeta,\eta) = &\frac{32}{\sqrt{6}}\pi C_F M_B^2\int_0^1 dxdydz\int_0^\infty b_1db_1b_2db_2\phi_B\psi_L \\
&\times\Big\{\big[-(\sqrt{\eta\bar{r^2}}\phi_s(r^2(\eta z-x)-z\bar{\eta}\bar{r^2})+\sqrt{\eta\bar{r^2}}\phi_t(r^2(2\bar{\eta}\bar{y}-x+\eta z)+z\bar{\eta}\bar{r^2}) \\
&-\phi_v(r^2-\bar{\eta})(\bar{r^2}(\bar{\eta}\bar{y}-x+2\eta z)+\eta r^2\bar{y}))\big]D_3(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_1,b_2) \\
&+\big[\sqrt{\eta\bar{r^2}}\phi_s(r^2(\eta z-x)-z\bar{\eta}\bar{r^2})-\sqrt{\eta\bar{r^2}}\phi_t(\bar{\eta}(z\bar{r^2}+2r^2y)+r^2(\eta z-x)) \\
&-\phi_v(\eta r^2-\bar{\eta}\bar{r^2})(z\bar{r^2}+y\bar{\eta}+r^2y-x+\eta z)\big] \\
&\times D_3(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_1,b_2)\Big\},
\end{aligned} \tag{16}$$

where $D_3 = \xi_u C_1 - \xi_t\,(C_3 + C_9)$ is for B^+ decays, $D_3 = [\xi_u C_2 - \xi_t(-C_3 + C_9/2 + 3C_{10}/2)]/\sqrt{2}$ is for B^0 decays associated with the $n\bar{n}$ component, and $D_3 = (\xi_u C_2 - 3\xi_t C_{10}/2)\,/\sqrt{2}$ with the $s\bar{s}$ component, $m_{\alpha 1} = M_B\sqrt{zx\bar{r^2}}$, $m_{\beta 1} = M_B\sqrt{(x-\bar{\eta}\bar{y})(z\bar{r^2}+r^2\bar{y})}$, $m_{\alpha 2} = m_{\alpha 1}$, $m_{\beta 2} = M_B\sqrt{(x-\bar{\eta}y)(z\bar{r^2}+r^2y)}$, $E_2(t) = \alpha_s(t)S_B(t)S_{2\pi}(t)S_\rho(t)|_{b_3=b_2}$.

$$
\begin{aligned}
M_e^{LR}(\zeta,\eta) = {} & \frac{32}{\sqrt{6}}\pi r M_B^2 C_F \int_0^1 dxdydz \int_0^\infty b_1db_1b_2db_2\phi_B \qquad (17)\\
& \times \Big\{\big[\phi_v(\bar{r^2}(\phi_\rho^s(\bar{\eta}\bar{y}-x+2\eta z)+\bar{\eta}\phi_\rho^t(\bar{\eta}\bar{y}-x))+\eta r^2\bar{y}(\phi_\rho^s-\bar{\eta}\phi_\rho^t))\\
& + \sqrt{\eta\bar{r^2}}\phi_s(\phi_\rho^s(\bar{y}(\bar{\eta}+r^2)+z\bar{r^2}-x+\eta z)\\
& + \bar{\eta}\phi_\rho^t(\bar{y}(\bar{\eta}-r^2)-z\bar{r^2}-x+\eta z))\\
& - \sqrt{\eta\bar{r^2}}\phi_t(\bar{\eta}\phi_\rho^t(\bar{y}(\bar{\eta}+r^2)+z\bar{r^2}-x+\eta z)\\
& - \phi_\rho^s(\bar{y}(r^2-\bar{\eta})+z\bar{r^2}+x-\eta z))\big]D_4(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_1,b_2)\\
& - \big[\phi_v(\phi_\rho^s(\bar{r^2}(y\bar{\eta}-x+2\eta z)+\eta r^2y)+\bar{\eta}\phi_\rho^t(\bar{r^2}(x-y\bar{\eta})+\eta r^2y))\\
& + \sqrt{\eta\bar{r^2}}\phi_s(\phi_\rho^s(z\bar{r^2}+y\bar{\eta}+r^2y-x+\eta z)\\
& + \bar{\eta}\phi_\rho^t(z\bar{r^2}-y\bar{\eta}+r^2y+x-\eta z))\\
& + \sqrt{\eta\bar{r^2}}\phi_t(\phi_\rho^s(z\bar{r^2}-y\bar{\eta}+r^2y+x-\eta z)\\
& + \bar{\eta}\phi_\rho^t(z\bar{r^2}+y\bar{\eta}+r^2y-x+\eta z))\big]\\
& \cdot D_4(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_1,b_2)\Big\},
\end{aligned}
$$

where $D_4 = -\xi_t\,(C_5+C_7)$ is for B^+ decays, $D_4 = +\xi_t\,(C_5-C_7/2)\,/\sqrt{2}$ is for B^0 decays associated with the $n\bar{n}$ component.

We obtain (S−P)(S+P) operators from (V−A)(V+A) ones,

$$
\begin{aligned}
M_e^{SP}(\zeta,\eta) = {} & \frac{32}{\sqrt{6}}\pi C_F M_B^2 \int_0^1 dxdydz \int_0^\infty b_1db_1b_2db_2\phi_B\psi_L \qquad (18)\\
& \times\Big\{\big[\sqrt{\eta\bar{r^2}}\phi_s(r^2(\eta z-x)-z\bar{\eta}\bar{r^2})-\sqrt{\eta\bar{r^2}}\phi_t(r^2(2\bar{\eta}\bar{y}-x+\eta z)+z\bar{\eta}\bar{r^2})\\
& - \phi_v(\eta r^2-\bar{\eta}\bar{r^2})(\bar{y}(\bar{\eta}+r^2)+z\bar{r^2}-x+\eta z)\big]\\
& \times D_5(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_1,b_2)\\
& - \big[\sqrt{\eta\bar{r^2}}\phi_s(r^2(\eta z-x)-z\bar{\eta}\bar{r^2})+\sqrt{\eta\bar{r^2}}\phi_t(\bar{\eta}(z\bar{r^2}+2r^2y)+r^2(\eta z-x))\\
& - \phi_v(r^2-\bar{\eta})(\bar{r^2}(y\bar{\eta}-x+2\eta z)+\eta r^2y)\big]\\
& \times D_5(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_1,b_2)\Big\},
\end{aligned}
$$

where $D_5 = -\xi_t\,(3C_8/2)\,/\sqrt{2}$ is for B^0 decays.

For the factorizable annihilation diagrams (c1) and (c2), we obtain (V−A)(V−A) currents contribution associated with the $n\bar{n}$ component,

$$
\begin{aligned}
&F_a^{LL}(\zeta,\eta)+F_a^{LR}(\zeta,\eta) \qquad (19)\\
&=8\pi f_B M_B^2 C_F\int_0^1 dydz\int_0^\infty b_2db_2b_3db_3\Big\{\big[r(\bar\eta(r\phi_\rho\phi_v(\eta-\bar{r^2})\\
&+2\sqrt{\eta\bar{r^2}}\phi_\rho^s(\phi_s+\phi_t))+2r^2\sqrt{\eta\bar{r^2}}\phi_\rho^s(\phi_s-\phi_t))+\bar z(\phi_\rho\phi_v(\eta^2r^2-\bar\eta\bar{r^2}^2)\\
&+2r\sqrt{\eta\bar{r^2}}\phi_\rho^s(\phi_s(\bar{r^2}+\eta)+\phi_t(\eta-\bar{r^2})))\big]D_1(t_1)E_3(t_1)h(m_{\alpha1},m_{\beta1},b_2,b_3)S_t(z)\\
&-\big[\phi_\rho\phi_v(\bar{r^2}(-\eta\bar\eta-y\bar\eta^2+\eta r^2)+\eta r^4y)+2r\sqrt{\eta\bar{r^2}}\phi_s(\phi_\rho^s(\bar{r^2}+y\bar\eta+\eta+r^2y)\\
&+\bar\eta\phi_\rho^t(-\bar{r^2}+y\bar\eta+\eta+r^2(-y)))\big]\cdot D_1(t_2)E_3(t_2)h(m_{\alpha2},m_{\beta2},b_3,b_2)S_t(y)\Big\},
\end{aligned}
$$

where $E_3(t)=\alpha_s(t)S_{2\pi}(t)S_\rho(t)$, $m_{\alpha1}=M_B\sqrt{-r^2-\bar z\bar{r^2}}$, $m_{\beta1}=M_B\sqrt{-(y\bar\eta+\eta)(r^2y+\bar z\bar{r^2})}$, and $m_{\alpha2}=M_B\sqrt{-(y\bar\eta+\eta)(r^2y+\bar{r^2})}$, $m_{\beta2}=m_{\beta1}$.

We get (S−P)(S+P) operators from (V−A)(V+A) ones.

$$
\begin{aligned}
F_a^{SP}(\zeta,\eta)&=-16\pi M_B^2C_F\int_0^1 dydz\int_0^\infty b_2db_2b_3db_3\Big\{\big[2r(\bar\eta(\bar{r^2}\phi_\rho^s\phi_v-r\phi_\rho\sqrt{\eta\bar{r^2}}\phi_t) \qquad (20)\\
&+\eta r^2\phi_\rho^s\phi_v)+\bar z(\phi_\rho\sqrt{\eta\bar{r^2}}\phi_s(\eta r^2-\bar\eta\bar{r^2})-\phi_\rho\sqrt{\eta\bar{r^2}}\phi_t(\bar\eta\bar{r^2}+\eta r^2)\\
&+4\eta r\bar{r^2}\phi_\rho^s\phi_v)\big]D_2(t_1)E_3(t_1)h(m_{\alpha1},m_{\beta1},b_2,b_3)S_t(z)\\
&+\big[2\phi_\rho\sqrt{\eta\bar{r^2}}\phi_s(\eta r^2-\bar\eta\bar{r^2})+r\phi_v(\phi_\rho^s(\bar{r^2}(y\bar\eta+2\eta)+\eta r^2y)\\
&+y\bar\eta\phi_\rho^t(\eta r^2-\bar\eta\bar{r^2}))\big]\cdot D_2(t_2)E_3(t_2)h(m_{\alpha2},m_{\beta2},b_3,b_2)S_t(y)\Big\},
\end{aligned}
$$

where $D_2=-\xi_t\,(d_6+d_8)$ is for B^+ decays, and $D_2=+\xi_t\,(d_6-d_8/2)\,/\sqrt{2}$ is for B^0 decays.

Similarly, we obtain all currents contributions for the unfactorizable annihilation diagrams (d1) and (d2),

$$
\begin{aligned}
M_a^{LL}(\zeta,\eta)&=-\frac{32}{\sqrt6}\pi C_FM_B^2\int_0^1 dxdydz\int_0^\infty b_1db_1b_2db_2\phi_B \qquad (21)\\
&\times\Big\{\big[\phi_\rho\phi_v(\eta(\bar\eta-r^2y\bar\eta+r^4y)\\
&-\bar{r^2}(\bar x(r^2-\bar\eta)+\bar\eta(y\bar\eta+r^2(-y)+1)+2\eta\bar z(\bar\eta-r^2)))\\
&+r\sqrt{\eta\bar{r^2}}(\phi_s(\phi_\rho^s(\bar z(\bar{r^2}+\eta)-\bar x+y\bar\eta+r^2y+3)\\
&+\bar\eta\phi_\rho^t(\bar z(\eta-\bar{r^2})-\bar x+y\bar\eta+r^2(-y)+1))\\
&+\phi_t(\phi_\rho^s(\bar z(\bar{r^2}-\eta)+\bar x-y\bar\eta+r^2y-1)\\
&-\bar\eta\phi_\rho^t(\bar z(\bar{r^2}+\eta)-\bar x+y\bar\eta+r^2y-1)))\big]D_3(t_1)E_2(t_1)h(m_{\alpha1},m_{\beta1},b_2,b_1)\\
&-\big[\phi_\rho\phi_v(\bar{r^2}+\eta)(\bar z(\eta r^2-\bar\eta\bar{r^2})-r^2x)
\end{aligned}
$$

$$+ r\sqrt{\eta \bar{r^2}}(\phi_s(\phi_\rho^s(\bar{z}(\bar{r^2}+\eta)+y\bar{\eta}+r^2y-x)$$
$$+\bar{\eta}\phi_\rho^t(\bar{z}(\bar{r^2}-\eta)-y\bar{\eta}+r^2y+x))-\phi_t(\phi_\rho^s(\bar{z}(\bar{r^2}-\eta)-y\bar{\eta}+r^2y+x)$$
$$+\bar{\eta}\phi_\rho^t(\bar{z}(\bar{r^2}+\eta)+y\bar{\eta}+r^2y-x)))\big]\cdot D_3(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_2,b_1)\Big\},$$

with $m_{\alpha 1} = M_B\sqrt{-(y\bar{\eta}+\eta)(r^2y+\bar{z}\bar{r^2})}$, $m_{\beta 1} = M_B\sqrt{(\bar{x}-\bar{\eta}y-\eta)(\bar{z}\bar{r^2}+r^2y-1)}$, $m_{\alpha 2} = m_{\alpha 1}$ and $m_{\beta 2} = M_B\sqrt{(x-\bar{\eta}y-\eta)(\bar{z}\bar{r^2}+r^2y)}$.

$$M_a^{LR}(\zeta,\eta) = \frac{32}{\sqrt{6}}\pi C_F M_B^2 \int_0^1 dxdydz \int_0^\infty b_1db_1b_2db_2\phi_B\Big\{\big[\phi_\rho\sqrt{\eta\bar{r^2}}\phi_s(2\bar{\eta}+r^2(-\bar{x}) \tag{22}$$
$$+\bar{z}(\eta r^2-\bar{\eta}\bar{r^2})-r^2)$$
$$+\phi_\rho\sqrt{\eta\bar{r^2}}\phi_t(-2\bar{\eta}+r^2(-\bar{x})+2r^2\bar{y}+\bar{z}(\bar{\eta}\bar{r^2}+\eta r^2)-r^2)$$
$$+r\phi_v(\phi_\rho^s(\bar{r^2}(\bar{x}-y\eta\bar{\eta}-2\eta\bar{z}+1)$$
$$+\eta(2-r^2y))+\bar{\eta}\phi_\rho^t(\bar{r^2}(\bar{x}-y\bar{\eta}+1)$$
$$+\eta(r^2y-2)))\big]D_4(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_2,b_1)$$
$$-\big[\phi_\rho\sqrt{\eta\bar{r^2}}\phi_s(\bar{z}(\eta r^2-\bar{\eta}\bar{r^2})-r^2x)$$
$$+\phi_\rho\sqrt{\eta\bar{r^2}}\phi_t(\bar{z}(\bar{\eta}\bar{r^2}+\eta r^2)-r^2(x-2y\bar{\eta}))$$
$$+r\phi_v(\bar{\eta}\phi_\rho^t(\bar{r^2}(x-y\bar{\eta})+\eta r^2y)$$
$$-\phi_\rho^s(\bar{r^2}(y\bar{\eta}+2\eta\bar{z}-x)+\eta r^2y))\big]D_4(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_2,b_1)\Big\},$$

$$M_a^{SP}(\zeta,\eta) = \frac{32}{\sqrt{6}}\pi C_F M_B^2 \int_0^1 dxdydz \int_0^\infty b_1db_1b_2db_2\phi_B\Big\{\big[\phi_\rho\phi_v(\eta(\bar{\eta}+r^2) \tag{23}$$
$$+r^2\bar{x}(-(\bar{r^2}+\eta))+\bar{z}(\bar{r^2}+\eta)(\eta r^2-\bar{\eta}\bar{r^2}))$$
$$+r\sqrt{\eta\bar{r^2}}(\phi_s(\phi_\rho^s(\bar{z}(\bar{r^2}+\eta)-\bar{x}+y\bar{\eta}+r^2y+3)$$
$$+\bar{\eta}\phi_\rho^t(\bar{z}(\bar{r^2}-\eta)+\bar{x}-y\bar{\eta}+r^2y-1))$$
$$-\phi_t(\phi_\rho^s(\bar{z}(\bar{r^2}-\eta)+\bar{x}-y\bar{\eta}+r^2y-1)$$
$$+\bar{\eta}\phi_\rho^t(\bar{z}(\bar{r^2}+\eta)-\bar{x}+y\bar{\eta}+r^2y-1)))\big]$$
$$\cdot D_5(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_2,b_1)$$
$$-\big[\phi_\rho\phi_v(r^2-\bar{\eta})(\bar{r^2}(y\bar{\eta}+2\eta\bar{z}-x)+\eta r^2y)$$
$$+r\sqrt{\eta\bar{r^2}}(\phi_s(\phi_\rho^s(\bar{z}(\bar{r^2}+\eta)+y\bar{\eta}+r^2y-x)$$
$$+\bar{\eta}\phi_\rho^t(\bar{z}(\eta-\bar{z}r^2)+y\bar{\eta}+r^2(-y)-x))$$
$$+\phi_t(\phi_\rho^s(\bar{z}(\bar{r^2}-\eta)-y\bar{\eta}+r^2y+x)$$
$$-\bar{\eta}\phi_\rho^t(\bar{z}(\bar{r^2}+\eta)+y\bar{\eta}+r^2y-x)))\big]D_5(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_2,b_1)\Big\}.$$

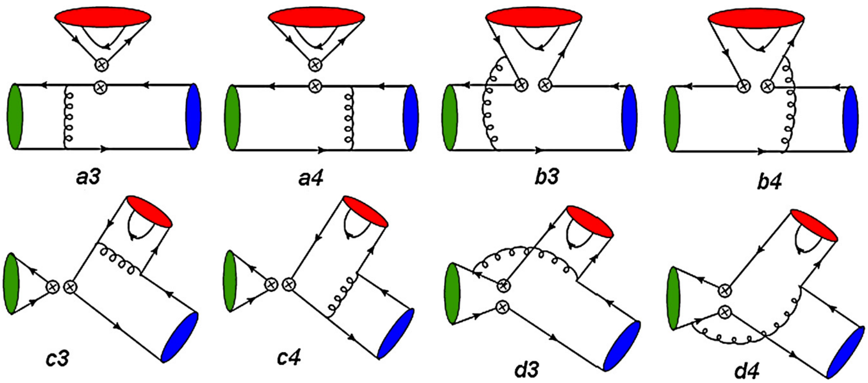

Fig. 2. Diagrams contributing to the $B \to \rho\pi\pi$ decays.

If we exchange the red bubble and blue one in Fig. 1, the other four analysis will be similar in Fig. 2. For the diagrams (a3) and (a4), we obtain (V−A)(V−A) and (S−P)(S+P) currents contributions,

$$F_{eV}^{LL}(\zeta,\eta)+F_{eV}^{LR}(\zeta,\eta)=8\pi F_s M_B^2 C_F\int_0^1 dxdy\int_0^\infty b_1db_1b_3db_3\phi_B \tag{24}$$
$$\times\Big\{\big[\psi_L(\bar\eta\overline{r^2}(3-y\bar\eta)+\eta r^2(r^2y-3))+r(\psi_s(\overline{r^2}(2y\bar\eta-3)+\eta(2r^2y-3))$$
$$+\bar\eta\psi_t(\overline{r^2}(2y\bar\eta-3)+\eta(3-2r^2y)))\big]D_6(t_1)E_4(t_1)h(m_{\alpha1},m_{\beta1},b_3,b_1)S_t(y)$$
$$+r\big[\eta r(2r\psi_s-\bar\eta\psi_L)-\overline{r^2}(x-\bar\eta)(r\psi_L+2\psi_s)\big]$$
$$\times D_6(t_2)E_4(t_2)h(m_{\alpha2},m_{\beta2},b_1,b_3)S_t(x)\Big\},$$

where $D_6=\xi_u d_1-\xi_t\,(2d_3+2d_5+(d_7+d_9)/2+d_4-d_{10}/2)$ is for B^0 or B^+ decays associated with the $n\bar n$ component, $D_6=-\xi_t\,(d_3+d_5-(d_7+d_9)/2)$ is for B^0 decays associated with the $s\bar s$ component. $E_4(t)=\alpha_s(t)S_B(t)S_\rho(t)$, $m_{\alpha1}=M_B\sqrt{y(\bar\eta+r^2-r^2y\bar\eta)}$, $m_{\beta1}=M_B\sqrt{y\bar\eta(x-r^2y)}$, and $m_{\alpha2}=M_B\sqrt{\bar\eta(x-r^2)}$, $m_{\beta2}=m_{\beta1}$.

$$F_{eV}^{SP}(\zeta,\eta)=16\pi F_s M_B^2 C_F\int_0^1 dxdy\int_0^\infty b_1db_1b_3db_3\phi_B\sqrt{\eta\overline{r^2}} \tag{25}$$
$$\times\Big\{\big[r(\psi_s(y\bar\eta+r^2y-6)+y\bar\eta\psi_t(r^2-\bar\eta))-3\psi_L(r^2-\bar\eta)\big]$$
$$\cdot D_7(t_1)E_4(t_1)h(m_{\alpha1},m_{\beta1},b_3,b_1)S_t(y)$$
$$+r\big[rx\psi_L-2\psi_s(\bar\eta+r^2-x)\big]D_7(t_2)E_4(t_2)h(m_{\alpha2},m_{\beta2},b_1,b_3)S_t(x)\Big\},$$

where $D_7=-\xi_t\,(d_6-d_8/2)$ is for $B\to\rho f_0(500)$ decays. We obtain (V−A)(V−A) and (V−A)(V+A) currents contribution for the unfactorizable diagrams (b3) and (b4),

$$
\begin{aligned}
M_{eV}^{LL}(\zeta,\eta) = {} & \frac{32}{\sqrt{6}}\pi M_B^2 C_F \int_0^1 dxdydz \int_0^\infty b_1 db_1 b_2 db_2 \phi_B \phi_v \\
& \times \Big\{ \big[\phi_\rho(\bar{r}^2+\eta)(\bar{z}(\eta r^2-\bar{\eta}\bar{r}^2)-r^2x)+r(\phi_\rho^s(\bar{r}^2(y\bar{\eta}+2\eta\bar{z}-x)+\eta r^2 y) \\
& +\bar{\eta}\phi_\rho^t(\bar{r}^2(x-y\bar{\eta})+\eta r^2 y))\big] D_8(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_1,b_2) \\
& -\big[\phi_\rho(\eta r^2-\bar{\eta}\bar{r}^2)(z\bar{r}^2+y\bar{\eta}+r^2y-x+\eta z) \\
& +r(\phi_\rho^s(\bar{r}^2(y\bar{\eta}-x+2\eta z)+\eta r^2 y) \\
& +\bar{\eta}\phi_\rho^t(\bar{r}^2(y\bar{\eta}-x)-\eta r^2 y))\big] D_8(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_1,b_2)\Big\},
\end{aligned} \tag{26}
$$

where $D_8 = \xi_u C_2 - \xi_t\,(C_3 + 2C_4 - C_9/2 + C_{10}/2)$ is for B^0 or B^+ decays associated with the $n\bar{n}$ component, and $D_8 = -\xi_t\,(C_4 - C_{10}/2)$ is for B^0 decays associated with the $s\bar{s}$ component. $m_{\alpha 1} = M_B\sqrt{y\bar{\eta}(x-r^2y)}$, $m_{\beta 1} = M_B\sqrt{(y\bar{\eta}+\eta)(x-r^2y-\bar{z}\bar{r}^2)}$, and $m_{\alpha 2} = m_{\alpha 1}$, $m_{\beta 2} = M_B\sqrt{y\bar{\eta}(x-r^2y-z\bar{r}^2)}$.

$$
\begin{aligned}
M_{eV}^{LR}(\zeta,\eta) = {} & \frac{32}{\sqrt{6}}\pi M_B^2 C_F \int_0^1 dxdydz \int_0^\infty b_1 db_1 b_2 db_2 \phi_B \sqrt{\eta\bar{r}^2}\Big\{ \\
& -\big[\phi_s(\phi_\rho(-(\bar{z}(\bar{\eta}\bar{r}^2-\eta r^2)+r^2x)) \\
& -r(\phi_\rho^s(\bar{z}(\bar{r}^2+\eta)+y\bar{\eta}+r^2y-x)+\bar{\eta}\phi_\rho^t(\bar{z}(\eta-\bar{r}^2)+y\bar{\eta}+r^2(-y)-x))) \\
& +\phi_t(\phi_\rho(r^2(x-2y\bar{\eta})-\bar{z}(\bar{\eta}\bar{r}^2+\eta r^2))+r(\bar{\eta}\phi_\rho^t(\bar{z}(\bar{r}^2+\eta)+y\bar{\eta}+r^2y-x) \\
& -\phi_\rho^s(\bar{z}(\bar{r}^2-\eta)-y\bar{\eta}+r^2y+x)))\big] D_9(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_1,b_2) \\
& +\big[\phi_s(\phi_\rho(-(z\bar{\eta}\bar{r}^2+r^2(x-\eta z)))-r(\phi_\rho^s(z\bar{r}^2+y\bar{\eta}+r^2y-x+\eta z) \\
& +\bar{\eta}\phi_\rho^t(-z\bar{r}^2+y\bar{\eta}+r^2(-y)-x+\eta z))) \\
& +\phi_t(\phi_\rho(\bar{\eta}(z\bar{r}^2+2r^2y)+r^2(\eta z-x)) \\
& +r(\phi_\rho^s(z\bar{r}^2-y\bar{\eta}+r^2y+x-\eta z)-\bar{\eta}\phi_\rho^t(z\bar{r}^2+y\bar{\eta}+r^2y-x+\eta z)))\big] \\
& \cdot D_9(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_1,b_2)\Big\},
\end{aligned} \tag{27}
$$

where $D_9 = -\xi_t\,(d_5 - d_7/2)$ is for $B \to \rho f_0(500)$ decays.

We obtain (S−P)(S+P) currents contribution,

$$
\begin{aligned}
M_{eV}^{SP}(\zeta,\eta) = {} & \frac{32}{\sqrt{6}}\pi M_B^2 C_F \int_0^1 dxdydz \int_0^\infty b_1 db_1 b_2 db_2 \phi_B \phi_v \Big\{ -\big[\phi_\rho(\eta r^2-\bar{\eta}\bar{r}^2) \\
& \cdot(\bar{z}(\bar{r}^2+\eta)+y\bar{\eta}+r^2y-x)+r(\phi_\rho^s(\bar{r}^2(y\bar{\eta}+2\eta\bar{z}-x)+\eta r^2y) \\
& +\bar{\eta}\phi_\rho^t(\bar{r}^2(y\bar{\eta}-x)-\eta r^2y))\big] D_8(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_1,b_2) \\
& +\big[r(\phi_\rho^s(\bar{r}^2(y\bar{\eta}-x+2\eta z)+\eta r^2y)+\bar{\eta}\phi_\rho^t(\bar{r}^2(x-y\bar{\eta})+\eta r^2y)) \\
& -\phi_\rho(\bar{r}^2+\eta)(z\bar{\eta}\bar{r}^2+r^2(x-\eta z))\big] D_8(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_1,b_2)\Big\},
\end{aligned} \tag{28}
$$

where $D_{10} = -\xi_t\,(2C_6 + C_8/2)$ is for B^0 or B^+ decays associated with the $n\bar{n}$ component, and $D_{10} = -\xi_t\,(C_6 - C_8/2)$ is for B^0 decays associated with the $s\bar{s}$ component.

For the annihilation diagrams (c3) and (c4), we get all currents contributions associated with the $n\bar{n}$ component below,

$$
\begin{aligned}
&F_{aV}^{LL}(\zeta,\eta) + F_{aV}^{LR}(\zeta,\eta) \qquad (29)\\
&= 8\pi f_B M_B^2 C_F \int_0^1 dydz \int_0^\infty b_2db_2b_3db_3 \Big\{\big[\bar{r}^2(\phi_\rho\phi_v(-\eta\bar{\eta} - \bar{\eta}^2\bar{y} + \eta r^2)\\
&\quad + 2r\sqrt{\eta\bar{r}^2}\phi_s(\bar{\eta}\phi_\rho^t + \phi_\rho^s)) + \bar{y}(2r\sqrt{\eta\bar{r}^2}\phi_s(\phi_\rho^s(\bar{\eta} + r^2) + \bar{\eta}\phi_\rho^t(r^2 - \bar{\eta}))\\
&\quad + \eta r^4\phi_\rho\phi_v) + 2\eta r\sqrt{\eta\bar{r}^2}\phi_s(\phi_\rho^s - \bar{\eta}\phi_\rho^t)\big]D_1(t_1)E_3(t_1)h(m_{\alpha 1}, m_{\beta 1}, b_3, b_2)S_t(y)\\
&\quad - \big[\phi_\rho\phi_v(\bar{\eta}(-z\bar{r}^{2^2} - r^2\bar{r}^2 + \eta r^2) + \eta^2r^2z) + 2r\sqrt{\eta\bar{r}^2}\phi_\rho^s(\phi_s(\bar{\eta} + z\bar{r}^2 + r^2 + \eta z)\\
&\quad + \phi_t(-\bar{\eta} + z\bar{r}^2 + r^2 - \eta z))\big]D_1(t_2)E_3(t_2)h(m_{\alpha 2}, m_{\beta 2}, b_2, b_3)S_t(z)\Big\},
\end{aligned}
$$

where $m_{\alpha 1} = M_B\sqrt{-(\eta + \bar{\eta}\bar{y})(\bar{r}^2 + r^2\bar{y})}$, $m_{\beta 1} = m_{\beta 2} = M_B\sqrt{-\bar{\eta}\bar{y}(z\bar{r}^2 + r^2\bar{y})}$, $m_{\alpha 2} = M_B\sqrt{-\bar{\eta}(z\bar{r}^2 + r^2)}$.

$$
\begin{aligned}
F_{aV}^{SP}(\zeta,\eta) &= 16\pi M_B^2 C_F \int_0^1 dydz \int_0^\infty b_2db_2b_3db_3 \Big\{\big[2\phi_\rho\sqrt{\eta\bar{r}^2}\phi_s(\eta r^2 - \bar{\eta}\bar{r}^2) \qquad (30)\\
&\quad + r\phi_v(\eta r^2\bar{y}(\phi_\rho^s - \bar{\eta}\phi_\rho^t) + \bar{r}^2(\phi_\rho^s(\bar{\eta}\bar{y} + 2\eta) + \bar{\eta}^2\bar{y}\phi_\rho^t))\big]\\
&\quad \times D_2(t_1)E_3(t_1)h(m_{\alpha 1}, m_{\beta 1}, b_3, b_2)S_t(y)\\
&\quad + \big[z\phi_\rho\sqrt{\eta\bar{r}^2}\phi_s(\eta r^2 - \bar{\eta}\bar{r}^2) + \phi_\rho\sqrt{\eta\bar{r}^2}\phi_t(\bar{\eta}(z\bar{r}^2 + 2r^2) + \eta r^2 z)\\
&\quad + 2r\phi_\rho^s\phi_v(\bar{r}^2(\bar{\eta} + 2\eta z) + \eta r^2)\big]D_2(t_2)E_3(t_2)h(m_{\alpha 2}, m_{\beta 2}, b_2, b_3)S_t(z)\Big\}.
\end{aligned}
$$

For the (V−A)(V−A), (V−A)(V+A) and (S−P)(S+P) operators, diagrams (d3) and (d4) give

$$
\begin{aligned}
M_{aV}^{LL}(\zeta,\eta) &= \frac{32}{\sqrt{6}}\pi M_B^2 C_F \int_0^1 dxdydz \int_0^\infty b_1db_1b_2db_2\phi_B \qquad (31)\\
&\quad \times \Big\{\big[\phi_\rho\phi_v(\bar{r}^2(r^2\bar{x} + \eta z(\bar{\eta} - r^2)) - \eta(\bar{\eta} - r^2\bar{x} + \eta r^2 z + r^2) + z\bar{\eta}\bar{r}^{2^2})\\
&\quad - r\sqrt{\eta\bar{r}^2}(\phi_s(\phi_\rho^s(r^2\bar{y} + z\bar{r}^2 - \bar{x} + \bar{\eta}\bar{y} + \eta z + 3)\\
&\quad + \bar{\eta}\phi_\rho^t(r^2(-\bar{y}) - z\bar{r}^2 - \bar{x} + \bar{\eta}\bar{y} + \eta z + 1))\\
&\quad - \phi_t(\phi_\rho^s(r^2(-\bar{y}) - z\bar{r}^2 - \bar{x} + \bar{\eta}\bar{y} + \eta z + 1)\\
&\quad + \bar{\eta}\phi_\rho^t(r^2\bar{y} + z\bar{r}^2 - \bar{x} + \bar{\eta}\bar{y} + \eta z - 1)))\big]D_3(t_1)E_2(t_1)h(m_{\alpha 1}, m_{\beta 1}, b_2, b_1)\\
&\quad + \big[\bar{r}^2(\phi_\rho\phi_v(r^2 - \bar{\eta})(\bar{\eta}\bar{y} - x + 2\eta z) + rz\sqrt{\eta\bar{r}^2}(\phi_s - \phi_t)(\bar{\eta}\phi_\rho^t + \phi_\rho^s))
\end{aligned}
$$

$$+\bar{y}(\eta r^2\phi_\rho\phi_v(r^2-\bar{\eta})+r\sqrt{\eta\bar{r^2}}(\phi_s(\phi_\rho^s(\bar{\eta}+r^2)+\bar{\eta}\phi_\rho^t(r^2-\bar{\eta}))$$
$$-\phi_t(\phi_\rho^s(r^2-\bar{\eta})+\bar{\eta}\phi_\rho^t(\bar{\eta}+r^2))))+r\sqrt{\eta\bar{r^2}}(\phi_s+\phi_t)(\eta z-x)(\phi_\rho^s-\bar{\eta}\phi_\rho^t)\big]$$
$$\cdot D_3(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_2,b_1)\Big\},$$

where $m_{\alpha 1}=M_B\sqrt{-\bar{\eta}\bar{y}(z\bar{r^2}+r^2\bar{y})}$, $m_{\beta 1}=M_B\sqrt{(\bar{x}-\bar{\eta}\bar{y})(z\bar{r^2}+r^2\bar{y}-1)}$, and $m_{\alpha 2}=m_{\alpha 1}$, $m_{\beta 2}=M_B\sqrt{-\bar{\eta}\bar{y}(z\bar{r^2}+r^2\bar{y})}$.

$$M_{aV}^{LR}(\zeta,\eta)=\frac{32}{\sqrt{6}}\pi M_B^2C_F\int_0^1 dxdydz\int_0^\infty b_1db_1b_2db_2\phi_B \tag{32}$$
$$\times\Big\{\big[\phi_\rho\sqrt{\eta\bar{r^2}}\phi_s(\bar{\eta}(2-z\bar{r^2})-r^2\bar{x}+r^2(\eta z-1))$$
$$+\phi_\rho\sqrt{\eta\bar{r^2}}\phi_t(r^2\bar{x}-\bar{\eta}(2r^2\bar{y}+z\bar{r^2}-2)+r^2(1-\eta z))$$
$$-r\phi_v(\phi_\rho^s(\bar{r^2}(-\bar{x}+\bar{\eta}\bar{y}+2\eta z-1)+\eta(r^2\bar{y}-2))$$
$$+\bar{\eta}\phi_\rho^t(\bar{r^2}(\bar{x}-\bar{\eta}\bar{y}+1)+\eta(r^2\bar{y}-2)))\big]D_4(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_2,b_1)$$
$$-\big[\phi_\rho\sqrt{\eta\bar{r^2}}\phi_s(r^2(\eta z-x)-z\bar{\eta}\bar{r^2})-\phi_\rho\sqrt{\eta\bar{r^2}}\phi_t(r^2(2\bar{\eta}\bar{y}-x+\eta z)+z\bar{\eta}\bar{r^2})$$
$$-r\phi_v(\bar{r^2}(\phi_\rho^s(\bar{\eta}\bar{y}-x+2\eta z)+\bar{\eta}\phi_\rho^t(x-\bar{\eta}\bar{y}))+\eta r^2\bar{y}(\bar{\eta}\phi_\rho^t+\phi_\rho^s))\big]$$
$$\cdot D_4(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_2,b_1)\Big\},$$

$$M_{aV}^{SP}(\zeta,\eta)=\frac{32}{\sqrt{6}}\pi M_B^2C_F\int_0^1 dxdydz\int_0^\infty b_1db_1b_2db_2\phi_B\Big\{\big[\bar{r^2}(\phi_\rho\phi_v(\bar{x}(\bar{\eta}-r^2) \tag{33}$$
$$+\bar{\eta}(r^2\bar{y}-2\eta z-1)-\bar{\eta}^2\bar{y}+2\eta r^2z)+rz\sqrt{\eta\bar{r^2}}(\phi_s-\phi_t)(\bar{\eta}\phi_\rho^t+\phi_\rho^s))$$
$$+\bar{y}(\eta r^2\phi_\rho\phi_v(r^2-\bar{\eta})+r\sqrt{\eta\bar{r^2}}(\phi_s(\phi_\rho^s(\bar{\eta}+r^2)+\bar{\eta}\phi_\rho^t(r^2-\bar{\eta}))$$
$$-\phi_t(\phi_\rho^s(r^2-\bar{\eta})+\bar{\eta}\phi_\rho^t(\bar{\eta}+r^2))))$$
$$+\eta\bar{\eta}\phi_\rho\phi_v-r\bar{\eta}\sqrt{\eta\bar{r^2}}\phi_s\phi_\rho^t-r\bar{x}\sqrt{\eta\bar{r^2}}\phi_\rho^s\phi_t+r\bar{\eta}\bar{x}\sqrt{\eta\bar{r^2}}\phi_s\phi_\rho^t$$
$$-\eta rz\bar{\eta}\sqrt{\eta\bar{r^2}}\phi_s\phi_\rho^t-r\bar{x}\sqrt{\eta\bar{r^2}}\phi_\rho^s\phi_s+r\bar{\eta}\sqrt{\eta\bar{r^2}}\phi_\rho^t\phi_t+r\bar{\eta}\bar{x}\sqrt{\eta\bar{r^2}}\phi_\rho^t\phi_t$$
$$-\eta rz\bar{\eta}\sqrt{\eta\bar{r^2}}\phi_\rho^t\phi_t+3r\sqrt{\eta\bar{r^2}}\phi_\rho^s\phi_s+r\sqrt{\eta\bar{r^2}}\phi_\rho^s\phi_t+\eta rz\sqrt{\eta\bar{r^2}}\phi_\rho^s\phi_t$$
$$+\eta rz\sqrt{\eta\bar{r^2}}\phi_\rho^s\phi_s\big]D_5(t_1)E_2(t_1)h(m_{\alpha 1},m_{\beta 1},b_2,b_1)$$
$$+\big[\phi_\rho\phi_v(\bar{r^2}+\eta)(z\bar{\eta}\bar{r^2}+r^2(x-\eta z))+r\sqrt{\eta\bar{r^2}}(\phi_s(\phi_\rho^s(-\bar{y}(\bar{\eta}+r^2)$$
$$-z\bar{r^2}+x-\eta z)+\bar{\eta}\phi_\rho^t(\bar{y}(r^2-\bar{\eta})+z\bar{r^2}+x-\eta z))+\phi_t(\bar{\eta}\phi_\rho^t(\bar{y}(\bar{\eta}+r^2)$$
$$+z\bar{r^2}-x+\eta z)-\phi_\rho^s(\bar{y}(r^2-\bar{\eta})+z\bar{r^2}+x-\eta z)))\big]$$
$$\times D_5(t_2)E_2(t_2)h(m_{\alpha 2},m_{\beta 2},b_2,b_1)\Big\}.$$

4. Numerical results and discussions

The total decay width is written as

$$\Gamma = \frac{G_F^2 M_B^5}{512\pi^4} \int_0^1 d\eta(1-\eta) \int_0^1 d\zeta |\mathcal{M}(\zeta,\eta)|^2 \tag{34}$$

with the decay amplitude $\mathcal{M}$ being the sum of all decay amplitudes from different diagrams.

The Flatté model for the $f_0(980)$ resonance and the Breit–Wigner formula for the $f_0(500)$, $f_0(1500)$, and $f_0(1790)$ resonances are adopted to parameterize the time-like scalar form factors. Considering the relative strengths and strong phases among different resonances, we write the timelike scalar form factor associated with the $s\bar{s}$ component as in Ref. [29]

$$\begin{aligned} F_s^{s\bar{s}}(\omega^2) = & \frac{c_1 m_{f_0(980)}^2 e^{i\theta_1}}{m_{f_0(980)}^2 - \omega^2 - i m_{f_0(980)}(g_{\pi\pi}\rho_{\pi\pi} + g_{KK}\rho_{KK})} \\ & + \frac{c_2 m_{f_0(1500)}^2 e^{i\theta_2}}{m_{f_0(1500)}^2 - \omega^2 - i m_{f_0(1500)}\Gamma_{f_0(1500)}(\omega^2)} \\ & + \frac{c_3 m_{f_0(1790)}^2 e^{i\theta_3}}{m_{f_0(1790)}^2 - \omega^2 - i m_{f_0(1790)}\Gamma_{f_0(1790)}(\omega^2)}. \end{aligned} \tag{35}$$

The $f_0(500)$ contribution is parameterized as a Breit–Wigner model

$$F_s(\omega^2) = \frac{c m_{f_0(500)}^2}{m_{f_0(500)}^2 - \omega^2 - i m_{f_0(500)}\Gamma_{f_0(500)}(\omega^2)}. \tag{36}$$

Considering the width of the resonance $f_0(500)$, we here also parameterize its contribution to the time-like scalar form factor in the Bugg's model [30]

$$R_{f_0(500)}(s) = m_r\Gamma_1(s) \Big/ \left[m_r^2 - s - g_1^2 \frac{s - s_A}{m_r^2 - s_A}\left[j_1(s) - j_1(m_r^2)\right] - i m_r \sum_{i=1}^{4} \Gamma_i(s) \right], \tag{37}$$

with the relevant parameters

$$\begin{aligned} m_r\Gamma_1(s) &= g_1^2 \frac{s - s_A}{m_r^2 - s_A}\rho_1(s), \\ m_r\Gamma_2(s) &= 0.6 g_1^2(s)(s/m_r^2)\exp(-\alpha|s - 4m_K^2|)\rho_2(s), \\ m_r\Gamma_3(s) &= 0.2 g_1^2(s)(s/m_r^2)\exp(-\alpha|s - 4m_\eta^2|)\rho_3(s), \\ m_r\Gamma_4(s) &= m_r g_{4\pi}\rho_{4\pi}(s)/\rho_{4\pi}(m_r^2), \\ g_1^2(s) &= m_r(b_1 + b_2 s)\exp(-(s - m_r^2)/A), \\ j_1(s) &= \frac{1}{\pi}\left[2 + \rho_1 \ln\left(\frac{1-\rho_1}{1+\rho_1}\right)\right], \\ \rho_{4\pi}(s) &= 1/\left[1 + \exp(7.082 - 2.845 s)\right]. \end{aligned} \tag{38}$$

We set $m_r = 0.953$ GeV, $s_A = 0.41\ m_\pi^2$, $b_1 = 1.302$ GeV, $b_2 = 0.340\ \text{GeV}^{-1}$, $A = 2.426\ \text{GeV}^2$ and $g_{4\pi} = 0.011$ GeV [30] in the numerical calculation. The phase-space factors of the decay channels $\pi\pi$, KK and $\eta\eta$ are defined as $\rho_i(s) = \sqrt{1 - 4m_i^2/s}$ with $i = 1, 2, 3$ for π, K and η respectively.

We use the following input parameters for numerical calculation [29,32,33]:

$$\begin{aligned} &M_{f_0(500)} = 0.50 \text{ GeV}, \quad M_{f_0(1500)} = 1.50 \text{ GeV}, \quad M_{f_0(1790)} = 1.81 \text{ GeV}, \\ &\Gamma_{f_0(500)} = 0.40 \text{ GeV}, \quad \Gamma_{f_0(1500)} = 0.12 \text{ GeV}, \quad \Gamma_{f_0(1790)} = 0.32 \text{ GeV}, \\ &M_B = 5.28 \text{ GeV}, \quad M_{B_s} = 5.367 \text{ GeV}, \quad M_{(f_0(980))} = 0.97 \text{ GeV}, \\ &f_B = 0.19 \text{ GeV}, \quad f_{B_s} = 0.236 \text{ GeV}, \quad g_{\pi\pi} = 0.167, \quad g_{KK} = 3.47 g_{\pi\pi}, \\ &\tau_{B^+} = 1.641 \text{ ps}, \quad \tau_{B^0} = 1.519 \text{ ps}, \quad \tau_{B_s} = 1.512 \text{ ps}. \end{aligned} \tag{39}$$

The mixing angle ϕ_m associated with $n\bar{n}$ and $s\bar{s}$ is given in Ref. [31], $tan^2\phi_m = 1.1 \times 10^{-2}$.

We adopt the Wolfenstein parametrization and the updated parameters for the CKM matrix elements [32]

$$\begin{aligned} &\lambda = 0.22535 \pm 0.00065, \quad A = 0.817 \pm 0.015, \\ &\bar{\rho} = 0.136 \pm 0.018, \quad \bar{\eta} = 0.348 \pm 0.014. \end{aligned} \tag{40}$$

The pQCD predictions for the branching ratios in $B \to f_0(500)\rho \to f_0(500)\pi^+\pi^-$ decays based on the BW model and the Bugg's model are the following

$$\mathcal{B}(B^+ \to \rho^+ f_0(500)[\pi^+\pi^-])_{(BW)} = [3.84 \pm 0.44(\omega_b)^{+0.30}_{-0.24}(a_2^{I=0}) \times 10^{-8}\,, \tag{41}$$

$$\mathcal{B}(B^0 \to \rho^0 f_0(500)[\pi^+\pi^-])_{(BW)} = [4.43 \pm 0.49(\omega_b)^{+0.34}_{-0.38}(a_2^{I=0}) \times 10^{-8}\,, \tag{42}$$

$$\mathcal{B}(B^+ \to \rho^+ f_0(500)[\pi^+\pi^-])_{(Bugg)} = [4.07 \pm 0.48(\omega_b)^{+0.32}_{-0.28}(a_2^{I=0}) \times 10^{-8}\,, \tag{43}$$

$$\mathcal{B}(B^0 \to \rho^0 f_0(500)[\pi^+\pi^-])_{(Bugg)} = [4.91 \pm 0.52(\omega_b) \pm 0.38(a_2^{I=0}) \times 10^{-8}\,. \tag{44}$$

We can see easily that the two PQCD predictions in different model are very similar.

For the decay $B_s \to f_0(X)\rho \to f_0(X)\pi^+\pi^-(X = 980, 1500, 1790)$, the predictions are given below

$$\mathcal{B}(B_s^0 \to \rho^0 f_0(980)[\pi^+\pi^-]) = [8.91^{+1.20}_{-1.12}(\omega_{b_s})^{+0.79}_{-0.78}(a_2^{I=0}) \times 10^{-8}\,, \tag{45}$$

$$\mathcal{B}(B_s^0 \to \rho^0 f_0(1500)[\pi^+\pi^-]) = [1.43^{+0.44}_{-0.35}(\omega_{b_s}) \pm 0.12(a_2^{I=0}) \times 10^{-8}\,, \tag{46}$$

$$\mathcal{B}(B_s^0 \to \rho^0 f_0(1790)[\pi^+\pi^-]) = [2.78^{+1.39}_{-1.33}(\omega_{b_s})^{+0.78}_{-0.74}(a_2^{I=0}) \times 10^{-9}\,, \tag{47}$$

where the two major errors come from the uncertainties of $\omega_b = 0.40 \pm 0.04$ GeV, $\omega_{b_s} = 0.50 \pm 0.05$ GeV and $a_2^{I=0} = 0.2 \pm 0.2$, respectively. The other errors are very small and negligible, which come from the inputs such as the mean lifetime of the $B_0(B_s)$ meson, Wolfenstein parameters and so on.

Taking into account of the interference between different scalars $f_0(X)$, we get the total branching ratio 1.13×10^{-7}. The interference between $f_0(980)$ and $f_0(1500)$, $f_0(980)$ and $f_0(1790)$, as well as $f_0(1500)$ and $f_0(1790)$, contributes 1.23×10^{-8}, 1.52×10^{-8} and -2.58×10^{-9} to the total branching ratio, respectively.

In Fig. 3(a), we illustrate the differential branching ratios $d\mathcal{B}/dM$ with respect to the dipion invariant mass from each resonance $f_0(980)$ (the solid curve), $f_0(1500)$ (the dashed curve) and $f_0(1790)$ (the dotted curve) in B_s decays. In Fig. 3(b), we also show the decay spectrum of $B^0 \to \rho^0 f_0(500)$ and $B^+ \to \rho^+ f_0(500)$ from the resonance $f_0(500)$ as a function of $\omega = M(\pi^+\pi^-)$.

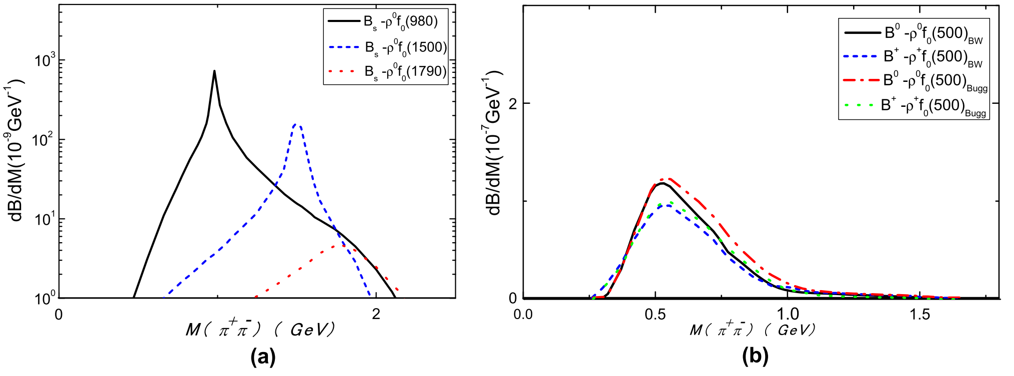

Fig. 3. S-wave resonance contribution to the $B \to \rho\pi^-\pi^+$ decay spectrum with respect to the dipion invariant mass $M(\pi^+\pi^-)$.

From the pQCD predictions, we can see the point below. For $B_s \to f_0(X)\rho \to f_0(X)\pi^+\pi^-$ decay, as shown clearly in Fig. 3(a), the contribution from the resonance $f_0(1790)$ is only about 3%, while the dominant contribution is from $f_0(980)$, about 75%. The interference between $f_0(980)$ and $f_0(1500)$, as well as $f_0(980)$ and $f_0(1790)$, are constructive and enhance the total decay rate up to 18%. However, the interference between $f_0(1500)$ and $f_0(1790)$ is destructive, but only -2%, very small in size.

The heavy quark physics is one of the main topics in LHC experiments. Especially, LHCb detector is designed to make precise studies on CP asymmetries and rare decays of b-hadron systems. The LHC gives access to high energy frontier at TeV scale. These decays might be hopefully detectable and the theoretical predictions will be tested in the running LHC-b experiments and forthcoming experiments.

5. Conclusions

In summary, we study the three-body decays $B_{(s)} \to \rho f_0(X) \to \rho\pi^-\pi^+$ in the pQCD factorization approach. We evaluate S-wave resonance contributions, and show the contribution to the $B \to \rho\pi^-\pi^+$ decay spectrums with respect to the dipion invariant mass $M(\pi^+\pi^-)$. We compute the branching ratios for such decays. Taking into account the interference between each other in $f_0(X)(X = 980, 1500, 1790)$ resonances, we give the total branching ratios in $B_s \to \rho f_0(X) \to \rho\pi^-\pi^+$ decays. These pQCD theoretical predictions will be tested by running LHC-b and the forthcoming Super-B experiments. The reliability of the formalism of two-meson DAs can also be tested by such experiments.

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